A Structural Approach to the Native-Immigrant Wage Gap Using Matched Employer-Employee Data*

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Abstract

In this paper we propose and estimate an equilibrium search model using matched employer-employee data to study the extent to which wage differentials between natives and immigrants can be explained by differences in ability, frictions pattern's disparity or wage discrimination. Structural models may provide an interpretation of observed wage gaps as a consequence of disparities in group specific fundamentals of labor market performance as skills, skill prices and job creation and destruction rates. Nevertheless structural estimation has not considerably advanced in this direction due to the identification problem of empirically distinguishing between skill differentials and wage discrimination. The availability of matched employer-employee data allows us to estimate marginal products for each group at the firm level providing a clear separation of productivity and skill prices. We estimate the micro-structural model with LIAB, a German matched employer-employee data set. We find that immigrants are more productive than natives in similar jobs. In general they are more mobile and they have lower bargaining power, which would mean that, in spite of having positive wage differentials, they are being discriminated.

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1 Introduction

In this paper we propose and estimate an equilibrium search model using matched employer-employee data to study the extent to which wage differentials between natives and immigrants can be explained by differences in ability, friction pattern's disparity or wage discrimination. The model features wage bargaining, on-the-job search and firm and individual heterogeneity in productivity. Its estimation involves several steps: firstly, we estimate group-specific productivity from firm-level production functions. Secondly we compute job-retention and job-finding rates using employee-level data. Finally, we calculate the wage-setting parameters (bargaining power) relying on individual wage data and the outside option and productivity measures estimated in previous stages.

There has been a large number of studies trying to estimate how much of the unconditional mean wage differential between groups may be understood as wage discrimination[1]. The traditional approach takes the unexplained gap in wage regressions as evidence of discrimination. This method estimates Mincer-type equations for both groups and then it decomposes the difference of mean wages into "explained" and "unexplained" components. The fraction of the gap that cannot be explained by differences in observable characteristics is considered as discrimination. This kind of analysis has been very informative from a descriptive perspective but the causal interpretation is not clear.

Discrimination refers to differences in wages that are caused by the fact of belonging to a given group, therefore causality is an essential issue in this context. Ideally, detecting discrimination would require to have measures of the effect of each wage determinant and then to test if the group effect is significant.

The availability of matched employer-employee data allowed a new approach pioneered in Hellerstein and Neumark (1999)[2]. Their method uses

[1]See Blau and Kahn (2003) and Altonji and Blank (1999) for good surveys.

[2]Some of the main papers in this brach are: Hellerstein and Neumark, (1999) with Israeli data, Hellerstein, Neumark and Troske, (1999) with U.S. data, Crepon, Deniau and Pérez-Duarte (2003) with french data, Kawaguchi (2007) with Japanese data and Van

firm level data to estimate relative marginal products of various worker types, which are then compared with their relative wages. This analysis implies a clear causality from productivity to wages. Whenever perfect competition holds in the labor market, there is a one-to-one mapping between both, therefore any difference in wages that is not driven by a difference in productivity may be considered as discrimination. But a frictionless scenario has been shown to not be very useful to understand the labor market. In a labor market with frictions the relationship between productivity and salaries is more obscure, and a direct comparison between both is less informative.

Moreover, wage differentials across groups are often accompanied by unemployment rate and job duration differentials. There is a vast literature estimating differentials in job-finding and job-retention rates across groups, directly observing duration in the unemployment and employment or with experiments in audit studies. Although there is an agreement in predicting an effect of frictions over wages, there is scarce empirical evidence on how much of the wage gap can be accounted for differences in friction patterns.

Estimated structural models may provide an interpretation of observed wage gaps as a consequence of disparities in group-specific fundamentals of labor market performance like ability, bargaining power and job creation and destruction rates. Nevertheless, progress in this direction has been slow mainly due to the difficulty of separately identifying the impacts of skill differentials and discrimination from worker-level survey data. The main references are Eckstein and Wolpin (1999) and Bowlus and Eckstein (2002). Both papers study racial discrimination in the U.S. and they deal with this empirical identification problem through structural assumptions. Eckstein and Wolpin (1999) proposed a method based on a two-sides, search-matching model that formally accounts for unobserved heterogeneity and unobserved offered wages. They showed that identification of the bargaining power parameter (their index of discrimination) depends on strong assumptions about the equality of unobserved productivity differences across groups. Bowlus and Eckstein (2002) also proposed a search model with heterogeneity in workers' productivity but including an appearance-based employer desutility

Biesebroeck, (2007) with subsaharian data.

factor. As there are firms that do not discriminate, they are able to identify between-group differences the skill distribution and thereafter the discrimination parameter, which in their case is the proportion of discriminatory employers[3].

In this paper we propose to estimate a fairly standard search matching model with on-the-job search, rent splitting, and productivity heterogeneity in firms and workers[4]. The availability of matched employer-employee data furthers identification by allowing us to disentangle differences in workers productivity across groups from differences in wage policies toward those groups. We combine the productivity measures estimated at the firm level *a la* Hellerstein et al, group specific friction patterns estimated from individual duration data and individual wages to estimate the structural model wage equation. This structural wage equation states the precise relationship between wages, ability, friction patterns and bargaining power, and therefore it allows us to undertake counterfactual analysis. One possibility, for example, is to compare wages of two ex-ante identical workers in terms of ability and outside options, which only differ in the wage-setting parameter corresponding to their migration status[5].

To differentiate which part of the wage gap is driven by differences in ability or differences in outside options and bargaining power is crucial for social policy. The first one, as Heckman (1998) points out, is probably due to differences in the skills workers bring to the market, and not to discrimination within the labor market and, therefore, it has to be tackled at the skill formation level. But differences in the job offer arrival rate, job duration and bargaining power are inequalities within the labor market and there should be specific policies or regulations to deal with each of them.

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[3]Flabbi (2005) and Mondal (2006) estimate similar models to study gender and racial wage differentials in the U.S..

[4]From now on, we will refer to worker productivity as ability.

[5]Note that a difference in the bargaining power between immigrant and natives is considered as wage discrimination. This has already been assumed in Eckstein and Wolpin (1999) and it is meaningfull in the sense that an inequality in bargaing power generates a difference in wages between two workers with the same ability and outside option that are working in similar jobs in terms of sector and qualification that only differ in terms their migration status.

We use a 1996-2005 panel of matched employer-employee data provided by the German Labor Agency called LIAB[6]. This dataset is especially useful for this study for two reasons. Firstly, it contains essential individual variables like nationality, wages, and occupation. Secondly, it is a panel that tracks firms as opposed to individuals, which is important in order to be able to estimate production functions using standard panel estimation methods. As far as we know, this is the first structural estimation that uses matched employer-employee data to study labor market discrimination.

The empirical analysis proceeds by first calculating non-structural decompositions of differences in productivity and wages, following the approach in Hellerstein et al for male-female gaps. Interestingly, we find positive differentials in favor of immigrants. When analyzing group-specific dynamics, we find that immigrants have higher job-creation rates than equivalent natives, and that skilled immigrants have lower job-retention rates than skilled natives. Surprisingly in spite of immigrants have positive differentials in wages, their estimated bargaining power is, in general, lower than the estimated for the natives, which would mean that they are being discriminated.

The rest of the paper is organized as follows. In the next section we describe the structural model. In section 3 data is described. In section 4 we estimate the structural model inputs, namely productivity measures and friction parameters, we present and discuss those intermediate results and finally we estimate the structural wage equation. In section 5 we perform and discuss some counterfactual experiments and we compare our empirical results with those resulting from other discrimination detecting strategies using the same data. A conclusion is offered in section 6.

2 Structural framework

In this section we describe the behavioral model of labor market search with matching and rent splitting. The main goal of estimating a structural model

[6] This dataset is subject to strict confidentiality restrictions. It is not direcly available but only after the IAB has approved the research project, The Research Data Center (FDZ) provides on site use or remote access to external researchers.

is to clearly state a wage setting equation that allows us to measure the effect of each wage determinant. Having this wage equation estimated, it is straightforward to obtain effect of discriminating wage policies.

There have been a lot of research trying to show the proficiency of this kind of models in describing the labor market outputs and dynamics. Trusting on these assessments, in this paper we are interested in accomplishing an empirical contribution using the structural model as a measurement tool that allows us to identify the effect of discrimination over wages. There are various papers that have used search-matching models as an instrument to solve empirical questions, as the already mentioned papers in the discrimination literature, but also there are interesting contributions in measuring the return to education as in Eckstein and Wolpin (1995) or in analyzing the effect of a change in the minimum wage in Flinn (2006).

