

# LABOUR MARKET EFFECTS OF PUBLIC SECTOR EMPLOYMENT AND WAGES\*

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## Abstract

I study the labour market effects of public sector employment and wages. I build a dynamic stochastic general equilibrium model with search and matching frictions and both public and private sectors. I discuss what is the public sector wage that achieves the social planner's solution and how it varies with different labour market parameters. Public sector wage and employment shocks have opposite effects on unemployment. Hiring more people reduces unemployment but increasing their pay raises it. Both shocks increase the wage in the private sector and induce the unemployed to search more for public sector jobs dampening private sector job creation. I then discuss the optimal public policy in response to technology shocks. It consists on a counter-cyclical vacancies posting and a procyclical public sector wage. Deviations from the optimal policy can increase the volatility of unemployment rate significantly. I employ Bayesian techniques to estimate the parameters of the model for the US, using quarterly data on government employment and wages, unemployment rate, private sector wages, job separation rate and job finding rate. The separation rate and the matching elasticity with respect to unemployment are much lower in the public sector than in the private sector. There is also evidence of a mildly counter-cyclical vacancy policy but public sector wages do not seem to respond to their private counterpart. To complete the empirical study, I estimate a structural VAR model. I find that public sector employment and wages positively affect private sector wage. The negative effect on private sector hours is only significant over the last 20 years. Both government employment and wages do not respond to innovations in productivity.

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**Keywords:** Public sector employment; Public sector wages, Fiscal shocks.

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# 1 Introduction

Traditionally, macroeconomists have studied the aggregate effects of government spending in the form of goods bought to the private sector.<sup>1</sup> However, the main element of government consumption is compensation to employees. As shown in Table 1, it represents between 50 to 60% of final government consumption expenditures in most OECD countries. But public sector employment is not just an important aspect of fiscal policy. It is also a sizable element of the labour market. In OECD economies, public sector employment ranges from 7% to 30% of total employment. Given its relevance, it seems reasonable that part of the transmission mechanism of fiscal policy occurs through the labour market.

Compared to the theoretical research that focusses on government spending as buying part of the production of the economy, the literature that studies the effects of public sector employment and wages is relatively scarce. Early references include Rotemberg and Woodford (1992) and Finn (1998) that find that, contrary to government purchases of goods and services, the purchase of hours raises real wages and reduces private output. More recently, Pappa (2005) and Cavallo (2005) also concluded that, in a perfectly competitive labour market, after an increase in government employment, private sector hours and output goes down. Ardagna (2007) and Algan, Cahuc, and Zylberberg (2002) study the issue in a unionized economy. In their setting, an increase in public sector employment, public sector wage or unemployment benefits, raises the wage in the private sector and therefore unemployment.

However, to genuinely understand the transmission mechanisms of fiscal policy through the labour market it is crucial to model the existing frictions. There have been some attempts to do it in a search and matching environment. In Holmlund and Linden (1993) an increase in public employment has a direct negative effect in unemployment but crowds out private employment due to an increase in wages. But, for all realistic calibrations the direct effect of reducing unemployment is always stronger than the indirect effect through wages. Quadrini and Trigari (2007) examine the impact of public sector employment on business cycle volatility and find that the presence of public sector increases the volatility of private and total employment. Hörner, Ngai, and Olivetti (2007) study the effect of public enterprises and conclude that a country with public sector firms has higher unemployment than if the companies were privately managed.

The aim of this work is to provide a comprehensive, yet simple, framework to study the macroeconomic effects of public sector employment and public sector wages. I build a dynamic stochastic general equilibrium model with search and matching frictions along the lines of Pissarides (1988) with both public and private sectors. The model shares many features with Quadrini and Trigari (2007). In a first stage, I solve the social planner's problem to find the optimal allocation. I then

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<sup>1</sup>For example: Barro (1990), Baxter and King (1993), Ludvigson (1996) study the effects of government spending in a Neo-Classical setting. Linnemann and Schabert (2003) extends the analyses to the standard New Keynesian model to fiscal policy and Galí, López-Salido, and Vallés (2007) introduces rule of thumb agents. These papers share the feature of considering government spending as buying goods to the private sector. Most of them do not explore the effects of government spending *per se*, but focus on the effects of alternative forms of financing.

solve the decentralized equilibrium and determine the level of public sector wage consistent with the optimal steady state allocation. The optimal public sector wage premium depends only on the differences of the labour market frictions parameters of the public sector relative to the private sector. If the government sets a higher wage, it raises private sector wages and induces too many unemployed to search for public sector jobs, reducing private sector job creation and increasing unemployment. Conversely, if it sets a low wage, few unemployed want a public sector job and the government faces recruitment problems.

The model allows us to disaggregate fiscal shocks into employment and wage shocks. The response to the two shocks is quite different. Hiring more workers reduces the unemployment rate, despite partially crowding out of private sector employment. By contrast, paying more to public sector workers raises unemployment. The effects of an increase in the public sector wage works through two channels. On the one hand, more unemployed will direct their search towards the public sector. On the other hand, the public sector wage spillover to the private sector wage as they increase the value of unemployment. The two channels lead to a reduction of job creation in the private sector.

The opposite effects of the different component of fiscal shocks is one of the key results of the paper. The extensive empirical literature on the macroeconomic effects of government spending finds opposite results on private consumption, real wage or employment.<sup>2</sup> As a consequence, the center of the debate has been on the technical methodology, particularly on the identification of fiscal shocks. I believe that the mixed evidence is more related to the data, rather than the methodological strategy used. Fiscal policy shocks can be different depending on the type of expenditure we are considering: employment, wages, purchases of privately produced goods, government investment or transfers. By including all components together or one in particular, we should expect different results. This argument is not entirely new. Finn (1998) distinguishes between the purchase of goods and services, and employment compensation components of spending and find that they have opposite effects on private output, private employment and private investment. I go a step further and show that disaggregating employment compensation into the level of employment and the per-employee wage also generates different effects. This hypothesis is consistent with recent evidence from Caldara and Kamps (2008) that, using the same variables and sample in a VAR, concluded that different identification strategies yield similar results.

Finally, I examine the properties of the model when subject to technology shocks. The optimal policy consists of a countercyclical public vacancy posting and a procyclical public sector wage. If the government follows the optimal policy, the presence of the public sector reduces unemployment volatility. Deviations from optimal policy can entail significant welfare losses. If, for instance, public sector wage does not respond to the cycle, unemployment volatility increases more than twice relative to the scenario under optimal policy.

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<sup>2</sup>See Caldara and Kamps (2008) for an overview.

Table 1: Public sector and the labour market

	Public wage bill (% gov. consumption)	Public Employment (% total employment)	Unemployment rate	Correlation ( $u_t, l_t^g$ )
Australia	52.2%	14.1%	6.3%	0.51
Austria	53.4%	13.1%	4.7%	0.34
Belgium	53.8%	17.9%	6.9%	0.91
Canada	59.8%	20.5%	6.8%	0.55
Denmark	67.8%	30.5%	4.4%	0.78
Finland	63.2%	24.8%	9.9%	0.76
France	58.4%	22.5%	9.4%	0.95
Germany	41.5%	11.6%	7.5%	0.82
Iceland	60.0%	19.0%	2.3%	0.74
Ireland	57.0%	12.7%	4.3%	0.84
Italy	55.6%	16.9%	10.7%	-0.40
Japan	37.7%	8.4%	4.7%	0.35
Luxembourg	49.1%	15.0%	2.6%	0.88
Netherlands	42.2%	10.9%	2.6%	0.80
Norway	63.1%	33.6%	3.4%	0.82
Portugal	72.8%	14.3%	4.0%	0.22
Spain	59.2%	14.1%	11.4%	0.13
Sweden	59.2%	31.1%	4.7%	0.33
United Kingdom	53.3%	18.0%	5.5%	0.19
United States	66.5%	15.2%	4.1%	0.66
Average	56.3%	18.2%	5.9%	0.49

*Note: Public wage bill, public employment and unemployment rate refer to the year 2000. The correlation between public sector employment and the unemployment rate is computed from quarterly data (1970 to 2007). Source: OECD.*

The calibration of the model poses a serious challenge, because there is no evidence to help us calibrate the friction parameters of the public sector. Therefore, in the empirical part, I employ Bayesian techniques to estimate the parameters of the model for the US, using quarterly data since 1966 on: government employment and wages, unemployment rate, private sector wages, job separation rate and job finding rate. I find that the separation rate and the matching elasticity with respect to unemployment are much lower in the public sector than in the private sector. Moreover, the matching efficiency is higher in the private sector. There is also evidence that the government follows a mildly counter-cyclical vacancy policy but public sector wages do not seem to respond to their private counterpart.

To complete the empirical part of the paper I estimate a structural VAR using US quarterly data since 1950. I find that government employment and wages do not respond to innovations in productivity. Both public sector employment and wages positively affect private sector wage. The negative effect on private sector hours is only significant over the last 20 years.

The paper proceeds as follows. Section 2 describes the model. Section 3 discusses how the government can attain the social planner's solution and what are the costs of moving away from the efficient allocation. Section 4 examines the response to fiscal shocks while Section 5 addresses the optimal fiscal policy in response to technology shocks. In the empirical part of the paper Section

6 shows the impulse responses from the VAR and Section 7 explains the details and displays the results of the model estimation. Section 8 concludes.

## 2 Model

### 2.1 General setting

The model is a dynamic stochastic general equilibrium model with public and private sectors. The only rigidities present in the model are due to search and matching frictions. Public sector variables are denoted with superscript  $\{g\}$  while private sector variables are denoted by  $\{p\}$ . Time is denoted by  $t = 0, 1, 2, \dots$

The labour force consists of many individuals  $j \in [0, 1]$ . Part of them are unemployed ( $u_t$ ), while the remaining are working either in the public ( $l_t^g$ ) or in the private ( $l_t^p$ ) sectors.

$$1 = l_t^p + l_t^g + u_t \quad (1)$$

The presence of search and matching frictions in the labour market prevents some unemployed from finding jobs. The evolution of public and private sector employment depends on the number of new matches  $m_t^p$  and  $m_t^g$  and on separations in each sectors. I consider that, in each period, a constant fraction of jobs are destroyed. This fraction might be different between the two sectors.

$$l_{t+1}^i = (1 - \lambda^i)l_t^i + m_t^i, i = p, g \quad (2)$$

I assume the unemployed choose which sector they want to search in, so  $u_t^i$  represents the number of unemployed searching in sector  $i$ . The number of matches formed in each period is determined by two matching functions.

$$m_t^i = m^i(u_t^i, v_t^i), i = p, g \quad (3)$$

The matching functions are assume to have constant returns to scale and constant elasticity with respect to unemployment and vacancies. An important part of the subsequent analysis focuses on the behaviour of the share of unemployed search for a public sector job, defined as:  $s_t = \frac{u_t^g}{u_t}$ .

From the matching functions we can define the probabilities of vacancies being filled  $q_t^i$ , the job finding rates conditional on searching in a particular sector,  $p_t^i$ , and the unconditional job finding rates,  $f_t^i$ :

$$q_t^i = \frac{m_t^i}{v_t^i}, p_t^i = \frac{m_t^i}{u_t^i}, f_t^i = \frac{m_t^i}{u_t}, i = p, g$$

The assumption of directed search implies that the number of vacancies posted in one sector only affects contemporarily the probability of filling a vacancy in the other sector through the endogenous reaction of  $s_t$ .

## 2.2 Households

In the presence of unemployment risk we would observe consumption differences across different individuals. As in Merz (1995), I assume all the income of the members is pooled so the private consumption is equalised across members.

The household is infinitely-lived and has preferences over private consumption good,  $c_t$ , and a public consumption good  $g_t$ .

$$\sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \quad (4)$$

Where  $\beta \in (0, 1)$  is the discount factor. The budget constraint in period  $t$  is given by:

$$c_t + B_t = (1 + r_{t-1})B_{t-1} + w_t^p l_t^p + w_t^g l_t^g + zu_t + \Pi_t \quad (5)$$

Where  $r_{t-1}$  is the real interest rate from period  $t-1$  to  $t$ ,  $B_{t-1}$  are the holdings of one period bonds.  $w_t^i l_t^i$  is the total wage income from the members working in sector  $i$ . The unemployed members receive unemployment benefits  $z$ . Finally,  $\Pi_t$  encompasses all lump sum transfers from the firm and taxes paid to the government.

The household chooses  $c_t$  and  $B_t$  to maximize the expected lifetime utility subject to the sequence of budget constraints, taking the public consumption good as given. The solution is the consumption Euler equation:

$$u_c(c_t, g_t) = \beta(1 + r_t)E_t[u_c(c_{t+1}, g_{t+1})] \quad (6)$$

## 2.3 Workers

The value to the household of each member depends on their current state. The value of being employed is given by:

$$W_t^i = w_t^i + E_t \beta_{t,t+1} [(1 - \lambda^i)W_{t+1}^i + \lambda^i U_{t+1}], i = p, g \quad (7)$$

The value of being employed depends on the current wage, as well as, the continuation value of the job, that depends on the separation probability in each sector. Under the assumption of directed search the agents are either searching in the private or in the public sector, with value functions given by:

$$U_t^i = z + E_t \beta_{t,t+1} [p_t^i W_{t+1}^i + (1 - p_t^i)U_{t+1}], i = p, g \quad (8)$$

The value of unemployment depends on the level of unemployment benefits and on the probabilities of finding a job in the two sectors. Optimality implies that there are movements between the two segments that guarantee there is no additional gain of searching in one sector *vis-a-vis* the other:

$$U_t^p = U_t^g = U_t \quad (9)$$

This equality determines the share of unemployed searching in each sector. The expression is given implicitly by:

$$\frac{m_t^p E_t[W_{t+1}^p - U_{t+1}]}{(1 - s_t)} = \frac{m_t^g E_t[W_{t+1}^g - U_{t+1}]}{s_t} \quad (10)$$

An increase of the value of being employment, driven by either an increase in the public sector wage, an increase in probability of being hired or a decrease in the separation rate, leads to an endogenous increase in  $s_t$ , the share of the unemployed searching for a public sector job, until there is no extra gain from searching in the public sector. Under the directed search assumption the public sector wage plays a key role in determining  $s_t$ . If we consider that search is random, public sector wages would not affect any decision of households.

## 2.4 Firms

The private sector representative firm hires labour to produce the private consumption good. The production function is linear in labour, but part of the resources produced have to be used to pay for the cost of posting vacancies  $\varsigma^p v_t^p$ .

$$y_t = a_t^p l_t^p - \varsigma^p v_t^p \quad (11)$$

The firm's objective is to maximize present discounted value of profits given by:

$$E_t \sum_{k=0}^{\infty} \beta_{t,t+k} [a_{t+k}^p l_{t+k}^p - w_{t+k}^p l_{t+k}^p - \varsigma^p v_{t+k}^p] \quad (12)$$

Where  $\beta_{t,t+k} = \beta^k \frac{u_c(c_{t+k}, g_{t+k})}{u_c(c_t, g_t)}$  is the stochastic discount factor. The firm faces the law of motion for private sector employment given by:

$$l_{t+1}^p = (1 - \lambda^p) l_t^p + q_t^p v_t^p \quad (13)$$

The firm takes the probability of filling a vacancy,  $q_t^p$ , as given. At time  $t$  the level of employment is predetermined and the firm can only control the number of vacancies it posts. The solution to the firm's problem is given by equation (14).

$$\frac{\varsigma^p}{q_t^p} = E_t \beta_{t,t+1} [a_{t+1}^p - w_{t+1}^p + (1 - \lambda^p) \frac{\varsigma^p}{q_{t+1}^p}] \quad (14)$$

The condition states that the expected cost of hiring a worker must equal its expected return. The benefits of hiring an extra worker is the discounted value of the expected difference between its marginal productivity and its wage and the continuation value, knowing that with some probability  $\lambda^p$  the match can be destroyed.

## 2.5 Private sector wage bargaining

I consider the private sector wage is the outcome of a Nash bargaining between workers and firms. The solution is given by:

$$(1 - b)(W_t^p - U_t) = bJ_t \quad (15)$$

Where  $J_t$  is the value of a job for the firm, given by the following expression:

$$J_t = a_t^p - w_t^p + E_t \beta_{t,t+1} [(1 - \lambda^p) J_{t+1}] \quad (16)$$

## 2.6 Government

The government produces its consumption good using a linear technology on labour. As in the private sector, the costs of posting vacancies are deduced from production.

$$g_t = a_t^g l_t^g - \varsigma^g v_t^g \quad (17)$$

It sets a lump sum tax to finance the wage bill and the unemployment benefits.

$$\tau_t = w_t^g l_t^g + z u_t \quad (18)$$

Finally, the government follows a policy for public sector vacancies and public sector wage  $\{v_t^g, w_{t+1}^g\}_{t=0}^{\infty}$ . I assume the government sets the wage one period in advance, at the time it posts the vacancies. As  $s_t$  is determined based on the expected value of both public and private sector wage in  $t + 1$ , the current period public sector wage does not affect any variable in the model. I assume the government commits to a certain future path for wages. There is no time inconsistency problem because, as taxes are lump sum, the government does not gain from setting a different current wage than promised.