2.1 Assumptions

We propose a continuos time, infinite horizon, stationary economy. The economy is populated by infinitely lived firms and workers. All the agents are risk neutral and discount future income at rate $\rho > 0$.

Workers: We normalize the measure of workers to one. Workers may belong to different groups (k) in terms of their occupation and their migration status. Workers have different abilities (ε) measured in terms of efficiency units they provide per unit of time. The distribution of ability in the population of workers is exogenous and specific for each group, with cumulative distribution function $L_k(\varepsilon)$. This source of heterogeneity is perfectly observable by every agent in the economy. Each worker may be unemployed or employed. Those workers from a generic group k that are not actually working receive a flow utility, proportional to their ability, $b_k\varepsilon$.

Firms: Every firm is characterized by its productivity (p). We Assume that there is only frictions in the labor market. Firms can adjust capital instantaneously in every period without adjustment costs. We assume that p is distributed across firms according to a given cumulative distribution function $H(p)$, which is continuously differentiable with support $[p_{\min}, p_{\max}]$.

This source of heterogeneity is perfectly observable by every agent in the economy. The opportunity cost of recruiting a worker is zero.

Each firm contacts a worker at the same constant rate, regardless of the firm's bargained wage, its productivity or how many filled job it has. Unemployed workers receive job offers at Poisson rate $\lambda_0 > 0$ and employed workers may also search for a better job while employed and they receive job offers at Poisson rate $\lambda_1 > 0$. We treat λ_0 and λ_1 as exogenous parameters. To search while unemployed as to search while employed has no cost. Employment relationships are exogenously destroyed at a constant rate $\delta > 0$, leaving the worker unemployed and the firm with nothing. The marginal product of a match between a worker with ability ε and a firm with productivity p is εp.

When the worker and the firm reach a wage agreement, this wage remains fixed for the duration of the match. Whenever an employed worker meets a new firm, the worker must choose an employer and then, if she switches employers, she bargains with the new employer with no possibility of recalling her old job. If she stays at her old job, nothing happens. Consequently when a worker negotiates with a firm, her alternative option is always the unemployment. The surplus generated by the match is split in proportion β and $(1-\beta)$, for the worker and the firm respectively, where $\beta \in (0,1)$. We will refer about β as the wage setting parameter. As in Wolpin and Eckstein (1999), we interpret β as an index of the level of discrimination in the labor market. A difference in β in the same kind of job and sector, reveals differential payments unrelated to productivity and outside options, that are only driven by the fact of belonging to a given group.

It is not clear if β can be interpreted as a Nash Bargaining power. Shimer (2006) argues that in a simple search-matching model with on-the-job search, the standard axiomatic Nash bargaining solution is inapplicable, because the set of feasible payoffs is not convex. This non-convexity arises because an increase in the wage has a direct negative effect over the firm's rents but an indirect positive effect raising the duration of the job.

In an environment where contracts cannot be written and wages are continuously negotiated, the alternative value of the match is always unemployment. If a worker receive an offer from a firm with higher productivity, she must switch. She cannot use this offer to renegotiate with her actual firm, because she knows than tomorrow this offer will be not available and then her future option will be again the unemployment.

In a world where agents can make contract and take commitment with other agents, this may also be an equilibrium because firms will commit to not bid workers to better firms, because this would only generate renegotiation and lower rents for the better firm[7]. Then, the Cahuc et al proposed environment is an intermediate option where firms cannot commit with other firms to follow an optimal strategy but they are allowed to sign contracts with workers that compel them to maintain wages also when the worker's option has become worse.

This setting rules out the possibility that a worker can exploit multiple job opportunities and then, it ensures the convexity of feasible payoffs because now transitions are only driven by productivity differences like in Cahuc et al[8]. With this assumption nothing changes in the model but we are allowed to named β as a Nash-negotiation power, because now we can justify the wage as determined as the outcome of a Rubinstein (1982) infinite-horizon game of alternating offers.

2.2 Value Functions

The expected value of income for a worker with ability ε, who belongs to a group k, currently employed at wage $w(p, \varepsilon, k)$ is denoted by $E(w(p, \varepsilon, k), \varepsilon, k)$ and it satisfies:

[7] Firms may commit with other firms to not renegotiate or in a model with search effort they may credibly commit to ignore outside offers to their employees, let them go without a counteroffer, and suffer the loss, in order to keep in line the other employees' incentives to not search on the job. This is analysed in Moscarini (2004)

[8] If wages are continuously negotiated, firms could increase the wage of the worker at the moment of the on-the-job offer to try to avoid the quit. If the alternative employer is more productive can force the transition also paying a premiun. This auction for the worker finishs when the actual firm cannot pay more than the whole productivity and transition holds. This premium may be considered as a hiring cost for the firm but we abstract from it becuase to model it would be outside the scope of this work.

$$
\begin{aligned}
& \rho E(w(p,\varepsilon,k),\varepsilon,k) && (1)\\
= \; & w(p,\varepsilon,k)+\delta_k(U(\varepsilon,k)-E(w(p,\varepsilon,k),\varepsilon,k))+ \\
& \lambda_{1k}\int_{w(p,\varepsilon,k)}^{w(\varepsilon,p\max,k)}\left[E(\tilde{w}(p,\varepsilon,k),\varepsilon,k)-E(\tilde{w}(p,\varepsilon,k),\varepsilon,k)\right]dF(\tilde{w}(p,\varepsilon,k))
\end{aligned}
$$

The expected value of being unemployed for a worker with ability ε, who belongs to a group k is given by:

$$
\begin{aligned}
& \rho U(\varepsilon,k) \\
= \; & b_k\varepsilon+\lambda_{0k}\int_{w(\varepsilon.k)\min}^{w(\varepsilon,p\max,k)}\left[E(\tilde{w}(p,\varepsilon,k),\varepsilon,k)-U(\varepsilon,k)\right]dF(\tilde{w}(p,\varepsilon,k))
\end{aligned}
$$

Finally, the value of the match with productivity $p\varepsilon$ for the firm when paying a wage $w(p,\varepsilon)$ to a worker of group k is given by:

$$
\begin{aligned}
& \rho J_p(w(p,\varepsilon,k),p\varepsilon,k) && (2)\\
= \; & p\varepsilon-w(p,\varepsilon,k)-(\delta_k+\lambda_{1k}\bar{F}(w(p,\varepsilon,k)|\varepsilon))J_p(w(p,\varepsilon,k))
\end{aligned}
$$

Where $\bar{F}(w(p,\varepsilon,k)|\varepsilon)=1-F(w(p,\varepsilon,k)|\varepsilon)$. Note that every parameter is group-specific. As the alternative value is always zero, the value of a match does not depend on alternative matches and therefore it is independent on other groups' parameters. Although every group is sharing the same labor market, all the value functions may be considered group by group as if they were in independent markets. For notation simplicity we then omit the k-index.

These expressions are equivalent to the value functions of the model with heterogenous firms in Shimer (2006) including heterogeneity in workers ability. But here, wages are determined by the following surplus splitting rule:

$$
(1-\beta)\left[E(w(p,\varepsilon),\varepsilon)-U(\varepsilon)\right]=\beta J_p(w(p,\varepsilon),\varepsilon) \tag{3}
$$

With some algebra (see the appendix for the whole proof) we can show that:

$$w(p,\varepsilon)=p\varepsilon-(\rho+\delta+\lambda_1\bar{F}(w(p,\varepsilon)|\varepsilon))\frac{(1-\beta)}{\beta}\int_{w(\varepsilon)_{\min}}^{w(p,\varepsilon)}\frac{1}{(\rho+\delta+\lambda\bar{F}(\tilde{w}(p,\varepsilon)|\varepsilon))}d(\tilde{w}(p,\varepsilon))$$

Noting that $\bar{F}(w(p,\varepsilon)|\varepsilon)=\bar{H}(p)$ and changing the variable within the integral, we have a first order differential equation.

$$w(p,\varepsilon)=p\varepsilon-(\rho+\delta+\lambda_1\bar{H}(p))\frac{(1-\beta)}{\beta}\int_{p_{\min}}^{p}\frac{1}{(\rho+\delta+\lambda\bar{H}(p'))}\frac{d(\tilde{w}(p,\varepsilon))}{dp'}dp'$$

Solving the differential equation and with some algebra the wage equation takes the following form:

$$w(p,\varepsilon)=\varepsilon p-\varepsilon(1-\beta)(\rho+\delta+\lambda_1\bar{H}(p))^{\beta}\int_{p\min}^{p}\left(\rho+\delta+\lambda_1\bar{H}(p')\right)^{-\beta}dp' \quad (4)$$

This expression states a clear relationship between wages $w(p,\varepsilon)$, workers' ability (ε), firm productivity (p), friction patterns (λ_1,δ) and the wage setting parameter (β) This wage equation is relatively similar to the one proposed by Cahuc, Postel-Vinay and Robin (2006) when the wage is bargained between a firm with productivity p and an unemployed worker with ability ε.