## 2.7 Social planner's solution

Up until now, I have described the competitive equilibrium. As a benchmark for the analysis of the model we are going to consider the efficient solution. The social planner's problem is to maximize the consumers lifetime utility (4) subject to the labour market and technology constraints (1)-(3), (11) and (17). The first order conditions are given by:

$$\frac{\varsigma^p}{q_t^p} = \beta E_t \left\{ \frac{u_c(c_{t+1}, g_{t+1})}{u_c(c_t, g_t)} \left[ (1 - \eta^p) a_{t+1}^p + (1 - \lambda^p) \frac{\varsigma^p}{q_{t+1}^p} - \frac{\eta^p \varsigma^p v_{t+1}^p}{(1 - s_{t+1}) u_{t+1}} \right] \right\} \quad (19)$$

$$\frac{\varsigma^g}{q_t^g} = \beta E_t \left\{ \frac{u_g(c_{t+1}, g_{t+1})}{u_g(c_t, g_t)} \left[ (1 - \eta^g) a_{t+1}^g + (1 - \lambda^g) \frac{\varsigma^g}{q_{t+1}^g} - \frac{\eta^g \varsigma^g v_{t+1}^g}{s_{t+1} u_{t+1}} \right] \right\} \quad (20)$$

$$\frac{u_g(c_t, g_t) \varsigma^g v_t^g \eta^g}{(1 - \eta^g) s_t} = \frac{u_c(c_t, g_t) \varsigma^p v_t^p \eta^p}{(1 - \eta^p) (1 - s_t)} \quad (21)$$



Where  $\eta^i$  is the matching elasticity with respect to unemployment. Conditions (19) and (20) describe the optimal level of private and public sector vacancies. On the left hand side we have the expected cost of hiring an extra worker. The right hand side give us the marginal benefit of hiring an additional worker. The social benefit of having an extra worker in a sector is its marginal productivity weighted by the matching elasticity with respect to unemployment, plus, given that the match is kept with probability  $(1 - \lambda^i)$ , its continuation value. The third element, that enters negatively in the expression, reflects the fact that hiring an additional worker makes it harder to recruit a worker in the future for both sectors.

The optimal share of unemployed searching in a particular sector, pinned down in (21), depends positively on the marginal utility of consumption of the respective good, on the number of vacancies and their cost, and on the matching elasticity with respect to unemployment in the sector.

### 3 Steady state analysis

#### 3.1 Parametrization

To solve the model, we have to make assumptions on the functional form of the utility function and the matching function. I assume the utility function has a CES form which allow us to address different elasticities of substitution between the private and public consumption good. As typical in the literature, the matching function is a Cobb-Douglas.

$$m_t^i = \bar{m}^i u_t^{i\eta^i} v_t^{i1-\eta^i}$$

$$u(c_t, g_t) = \frac{1}{\gamma} \ln[c_t^\gamma + \zeta g_t^\gamma]$$

Table 2 shows the baseline parametrization. I calibrate the model at a quarterly frequency to be, in general, representative of an OECD economy. I normalize the technology in both sectors to 1 and the discount factor to 0.99. I consider the case with no unemployment benefits. The separation rate in the private sector is 0.06. I set the private sector matching elasticity with respect to unemployment,  $\eta^i$ , to 0.5 and I give the same value to the worker's share in the Nash bargaining, so the model satisfies the Hosios condition.

Unfortunately, there are not many studies that help us assign realistic values to the public sector parameters. A notable exception is a recent study on labour market flows in the United Kingdom by Gomes (2008). He finds that the separation rate in the public sector is less than half the separation rate in the private sector (1.5% in the private sector and 0.7% in the public sector, quarterly flows). Furthermore, he finds that the job finding rate is 6 times higher in the private sector (22% in the private sector against 3.6% in the public sector).

Taking an agnostic perspective, I consider that  $\eta^i, c^i$  and  $\bar{m}^i$  are equal for both sectors. I then distinguish two cases: one where both sectors have equal separation probabilities and one where the separation rate of the public sector is half the one of the private sector.

I distinguish three cases regarding the relation between the private and government consumption: if they are close substitutes ( $\gamma = 0.8$ ), complements ( $\gamma = -1.0$ ) and a case with an elasticity of substitution of 1 ( $\gamma = 0.0$ ). In each case, the parameter  $\zeta$  is chosen such that the optimal level of public sector employment is 0.15.

The parameter of vacancy cost and the scale of the matching functions are chosen such that the optimal unemployment rate is around 6.5% and the overall unconditional job finding probability is around 0.85.

Table 2: Parametrization

	$a^p$	1	$\eta^p$	0.5	$\zeta^p$	0.5	$\bar{m}^p$	0.65	$\lambda^p$	0.06	$z$	0.0
	$a^g$	1	$\eta^g$	0.5	$\zeta^g$	0.5	$\bar{m}^g$	0.65	$b$	0.5	$\beta$	0.99
	$\gamma = 0$				$\gamma = 0.8$				$\gamma = -1.0$			
	$\zeta = 0.191$				$\zeta = 0.718$				$\zeta = 0.037$			
$\lambda^g = 0.06$	$l^g$	0.150	$u$	6.58	$l^g$	0.150	$u$	6.58	$l^g$	0.151	$u$	6.58
	$q^g$	0.496	$q^p$	0.496	$q^g$	0.496	$q^p$	0.496	$q^g$	0.496	$q^p$	0.496
	$f^g$	0.091	$f^p$	0.761	$f^g$	0.137	$f^p$	0.715	$f^g$	0.182	$f^p$	0.670
$\lambda^g = 0.03$	$l^g$	0.155	$u$	6.23	$l^g$	0.192	$u$	6.05	$l^g$	0.152	$u$	5.88
	$q^g$	0.480	$q^p$	0.496	$q^g$	0.480	$q^p$	0.496	$q^g$	0.480	$q^p$	0.496
	$f^g$	0.050	$f^p$	0.804	$f^g$	0.077	$f^p$	0.777	$f^g$	0.106	$f^p$	0.750

### 3.2 Attaining the steady-state efficient allocation

The efficient steady state allocation consists on the triplet of  $\{\bar{v}^p, \bar{v}^g, \bar{s}\}$ .<sup>3</sup> In order to achieve it, the government can post a number of vacancies consistent with the optimal allocation directly, but it has still to induce an optimal share of the unemployed searching for public sector jobs. The government can do so by choosing an appropriate level of the public sector wage. The Nash bargaining in the private sector, provided that Hosios condition is satisfied, insures an efficient posting of private sector vacancies.<sup>4</sup>

Let us assume that the government sets the public sector wage, giving a premium over the private sector wage:

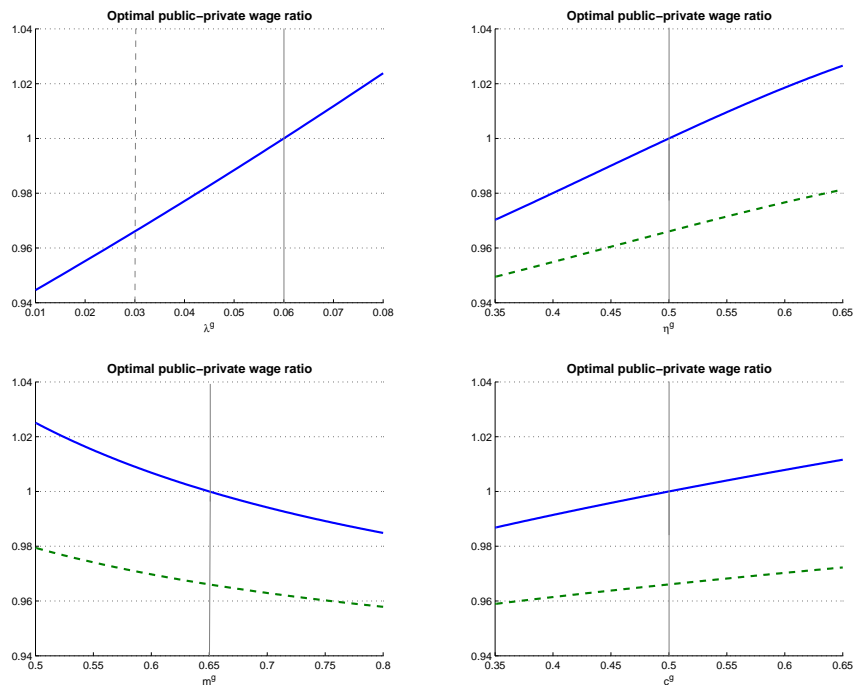
$$w^g = \psi w^p$$

<sup>3</sup>The first natural question to ask is whether efficiency can be achieved if the government privatizes the sector. There are two possible answers depending on whether we consider the frictions in the public sector as intrinsic to the sector itself or to the fact that they are publicly run. Consider first, that the frictions are different across sectors because one of them is run by the government (for instance, it has lower separation rate and higher matching efficiency). If the sector is privatized, it is not clear that, with a different set of friction parameters, the representative household would be better off. On the other hand, if we consider the frictions are intrinsic to the sector itself, then it is easy to understand why the competitive equilibrium would not necessarily be efficient. Even if the Hosios condition was satisfied in both sectors, the resulting wages would not necessarily be consistent with the optimal  $s_t$ .

<sup>4</sup>This is a result from simulation. If the government set their vacancies optimally and sets a wage such that the optimal share of unemployed searching for public jobs, the decentralized equilibrium level of private sector vacancies, if the Hosios condition is satisfied, will be efficient.

Even though we cannot find an analytical solution for the optimal ratio as a function of the model's parameters, we can find it numerically. The optimal public-private wage ratio does not depend on the parameters of the utility function or on technology. We can see in Figure 1 how it depends on the different labour market parameters.

Figure 1: Optimal public-private wage ratio



*Note: Solid line corresponds to the case  $\lambda^g = 0.06$ ; dash line corresponds to the case  $\lambda^g = 0.03$*

In the symmetric baseline scenario, the optimal public-private wage ratio is 1. When the separation rate in the public sector is half the one in the private sector, the government reaches the optimal allocation with a public sector wage gap of 3.5%. The lower the separation rate of the public sector relative to the private sector, the higher the public sector wage gap should be. As the separation rate decreases, the value of a public job for the unemployed increases so they increase the search of public jobs above optimum. The public sector wage should decline relative to the private sector to ensure that the search effort is optimal.

When the public sector matching process depends more on vacancies than the private sector (smaller  $\eta^g$ ), the government should also pay less to its workers. This is also the case when the matching efficiency is higher in the public sector or when the cost of posting a public sector vacancy is lower than in the private sector.

### 3.3 Welfare cost of public sector wages

Estimates of public sector wage premium have proved quite sensitive to the country choice, empirical specification, education and sex of worker or even the sub-sector of the government. In

general, people have found a positive wage premium in the public sector ranging from 3% and 11% (Gregory and Borland (1999)). These values can only be reconciled with the optimal allocation if the separation rate, the coefficient on unemployment on the matching function and the cost of posting a vacancy are higher than in the private sector, or if the matching efficiency in the public is much lower than in the private sector.

The only available evidence suggests that the separation rate in the public sector is lower than in the private sector. Furthermore, as the recruitment process is more centralized, it is not likely that the weight on vacancies in the matching function is lower in the public sector. Therefore, there is a clear possibility that governments are paying more to their employees than the optimum.

To investigate the welfare consequences of paying more to public sector employees, I compare the unemployment rate and household's welfare when the public sector wage is optimal, with the case where the government sets exogenously a premium of 5% over the private sector wage. If the wages are out of the optimum, we have to make an assumption on how the public employment is set. I consider that the government either sets the optimal level of vacancies or it sets the optimal level of employment. When the government sets the optimal level of vacancies, as more unemployed search for public jobs because of higher wages, the level of public sector employment is higher than the optimum.

Table 3: Welfare costs of high public sector wage

		$u^{opt}$		Optimal $l^g$		Optimal $v^g$	
		$u^{opt}$	$\frac{w^g}{w^p}^{opt}$	$u^{\psi=1.05}$	$Cost^{\psi=1.05}$	$u^{\psi=1.05}$	$Cost^{\psi=1.05}$
$\lambda^g = 0.06$	$\gamma = 0$	6.58%	1.000	7.23%	0.35%	7.65%	4.75%
	$\gamma = 0.8$	6.58%	1.000	7.23%	0.34%	7.65%	1.51%
	$\gamma = -1.0$	6.58%	1.000	7.23%	0.36%	7.66%	7.70%
$\lambda^g = 0.03$	$\gamma = 0$	6.05%	0.966	7.04%	0.85%	7.95%	27.99%
	$\gamma = 0.8$	5.93%	0.966	7.15%	1.10%	8.28%	13.19%
	$\gamma = -1.0$	6.07%	0.966	7.03%	0.84%	7.92%	35.80%

*Note:  $u^{opt}$  and  $\frac{w^g}{w^p}^{opt}$  indicate the optimal unemployment rate and the public-private wage ratio.  $u^{\psi=1.05}$  is the unemployment rate with a public sector wage premium of 5% and  $Cost^{\psi=1.05}$  give the welfare lost as a percentage of steady state private consumption good.*

Table 3 shows the optimal level of unemployment, the optimal public-private wage ratio, the unemployment if the government sets exogenously a wage premium of 5% and the welfare costs as a percentage of steady state consumption loss.

In the symmetric case, the optimal level of unemployment is around 6.5%, independently of the parameters of the utility function. When the government sets its wage 5% above the optimum, the unemployment rate is between 0.7 and 1.0 percentage points higher, depending on the type of policy the government follows. The welfare costs are substantial higher if the government sets the optimal level of vacancies. If it keeps public sector employment constant, the only source of inefficiency is a lower private sector employment which generates a welfare loss of around 0.35% of steady state

private consumption. If it keeps the public sector vacancies constant, as more unemployed search for public sector jobs, total public sector employment will be higher and private sector employment even lower, so we would be further deviating from the efficient allocation. The welfare loss under this scenario varies between 1.5% to 7.5% of steady state private consumption, depending on the substitutability or complementarity between the two goods.

When the separation rate in the public sector is half the one in the private sector, the optimal unemployment rate depends on  $\gamma$  but the optimal public sector wage gap is constant at around 3.3%. By increasing the public sector wage premium to 5%, the unemployment rate increases between 1.0% and 2.3% of the labour force. The welfare cost can be as high as 35% of steady state consumption for the particular case of constant public sector vacancies and strong complementarity between the two goods. The exact values of the welfare lost are particularly sensitive to the different scenarios and the employment policy. However it seems that public sector wage can significantly affect the equilibrium unemployment rate.

## 4 The effects of fiscal shocks

### 4.1 Response to a public sector employment shock

This framework allows us to disentangle the effects of increasing public sector employment from increasing the public sector wage. Starting from the efficient steady state, I explore the effects of a persistent shock to public sector vacancies.

$$\ln(v_t^g) = \ln(\bar{v}^g) + \epsilon_t^v$$

where  $\epsilon_t$  follows an AR(1) process with autocorrelation coefficient of 0.8. I assume the government holds the public sector wage constant at its optimal steady state level.<sup>5</sup> Figure 2 shows the response of the variables to a public sector vacancies shock under the symmetric calibration.

Independently of the  $\gamma$ , the impact of a public employment shock on unemployment is negative. After the shock, private sector employment does decrease but the crowding out is not complete. There are three channels through which hiring more employees in the public sector reduces the job creation in the private sector. First, as the pool of unemployed goes down, the cost of hiring an extra worker increases. Second, more unemployed search for public sector jobs which further reduces the probability of filling a vacancy for the firm. Finally, as the overall job finding probability increases so does the value of being unemployed, which increases the private sector wage through the wage bargaining. These three elements decrease the job creation in the private sector but not enough to overcome the direct impact of public employment on unemployment. If the goods are strong substitutes the effect on unemployment is smaller. Households reduce more the consumption of the private good, leading to a bigger crowding out of the private sector employment. If the goods

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<sup>5</sup>I have also analysed the case where the government sets the premium over the private sector wage as constant which would allow the public sector wage to move after the shock. The results are very similar.

are complements the overall effect on unemployment is stronger, as the increase in government consumption good raises the marginal utility of private consumption.

## 4.2 Response to a public sector wage shock

I consider the effects of a AR(1) shock to the public sector wage with a autocorrelation coefficient equal to 0.8. I consider that the government sets the level of employment at the optimal level ( $l_t^g = \bar{l}^g$ ).<sup>6</sup>

$$\ln(w_t^g) = \ln(\bar{w}^g) + \epsilon_t^w$$

Figure 3 shows the impulse responses. An increase in public sector wage leads to a reduction of private sector employment *via* two channels. On the one hand, the increase of the public sector wage spillover to the private sector wage. On the other hand, it induces more unemployed to search for a job in the government, which reduces the probability of filling a vacancy for firm in the private sector. As a consequence, the firm posts less vacancies and unemployment rises.