Note that, as expected, if $\beta=1\Rightarrow w(p,\varepsilon)=p\varepsilon$, the maximum wage that a firm with productivity p can pay to a worker with ability ε is the whole productivity. If $\beta=0\Rightarrow w(p,\varepsilon)=p_{\min}\varepsilon$, that is the minimum wage that a worker would accept to leave the unemployment..

The stationary equilibrium conditions are the standard ones. The inflow must balance the outflow for every stock of workers, defined in terms of individual ability, employment status and for those that are employed, firm's productivity. The relevant flow-balance conditions that we will exploit are:

- The inflow to the unemployment must be equal to its outflow, $\lambda_0\mu=\delta(1-\mu)$., where μ is the unemployment rate and is given by:

$$\mu=\frac{\delta}{\delta+\lambda_0} \quad (5)$$

- The inflow to jobs in firms with productivity p or lower than p must be equal to its outflow:

$$\lambda_0 H(p)\mu = \left(\lambda_1 \bar{H}(p) + \delta\right) G(p)$$

Where $G(p)$ is the fraction of workers employed at a firm with productivity p or lower than p. Then using condition (5) and rearranging:

$$G(p) = \frac{H(p)}{1 + \kappa_1 \bar{H}(p)} \tag{6}$$

This stationary condition, (or the same one in terms of wages) is quite standard and it has been broadly used after Burdett and Mortensen (1998) to make inference about the primitive distribution of productivity (or the primitive distribution of wages) when we only observe the distribution of productivity (or distribution of wages) within employed workers. As here we have matched employer-employee data we directly observe the empirical distribution of productivity at the firm level. We only use this stationary condition to construct the likelihood of the duration analysis in section 4.

- The fraction of employed workers with ability ε or lower than ε that are working in firms with productivity p or lower than p are $(1-\mu)\tilde{F}(\varepsilon, p)$. Those workers leave this group due to a better offer or because they go to the unemployment, this event occurs with probability $(\delta + \lambda_1 \bar{H}(p))$. The inflow to this group is given by those unemployed workers with ability ε or lower than ε $(L(\varepsilon)\mu)$ who receive an offer from a firm with productivity p or lower than p, this last event occurs with probability $\lambda_0 H(p)$. Then we have the following condition:

$$(1-\mu)(\delta + \lambda_1 \bar{H}(p))F(\varepsilon, p) = \lambda_0 H(p) L(\varepsilon)\mu$$

Then using conditions (5) and (6) and rearranging:

$$F(\varepsilon, p) = \frac{H(p)}{(1 + \kappa_1 \bar{H}(p))} L(\varepsilon) = G(p)L(\varepsilon) \tag{7}$$

This expression says that there is not sorting between firm's productivity and worker‘s ability. This statement is controversial, and there is an active debate in the assortative matching literature. Becker (1973) showed that in a model without search frictions but with transferable utility, if there are supermodular production functions, any competitive equilibrium exhibits positive assortative matching. In more recent papers, Shimer and Smith (2000) and Atakan (2006) show that in search models, complementaries in production function are not sufficient to ensure assortative matching. Assuming different cost functions the first one predicts a negative correlation while the second one the contrary. The empirical literature, after Abowd, Kramarz, and Margolis (1999) mainly focused on estimating the correlation between individual and firm fixed effects using matched employer-employee data. Still there has not been definitive results. For example, Abowd et al found negative and small or zero correlation between firms and workers fixed effects while Mendez, Van Den Berg and Lindeboom (2007), using a Portuguese matching employer-employee dataset find that there is positive assortative matching.

3 Data

Linked Employer-Employee Data from the IAB (LIAB)

We use the linked employer-employee dataset of the IAB (LIAB) covering the period 1996-2005. LIAB is created by matching the data of the IAB establishment panel and the process-produced data of the Federal Employment Services (Social security records). The distinctive feature of this data is the combination of information about individuals and details concerning the firms in which these people work. The workers source contains valuable data on age, sex, nationality, daily wage (censored at the upper earnings limit for social security contributions), schooling/training, the establishment number and occupation based on a 3-digit code that is collapsed into two distinctive cells: skilled and unskilled jobs.[9]

[9] We have considered unskilled jobs to the following groups: Agrarian occupations, manual occupations, services and simple comercial or administrative occupations. While we have considered skilled jobs to: Engineers, professional or semi-profesional occupations, qualified comercial or administrative occupations, and managerial occupations.

The firms data gives details on total sales, value added, investment, depreciation, number of workers and sector[10]. In particular only firms with more than 10 workers, positive output and positive depretiated capital had been included in our subsample. As firms of different sector do not share the same market we define independent samples for each sector. LIAB has a very detailed classification of industry sector. We focus on four main sectors: Manufacturing, Construction, Trade and Services, which collapses industrial services, transport and communication, and other services. Participation of establishments is voluntary, but the response rates are high, they exceed 70 per cent. The response rate in some key-variables for our inquiry is lower. Between compliers to the survey, in those four sector only 60% of the firm have valid responses for value-added and only 43% have valid responses for depretiated capital, see Table 1. To estimate productivity we need data on value added, number of workers and depretiated capital, the number of firms with estimated productivity falls to 35.088 observations.

Table 1: Firm's data

	Observations	Mean	std. Dev	Maximum
Output*	66170	63.9	4.180	152,000
Workers surveyed	111,097	191	754	53,425
Depreciated capital*	47742	1.64	24.9	4,800

* in millions of euros

The employee data is matched to those firms for which we have valid estimates of productivity through a unique firm identifier. Between 1996 and 2004 there are originally 21,246,022 observations but after this final trimming we have a 9-year imbalance panel, involving a total of 7,349,044 workers' observations distributed into 35,088 firms' observations (see Table 2).

The main goal of this study is to understand the native-immigrant wage gap. The unconditional Immigrant-Native Wage Gap is -4.49%[11]. This would

[10]For a more presice description of this dataset, see Alda et al (2005)

[11]This results is mainly driven by the fact that Manufacturing is the most populated

Table 2: Subsample Details

	n^o of firms	n^o of workers	Immigrants (%) Unsk.	Immigrants (%) Skill	Natives (%) Unsk.	Natives (%) Skill
Manufacturing	14,816	5,045,284	8.9	1.0	59.1	30.9
Construction	4,464	305,147	5.3	0.8	68.9	25.0
Trade	4,427	328,104	4.3	0.9	63.5	31.3
Services	11,381	1,670,509	5.8	1.1	52.3	40.8

mean that immigrant, on average, have salaries 4% higher than natives, see Tables 8 and 9 in the appendix. This difference is not stable across sectors and occupations, see Table 3. If we estimate mean-wages across sectors and occupations, we find that the gap ranges between -18% and 12%. In general wage gaps is negative in unskilled occupations.

Table 3: Native-Immigrant Wage Gap

		Mean Daily-Wage		**Wage**
		Natives	**Immigrants**	**Gap**
manufacturing	**Unskilled**	76.79	83.75	-9.05%
		(2.73)	(1.98)	
	Skilled	129.04	152.98	-18.55%
		(3.53)	(9.86)	
Construction	**Unskilled**	59.32	65.08	-9.71%
		(2.52)	(2.66)	
	Skilled	82.52	72.55	12.08%
		(3.85)	(9.10)	
Trade	**Unskilled**	49.61	50.91	-2.61%
		(3.65)	(3.34)	
	Skilled	76.59	71.35	6.84%
		(3.25)	(6.92)	
Services	**Unskilled**	46.44	45.22	2.62%
		(5.23)	(5.07)	
	Skilled	77.28	85.10	-10.12%
		(3.60)	(7.91)	

Note: Standard errors are given in parentheses.

sector (see Table 2)

German Socio-Economic Panel

This version of LIAB is a panel of firms complemented with workers data To correct for panel mortality, exits, and newly-founded units, the data are augmented regularly, yielding an unbalanced panel. As it does not track workers, it is not possible to distinguish between attrition and job-termination[12]. For that reason we use GSOEP (German Socio-Economic Panel) to estimate group-specific transition parameters[13]. The German Socio-economic panel is a representative repeating study of private households in Germany. This survey has been carried out annually with the same people and families in Germany since 1984 (but we only use 1996-2005). This dataset is the German equivalence to the American PSID.