The opposite effects of the different types of fiscal shocks is one of the key results of the paper. The extensive literature that tries to understand the macroeconomic effects of fiscal policy tends to find contradictory effects on real wages, employment and private consumption. Rotemberg and Woodford (1992) find that after a military expenditure shock (government military purchases and military employment) real wages go up but Edelberg, Eichenbaum, and Fisher (1999) and Ramey and Shapiro (1998) find that after a government military purchases shock real wages go down. Blanchard and Perotti (2002), Fatás and Mihov (2001) find that private consumption increases after a government consumption shock but Mountford and Uhlig (2008), Ramey (2008) and Tenhofen and Wolff (2007) report a negative or zero response of private consumption. Most of the discussion has focused particularly on the technical methodology, particularly on the identification of fiscal shocks. In light of my results, I believe the mixed evidence is not due to methodological issues, but it is related to the data used. Fiscal policy shocks can be different depending on the type of expenditure we are considering. By including all components together or different samples, as in most studies, we are mixing the effects.

## 5 Public sector policies and the business cycle

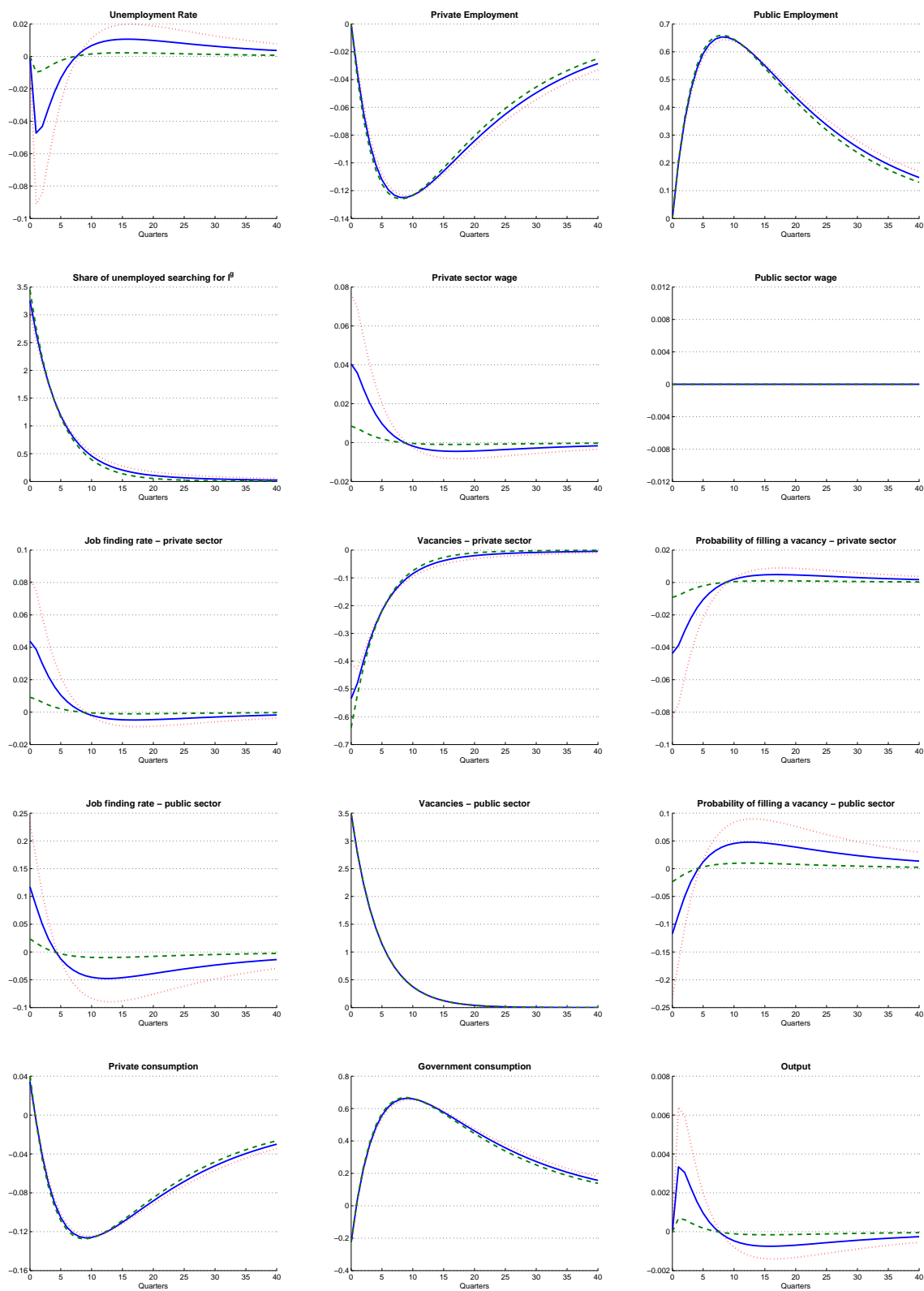
### 5.1 Response under the optimal rule

One of the main conclusion of the Real Business Cycle theory is that governments should not try to pursue active business cycle policies. Although the model presented is in its essence a real business cycle model with only real frictions, the policy prescription is the opposite. Let's examine

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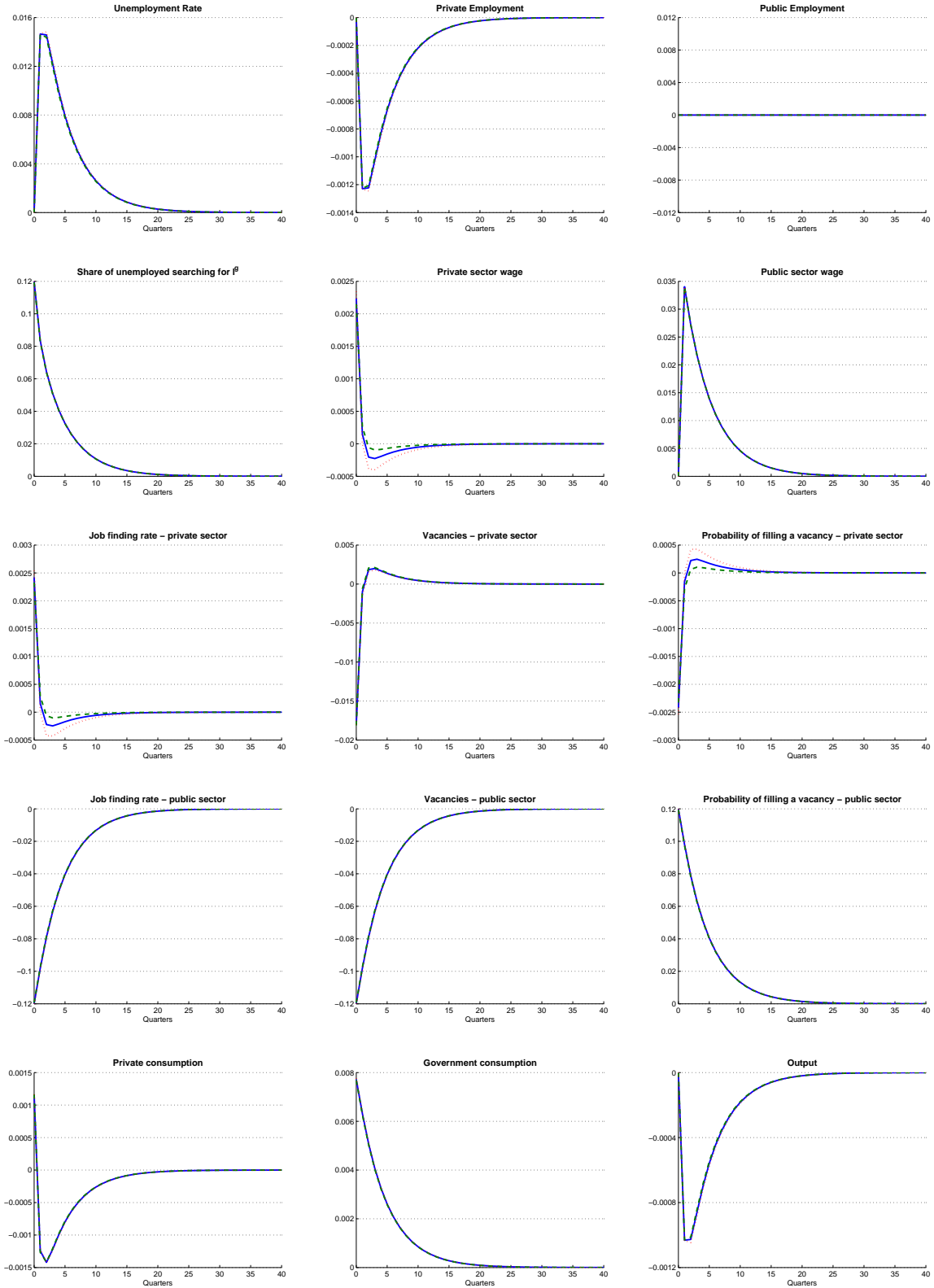
<sup>6</sup>I could alternatively assume that the government keeps the level of vacancies constant. If it sets the level of public employment, the government posts less vacancies to attract the same number of workers. If it maintains the same number of vacancies, as more unemployed search for government jobs, public sector employment increases after a wage shock. Under this policy, the shock to wages also incorporates a shock to employment. Although this does not change the results qualitatively, the overall positive effect on unemployment is not as strong.

Figure 2: Response to a public sector vacancies shock



Note: Baseline case with  $\lambda^g = 0.06$ . Solid line ( $\gamma = 0.0$ ); dash line ( $\gamma = 0.8$ ) and dotted line ( $\gamma = -1.0$ ). Variables in logs.

Figure 3: Response to a public sector wage shock



Note: Baseline case with  $\lambda^g = 0.06$ . Solid line ( $\gamma = 0.0$ ); dash line ( $\gamma = 0.8$ ) and dotted line ( $\gamma = -1.0$ ). Variables in logs.



the effects of a persistent private technology shock on the economy, under different government policies. I again consider an AR(1) shock with autoregressive coefficient of 0.8.

$$\ln(a_t^p) = \ln(\bar{a}^p) + \epsilon_t^a$$

The impulse responses when the government follows the optimal rule are shown in Figure 4. After the productivity shock, the private sector posts more vacancies, the probability of finding a job in the private sector increases so the unemployed increase their search of private sector jobs. The unemployment rate decreases.

We can see from the graphs that the optimal response of the government is to reduce the number of public vacancies and consequently of public employment. This result is overturn only in the case where the goods are strong complements. In that case, the government should increase its vacancies. The argument for hiring more people in recessions to work in the public sector is different from the traditional demand argument dated back to Keynes. If the private sector has lower productivity, it is better for the economy to absorb part of the unused labour force into the public sector. If, for example, the government jobs were not productive (the famous metaphor of digging holes and covering them) it would not be optimal to hire anyone at the first place.

On the other hand, under the three scenarios, public sector wage should follow closely the rise of the private sector wage. In recessions, if public sector wage is constant, they become more attractive relative to the private sector wage, therefore increasing the number of unemployed searching for public sector jobs. This further dampens private job creation and amplifies the business cycle. The optimal fiscal policy is, therefore, to have a leaning-against-the-wind countercyclical vacancies policy and a procyclical wage policy.

In their paper, Quadrini and Trigari (2007) concluded that the best policy to stabilize total employment is to have a procyclical public sector employment. This appears counter-intuitive and it is clearly at odds with the optimal policy.<sup>7</sup> In their model, the government does not choose vacancies and wages optimally. Instead, it sets a constant wage premium exogenously. As we have seen previously, this potentially generates significant deviations from the efficient steady-state but it might also distort the response of the model to shocks. Under a high public sector wage premium, the crowding out of private sector employment after a shock to public sector employment can be more than complete, so it can increase unemployment. This switch in the effects of public sector employment on unemployment completely alters the policy recommendations.

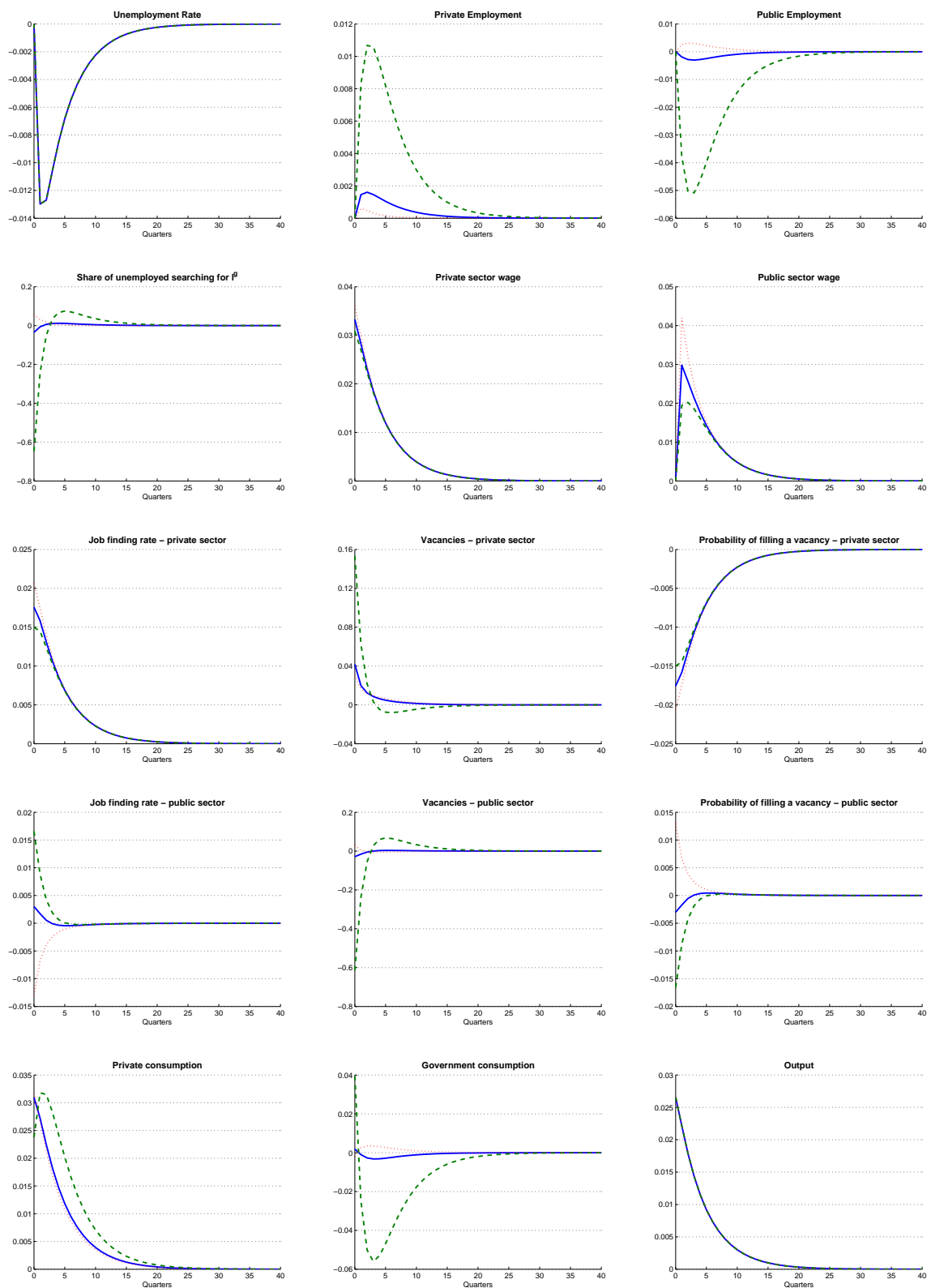
## 5.2 Response under simple rules

Existing evidence by Lane (2003); Lamo, Pérez, and Schuknecht (2007) suggest that public sector wage are less procyclical than private sector wages, which implies that governments might not

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<sup>7</sup>Unless there are strong complementarities between the two goods, which was not their case

Figure 4: Response to a private sector technology shock under the optimal policy



Note: Baseline case with  $\lambda^g = 0.06$ . Solid line ( $\gamma = 0.0$ ); dash line ( $\gamma = 0.8$ ) and dotted line ( $\gamma = -1.0$ ). Variables in logs.

be following the optimal policy. I now compare the responses to a technology shock when the government follows simple rules for vacancies and public sector wage of the type:

$$\log(v_t^g) = \log(\bar{v}^g) + \zeta^v [\log(v_t^p) - \log(\bar{v}^p)] \quad (22)$$

$$\log(w_t^g) = \log(\bar{w}^g) + \zeta^w [\log(w_t^p) - \log(\bar{w}^p)] \quad (23)$$

I consider three rules. The first where the public sector vacancies decline proportionally to the private sector vacancies and the public sector wage moves one to one with private sector wages ( $\zeta^v = -1$  and  $\zeta^w = 1$ ), which should mimic closely the optimal policy. The second rule is a countercyclical public sector vacancies but with constant public sector wage ( $\zeta^v = -1$  and  $\zeta^w = 0$ ). In addition, the case where the governments do not respond to the cycle ( $\zeta^v = 0$  and  $\zeta^w = 0$ ). Figure 5 shows the impulse response functions.