4 Empirical Strategy and results

As it has become usual in this kind of micro-estimated models[14], the discrete nature of annual data implies a complicated censoring of the continuous-time trajectories generated by the theoretical model. Then, a potentially efficient, full information maximum likelihood is not considered as a candidate for the estimation. We perform a multi-step estimation procedure.

Even though it may be inefficient, we prefer this step-by-step method, firstly because, the efficiency of full information maximum likelihood is only guaranteed in case of correct specification of the model. We are interested in having productivity difference and transition parameter estimates robust to misspecification in other parts of the model, and secondly because transition parameters are better estimated with a standard labor force survey due to a severe atrittion problem in the basic dataset.

This multi-step estimation procedure allows us to have control of the source of variation that is identifying each parameter. The empirical identification of productivity differences with firm level data is weak and imprecise.

[12] Unless the worker leave the firm and moves to another firm within the panel.

[13] Cahuc, Postel-Vinay and Robin (2006) follows the same strategy estimating transition parameters with the French Labor Force Survey. .

[14] This has been done firstly in Bontemps, Robin, and Van den Berg (2000), but also in Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006).

Full-information maximum likelihood may have helped empirically because we would use data on wages to improve the productivity estimates but on the other hand we would not be able to guarantee that those estimates are solely revealing productivity differences but also wage setting inequalities. If the model were the true data generating process this caution would not be necessary, because the model do not imply any reverse causality from wages to productivity and the noise in productivity estimates would be only due to the contemporary productivity shock uncorrelated with wages.

As in Postel-Vinay and Robin (2002), the wage equation uses results of preceding stages. Estimated errors should take into account that we are including pre-estimated parameters but the complexity of the whole procedure makes the estimation of appropriate standard errors intractable. Fortunately, the huge sizes of the samples attenuates the severity of this problem.

4.1 Productivity

The production function specification chosen in the empirical section, is a standard Cobb-Douglas function with constant return to scale and Quality Adjusted Labor Input. This function has already been used in the discrimination literature to estimate between-group productivity differences and it is also consistent with the theory proposed in the preceding section. The value added Y_{jt} produced by j in period t, is given by:

$$Y_{jt} = A_j K_{jt}^{(1-\alpha)} Ql_{jt}^{\alpha} e^{u_{jt}} \tag{8}$$

Where K_{jt} is the total capital, A_j is a firm specific productivity parameter, u_{jt} is a zero mean stationary productivity shock and Ql_{jt} is the total amount of labor in efficiency units given by:

$$Ql_{jt} = \sum_{K} \tilde{\gamma}_k L_{k,jt}.$$

We normalize $\gamma_1 = 1$ considering the native skill group as the reference group[15]. Now $\gamma_k = \tilde{\gamma}_k / \tilde{\gamma}_1$ is the proportional productivity of group k relative

[15] Due to this normalization, the firm specific productivity $\tilde{A}_j$ is redefined as $A_j \gamma_1^{\alpha_l}$.

to the productivity of natives skilled workers.

As it has been assumed in the theory, the labor input is given for the firm because it has not control over job-creation and job-destruction Poisson processes but capital is chosen to maximize profit. As there are not frictions neither adjustment cost in the capital market, when a firm knows the total labor QL_{jt} it will have in the present period, it solves the following problem:

$$\max_{k_{jt}}(A_j K_{jt}^{(1-\alpha)} Ql_{jt}^{\alpha} - r_t K_{jt})$$

Substituting the first order condition into the production function and rearranging, we have that:

$$Y_{jt} = \left[\left(A_j^{\frac{1}{1-\alpha}} \frac{1-\alpha}{r_t} \right)^{\frac{1-\alpha}{\alpha}} \right] Ql_{jt} = p_{jt} Ql_{jt}$$

Where r_t is the cost of the capital. Note that this production function is equivalent to $p\varepsilon$, the production function assumed in the theory, where p is time and firm specific: $p_{jt} = A_j^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{r_t} \right)^{\frac{1-\alpha}{\alpha}}$.

Using the panel of firm data on value-added, depreciated capital and number of workers in each category, we estimate (8) in logs without imposing constant returns to scale but forcing constant proportionality between skilled and unskilled workers.

$$log(Y_{jt}) = \log(\breve{A}_j) + \alpha_k \log(K_{jt}^d) + \alpha_l \log(L_{jt}^{ns} + \gamma_i L_{jt}^{is} + \gamma_u L_{jt}^{nu} + \gamma_i \gamma_u L_{jt}^{iu}) + u_{jt} \tag{9}$$

Where L_{jt}^{ns} and L_{jt}^{is} are, respectively, the number of natives and immigrants in skilled occupations in firm j in time t while L_{jt}^{nu} and L_{jt}^{iu} are, respectively, the number of natives and immigrants in unskilled occupations in firm j in time t. We estimate (9)[16] by SYSTEM-GMM, using lagged levels to instrument the equation in differences and lagged differences to instrument

[16] Where K_{jt}^d is the depretiated capital ($K_{jt}^d = \delta K_{jt}$) and $\breve{A}_j$ is the new fixed effect that now is also capturing the depreciation rate ($\breve{A}_j = \delta A_j$)

the equation in levels as proposed in Arellano and Bover (1995)[17]. GMM is the natural candidate for this estimation: firstly because the model predict that labor input will be positively correlated with the firm fixed effects, because the higher the productivity the lower the incentives to job-to-job transitions. Secondly, because we have assumed that there is not friction in the labor market and therefore the optimal capital input is also function of the firm fixed effect.

We have a very severe problem of lack of precision in the GMM $\gamma's$ estimates. This problem is quite normal in this kind of production function's specification. Cahuc et al have opted to estimate the productivity parameters and the wage equation parameter simultaneously by an iterated non-linear least squares procedure. Here we prefer to maintain the productivity estimation *aseptic* from the wage equation estimation. Therefore, we estimate the production function by non-linear least squares.

Results are shown in Table 4. Surprisingly we observe that immigrants are more productive than natives in similar jobs. Although this difference is not significant in trade and services, this fact is fairly stable across groups and across estimation method. Capital coefficient are positive and significant[18]. The precision in the $\gamma's$ GMM estimates is disappointing. We have tried with different set of instrument without been able to solve it. It is also noteworthy the low productivity of unskilled worker that is almost a third of the skilled's one and the good fit of NLLS with R^2 that ranges between 71% and 86%.

4.2 Labor Market Dynamics

Given that job termination occurs due to job-to-job transitions and exogenous job destruction and that both processes are Poisson, the model defines the precise distribution of job durations t conditional on the firm productivity p.

[17]We used all available lag-differences and the first lag-level as instruments. Results reported correspond to the second stage where the optimal weighting-moment matrix has been estimated with the first stage residuals.

[18]SYSTEM-GMM has been broadly used in production function estimations mainly due to the low significance of capital coefficients when using Arellano-Bond or withing-groups estimators. See Blundell and Bond (1999).

Table 4: Production Function Estimates

Non Linear Least Squares

	α_k	α_l	γ_i	γ_u	R^2
Manufacturing	0.215	0.844	2.279	0.334	86%
	(0.005)	(0.007)	(0.172)	(0.014)	
Construction	0.182	0.883	1.783	0.451	80%
	(0.010)	(0.013)	(0.234)	(0.032)	
Trade	0.182	0.853	1.184	0.379	71%
	(0.013)	(0.016)	(0.241)	(0.029)	
Services	0.223	0.757	1.101	0.329	79%
	(0.006)	(0.008)	(0.139)	(0.013)	

Non Linear SYSTEM-GMM

	α_k	α_l	γ_i	γ_u	Sargan p-value
Manufacturing	0.097	0.829	3.073	0.246	11%
	(0.029)	(0.051)	(2.137)	(0.129)	
Construction	0.108	0.669	12.523	1.020	46%
	(0.002)	(0.031)	(14.227)	(0.158)	
Trade	0.079	0.780	0.799	0.377	90%
	(0.010)	(0.065)	(0.633)	(0.129)	
Services	0.182	0.594	2.866	0.137	93%
	(0.025)	(0.067)	(1.863)	(0.064)	

Note: Time dummies included. Standard errors are given in parentheses.

$$\mathcal{L}(t|p) = \left[\delta + \lambda_1 \bar{H}(p)\right] e^{-\left[\delta+\lambda_1 \bar{H}(p)\right]t} \tag{10}$$

As we use GSOEP to estimate transition parameters and it does not have productivity measures, λ_1 and δ are estimated treating p as an unobservable. We then maximize the unconditional likelihood $\mathcal{L}(t) = \int \mathcal{L}(t|p)g(p)dp$..

Taking derivatives with respect to p in equation (6), we get the density of firm's productivity in the population of workers:

$$g(p) = \frac{(1+\kappa_1)h(p)}{1+\kappa_1 \bar{H}(p)} \tag{11}$$

In the appendix we show that replacing (11) in (10), and with some algebra it becomes simple enough to be estimated. The non-censored individual contribution to the likelihood is given by:

$$\mathcal{L}(t) = \frac{\delta(1+\kappa_1)}{\kappa_1}\left[\int_{\delta t}^{(1+\kappa_1)\delta t} \frac{e^{-x}}{x} dx\right]$$

As it is noted in Cahuc et al (2006) Integrating unobserved productivity out of the conditional likelihood removes p and all reference to the sampling distribution $H(p)$. This method is robust to any misspecification in the wage bargaining. The only property of the structural model that is required, is that there exist a scalar firm index, in this case p, which monotonously defines transitions.

In the appendix, we show how to obtain the exact form of the likelihood that takes into account that some duration are right-censored and some others started before the survey started. Finally the individual contribution to the log-likelihood is:

$$\begin{aligned} l(t_i) \quad &= \quad (1-c_i)\log\left(\frac{\int_{\delta t}^{(1+\kappa_1)\delta t} \frac{e^{-x}}{x} dx}{\frac{e^{-\delta H_i}}{\delta} - \frac{e^{-\delta(1+\kappa_1)H_i}}{\delta(1+\kappa_1)} - H_i \int_{\delta H_i}^{(1+\kappa_1)\delta H_i} \frac{e^{-x}}{x} dx}\right) + \qquad (12) \\ & c_i \log\left(\frac{\frac{e^{-\delta t_i}}{\delta} - \frac{e^{-\delta(1+\kappa_1)t_i}}{\delta(1+\kappa_1)} - t_i \int_{\delta t_i}^{(1+\kappa_1)\delta t_i} \frac{e^{-x}}{x} dx}{\frac{e^{-\delta H_i}}{\delta} - \frac{e^{-\delta(1+\kappa_1)H_i}}{\delta(1+\kappa_1)} - H_i \int_{\delta H_i}^{(1+\kappa_1)\delta H_i} \frac{e^{-x}}{x} dx}\right) \end{aligned}$$

Where c_i is a right-censored spell indicator and H_i is the time period elapsed before the sample started.

Maximum likelihood estimates of (12) are reported in Table 5. The average duration of an employment spell (possibly changing employer) is between 8 and 46 year, but the mean-duration between sectors is 15.6 years. The average time between two outside offers range from 2 to 7 years. These results seem to be fairly large but they are compatible with the rest of the literature[19]. Skilled workers have in general lower transition rates to unemployment but there is not a clear pattern in terms of job-to-job transitions. Immigrant are more mobile than natives in terms of job-to-job transitions and, in general they have also higher job-destruction rates. Considering κ_i as

[19] See Jolivet, Postel-Vinay and Robin (2006) for similar estimates using the European Community Huosehold Panel.

Table 5: Transition Parameters - Maximum Likelihood Estimates

	Unskilled					
	Natives			**Immigrant**		
	λ_1	δ	κ_1	λ_1	δ	κ_1
Manufacturing	**0.371**	**0.037**	**9.915**	**0.500**	**0.021**	**23.333**
	(0.035)	(0.002)	(0.734)	(0.067)	(0.003)	(0.179)
Construction	**0.151**	**0.123**	**1.229**	**0.267**	**0.106**	**2.532**
	(0.001)	(0.001)	(0.002)	(0.045)	(0.017)	(0.082)
Trade	**0.201**	**0.092**	**2.185**	**0.269**	**0.117**	**2.304**
	(0.018)	(0.007)	(0.114)	(0.048)	(0.020)	(0.133)
Services	**0.242**	**0.093**	**2.616**	**0.333**	**0.136**	**2.453**
	(0.017)	(0.006)	(0.052)	(0.043)	(0.017)	(0.041)
	Skilled					
	Natives			**Immigrant**		
	λ_1	δ	κ_1	λ_1	δ	κ_1
Manufacturing	**0.261**	**0.036**	**7.293**	**0.369**	**0.055**	**6.723**
	(0.016)	(0.002)	(0.146)	(0.072)	(0.011)	(0.119)
Construction	**0.149**	**0.0806**	**1.849**	**0.314**	**0.126**	**2.499**
	(0.043)	(0.009)	(0.489)	(0.186)	(0.074)	(0.045)
Trade	**0.220**	**0.0571**	**3.860**	**0.253**	**0.107**	**2.357**
	(0.012)	(0.003)	(0.093)	(0.051)	(0.016)	(0.311)
Services	**0.273**	**0.059**	**4.632**	**0.219**	**0.091**	**2.397**
	(0.013)	(0.003)	(0.049)	(0.032)	(0.012)	(0.145)

Note: Per annum estimates. Standard errors are given in parentheses.

an index of frictions it is noteworthy the difference between manufacturing and the rest of the sectors.

4.3 The wage equation: Closing the model

Using the structural wage equation (4):

$$w_{j,t,i} = \varepsilon_i w^p_{j,t,k(i)}(p_{j,t}, \beta_{k(i)})$$

where $w_{j,t,i}$ is the daily wage of a worker i, who belongs to a group $k(i)$, in a firm j with productivity p_j in time t, and:

$$
\begin{aligned}
& w^{p}_{j,t,k(i)}(\beta) \\
= \; & p_{j,t} - (1-\beta_{k(i)})(\rho + \delta_{k(i)} + \lambda_{1,k(i)}\bar{H}(p_{j,t}))^{\beta_{k(i)}} \\
& * \int_{p\min}^{p_{j,t'}} \left(\rho + \delta_{k(i)} + \lambda_{1,k(i)}\bar{H}(p')\right)^{-\beta_{k(i)}} dp'
\end{aligned}
$$

As shown in equation (7) ε is independent of p, then:

$$E(w_{j,t,i}) = E(\varepsilon_i w^{p}_{j,t,k(i)}(p_{j,t},\beta)) = E_k(\varepsilon)E(w^{p}_{jtk}(p_{jt},\beta)))$$

$$E(w_{jtk}) = \gamma_k E(w^{p}_{jtk}(p_{jt},\beta))) \tag{13}$$

Where $E_k(\varepsilon) = \gamma_k$ is mean efficiency units of workers of group k in that market, relative to native skilled group[20].

For each firm in the sample we estimate the average daily wage $\bar{w}_{jtk}$ paid to workers of group k in time t. As wages are top-coded we estimate the firm mean-wage by maximum likelihood at the firm level assuming that wages

[20] The group chosen for normalization is trivial. Changing this group to a generic group k, we would change our measure of productivity. Instead p_j, that is the productivity measured in term of efficiency units of natives skilled we would have $p^k_j = \gamma_k p_j$, that is the productivity measured in term of efficiency units of group k. In fact to define (13) in term of productivity of group k, we only put γ_k inside the expectation operator:

$$
\begin{aligned}
E(w_{j,t,i}) \;=\; & E(\gamma_k p_{j,t} - \\
& (1-\beta_{k(i)})(\rho + \delta_{k(i)} + \lambda_{1,k(i)}\bar{H}(p_{j,t}))^{\beta_{k(i)}} \\
& * \int_{p_{\min}}^{p_{j,t}} \left(\rho + \delta_{k(i)} + \lambda_{1,k(i)}\bar{H}(p')\right)^{-\beta_{k(i)}} \gamma_k dp')
\end{aligned}
$$

Noting that $\frac{dp}{dp^k} = \frac{1}{\gamma_k}$, that $\bar{H}(p_{j,t}) = \bar{H}(p^k_{j,t})$ and changing the variable within the integral

$$
\begin{aligned}
E(w_{j,t,i}) \;=\; & E(p^k_{j,t} - \\
& (1-\beta_{k(i)})(\rho + \delta_{k(i)} + \lambda_{1,k(i)}\bar{H}(p^k_{j,t}))^{\beta_{k(i)}} \\
& * \int_{p^k_{\min}}^{p^k_{j,t}} \left(\rho + \delta_{k(i)} + \lambda_{1,k(i)}\bar{H}(p^{k\prime})\right)^{-\beta_{k(i)}} dp^{k\prime})
\end{aligned}
$$

are $log - normal$. Under the steady state assumption and according to the theory presented in section 2, $\bar{w}_{jtk}$ exhibits stationary fluctuation around the steady state mean wage $E(w_{jtk})$ paid by firm j with productivity p_j.