When the government does not adjust public sector wages, the response of unemployment is much stronger. The intuition is the following. In recessions, if public sector wage is constant, they become more attractive relative to the private sector wage, so the unemployed will increase their search of public sector jobs. This further dampens private job creation and increases unemployment. We can see that, when the public sector wages are constant, the fall of the search of public sector jobs is almost four times as high as in the case with procyclical wage.

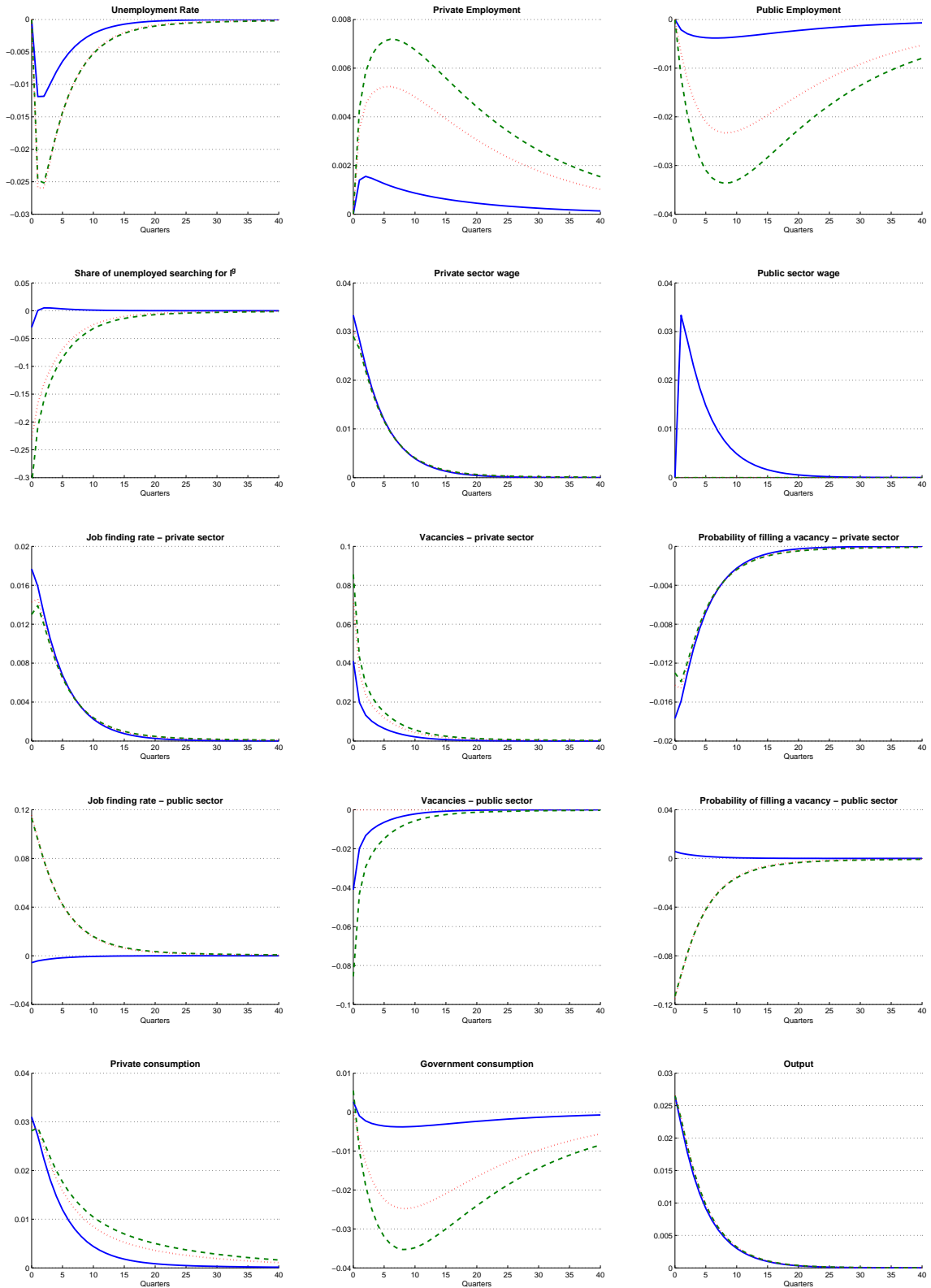
Another conclusion of Quadrini and Trigari (2007) was that the presence of public employment in the economy increases the volatility of unemployment. In our model this is not necessarily true. Table 4 compares the standard deviation of the key variables under the different policies, as well as, when there is no public employment. Under the optimal rule, the presence of public sector employment decreases the volatility of unemployment relative to the case where the public sector is absent. This also happens under the simple rule of countercyclical vacancies and procyclical wages. Nevertheless, under the two policies where the public sector wages are acyclical, the volatility of unemployment increases by twofold. The effects of the presence of public sector employment on the volatility of unemployment depends crucially on the government policy.

The last column of the table gives the welfare costs of business cycles under different rules.<sup>8</sup> When the public sector is absent, the welfare costs of fluctuations are very small. This is a well known result from the literature. When the public sector is present and the government follows the optimal policy the welfare costs of fluctuations are lower than when the public sector is absent. However, when the government does not follow the optimal policy, the welfare cost of fluctuations increases significantly, particularly if the public sector wages are constant. The cost can be up to three times higher compared to the optimal policy scenario.

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<sup>8</sup>See appendix for details.

Figure 5: Response to a private sector technology shock under alternative rules



Note: Baseline case with  $\gamma = 0.0$  and  $\lambda^g = 0.06$ . Variables in logs. Solid line (countercyclical vacancies and procyclical wages), dash line (countercyclical vacancies and constant wages) and dotted line (constant vacancies and wages).

Table 4: Business cycle properties under the different policies

	Policy	Standard deviations					Correl ( $l_t^g, u_t$ )	Welfare cost
		$l_t^p$	$l_t^g$	$u_t$	$w_t^p$	$y_t$		
$\zeta = 0$		0.002	—	0.030	0.059	0.054	—	0.153%
$\gamma = 0.0$	Opt.	0.003	0.007	0.025	0.058	0.046	0.95	0.126%
	Rule 1	0.005	0.016	0.024	0.058	0.046	0.69	0.127%
	Rule 2	0.030	0.145	0.051	0.054	0.047	0.65	0.376%
	Rule 3	0.022	0.099	0.053	0.055	0.047	0.63	0.266%
$\gamma = 0.8$	Opt.	0.024	0.117	0.025	0.056	0.046	0.94	0.091%
	Rule 1	0.004	0.015	0.023	0.057	0.046	0.69	0.126%
	Rule 2	0.029	0.137	0.052	0.056	0.047	0.64	0.164%
	Rule 3	0.021	0.094	0.052	0.056	0.047	0.62	0.157%
$\gamma = -1.0$	Opt.	0.001	0.007	0.025	0.061	0.046	-0.92	0.173%
	Rule 1	0.005	0.017	0.025	0.059	0.046	0.69	0.185%
	Rule 2	0.032	0.153	0.051	0.053	0.047	0.67	0.855%
	Rule 3	0.023	0.105	0.053	0.055	0.047	0.64	0.546%

## 6 Evidence from structural VAR

In this section I estimate a VAR model to understand how does the economy respond to exogenous shocks in public sector employment and wages, and whether or not the response of the government policy to a technology shocks is close to the optimal policy.

### 6.1 VAR Setting

The structural VAR is given by

$$A\mathbf{Y}_t = C(L)\mathbf{Y}_{t-1} + B\mathbf{v}_t$$

$$\mathbf{Y}_t = \begin{bmatrix} \pi_t \\ l_t^g \\ w_t^g \\ w_t^p \\ h_t^p \end{bmatrix}$$

Where  $\mathbf{Y}_t$  is the vector of macroeconomic variables. It includes five variables: private sector hours  $h_t^p$ , private sector wage  $w_t^p$ , government wage  $w_t^g$ , government employment  $l_t^g$  and productivity  $\pi_t$ . Matrix  $A$  describes the contemporaneous relation among the variables and  $C(L)$  is a matrix finite-order lag polynomial.  $\mathbf{v}_t$  is the vector of structural disturbances and matrix  $B$  reflect the disturbances variance and possible covariance. From the reduce form estimation residuals  $\boldsymbol{\mu}_t$  by imposing restrictions on matrices  $A$  and  $B$  we can back up the structural innovations.

$$A\boldsymbol{\mu}_t = B\mathbf{v}_t$$

I consider the Cholesky recursive decomposition where:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \alpha_{21} & 1 & 0 & 0 & 0 \\ \alpha_{31} & \alpha_{32} & 1 & 0 & 0 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & 1 & 0 \\ \alpha_{51} & \alpha_{51} & \alpha_{52} & \alpha_{53} & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 & 0 \\ 0 & 0 & b_{33} & 0 & 0 \\ 0 & 0 & 0 & b_{44} & 0 \\ 0 & 0 & 0 & 0 & b_{55} \end{bmatrix}$$

As we are interested in identifying productivity, government employment and government wages shocks, only the ordering on the first three variables matter. I order government wages after employment, because a shock to employment affects the average wage by an definition. I allow both of them to respond contemporaneously to a productivity shock.

The data are taken mostly from the Bureau of Labour Statistics and the Bureau of economic analysis. The baseline variables are “business sector: hours of all persons”  $h_t^p$ , “business sector: compensation per hour”  $w_t^p$ , “government per employee nominal wage”  $w_t^g$ , government employment  $l_t^g$  and “business sector: output per hour of all persons”  $\pi_t$ .

The overall sample starts in 1950. Under the baseline case I take the natural logarithm of the variables and I HP filter the data so all variables enter as percentage deviations from trend. I estimate the VAR with two lags (which was pointed by final prediction error, Akaike’s, Schwarz’s Bayesian and the Hannan and Quinn information criterion).

## 6.2 Impulse responses

Figure 6 shows the impulse responses to the three shocks: productivity shock, government employment shock and government wage shock. The error bands correspond to a 90% confidence interval.

After a productivity shock, private sector wage increase contemporaneously (with an elasticity of 1/6) but the effect dies out quickly. The contemporaneous response of private hours is not statistically significant. However, it picks up in the following quarters and stay above zero for more than 2 years. Government variables do not respond significantly to productivity shocks. Government employment increases after a productivity shock but the error bands are quite large. Government wage, if any, responds negatively after the shock.

A government employment shock has the expected positive significant effect on private sector wage. Nevertheless, the effect on private hours is not negative as expected - it is statistically not different from zero. One possible explanation for this is that the government services produced with public employment are complement to the private sector. Either because the goods are complements in the utility function or because public employment raises the productivity of the private sector. The positive response of productivity to a government employment shock gives some support for this hypotheses. Alternatively, there might be a demand channel which pushes the private sector

production. At the time of the government employment shock, average government wage tends to diminished but soon picks up and stays above level for around 4 years.

A government wage shock does impinge into private sector wage, with an elasticity of around 0.11%. It has also a negative effect on private sector total hours although it is not statistically significant. Overall, the model's responses to both public employment and wages seem consistent with the data.

### 6.3 Robustness check

To access the robustness of the results I run several alternative models. I use alternative variables: real wages instead of nominal wages and, as an alternative to total private hours, I use both unemployment rate and vacancies. I consider different subsamples: splitting the sample in two, as well as restricting it to the last 20 years. I try a different ordering of the variables, with productivity lining up after government employment and government wages. As an alternative to the HP filter, I use other detrending methods: I estimate the VAR with the variables in first differences and compute the cumulative impulse responses and I use a linear and quadratic detrending of the variables. Finally, I have also estimated the VAR with different lag lengths. The impulse responses of selected variables are shown in Appendix.

In general, the results are in line with those of the baseline case. There are, however, some results worth mentioning. The effect of a government employment shock on private real wages is not statistical different from zero, suggesting that the increase in nominal wage from the baseline case is somehow deteriorated by increasing inflation. As in the baseline case, government real wage shock impinges on private sector real wages but the effect on hours in not significant.

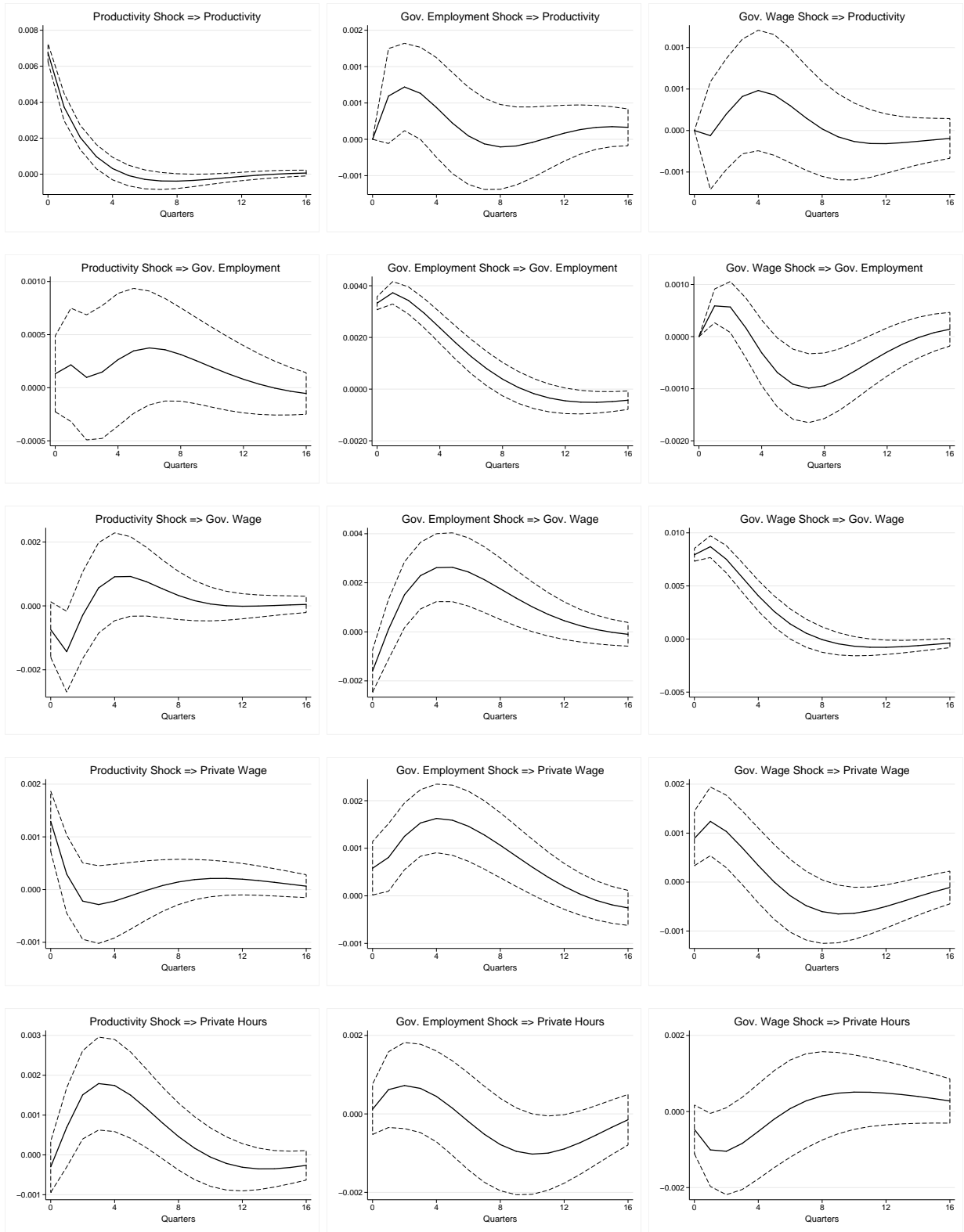
When we restrict the sample to last 20 year, total hours decline after both a government employment and wage shocks, which is more in line with the predictions of model. The properties of government employment might be intrinsically different now from the ones in previous decades. Also the effect of government wage on private wage is much stronger (elasticity of around 0.5%).

When we use the unemployment rate, instead of private hours, it goes down after both productivity and a government employment shock. If we use vacancies, they increase after a government employment shock, and also after a government wage shock. This mixed evidence on the effect of public sector employment and wages on private sector might indicate that there are other channels playing a role.

Estimating the VAR with different lag lengths, running it in differences or after different detrending methods and ordering of the variables does not alter the main results.

## 7 Bayesian Estimation of the model

Figure 6: Baseline VAR





One of the caveats of the model is that the quantitative predictions and policy prescriptions of the model are closely tied to a number of parameters for which not much evidence exists, namely the friction parameters in the public sector and the parameters of the utility function. As a further contribution I estimate a log-linearized version of the model using Bayesian methods as in Smets and Wouters (2007) and Sala, Söderström, and Trigari (2008). I use quarterly data on US from 1966Q1 to 2007Q3 for 6 variables: (1) unemployment rate, (2) government employment (% of labour force), (3) government per employee real wage, (4) private sector per hour real wage, (5) employment unemployment transition probability and (6) unemployed job finding rate.

As in the theoretical section, I assume two simple rules for public sector wages and vacancies, where each variable responds to its private counterpart:

$$\begin{aligned}\ln(v_t^g) &= \ln(\bar{v}^g) + \psi^v[\ln(v_t^p) - \ln(\bar{v}^p)] + \ln(\omega_t^v) \\ \ln(w_t^g) &= \ln(\bar{w}^g) + \psi^w[\ln(w_t^p) - \ln(\bar{w}^p)] + \ln(\omega_t^w)\end{aligned}$$

I include 6 different shocks: a shock to government vacancies, to government wages, to private and public separation rates, private sector bargaining power and to technology. These shocks are described in the following equations:

$$\begin{aligned}\ln(\omega_t^v) &= \rho^v \ln(\omega_{t-1}^v) + \epsilon_t^v \\ \ln(\omega_t^w) &= \rho^w \ln(\omega_{t-1}^w) + \epsilon_t^w \\ \ln(\lambda_t^g) &= (1 - \rho^{lg}) \ln(\bar{\lambda}^g) + \rho^{lg} \ln(\lambda_{t-1}^g) + \epsilon_t^{lg} \\ \ln(\lambda_t^p) &= (1 - \rho^{lp}) \ln(\bar{\lambda}^p) + \rho^{lp} \ln(\lambda_{t-1}^p) + \epsilon_t^{lp} \\ \ln(b_t) &= (1 - \rho^b) \ln(\bar{b}) + \rho^b \ln(b_{t-1}) + \epsilon_t^b \\ \ln(a_t^p) &= (1 - \rho^a) \ln(\bar{a}^p) + \rho^a \ln(a_{t-1}^p) + \epsilon_t^a\end{aligned}$$

With the exception of the wages, all other variables are stationary. I take advantage of this and use four different transformations of the data to estimate the model. In the first version the stationary variables enter in levels and the wages enter in log differences. In the second version all variables enter in log differences. In the third version, all stationary variables are previously HP filtered and the wages enter in log differences. Finally, in the last version, all variables are HP filtered. In Appendix I state all the equations of the model in its log-linearized form and the relation of the observable variables with the model variables in each version.

I use Shimer's data on transition probabilities to calculate the job finding and job separation rates. The available data are at monthly frequency but we estimate the model at a quarterly frequency. To retrieve the quarterly separation rate I compute the probability of multiple monthly transitions between all states (employment, unemployment and inactivity) starting from employment and ending up in unemployment. A similar procedure is done for the job finding rate.

The model contains 18 parameters plus 12 parameters of the shock processes. I normalise the steady state technology to 1 and I calibrate  $\beta$  to 0.99 and the utility function parameter

$\zeta$  to be equal to 1.92. I estimate all other parameters. I assume the matching elasticity with respect to unemployment, the flow value of unemployment, the steady-state bargaining power of the unemployed and the autoregressive coefficients of the shock process, to have a Beta distribution. I assume that the standard deviations of the shock process have a inverse gamma distribution. All other parameters are assumed to be normally distributed.

The most interesting parameters to estimate are the ones related to the matching frictions of the two sectors. Although there are some empirical evidence that the separation rates are different across sector and introspection would suggest differences in the other parameters, I start with an agnostic perspective. I consider equal priors across sectors. The prior distribution of the business cycle policy parameters is centered around 0.

There are two parameters difficult to identify. The cost of posting a vacancy in the public sector only affects the model through the the stochastic discount factor. The scale parameter for the public sector matching function is also difficult to identify because, in each round of new draws of the parameters  $\bar{v}^g$  is determined such that the steady state public employment is always 0.15.

I estimate the model with Bayesian methods (see An and Schorfheide (2007) for a review). The likelihood function of the model is combined with the prior distribution of the parameters, to obtain the posterior distribution. Then, 500,000 draws of the posterior are generated with the Metropolis Hastings algorithm, where the step size is chosen such that the acceptance rate is equal to 1/3. The draws are divided into two chains of 250.000 draws each in order to evaluate the stability of the sample. Table 5 and 6 report the prior distribution and the mean, the 5th and the 95th percentile of the posterior distribution of parameters.

The posterior distribution of the parameters are quite similar across the 4 alternative versions. The distribution of the separation rate in the public sector is quite small - around 1/6 of the one in the private sector. Another clear-cut result is that the elasticity of the matching function with respect to unemployment is much lower in the public sector. The estimated mean value for the private sector is around 0.75, but only 0.3 in the public sector. For the private sector, the mean value for the cost of posting a vacancy is lower and the matching efficiency is higher than the prior. The posterior distribution of the corresponding parameters in the public sector is very close to the prior distribution, which confirms that it is hard to identify them.

On the policy side, there is a mild countercyclical policy in vacancies with an estimated mean of  $-0.35$ . On the other hand, public sector wage policy is close to acyclical. The average mean of the distribution is only 0.06.

The estimated mean for the bargaining power is around 0.7, lower than the value found in Sala, Söderström, and Trigari (2008). The posterior distribution of the flow value of unemployment depends on the estimated version of the model. The mean of the distribution is between 0.28 and 0.68. The posterior distribution of  $\gamma$  suggests that there are no particular complementarity or substitutability between the two goods in the utility function.

Table 5: Prior and posterior distribution of structural parameters

		Prior	Levels	Posterior distribution		
		distribution		Dif	HP	HPw
<b>a) Structural parameters</b>						
Cost of posting vacancy - private sector	$\zeta^p$	Normal (0.6,0.10)	0.51 (0.40,0.62)	0.51 (0.38,0.64)	0.58 (0.43,0.74)	0.52 (0.39,0.63)
Cost of posting vacancy - public sector	$\zeta^g$	Normal (0.6,0.10)	0.61 (0.52,0.68)	0.65 (0.53,0.80)	0.61 (0.46,0.74)	0.55 (0.35,0.73)
Matching efficiency (private sector)	$m^p$	Normal (0.6,0.10)	0.93 (0.86,0.99)	0.70 (0.63,0.78)	0.76 (0.60,0.91)	0.80 (0.68,0.91)
Matching efficiency - public sector	$m^g$	Normal (0.6,0.10)	0.64 (0.50,0.80)	0.60 (0.48,0.76)	0.62 (0.42,0.80)	0.65 (0.53,0.79)
Matching elasticity w.r.t unemployment - private sector	$\eta^p$	Beta (0.5,0.10)	0.76 (0.67,0.85)	0.75 (0.68,0.82)	0.75 (0.66,0.84)	0.72 (0.66,0.80)
Matching elasticity w.r.t unemployment - public sector	$\eta^g$	Beta (0.5,0.10)	0.32 (0.20,0.44)	0.30 (0.21,0.38)	0.31 (0.19,0.43)	0.35 (0.21,0.46)
Separation rate - private sector	$\lambda^p$	Normal (0.06,0.01)	0.037 (0.034,0.040)	0.066 (0.060,0.072)	0.064 (0.051,0.076)	0.070 (0.054,0.087)
Separation rate - public sector	$\lambda^g$	Normal (0.06,0.01)	0.005 (0.004,0.006)	0.009 (0.007,0.012)	0.008 (0.005,0.011)	0.013 (0.006,0.019)
Relative flow value of unemployment	$z$	Beta (0.5,0.15)	0.28 (0.14,0.42)	0.49 (0.38,0.59)	0.47 (0.33,0.62)	0.68 (0.59,0.76)
Elasticity of substitution (public and private goods)	$\gamma$	Normal (0.00,0.10)	0.05 (-0.06, 0.16)	-0.04 (-0.13, 0.04)	-0.01 (-0.17, 0.15)	-0.01 (-0.12, 0.11)
Bargaining power	$b$	Beta (0.5,0.10)	0.69 (0.62,0.76)	0.73 (0.66,0.81)	0.74 (0.64,0.83)	0.64 (0.53,0.75)
Public sector wage premium	$\psi$	Normal (1.05,0.01)	1.052 (1.039,1.066)	1.057 (1.040,1.0718)	1.067 (1.051,1.083)	1.063 (1.051,1.076)
Business cycle response of public sector wages	$\psi^w$	Normal (0.0,0.2)	0.05 (-0.12, 0.20)	0.08 (-0.12, 0.25)	0.12 (-0.06, 0.30)	0.01 (-0.18, 0.18)
Business cycle response of public sector vacancies	$\psi^v$	Normal (0.0,0.2)	-0.35 (-0.54, -0.15)	-0.39 (-0.56, -0.24)	-0.32 (-0.55, -0.08)	-0.34 (-0.55, -0.13)

In appendix, I present tables of the posterior distribution when I restrict the sample to the last 20 years. The estimated means of the distribution do not change much, although the standard deviation of the posteriors are bigger.

## 8 Conclusion

This paper sheds some light on the links between public and private sectors focussing in the labour market. I have built a dynamic stochastic general equilibrium model with search and matching frictions, to analyse the effects of fiscal policies in the labour market, as well as to determine the optimal policy following technological shocks. I want to highlight three main conclusions.

Firstly, public sector wage play an important role in archiving the steady-state efficient allocation. If the government sets a very high wage, more unemployed direct their search towards public sector jobs, private sector wage will be higher and private sector job creation lower. As a consequence, equilibrium unemployment will be higher and the welfare of the representative consumer lower.

The second conclusion is that the response of unemployment to fiscal shocks depends on whether

Table 6: Prior and posterior distribution of shock parameters

		Prior	Levels	Posterior distribution		
		distribution		Dif	HP	HPw
<b>b) Autoregressive parameters</b>						
Productivity	$\rho^a$	Beta	0.95	0.94	0.94	0.66
		(0.5,0.15)	(0.93,0.97)	(0.92,0.97)	(0.92,0.96)	(0.57,0.76)
Public sector wage	$\rho^w$	Beta	0.93	0.93	0.93	0.74
		(0.5,0.15)	(0.91,0.95)	(0.91,0.96)	(0.91,0.95)	(0.67,0.81)
Public sector vacancies	$\rho^v$	Beta	0.34	0.52	0.39	0.32
		(0.5,0.15)	(0.18,0.51)	(0.34,0.68)	(0.16,0.62)	(0.17,0.49)
Private sector separation rate	$\rho^{lp}$	Beta	0.89	0.90	0.72	0.69
		(0.5,0.15)	(0.85,0.92)	(0.87,0.93)	(0.64,0.79)	(0.62,0.77)
Public sector separation rate	$\rho^{lg}$	Beta	0.21	0.16	0.13	0.13
		(0.5,0.15)	(0.12,0.30)	(0.08,0.23)	(0.06,0.20)	(0.06,0.20)
Bargaining power	$\rho^b$	Beta	0.92	0.91	0.88	0.71
		(0.5,0.15)	(0.88,0.96)	(0.87,0.95)	(0.83,0.93)	(0.61,0.80)
<b>c) Standard deviations</b>						
Productivity	$\sigma^a$	IGamma	0.008	0.008	0.007	0.007
		(0.05,0.15)	(0.007,0.008)	(0.007,0.008)	(0.007,0.008)	(0.006,0.007)
Public sector wage	$\sigma^w$	IGamma	0.010	0.010	0.010	0.009
		(0.05,0.15)	(0.009,0.011)	(0.009,0.011)	(0.009,0.011)	(0.008,0.010)
Public sector vacancies	$\sigma^v$	IGamma	0.046	0.040	0.053	0.038
		(0.05,0.15)	(0.015,0.076)	(0.013,0.072)	(0.016,0.086)	(0.018,0.057)
Private sector separation rate	$\sigma^{lp}$	IGamma	0.081	0.073	0.068	0.068
		(0.05,0.15)	(0.071,0.090)	(0.066,0.080)	(0.061,0.074)	(0.062,0.075)
Public sector separation rate	$\sigma^{lg}$	IGamma	0.902	0.496	0.578	0.373
		(0.05,0.15)	(0.724,1.042)	(0.369,0.637)	(0.343,0.821)	(0.167,0.556)
Bargaining power	$\sigma^b$	IGamma	0.081	0.073	0.068	0.068
		(0.05,0.15)	(0.071,0.090)	(0.066,0.080)	(0.061,0.074)	(0.062,0.075)

it is a shock to employment or to wages. A shock to public sector employment decreases unemployment while a shock to public sector wages increases unemployment. The opposite effects of the different components of government consumption on unemployment, might be one reason why many empirical studies on the effects of government spending find ambiguous results.

Finally, the optimal fiscal response to business cycles is to have a "leaning against the wind" public employment policy and a procyclical public sector wage. Empirical evidence on the US suggest that, particularly, public sector wages do not follow the optimal policy. Deviations from the optimal policy can substantially increase unemployment volatility and the welfare costs of fluctuations.

Recent research has emphasised the importance of the interaction between labour market friction and nominal rigidities for the understanding of business cycles and the effects of monetary policy (For instance: Thomas (2008), Krause, Lopez-Salido, and Lubik (2008), Sveen and Weinke (2008) and Blanchard and Gali (2008)). I show that the presence of labour market frictions also increases the scope of action for governments and the effects of fiscal policy, even in the absence of nominal rigidities. Future work includes introduction of sticky prices to better understand the interaction between fiscal and monetary policy.

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## Appendix I - Derivations

### Social planner's problem

The social planner's problem is to maximize the consumers lifetime utility (4) subject to the technology constraints (17) and (11) and the labour market conditions: (1)-(3). Setting up the lagrangean:

$$\begin{aligned} & \sum_{k=0}^{\infty} \beta^{t+k} \{u(a_{t+k}^p l_{t+k}^p - \varsigma^p v_{t+k}^p, a_{t+k}^g l_{t+k}^g - \varsigma^g v_{t+k}^g) \\ & - \Omega_{t+k}^1 [l_{t+k+1}^p - (1 - \lambda^p) l_{t+k}^p - m((1 - s_{t+k})(1 - l_{t+k}^p - l_{t+k}^g), v_{t+k}^p)] \\ & - \Omega_{t+k}^2 [l_{t+k+1}^g - (1 - \lambda^g) l_{t+k}^g - m(s_{t+k}(1 - l_{t+k}^p - l_{t+k}^g), v_{t+k}^g)]\} \end{aligned}$$

The first order conditions are given by:

$$\begin{aligned} v_t^c & : u_c(c_t, g_t) \varsigma^p = \Omega_t^1 (1 - \eta^p) q_t^p \\ v_t^g & : u_g(c_t, g_t) \varsigma^g = \Omega_t^2 (1 - \eta^g) q_t^g \\ s_t & : \frac{\Omega_t^2 \eta^g m_t^g}{s_t} = \frac{\Omega_t^1 \eta^c m_t^c}{1 - s_t} \\ l_{t+1}^p & : \Omega_t^1 = \beta \{a_{t+1}^p u_c(c_{t+1}, g_{t+1}) + \Omega_{t+1}^1 (1 - \lambda^p) - \Omega_{t+1}^1 \eta^p \frac{m_{t+1}^c}{u_{t+1}} - \Omega_{t+1}^2 \eta^g \frac{m_{t+1}^g}{u_{t+1}}\} \\ l_{t+1}^g & : \Omega_t^2 = \beta \{a_{t+1}^g u_g(c_{t+1}, g_{t+1}) + \Omega_{t+1}^2 (1 - \lambda^g) - \Omega_{t+1}^1 \eta^c \frac{m_{t+1}^c}{u_{t+1}} - \Omega_{t+1}^2 \eta^g \frac{m_{t+1}^g}{u_{t+1}}\} \end{aligned}$$

Plugging the first two equations on the third gives the implicit expression for optimal level of search in the each sector:

$$\frac{u_g(c_t, g_t) \varsigma^g \eta^g v_t^g}{(1 - \eta^g) s_t} = \frac{u_c(c_t, g_t) \varsigma^p \eta^p v_t^p}{(1 - \eta^p) (1 - s_t)}$$