Then we estimate:

$$\begin{aligned}\log \bar{w}_{jtk} &= \ln(\gamma_k) + \\ &\ln \Big(p_{j,t} - (1-\beta_{k(i)})(\rho + \delta_{k(i)} + \lambda_{1,k(i)}\bar{H}(p_{j,t}))^{\beta_{k(i)}} \\ &* \int_{p\min}^{p_{j,t'}} \left(\rho + \delta_{k(i)} + \lambda_{1,k(i)}\bar{H}(p')\right)^{-\beta_{k(i)}} dp'\Big) \\ &+ v_{jtk}\end{aligned}$$

by Non-Linear Least Squares at the firm level, where v_{jtk} is a transitory shock with unrestricted variance. As usual the discount factor has been set to an annual rate of 5% (daily rate of 0.0134%).

Results are presented in Table 6. In spite of having positive wage differentials, immigrants have lower bargaining power than natives. This is observed in every sector for skilled workers as also for unskilled workers, there is only an exception in skilled workers in services. The test for equality in bargaining power ($\beta_{immigrant} = \beta_{naitve}$) is rejected in every cell.

Contrary to what it have been found in Cahuc et all (2006), here we find that workers in low-qualification occupation have more bargaining power than workers in high-qualification occupations, this finding is also stable across sectors. These differences as differences between sectors may be understood as indirect evidence of compensating differentials.

These estimators are remarkably precise. This is very important because, as it is pointed out in section 3, standard errors are not taking into account the presence of nuisance parameters and therefore they can be considered as a lower bound for the true standard errors.

5 Discussion:

5.1 Counterfactual analysis:

What can we say about labour market discrimination?

Table 6: Bargaining Power Estimates

		Native	Immigrant
		β	β
Manufacturing	**Unskilled**	**0.675**	**0.239**
		(0.006)	(0.003)
	Skilled	**0.269**	**0.186**
		(0.002)	(0.008)
Construction	**Unskilled**	**0.524**	**0.228**
		(0.008)	(0.008)
	Skilled	**0.216**	**0.119**
		(0.004)	(0.029)
Trade	**Unskilled**	**0.467**	**0.286**
		(0.010)	(0.011)
	Skilled	**0.211**	**0.138**
		(0.005)	(0.013)
Services	**Unskilled**	**0.795**	**0.708**
		(0.012)	(0.018)
	Skilled	**0.435**	**0.571**
		(0.006)	(0.027)

Note: Standard errors are given in parentheses.

The estimated structural wage equation provides us a very appealing way to decompose wage differentials:

- Which proportion of the wage gap is due to differences in productivity? This question connects directly with a branch on the literature initiated in Hellerstein et al. In those papers they considered that wages and productivity had a one-to-one mapping and therefore any inequality in wages that are not driven by differences in productivity may be considered as discrimination. Here we have a more sophisticated way to connect wage and productivity, in fact this relationship has been shown to not be an equality, neither for the non-discriminated group.

- Which proportion of the differences in the wage may be due to differences in the mobility patterns. If we only change the offer arrival rate or the destruction rate we can have a monetary measure of the effect of friction pattern between-group differences. This experiment connect directly with audit studies that have attempted to measure difference in probabilities of being hired for the same job between two *a priori* identical workers. This experiment provides a clear causal affect over the hiring probability but it has no prediction over the wage gap, with our wage equation, this could be solved.

- Which proportion of the wage gap is driven by differences in bargaining power. In this context, we have considered this difference as a index of discrimination. because it implies that the wage of an immigrant would differ from the wage of an equivalent native in terms of ability and outside options who is working in an equivalent job in term of occupation, firm's productivity and industry. This is a direct measure of wage discrimination provided by this structural estimation. It is an hypothetical *ceteris paribus* experiment where we only vary migration status but keeping all the other wage determinant constant.

6 Concluding Remarks

This paper is the first attempt to estimate an equilibrium search model using matched employer-employee data to study the extent to which between-group wage differentials can be explained by differences in ability, friction pattern's disparity or wage discrimination. The structural estimation involved several steps: Firstly, we estimated group-specific productivity by production functions estimation at the firm-level using LIAB. Secondly we compute job-retention and job-finding rates using GSOEP employee-level data. Finally, we calculated the wage-setting parameters (bargaining power) using individual wage records in LIAB and those pre-estimated outside options and productivity measures specific for each firm.

When analyzing productivity, we observe that surprisingly immigrant are more productive than natives in similar jobs. This finding has been stable across groups and estimation method. The main finding in terms of friction patterns are that immigrant are more mobile than natives in terms of job-to-job transitions and, in general, they have also higher job-destruction rates. In spite of having positive wage differentials, immigrants have lower bargaining power than natives in similar jobs. This is observed in almost every sector for low-qualification and high-qualification jobs and it would be telling us that immigrant are being discriminated.

There are two main desirable extensions that we would like to perform. Firstly, when estimating the model we have fixed the discount factor. To directly estimate this parameter and moreover to measure the effect of different discount rates over the wage gap would be very informative. Secondly, we have considered the skilled-unskilled separation as exogenous The fact that immigrant has been found to be more productive than similar natives may be revealing some kind of selection between occupations that our model is not able to capture. To endogeneize occupation choice is not trivial but it may give us very valuable information about job segregation.

7 Appendix

7.1 Model Equations Derivation

Here we derive analytically the close form of the equilibrium wage equation. The first step is to find the partial derivative with respect to the wage of the value of a job in a firm with productivity p for a worker with ability ε.

Applying the Leibniz integral rule in (1).

$$\frac{\partial\left[E(w(p,\varepsilon))\right]}{\partial w(p.\varepsilon)} = \frac{1}{(r+\delta+\lambda_1\bar{F}(w(p,\varepsilon)|\varepsilon))} \tag{14}$$

Integrating 14 between $w(\varepsilon)_{\min}$ and $w(p,\varepsilon)$.

$$\begin{aligned}\int_{w(\varepsilon)_{\min}}^{w(p,\varepsilon)} \frac{1}{(r+\delta+\lambda\bar{F}(\tilde{w}(p,\varepsilon)|\varepsilon))}d(\tilde{w}(p,\varepsilon)) &= \int_{w(\varepsilon)_{\min}}^{w(p,\varepsilon)} \frac{\partial\left[E(\tilde{w}(p,\varepsilon),\varepsilon)\right]}{\partial\tilde{w}(p,\varepsilon)}d(\tilde{w}(p,\varepsilon)) \\ E(w(p,\varepsilon),\varepsilon)-E(w(\varepsilon)_{\min},\varepsilon) &= E(w(p,\varepsilon),\varepsilon)-U(\varepsilon)\end{aligned}$$

Using the surplus splitting rule (3), the value of the job for the worker (1), the value of the job for the firm (2) and rearranging:

$$
\begin{aligned}
w(p,\varepsilon) &= p\varepsilon - && (15)\\
& (\rho+\delta+\lambda_1\bar{F}(w(p,\varepsilon)|\varepsilon))\frac{(1-\beta)}{\beta} \\
& * \int_{w(\varepsilon)_{\min}}^{w(p,\varepsilon)} \frac{1}{(\rho+\delta+\lambda\bar{F}(\tilde{w}(p,\varepsilon)|\varepsilon))} d(\tilde{w}(p,\varepsilon)) && (16)
\end{aligned}
$$

Noting that

$$
\begin{aligned}
& \int_{w(\varepsilon)_{\min}}^{w(p,\varepsilon)} \frac{1}{(\rho+\delta+\lambda\bar{F}(\tilde{w}(p,\varepsilon)|\varepsilon))} d(\tilde{w}(p,\varepsilon)) \\
= & \int_{p_{\min}}^{p} \frac{1}{(\rho+\delta+\lambda\bar{H}(p'))} \frac{d(w(p,\varepsilon))}{dp'} dp'
\end{aligned}
$$

and taking derivatives with respect to p.

$$
\begin{aligned}
\frac{d(w(p,\varepsilon))}{dp'} &= \varepsilon - \frac{(1-\beta)}{\beta}\frac{d(w(p,\varepsilon))}{dp'} \\
& +\lambda_1 h(p)\frac{(1-\beta)}{\beta}\int_{p_{\min}}^{p} \frac{1}{(\rho+\delta+\lambda\bar{H}(p'))}\frac{d(w(p,\varepsilon))}{dp'}dp'
\end{aligned}
$$

Then, plugging equation (15):

$$
\begin{aligned}
\frac{d(w(p,\varepsilon))}{dp'} &= \varepsilon + \\
& \lambda_1 h(p)\frac{w(p,\varepsilon)-p\varepsilon}{(\rho+\delta+\lambda\bar{H}(p'))} - \frac{(1-\beta)}{\beta}\frac{d(w(p,\varepsilon))}{dp'}
\end{aligned}
$$