If we rewrite the third first order condition as  $\Omega_t^2 \eta^g M_t^g + \Omega_t^1 \eta^c M_t^c = \frac{\Omega_t^2 \eta^g M_t^g}{s_t} = \frac{\Omega_t^1 \eta^c M_t^c}{1 - s_t}$  we can use it to simplify the last two conditions and get:

$$\begin{aligned} \frac{\varsigma^p}{q_t^p} & = \beta \frac{u_c(c_{t+1}, g_{t+1})}{u_c(c_t, g_t)} \left\{ (1 - \eta^p) a_{t+1}^p + (1 - \lambda^p) \frac{\varsigma^p}{q_{t+1}^p} - \frac{\eta^p \varsigma^p v_{t+1}^p}{(1 - s_{t+1}) u_{t+1}} \right\} \\ \frac{\varsigma^g}{q_t^g} & = \beta \frac{u_g(c_{t+1}, g_{t+1})}{u_g(c_t, g_t)} \left\{ (1 - \eta^g) a_{t+1}^g + (1 - \lambda^g) \frac{\varsigma^g}{q_{t+1}^g} - \frac{\eta^g \varsigma^g v_{t+1}^g}{s_{t+1} u_{t+1}} \right\} \end{aligned}$$

### Welfare costs of high public sector wages

Let  $\{c^{opt}, g^{opt}\}$  be the steady state private and government consumption under the optimal public sector wage and  $\{\bar{c}, \bar{g}\}$  the allocation under an exogenous public sector wage. We want to find out what is the welfare cost as a percentage of steady state private consumption of having public sector wage different from the optimum. This is given by  $x$  that solves the following equation:

$$u((1 - x)c^{opt}, g^{opt}) = u(\bar{c}, \bar{g})$$



Using the CES function,

$$\begin{aligned}
& [((1-x)c^{opt})^\gamma + \zeta g^{opt\gamma}] = [\bar{c}^\gamma + \zeta \bar{g}^\gamma] \\
& x = 1 - \frac{[\bar{c}^\gamma + \zeta \bar{g}^\gamma - \zeta g^{opt\gamma}]^{\frac{1}{\gamma}}}{c^{opt}}, \gamma \neq 0
\end{aligned}$$

If  $\gamma = 0$ , the utility function is not defined, so I use the equivalent  $u(c_t, g_t) = \ln(c_t) + \zeta \ln(g_t)$ , so the welfare cost in terms of steady state consumption is given by:

$$x = 1 - \frac{\exp[\ln(\bar{c}) + \zeta \ln(\bar{g}) - \zeta \ln(g^{opt})]}{c^{opt}}, \gamma = 0$$

## Welfare costs of business cycles

I want to calculate the welfare costs of business cycles, when the economy is subject to technology shocks, under different policies for  $\{v_t^g, w_t^g\}$ . Let us start by defining the variables in log deviations from the steady state:

$$\begin{aligned}
\tilde{c}_t &= \log\left(\frac{c_t}{\bar{c}}\right) \\
\tilde{g}_t &= \log\left(\frac{g_t}{\bar{g}}\right)
\end{aligned}$$

so

$$\begin{aligned}
c_t &= \bar{c} \exp(\tilde{c}_t) \\
g_t &= \bar{g} \exp(\tilde{g}_t)
\end{aligned}$$

If we do a second order approximation to the variables around the steady state  $\{\bar{c}, \bar{g}\}$

$$\begin{aligned}
c_t &= \bar{c}(1 + \tilde{c}_t + \frac{1}{2}\tilde{c}_t^2) + o(3) \\
g_t &= \bar{g}(1 + \tilde{g}_t + \frac{1}{2}\tilde{g}_t^2) + o(3)
\end{aligned}$$

The second order approximation of the utility function gives:

$$\begin{aligned}
u(c_t, g_t) &= u(\bar{c}, \bar{g}) + u_c(\bar{c}, \bar{g})[c_t - \bar{c}] + u_g(\bar{c}, \bar{g})[g_t - \bar{g}] \\
&+ \frac{1}{2}u_{cc}(\bar{c}, \bar{g})[c_t - \bar{c}]^2 + \frac{1}{2}u_{gg}(\bar{c}, \bar{g})[g_t - \bar{g}]^2 + u_{cg}(\bar{c}, \bar{g})[c_t - \bar{c}][g_t - \bar{g}] + o(3)
\end{aligned}$$

But for it to be a correct second order approximation we have to plug in the second order approximation of the variables.

$$\begin{aligned}
u(c_t, g_t) &= u(\bar{c}, \bar{g}) \\
&+ u_c(\bar{c}, \bar{g})[\bar{c}(1 + \tilde{c}_t + \frac{1}{2}\tilde{c}_t^2) - \bar{c}] \\
&+ u_g(\bar{c}, \bar{g})[\bar{g}(1 + \tilde{g}_t + \frac{1}{2}\tilde{g}_t^2) - \bar{g}] \\
&+ \frac{1}{2}u_{cc}(\bar{c}, \bar{g})[\bar{c}(1 + \tilde{c}_t + \frac{1}{2}\tilde{c}_t^2) - \bar{c}]^2 \\
&+ \frac{1}{2}u_{gg}(\bar{c}, \bar{g})[\bar{g}(1 + \tilde{g}_t + \frac{1}{2}\tilde{g}_t^2) - \bar{g}]^2 \\
&+ u_{cg}(\bar{c}, \bar{g})[\bar{c}(1 + \tilde{c}_t + \frac{1}{2}\tilde{c}_t^2) - \bar{c}][\bar{g}(1 + \tilde{g}_t + \frac{1}{2}\tilde{g}_t^2) - \bar{g}] + o(3)
\end{aligned}$$

If we collect terms,

$$u(c_t, g_t) = u(\bar{c}, \bar{g}) + u_c \bar{c} \tilde{c}_t + u_g \bar{g} \tilde{g}_t + \frac{\bar{c}}{2} (\bar{c} u_{cc} + u_c) \tilde{c}_t^2 + \frac{\bar{g}}{2} (\bar{g} u_{gg} + u_g) \tilde{g}_t^2 + u_{cg}(\bar{c}, \bar{g}) \bar{c} \tilde{c}_t \tilde{g}_t + o(3)$$

and take the unconditional expectation, we can write the welfare loss in terms of the moments of the variables:

$$E[u(c_t, g_t) - u(\bar{c}, \bar{g})] \approx u_c \bar{c} E[\tilde{c}_t] + u_g \bar{g} E[\tilde{g}_t] + \frac{\bar{c}}{2} (\bar{c} u_{cc} + u_c) E[\tilde{c}_t^2] + \frac{\bar{g}}{2} (\bar{g} u_{gg} + u_g) E[\tilde{g}_t^2] + u_{cg}(\bar{c}, \bar{g}) \bar{c} \bar{g} E[\tilde{c}_t \tilde{g}_t] \equiv \Xi$$

I solve the model up to a second order, using perturbation methods, simulate it for 50000 periods and compute the moments of the variables to find the value of  $\Xi$ . If we want to express the welfare costs in terms of percentage of steady state consumption we solve the following equation:

$$u((1-x)\bar{c}, \bar{g}) - u(\bar{c}, \bar{g}) = \Xi$$

For the CES function, the derivatives are given by:

$$\begin{aligned} u_c(\bar{c}, \bar{g}) &= \frac{\bar{c}^{\gamma-1}}{\bar{c}^\gamma + \zeta \bar{g}^\gamma} \\ u_g(\bar{c}, \bar{g}) &= \frac{\zeta \bar{g}^{\gamma-1}}{\bar{c}^\gamma + \zeta \bar{g}^\gamma} \\ u_{cc}(\bar{c}, \bar{g}) &= \frac{(\gamma-1)\bar{c}^{\gamma-2}}{\bar{c}^\gamma + \zeta \bar{g}^\gamma} - \frac{\gamma \bar{c}^{2\gamma-2}}{(\bar{c}^\gamma + \zeta \bar{g}^\gamma)^2} \\ u_{gg}(\bar{c}, \bar{g}) &= \frac{(\gamma-1)\zeta \bar{g}^{\gamma-2}}{\bar{c}^\gamma + \zeta \bar{g}^\gamma} - \frac{\zeta^2 \gamma \bar{g}^{2\gamma-2}}{(\bar{c}^\gamma + \zeta \bar{g}^\gamma)^2} \\ u_{cg}(\bar{c}, \bar{g}) &= \frac{-\gamma \zeta \bar{g}^{\gamma-1} \bar{c}^{\gamma-1}}{(\bar{c}^\gamma + \zeta \bar{g}^\gamma)^2} \end{aligned}$$

And the expression for the welfare cost is:

$$\begin{aligned} \frac{1}{\gamma} \ln[((1-x)\bar{c})^\gamma + \zeta \bar{g}^\gamma] - \frac{1}{\gamma} \ln[\bar{c}^\gamma + \zeta \bar{g}^\gamma] &= \Xi \\ x = 1 - \frac{\{\exp[\gamma \Xi + \ln(\bar{c}^\gamma + \zeta \bar{g}^\gamma)] - \zeta \bar{g}^\gamma\}^{\frac{1}{\gamma}}}{\bar{c}}, \gamma \neq 0 \end{aligned}$$

If  $\gamma = 0$  the solution is given by:

$$x = 1 - \frac{\exp\{\Xi + \ln \bar{c}\}}{\bar{c}}$$

## Appendix II - Data

Figure A1: Looking at the data

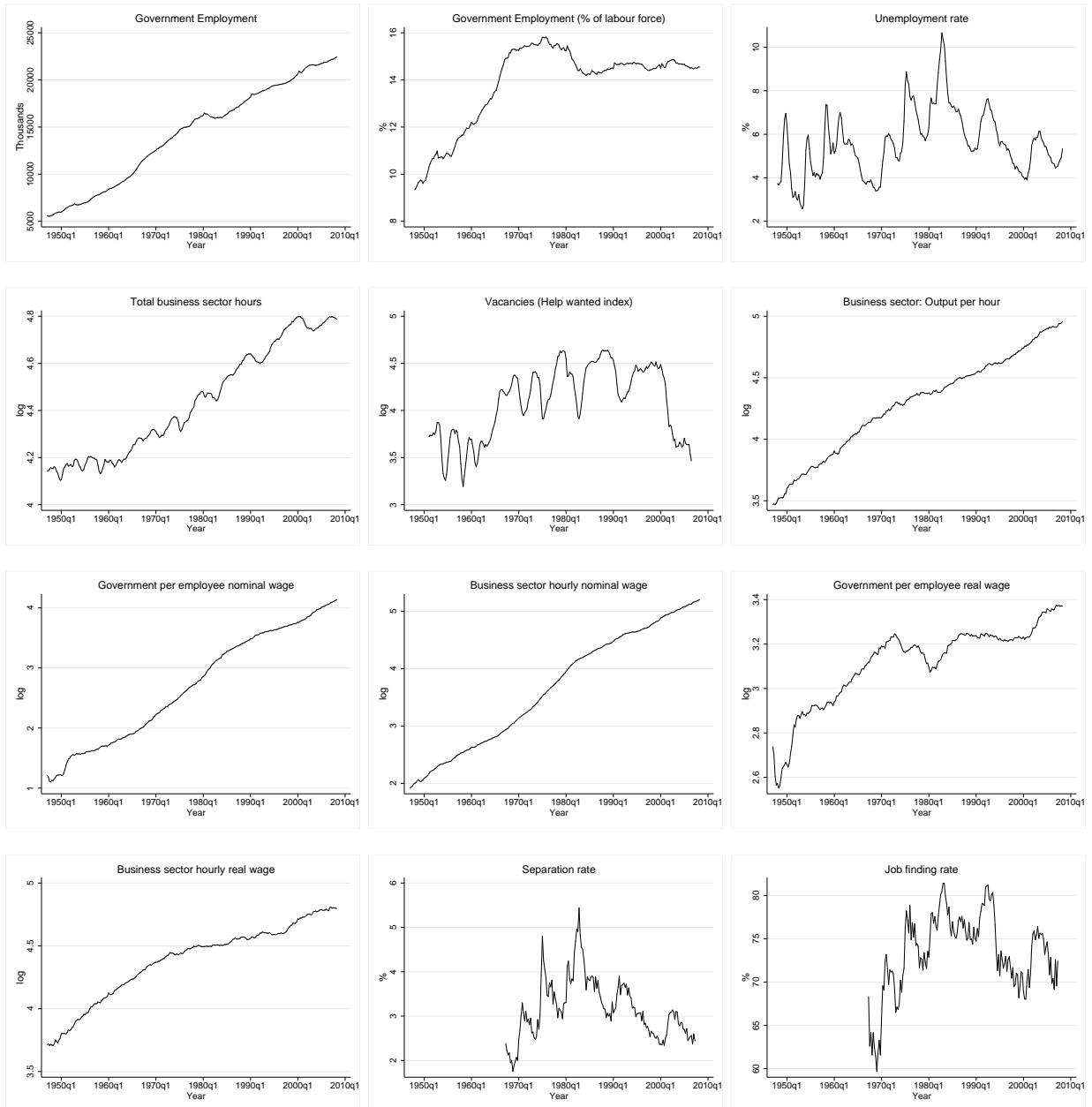


Table A1: Data - definition and sources

	<b>Variable</b>	<b>Definition and source</b>	<b>Availability</b>
$\pi_t$	Productivity	Business Sector: Output Per Hour of All Persons (BLS)	1947q1 2008q2
$L_t^g$	Government employment	All Employees: Government (BLS)	1939q1 2008q3
$w_t^g$	Government per employee nominal wage	Government consumption expenditures: Compensation of general government employees / government employees (BEA-NIPA Tables and own calculation)	1947q1 2008q2
$w_t^p$	Business sector hourly nominal wage	Business Sector: Compensation Per Hour (BLS)	1947q1 2008q2
$H_t^p$	Total business sector hours	Business Sector: Hours of All Persons (BLS)	1947q1 2008q2
$u_t$	Unemployment rate	Civilian Unemployment Rate (BLS)	1948q1 2008q3
$v_t$	Vacancies	Index of Help Wanted Advertising in Newspapers (The Conference Board)	1951q1 2006q2
$wr_t^g$	Government per employee real wage	Government per employee nominal wage deflated by CPI (BEA-NIPA Tables and own calculation)	1947q1 2008q2
$wr_t^p$	Business sector hourly real wage	Business Sector: Real Compensation Per Hour (BLS)	1947q1 2008q2
$\Lambda_t$	Separation rate	Employment to unemployment transition probability (Shimer, own calculation for quarterly aggregation)	1967q1 2007q2
$f_t$	Job finding rate	Unemployment to employment transition probability (Shimer, own calculation for quarterly aggregation)	1967q1 2007q2

Figure A2: VAR with real wages

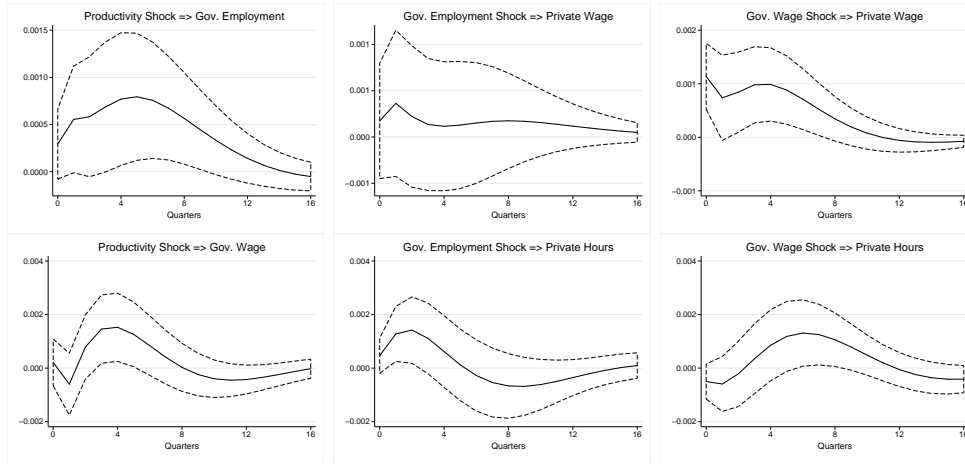


Figure A3: VAR with unemployment rate

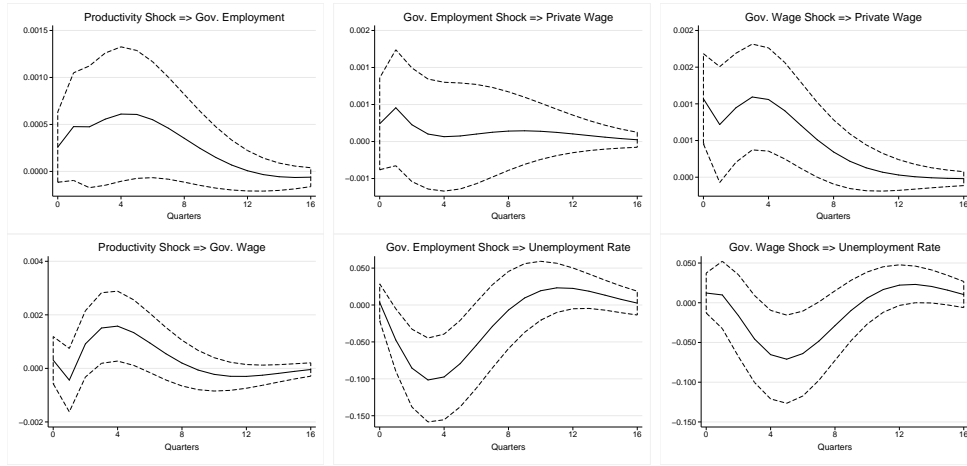


Figure A4: VAR with vacancies

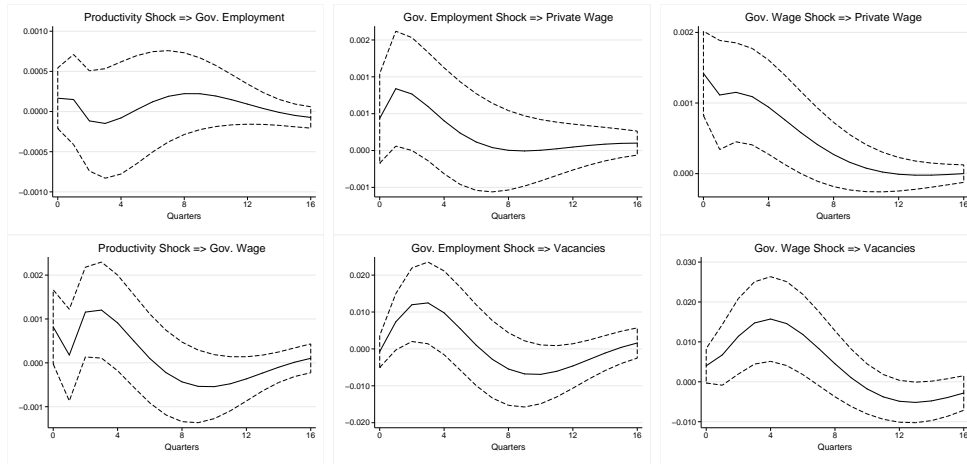


Figure A5: VAR over last 20 year

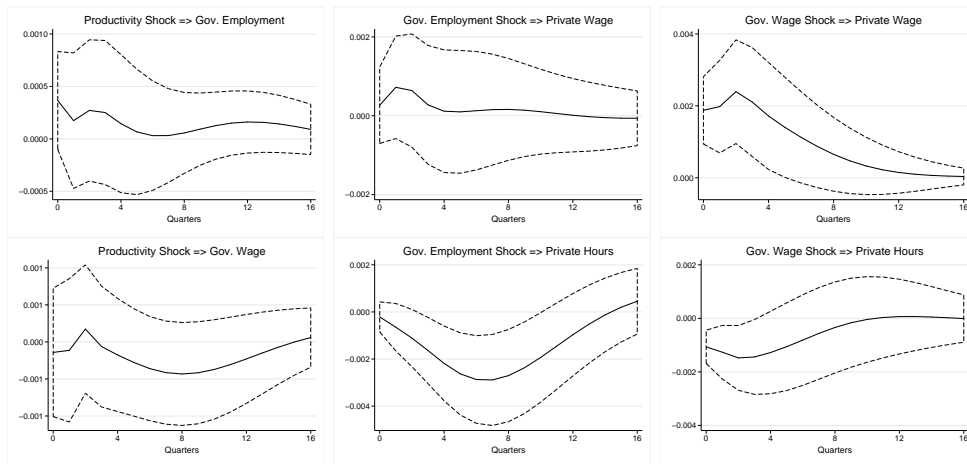


Figure A6: VAR in differences

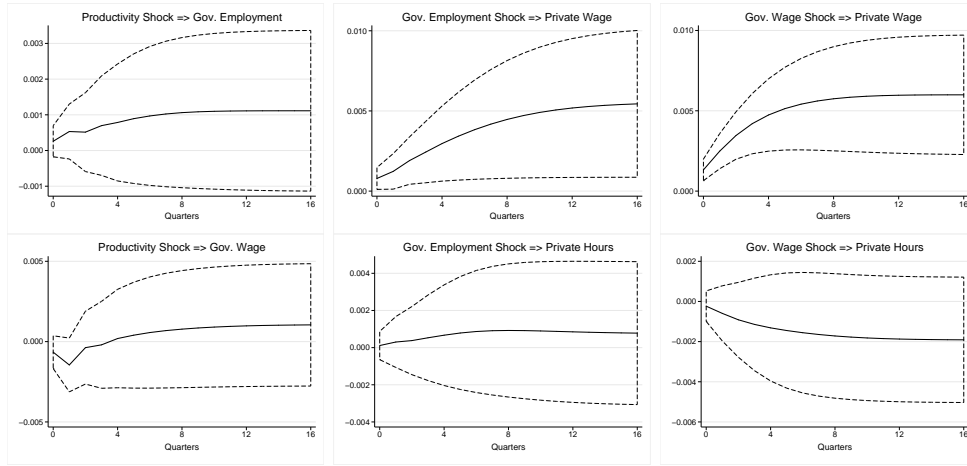


Figure A7: Fig: VAR with alternative ordering

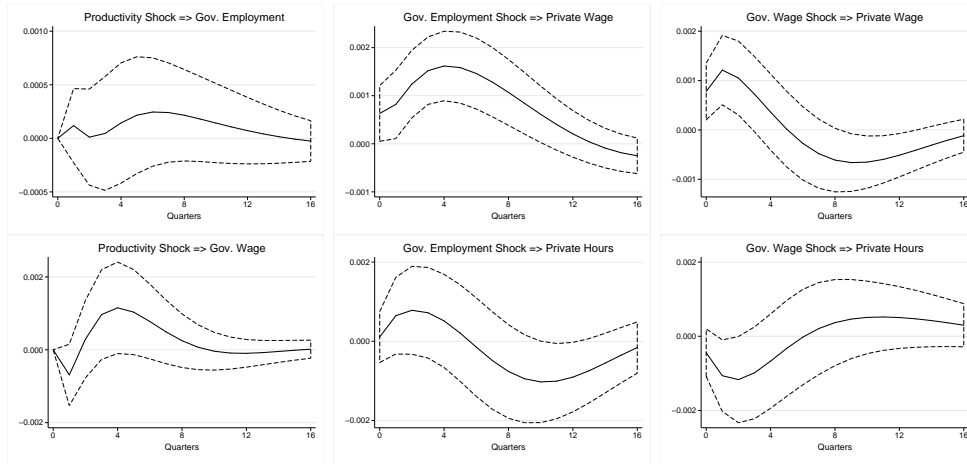


Figure A8: VAR with different detrending

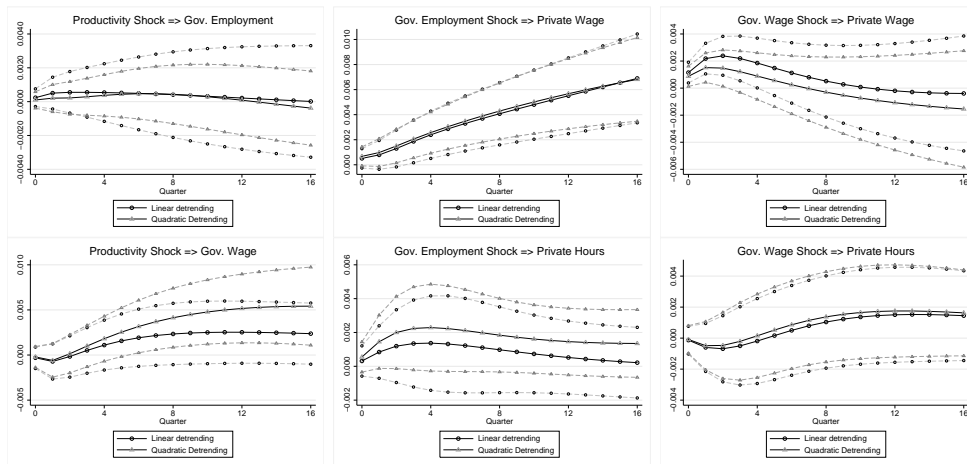


Figure A9: VAR with different lag length

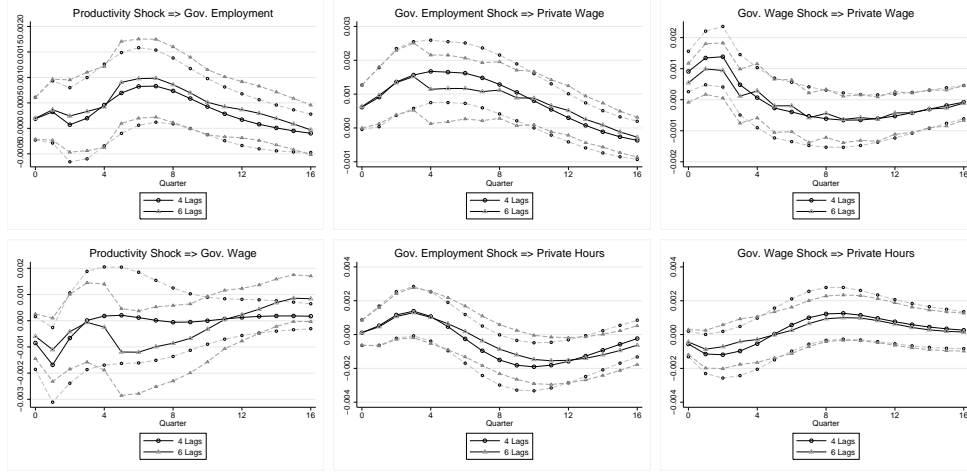
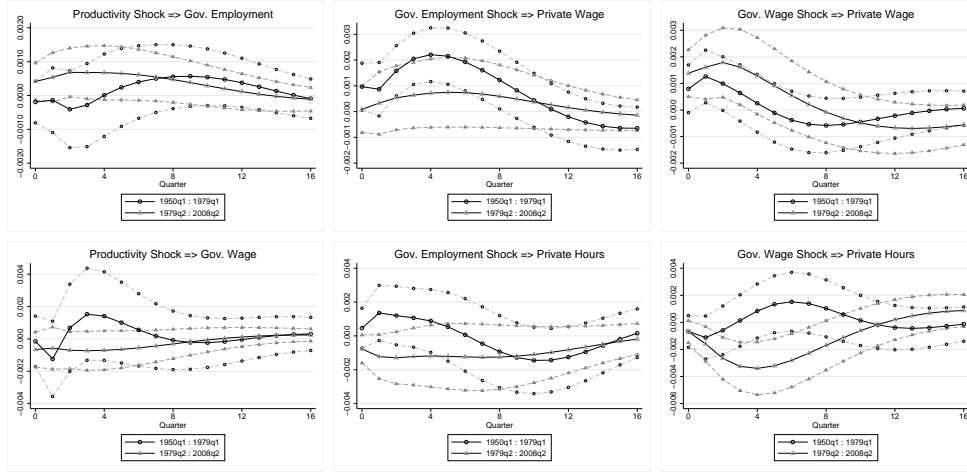


Figure A10: VAR over different sub-samples



## Appendix III - Bayesian estimation

### Estimated model in levels

The labour market is described by the following equations:

$$1 = l_t^p + l_t^g + u_t \quad (\text{A1})$$

$$l_{t+1}^p = (1 - \lambda_t^p) l_t^p + m_t^p \quad (\text{A2})$$

$$l_{t+1}^g = (1 - \lambda_t^g) l_t^g + m_t^g \quad (\text{A3})$$

$$m_t^p = \mu^p ((1 - s_t) u_t)^{\eta^p} (v_t^p)^{1 - \eta^p} \quad (\text{A4})$$

$$m_t^g = \mu^g (s_t u_t)^{\eta^g} (v_t^g)^{1-\eta^g} \quad (\text{A5})$$

$$q_t^p = \frac{m_t^p}{v_t^p} \quad (\text{A6})$$

$$p_t^p = \frac{m_t^p}{(1-s_t)u_t} \quad (\text{A7})$$

$$p_t^g = \frac{m_t^g}{s_t u_t} \quad (\text{A8})$$

The marginal utility of consumption and the stochastic discount factor.

$$u_c(c_t, g_t) = \frac{c_t^{\gamma-1}}{c_t^\gamma + \zeta g_t^\gamma} \quad (\text{A9})$$

$$\beta_{t,t+1} = \beta \frac{u_c(c_{t+1}, g_{t+1})}{u_c(c_t, g_t)} \quad (\text{A10})$$

I define a new variable that is the difference between the value of working and being unemployed  $x_t^i$ . I use it to re-write the equation that pins down the  $s_t$  and the Nash bargaining equation.

$$x_t^p = W_t^p - U_t^p = w_t^p - z + E_t \beta_{t,t+1} (1 - \lambda_t^p - p_t^p) x_{t+1}^p \quad (\text{A11})$$

$$x_t^g = W_t^g - U_t^g = w_t^g - z + E_t \beta_{t,t+1} (1 - \lambda_t^g - p_t^g) x_{t+1}^g \quad (\text{A12})$$

$$J_t = a_t^p - w_t^p + E_t \beta_{t,t+1} [(1 - \lambda_t^p) J_{t+1}] \quad (\text{A13})$$

$$\frac{m_t^p E_t [x_{t+1}^p]}{(1-s_t)} = \frac{m_t^g E_t [x_{t+1}^g]}{s_t} \quad (\text{A14})$$

$$(1-b_t)(x_t^p) = b_t J_t \quad (\text{A15})$$

Finally we have the production functions, the equation that determines the firm's optimal vacancy posting and the policy equations.

$$c_t = a_t^p l_t^p - \varsigma^p v_t^p \quad (\text{A16})$$

$$g_t = a_t^g l_t^g - \varsigma^g v_t^g \quad (\text{A17})$$

$$\frac{\varsigma^p}{q_t^p} = E_t \beta_{t,t+1} [a_{t+1}^p - w_{t+1}^p + (1 - \lambda_t^p) \frac{\varsigma^p}{q_{t+1}^p}] \quad (\text{A18})$$



$$\ln(v_t^g) = \ln(\bar{v}^g) + \psi^v [\ln(v_t^p) - \ln(\bar{v}^p)] + \ln(\omega_t^v) \quad (\text{A19})$$

$$\ln(w_t^g) = \ln(\bar{w}^g) + \psi^w [\ln(w_t^p) - \ln(\bar{w}^p)] + \ln(\omega_t^w) \quad (\text{A20})$$

I include 6 different shocks: a shock to government vacancies, to government wages, to private and public separation rates, private sector bargaining power and to technology. These shocks are described in the following equations:

$$\ln(\omega_t^v) = \rho^v \ln(\omega_{t-1}^v) + \epsilon_t^v \quad (\text{A21})$$

$$\ln(\omega_t^w) = \rho^w \ln(\omega_{t-1}^w) + \epsilon_t^w \quad (\text{A22})$$

$$\ln(\lambda_t^g) = (1 - \rho^{lg}) \ln(\bar{\lambda}_t^g) + \rho^{lg} \ln(\lambda_{t-1}^g) + \epsilon_t^{lg} \quad (\text{A23})$$

$$\ln(\lambda_t^p) = (1 - \rho^{lp}) \ln(\bar{\lambda}_t^p) + \rho^{lp} \ln(\lambda_{t-1}^p) + \epsilon_t^{lp} \quad (\text{A24})$$

$$\ln(b_t) = (1 - \rho^b) \ln(\bar{b}) + \rho^b \ln(b_{t-1}) + \epsilon_t^b \quad (\text{A25})$$

$$\ln(a_t^p) = (1 - \rho^a) \ln(\bar{a}^p) + \rho^a \ln(a_{t-1}^p) + \epsilon_t^a \quad (\text{A26})$$

Finally, I define that overall separation rate and job finding rates:

$$f_t = \frac{m_t^p + m_t^g}{u_t} \quad (\text{A27})$$

$$\Lambda_t = \frac{\lambda_t^p l_t^p + \lambda_t^g l_t^g}{l_t^p + l_t^g} \quad (\text{A28})$$