Rearranging, we have a first order differential equation,

$$
\frac{d(w(p,\varepsilon))}{dp'} + \frac{\beta\lambda_1 h(p)}{\rho+\delta+\lambda_1\bar{H}(p)} w(p,\varepsilon) = \varepsilon\beta\left[\frac{\rho+\delta+\lambda_1\bar{H}(p)+\lambda_1 h(p)p}{\rho+\delta+\lambda_1\bar{H}(p)}\right] \quad (17)
$$

To solve this differential equation, note that:

$$
\frac{d(\rho+\delta+\lambda_1\bar{H}(p))^{-\beta}}{dp} = (\rho+\delta+\lambda_1\bar{H}(p))^{-\beta}\frac{\beta\lambda_1 h(p)}{\rho+\delta+\lambda_1\bar{H}(p)}
$$

Then, multiplying both sides of equation (17) by $(\rho+\delta+\lambda_1\bar{H}(p))^{-\beta}$ and rearranging

$$\frac{d\left[w(p,\varepsilon)(\rho+\delta+\lambda_1\bar{H}(p))^{-\beta}\right]}{dp}=\varepsilon\beta\left[\frac{\rho+\delta+\lambda_1\bar{H}(p)+\lambda_1h(p)p}{\left(\rho+\delta+\lambda_1\bar{H}(p)\right)^{1+\beta}}\right] \tag{18}$$

Integrating (18) between $p_{\min}$ and p, and noting that the lowest productivity firm will produce no surplus $\Leftrightarrow$ $w(p_{\min},\varepsilon)=p_{\min}\varepsilon$, straightforward algebra shows that:

$$\begin{aligned}&w(p,\varepsilon)(\rho+\delta+\lambda_1\bar{H}(p))^{-\beta}\\=\;&(\rho+\delta+\lambda_1)^{-\beta}p_{\min}\varepsilon+\varepsilon\beta\int_{p\min}^{p}\left[\frac{\rho+\delta+\lambda_1\bar{H}(p')+\lambda_1h(p')p'}{\left(\rho+\delta+\lambda_1\bar{H}(p')\right)^{1+\beta}}\right]dp'\end{aligned}$$

Separating the integral in a convenient way and noting that:

$$\frac{\partial\left(\left(\rho+\delta+\lambda_1\bar{H}(p')\right)^{-\beta}p'\right)}{\partial p'}=\left(\rho+\delta+\lambda_1\bar{H}(p')\right)^{-\beta}+\frac{\beta\lambda_1h(p')p'}{\left(\rho+\delta+\lambda_1\bar{H}(p')\right)^{1+\beta}}dp'$$

It solves as:

$$\begin{aligned}w(p,\varepsilon)\;=\;&\frac{(\rho+\delta+\lambda_1\bar{H}(p))^{\beta}}{(\rho+\delta+\lambda_1)^{\beta}}p_{\min}\varepsilon-\\&\varepsilon(1-\beta)(\rho+\delta+\lambda_1\bar{H}(p))^{\beta}\int_{p\min}^{p}\left(\rho+\delta+\lambda_1\bar{H}(p')\right)^{-\beta}dp'+\\&\varepsilon(\rho+\delta+\lambda_1\bar{H}(p))^{\beta}\int_{p\min}^{p}\frac{\partial\left(\left(\rho+\delta+\lambda_1\bar{H}(p')\right)^{-\beta}p'\right)}{\partial p'}dp'\end{aligned}$$

Rearranging we get the wage equation as a function of individual skill (ε), friction patterns (δ *and* λ_1) and firm's productivity (p).

$$w(p,\varepsilon)=\varepsilon p-\varepsilon(1-\beta)(\rho+\delta+\lambda_1\bar{H}(p))^{\beta}\int_{p\min}^{p}\left(\rho+\delta+\lambda_1\bar{H}(p')\right)^{-\beta}dp'$$

Now we show that p_{min} is independent of $\varepsilon . p_{\min}$ is the minimum observed productivity level. Firms with productivity $p_{\min}$ make zero profit, and therefore the whole productivity goes to the worker, who receive $\varepsilon p_{\min}$ this wage exactly compensate the worker to leave the unemployment, Therefore:

$$E(p_{\min}\varepsilon, \varepsilon) = U(\varepsilon)$$

$$\begin{aligned} & p\varepsilon + \lambda_1 \int_{w(p_{\min},\varepsilon)}^{w(p_{\max},\varepsilon)} \left[E(w(p',\varepsilon),\varepsilon) - U(\varepsilon)\right] dF(W(p',\varepsilon)) \\ = \; & b\varepsilon + \lambda_0 \int_{w(p_{\min},\varepsilon)}^{w(p_{\max},\varepsilon)} \left[E(w(p',\varepsilon),\varepsilon) - U(\varepsilon)\right] dF(W(p',\varepsilon)) \end{aligned}$$

$$p_{\min}\varepsilon = b\varepsilon + (\lambda_0 - \lambda_1) \int_{w(p_{\min},\varepsilon)}^{w(p_{\max},\varepsilon)} \left[E(w(p',\varepsilon),\varepsilon) - U(\varepsilon)\right] dF(W(p',\varepsilon))$$

The intuition in discrete time is clear because the value of being employed and the value of being unemployed are infinite additions of flows linear on ε ($w(\varepsilon, p)$ and $b\varepsilon$) multiplied by the discount rate and the probability of being in each state, that do not depend on ε. The value of being employed and the value of being unemployed are both linear in ε, then $p_{\min}$ is a function of the parameters of the model and it does not depend on ε ($p \min \varepsilon = \varepsilon\Psi(\lambda_0, \lambda_1, \delta, \rho, b, p_{\max})$). This condition must hold in order to avoid sorting between p and ε..Formal proof must to be done.

7.2 Empirical strategy: Duration model - Maximum Likelihood details

The unconditional likelihood of job spell durations is:

$$\mathcal{L}(t) = \int \mathcal{L}(t|p)g(p)dp.$$

$$\mathcal{L}(t) = \int_{p_{\min}}^{p_{max}} \frac{(1+\kappa_1)h(p)}{1+\kappa_1\bar{H}(p)} \left[\delta + \lambda_1 H(p)\right] e^{-[\delta+\lambda_1 H(p)]t} dp$$

Rearranging and remembering that $\kappa_1 = \lambda_1/\delta$.

$$\mathcal{L}(t)=\frac{(1+\kappa_1)\delta}{\kappa_1}\int_{p_{\min}}^{p_{max}}\frac{1}{\delta+\lambda_1\bar{H}(p)}e^{-[\delta+\lambda_1 H(p)]t}\lambda_1 h(p)dp$$

Changing the variable within the integral, $x=\left[\delta+\lambda_1\bar{H}(p)\right]t$. with straightforward algebra we get:

$$\mathcal{L}(t)=\frac{(1+\kappa_1)\delta}{\kappa_1}\left[E_1(\delta t)-E_1(\delta(1+\kappa_1)t)\right]$$

Where $E_1(t)=\int_t^\infty \frac{e^{-x}}{x}dx$ is the exponential integral function. Our sample covers a fixed number of period so that some job duration are right censored and other job spell have started before our panel's beginning. Then, the exact likelihood function that takes into account those events is:

$$l(t_i)=(1-c_i)\log\left(\frac{\mathcal{L}(t_i)}{\int_{H_i}^\infty \mathcal{L}(t)dt}\right)+c_i\log\left(\frac{\int_{t_i}^\infty \mathcal{L}(t)dt}{\int_{H_i}^\infty \mathcal{L}(t)dt}\right)$$

Where c_i is a truncated spell indicator and H_i is the time period elapsed before the sample.

$$\begin{aligned} l(t_i) &= (1-c_i)\log\left(\frac{\left[E_1(\delta t)-E_1(\delta(1+\kappa_1)t)\right]}{\int_{H_i}^\infty \left[E_1(\delta t)-E_1(\delta(1+\kappa_1)t)\right]dt}\right) \\ &\quad +c_i\log\left(\frac{\int_{t_i}^\infty \left[E_1(\delta t)-E_1(\delta(1+\kappa_1)t)\right]dt}{\int_{H_i}^\infty \left[E_1(\delta t)-E_1(\delta(1+\kappa_1)t)\right]dt}\right) \end{aligned}$$

As we know that $\int E_1(at)dt=-\int E_i(-at)dt=-\left(tE_i(-at)+\frac{e^{-at}}{a}\right)$.(See Abramowitz and Stegun (1972)). And Noting that $E_1(-\infty)=0$

$$\begin{aligned} \int_{t_i}^\infty \left[E_1(\delta t)-E_1(\delta(1+\kappa_1)t)\right]dt &= \int_{t_i}^\infty E_1(\delta t)dt-\int_{t_i}^\infty E_1(\delta(1+\kappa_1)t)dt \\ &= -\,tE_i(-\delta t)+\left.\frac{e^{-\delta t}}{\delta}\right|_{t_i}^\infty + \\ &\quad tE_i(-(1+\kappa_1)\delta t)+\left.\frac{e^{-\delta t}}{(1+\kappa_1)\delta}\right|_{t_i}^\infty \end{aligned}$$

$$
\begin{aligned}
& \int_{t_i}^{\infty} \left[E_1(\delta t) - E_1(\delta(1+\kappa_1)t) \right] dt \\
= \;& t_i E_i(-\delta t_i) + \frac{e^{-\delta t_i}}{\delta} - t_i E_i(-(1+\kappa_1)\delta t) - \frac{e^{-\delta t_i}}{(1+\kappa_1)\delta}
\end{aligned}
$$

As $E_i(-at) = -E_1(at) = -\int_{at_i}^{\infty} \frac{e^{-x}}{x} dx$.

$$
\begin{aligned}
& \int_{t_i}^{\infty} \left[E_1(\delta t) - E_1(\delta(1+\kappa_1)t) \right] dt \\
= \;& \frac{e^{-\delta t_i}}{\delta} - t_i \int_{\delta t_i}^{\delta(1+\kappa_1)t} \frac{e^{-x}}{x} dx - \frac{e^{-\delta t_i}}{(1+\kappa_1)\delta}
\end{aligned}
$$

The same for $\int_{H_i}^{\infty} \mathcal{L}(t)dt$, then the likelihood takes the following form:

$$
\begin{aligned}
l(t_i) \;=\; & (1-c_i) \log \left(\frac{\int_{\delta t}^{(1+\kappa_1)\delta t} \frac{e^{-x}}{x} dx}{\frac{e^{-\delta H_i}}{\delta} - \frac{e^{-\delta(1+\kappa_1)H_i}}{\delta(1+\kappa_1)} - H_i \int_{\delta H_i}^{(1+\kappa_1)\delta H_i} \frac{e^{-x}}{x} dx} \right) + \\
& c_i \log \left(\frac{\frac{e^{-\delta t_i}}{\delta} - \frac{e^{-\delta(1+\kappa_1)t_i}}{\delta(1+\kappa_1)} - t_i \int_{\delta t_i}^{(1+\kappa_1)\delta t_i} \frac{e^{-x}}{x} dx}{\frac{e^{-\delta H_i}}{\delta} - \frac{e^{-\delta(1+\kappa_1)H_i}}{\delta(1+\kappa_1)} - H_i \int_{\delta H_i}^{(1+\kappa_1)\delta H_i} \frac{e^{-x}}{x} dx} \right)
\end{aligned}
$$

7.3 Robustness check

We are constrained in terms of the number variables in which we can condition the analysis. We have only chosen skilled-unskilled because the skilled-unskilled composition is very different between the natives and the immigrants (see Table 2). Other variable that is expected to have a very different composition between both population is gender. We cannot undertake the whole analysis also considering this variable because we do not have enough observation in the GSOEP to estimate transition parameter.

As a robustness check, we present results of the productivity estimates also including gender (we have 8 groups now). Using the panel of firm data on value-added, depreciated capital and number of workers in each category, we estimate (19) by Non linear Least Squares.

Table 7: Production Function Estimates

Non Linear Least Squares

	$\boldsymbol{\alpha}_k$	$\boldsymbol{\alpha}_l$	$\boldsymbol{\gamma}_w$	$\boldsymbol{\gamma}_i$	$\boldsymbol{\gamma}_u$
Manufact.	0.209	0.828	0.418	2.151	0.301
	(0.005)	(0.007)	(0.023)	(0.165)	(0.012)
Construction	0.176	0.869	0.542	1.680	0.336
	(0.010)	(0.013)	(0.052)	(0.228)	(0.026)
Trade	0.160	0.870	0.449	1.120	0.415
	(0.013)	(0.015)	(0.033)	(0.218)	(0.030)
Services	0.212	0.751	0.606	0.971	0.327
	(0.006)	(0.076)	(0.028)	(0.126)	(0.035)

Note: Standard errors are given in parentheses.

$$
\begin{aligned}
log(Y_{jt}) \quad = \quad & \log(\check{A}_j) + \alpha_k \log(K^d_{jt}) + \\
& \alpha_l \log(L^{mns}_{jt} + \gamma_i L^{mis}_{jt} + \gamma_u L^{mnu}_{jt} + \gamma_i \gamma_u L^{miu}_{jt} + \\
& \gamma_f L^{fns}_{jt} + \gamma_f \gamma_i L^{fis}_{jt} + \gamma_f \gamma_u L^{fnu}_{jt} + \gamma_f \gamma_i \gamma_u L^{iu}_{jt}) \\
& + u_{jt}
\end{aligned} \tag{19}
$$

Where L^{mns}_{jt} and L^{mis}_{jt} are, respectively, the number of men and natives, and men and immigrants in skilled occupations in firm j in time t while L^{mnu}_{jt} and L^{miu}_{jt} are, respectively, the number of men and natives, and men and immigrants in unskilled occupations in firm j in time t.The same for female with $f - subindex$.

As it can be seen in Table 7 punctual estimates, as standard errors are quite similar to those presented in Table 4. We cannot reject any test of equality in the punctual estimates of $\gamma_i.\gamma_u$ are also similar but test of equality is rejected in Trade and Construction.

7.4 Detecting Discrimination - Traditional Approach

In order to compare different strategies to detect wage discrimination. We perform the traditional approach using Mincer-type wage equations. As it can be seen in Table 8, immigrants have positive wage differentials. Controlling for observed characteristics, they receive wages, on average, 9.8% higher than natives.

Oaxaca-Blinder Decomposition

Using results presented on Table 8, we perform a Oaxaca-Blinder decomposition which is to simply decompose the wage-gap between differences in observable and unobservable characteristic.

Oaxaca-Blinder decomposition's results are presented in Table 9. The counterfactual immigrant mean-wage has to be interpreted as the mean-wage that immigrants would have if they have the native's distribution of observable characteristics. Therefore the difference between the counterfactual immigrants mean-wage and the observed immigrant mean-wage is the portion of the gap that is due to differences in observable characteristics.

The portion of the unconditional wage-gap that is not accounted for observable characteristics has usually been interpreted as wage discrimination. In this case we would have that immigrant are not being discriminated. They are receiving wages 1.24% higher than similar natives.

These results are fairly different to those obtained in this paper and our hypotheses is that this difference is due to the fact that the traditional approach is not able to control for non-observable differences in productivity between groups.

Table 8: Mincer Wage Equations - Censored-Normal Regression. Maximum Likelihood Estimates

	General	Natives	Immigrants
Sex	-0.188	-0.188	-0.182
	(0.0003)	(0.0003)	(0.0009)
Immigrant	0.098	0.000	1.000
	(0.0004)	(0.0000)	(0.0000)
Age	0.072	0.075	0.035
	(0.0001)	(0.0001)	(0.0003)
Primary Education	0.228	0.229	0.184
	(0.0008)	(0.0006)	(0.0025)
College	0.505	0.505	0.400
(incomplete)	(0.0007)	(0.0006)	(0.0024)
Technical College	0.634	0.638	0.534
(completed)	(0.0008)	(0.0009)	(0.0034)
College	0.797	0.780	0.738
	(0.0009)	(0.0007)	(0.0039)
University Degree	0.916	0.918	0.850
	(0.0008)	(0.0007)	(0.0029)
Tenure	0.029	0.030	0.020
	(0.0001)	(0.0001)	(0.0002)
Experience	0.029	0.028	0.037
	(0.0001)	(0.0001)	(0.0002)
Skilled	0.407	0.407	0.357
	(0.0010)	(0.0010)	(0.0055)
Part-time jobs	-0.666	-0.665	-0.684
	(0.0003)	(0.0003)	(0.0013)
West Germany	0.261	0.263	0.299
	(0.0003)	(0.0003)	(0.0024)
Constant	1.634	2.381	2.467
	(0.0019)	(0.0020)	(0.0081)
Pseudo R2	43.6%	43.7%	45.9%
Observations	18,764,153	17,607,925	1,156,228

Note: Standard errors are given in parentheses. Time Dummies included.

Table 9: Oaxaca-Blinder Decomposition

(a) Observed Natives Mean LogWage	(b) Observed Immigrants Mean-LogWage	(c) Counterfactual Immigrants Mean-LogWage
4.346	4.391	4.358
Unconditional LogWage Difference (a)-(b)	**Explained Difference (c)-(b)**	**Unexplained Difference (c)-(a)**
-0.0449	-0.0326	-0.01239445

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