## Estimated model - steady state

I determine that the government employment in steady state is 0.15. As there is no recurrent way to write the steady state values of the equations, they solve the following non linear system of

equations:

$$\begin{aligned}
\bar{l}^g &= 0.15 \\
\bar{l}^p &= 1 - \bar{l}^g - \bar{u} \\
\bar{m}^p &= \lambda^p \bar{l}^p \\
\bar{m}^g &= \lambda^g \bar{l}^g \\
\bar{m}^p &= \mu^p ((1 - \bar{s})\bar{u})^{\eta^p} (\bar{v}^p)^{1-\eta^p} \\
\bar{m}^g &= \mu^g (\bar{s}\bar{u})^{\eta^g} (\bar{v}^g)^{1-\eta^g} \\
p_t^p &= \frac{\bar{m}^p}{(1 - \bar{s})\bar{u}} \\
p_t^g &= \frac{\bar{m}^g}{(\bar{s})\bar{u}} \\
\bar{q}^p &= \frac{\bar{m}^p}{v^p} \\
\bar{x}^g &= \frac{\bar{w}^g - z}{1 - \beta(1 - \lambda^g - \bar{p}^g)} \\
\bar{x}^p &= \frac{\bar{w}^p - z}{1 - \beta(1 - \lambda^p - \bar{p}^p)} \\
\bar{m}^p \bar{x}^p \bar{s} &= \bar{m}^g \bar{x}^g (1 - \bar{s}) \\
(1 - b)(\bar{x}^p) &= b\bar{J} \\
\bar{J} &= \frac{\bar{a}^p - \bar{w}^p}{1 - \beta(1 - \lambda^p)} \\
\frac{\zeta^p}{\bar{q}^p} (1 - \beta(1 - \lambda^p)) &= \beta(\bar{a}^p - \bar{w}^p) \\
\bar{w}^g &= \pi \bar{w}^p \\
\bar{c} &= \bar{a}^p \bar{l}^p - \zeta^p \bar{v}^p \\
\bar{g} &= \bar{a}^g \bar{l}^g - \zeta^g \bar{v}^g \\
u_c(\bar{c}, \bar{g}) &= \frac{\bar{c}^{\gamma-1}}{\bar{c}^\gamma + \zeta \bar{g}^\gamma} \\
\bar{f} &= \frac{\bar{m}^p + \bar{m}^g}{\bar{u}} \\
\bar{\Lambda} &= \frac{\lambda^p \bar{l}^p + \lambda^g \bar{l}^g}{\bar{l}^p + \bar{l}^g}
\end{aligned}$$

### Estimated log-linearized model

The variables with tilde are expressed in deviations from steady state.

$$0 = \bar{l}^p \tilde{l}_t^p + \bar{l}^g \tilde{l}_t^g + \bar{u} \tilde{u}_t \quad (\text{L1})$$

$$\tilde{l}_{t+1}^p = (1 - \bar{\lambda}^p) \tilde{l}_t^p - \bar{\lambda}^p \tilde{\lambda}_t^p + \bar{\lambda}^p \tilde{m}_t^p \quad (\text{L2})$$

$$\tilde{l}_{t+1}^g = (1 - \bar{\lambda}^g)\tilde{l}_t^g - \bar{\lambda}^g\tilde{\lambda}_t^g + \bar{\lambda}^g\tilde{m}_t^g \quad (\text{L3})$$

$$\tilde{m}_t^p = \eta^p(\tilde{u}_t - \frac{\bar{s}}{1 - \bar{s}}\tilde{s}_t) + (1 - \eta^p)\tilde{v}_t^p \quad (\text{L4})$$

$$\tilde{m}_t^g = \eta^g(\tilde{u}_t + \tilde{s}_t) + (1 - \eta^g)\tilde{v}_t^g \quad (\text{L5})$$

$$\tilde{q}_t^p = \tilde{m}_t^p - \tilde{v}_t^p \quad (\text{L6})$$

$$\tilde{p}_t^p = \tilde{m}_t^p + \frac{\bar{s}}{1 - \bar{s}}\tilde{s}_t - \tilde{u}_t \quad (\text{L7})$$

$$\tilde{p}_t^g = \tilde{m}_t^g - \tilde{s}_t - \tilde{u}_t \quad (\text{L8})$$

$$\tilde{u}_c(\tilde{c}_t, \tilde{g}_t) = \tilde{c}_t(\gamma - 1 - \frac{\gamma\bar{c}^\gamma}{\bar{c}^\gamma + \zeta\bar{g}^\gamma}) - \tilde{g}_t(\frac{\zeta\gamma\bar{g}^\gamma}{\bar{c}^\gamma + \zeta\bar{g}^\gamma}) \quad (\text{L9})$$

$$\tilde{\beta}_{t,t+1} = E_t[\tilde{u}_c(\tilde{c}_{t+1}, \tilde{g}_{t+1}) - \tilde{u}_c(\tilde{c}_t, \tilde{g}_t)] \quad (\text{L10})$$

$$\tilde{x}_t^p = \frac{\bar{w}^p}{\bar{x}^p}\tilde{w}_t^p - \beta(\bar{\lambda}^p\tilde{\lambda}_t^p + \bar{p}^p\tilde{p}_t^p) + \beta(1 - \bar{\lambda}^p - \bar{p}^p)E_t(\tilde{x}_{t+1}^p + \tilde{\beta}_{t,t+1}) \quad (\text{L11})$$

$$\tilde{x}_t^g = \frac{\bar{w}^g}{\bar{x}^g}\tilde{w}_t^g - \beta(\bar{\lambda}^g\tilde{\lambda}_t^g + \bar{p}^g\tilde{p}_t^g) + \beta(1 - \bar{\lambda}^g - \bar{p}^g)E_t(\tilde{x}_{t+1}^g + \tilde{\beta}_{t,t+1}) \quad (\text{L12})$$

$$\tilde{J}_t = \frac{\bar{a}^p}{\bar{J}}\tilde{a}_t^p - \frac{\bar{w}^p}{\bar{J}}\tilde{w}_t^p + \beta E_t((1 - \bar{\lambda}^p)\tilde{\beta}_t + (1 - \bar{\lambda}^p)\tilde{J}_{t+1} - \bar{\lambda}^p\tilde{\lambda}_t^p) \quad (\text{L13})$$

$$\frac{1}{1 - \bar{s}}\tilde{s}_t + \tilde{m}_t^p - \tilde{m}_t^g = E_t(\tilde{x}_{t+1}^g - \tilde{x}_{t+1}^p) \quad (\text{L14})$$

$$\tilde{J}_t + \frac{1}{1 - \bar{b}}\tilde{b}_t = \tilde{x}_t^p \quad (\text{L15})$$

$$\tilde{c}_t = \frac{\bar{a}^p\bar{l}^p}{\bar{c}}(\tilde{a}_t^p + \tilde{l}_t^p) - \frac{\zeta^p\bar{v}^p}{\bar{c}}\tilde{v}_t^p \quad (\text{L16})$$

$$\tilde{g}_t = \frac{\bar{a}^g\bar{l}^g}{\bar{g}}(\tilde{a}_t^g + \tilde{l}_t^g) - \frac{\zeta^g\bar{v}^g}{\bar{g}}\tilde{v}_t^g \quad (\text{L17})$$

$$-\frac{\zeta^p}{\bar{q}^p}\tilde{q}_t^p = \beta[\bar{a}^p\tilde{a}_{t+1}^p - \bar{w}^p\tilde{w}_{t+1}^p - (1 - \bar{\lambda}^p)\frac{\zeta^p}{\bar{q}^p}\tilde{q}_{t+1}^p - \bar{\lambda}^p\frac{\zeta^p}{\bar{q}^p}\tilde{\lambda}_t^p + (\bar{a}^p - \bar{w}^p + (1 - \bar{\lambda}^p)\frac{\zeta^p}{\bar{q}^p})\tilde{\beta}_t] \quad (\text{L18})$$

$$\tilde{v}_t^g = \psi^v \tilde{v}_t^p + \tilde{\omega}_t^v \quad (\text{L19})$$

$$\tilde{w}_{t+1}^g = \psi^w \tilde{w}_t^p + \tilde{\omega}_t^w \quad (\text{L20})$$

$$\tilde{\omega}_t^v = \rho^v \tilde{\omega}_{t-1}^v + \epsilon_t^v \quad (\text{L21})$$

$$\tilde{\omega}_t^w = \rho^v \tilde{\omega}_{t-1}^w + \epsilon_t^w \quad (\text{L22})$$

$$\tilde{\lambda}_t^g = \rho^{lg} \tilde{\lambda}_{t-1}^g + \epsilon_t^{lg} \quad (\text{L23})$$

$$\tilde{\lambda}_t^p = \rho^{lp} \tilde{\lambda}_{t-1}^p + \epsilon_t^{lp} \quad (\text{L24})$$

$$\tilde{b}_t = \rho^b \tilde{b}_{t-1} + \epsilon_t^b \quad (\text{L25})$$

$$\tilde{a}_t^p = \rho^b \tilde{a}_{t-1}^p + \epsilon_t^a \quad (\text{L26})$$

$$f_t = \tilde{m}_t^p \frac{\tilde{m}^p}{\tilde{m}^p + \tilde{m}^g} + \tilde{m}_t^g \frac{\tilde{m}^g}{\tilde{m}^p + \tilde{m}^g} - \tilde{u}_t \quad (\text{L27})$$

$$\tilde{\Lambda}_t = (\tilde{\lambda}_t^p + \tilde{l}_t^p) \frac{\bar{\lambda}^p \bar{l}^p}{\bar{\lambda}^p \bar{l}^p + \bar{\lambda}^g \bar{l}^g} + (\tilde{\lambda}_t^g + \tilde{l}_t^g) \frac{\bar{\lambda}^g \bar{l}^g}{\bar{\lambda}^p \bar{l}^p + \bar{\lambda}^g \bar{l}^g} + \tilde{u}_t \frac{\bar{u}}{1 - \tilde{u}} \quad (\text{L28})$$

## Definition of observable variables

$$\begin{array}{cccc}
 \text{Model 1} & & \text{Model 2} & & \text{Model 3} & & \text{Model 4} \\
 \left[ \begin{array}{cc} l_t^{gOb} & \bar{l}^g(1 + \tilde{l}_t^g) \\ u_t^{Ob} & \bar{u}(1 + \tilde{u}_t) \\ w_t^{gOb} & \tilde{w}_t^g - \tilde{w}_{t-1}^g \\ w_t^{pOb} & \tilde{w}_t^p - \tilde{w}_{t-1}^p \\ \Lambda_t^{Ob} & \bar{\Lambda}(1 + \tilde{\Lambda}_t) \\ f_t^{Ob} & \bar{f}(1 + \tilde{f}_t) \end{array} \right] & = & \left[ \begin{array}{cc} l_t^{gOb} & \tilde{l}_t^g - \tilde{l}_{t-1}^g \\ u_t^{Ob} & \tilde{u}_t - \tilde{u}_{t-1} \\ w_t^{gOb} & \tilde{w}_t^g - \tilde{w}_{t-1}^g \\ w_t^{pOb} & \tilde{w}_t^p - \tilde{w}_{t-1}^p \\ \Lambda_t^{Ob} & \tilde{\Lambda}_t - \tilde{\Lambda}_{t-1} \\ f_t^{Ob} & \tilde{f}_t - \tilde{f}_{t-1} \end{array} \right] & = & \left[ \begin{array}{cc} l_t^{gOb} & \tilde{l}_t^g \\ u_t^{Ob} & \tilde{u}_t \\ w_t^{gOb} & \tilde{w}_t^g - \tilde{w}_{t-1}^g \\ w_t^{pOb} & \tilde{w}_t^p - \tilde{w}_{t-1}^p \\ \Lambda_t^{Ob} & \tilde{\Lambda}_t \\ f_t^{Ob} & \tilde{f}_t \end{array} \right] & = & \left[ \begin{array}{cc} l_t^{gOb} & \tilde{l}_t^g \\ u_t^{Ob} & \tilde{u}_t \\ w_t^{gOb} & \tilde{w}_t^g \\ w_t^{pOb} & \tilde{w}_t^p \\ \Lambda_t^{Ob} & \tilde{\Lambda}_t \\ f_t^{Ob} & \tilde{f}_t \end{array} \right]
 \end{array}$$

## Alternative estimation results

Table A2: Prior and posterior distribution of structural parameters for the sample 1988-2007

		Prior distribution		Posterior distribution		
			Levels	Dif	HP	HPw
<b>a) Structural parameters</b>						
Cost of posting vacancy - private sector	$\zeta^p$	Normal	0.54	0.51	0.57	0.57
		(0.6,0.10)	(0.39,0.70)	(0.36,0.65)	(0.40,0.73)	(0.37,0.77)
Cost of posting vacancy - public sector	$\zeta^g$	Normal	0.59	0.58	0.55	0.60
		(0.6,0.10)	(0.43,0.78)	(0.43,0.72)	(0.38,0.72)	(0.47,0.72)
Matching efficiency (private sector)	$m^p$	Normal	0.81	0.69	0.65	0.73
		(0.6,0.10)	(0.69,0.92)	(0.51,0.84)	(0.49,0.80)	(0.59,0.88)
Matching efficiency - public sector	$m^g$	Normal	0.66	0.59	0.59	0.62
		(0.6,0.10)	(0.50,0.85)	(0.41,0.76)	(0.43,0.74)	(0.43,0.79)
Matching elasticity w.r.t unemployment - private sector	$\eta^p$	Beta	0.51	0.65	0.66	0.68
		(0.5,0.10)	(0.39,0.63)	(0.52,0.77)	(0.56,0.77)	(0.58,0.80)
Matching elasticity w.r.t unemployment - public sector	$\eta^g$	Beta	0.19	0.32	0.34	0.36
		(0.5,0.10)	(0.12,0.27)	(0.21,0.44)	(0.21,0.46)	(0.23,0.49)
Separation rate - private sector	$\lambda^p$	Normal	0.046	0.068	0.069	0.066
		(0.06,0.01)	(0.026,0.064)	(0.052,0.083)	(0.056,0.081)	(0.054,0.080)
Separation rate - public sector	$\lambda^g$	Normal	0.047	0.016	0.022	0.028
		(0.06,0.01)	(0.033,0.060)	(0.005,0.025)	(0.009,0.032)	(0.015,0.039)
Relative flow value of unemployment	$z$	Beta	0.27	0.50	0.44	0.64
		(0.5,0.15)	(0.13,0.42)	(0.36,0.62)	(0.28,0.59)	(0.59,0.73)
Elasticity of substitution (public and private goods)	$\gamma$	Normal	0.08	0.01	0.06	0.07
		(0.00,0.10)	(-0.09,0.25)	(-0.13,0.17)	(-0.12,0.23)	(-0.06,0.21)
Bargaining power	$b$	Beta	0.61	0.60	0.66	0.55
		(0.5,0.10)	(0.51,0.71)	(0.48,0.73)	(0.53,0.78)	(0.41,0.69)
Public sector wage premium	$\psi$	Normal	1.042	1.055	1.058	1.063
		(1.05,0.01)	(1.029,1.054)	(1.042,1.067)	(1.043,1.073)	(1.048,1.077)
Business cycle response of public sector wages	$\psi^w$	Normal	-0.03	0.03	-0.02	-0.06
		(0.0,0.2)	(-0.21,0.15)	(-0.16,0.21)	(-0.20,0.17)	(-0.25,0.13)
Business cycle response of public sector vacancies	$\psi^v$	Normal	-0.10	-0.22	-0.13	-0.21
		(0.0,0.2)	(-0.24,0.03)	(-0.49,0.02)	(-0.37,0.10)	(-0.48,0.04)

Table A3: Prior and posterior distribution of shock parameters for the sample 1988-2007

		Prior	Posterior distribution			
		distribution	Levels	Dif	HP	HPw
<b>b) Autoregressive parameters</b>						
Productivity	$\rho^a$	Beta (0.5,0.15)	0.91 (0.87,0.96)	0.90 (0.86,0.95)	0.89 (0.85,0.94)	0.62 (0.50,0.74)
Public sector wage	$\rho^w$	Beta (0.5,0.15)	0.90 (0.86,0.93)	0.90 (0.86,0.94)	0.90 (0.86,0.93)	0.58 (0.47,0.70)
Public sector vacancies	$\rho^v$	Beta (0.5,0.15)	0.31 (0.10,0.53)	0.49 (0.24,0.73)	0.35 (0.16,0.54)	0.33 (0.15,0.52)
Private sector separation rate	$\rho^{lp}$	Beta (0.5,0.15)	0.96 (0.92,1.00)	0.90 (0.85,0.95)	0.68 (0.57,0.78)	0.66 (0.55,0.77)
Public sector separation rate	$\rho^{lg}$	Beta (0.5,0.15)	0.40 (0.12,0.71)	0.18 (0.07,0.29)	0.16 (0.06,0.26)	0.14 (0.05,0.22)
Bargaining power	$\rho^b$	Beta (0.5,0.15)	0.93 (0.88,0.99)	0.88 (0.82,0.94)	0.84 (0.76,0.91)	0.68 (0.55,0.81)
<b>c) Standard deviations</b>						
Productivity	$\sigma^a$	IGamma (0.05,0.15)	0.009 (0.009,0.010)	0.009 (0.008,0.010)	0.009 (0.008,0.010)	0.008 (0.007,0.009)
Public sector wage	$\sigma^w$	IGamma (0.05,0.15)	0.009 (0.008,0.011)	0.009 (0.008,0.011)	0.009 (0.008,0.011)	0.008 (0.007,0.009)
Public sector vacancies	$\sigma^v$	IGamma (0.05,0.15)	0.064 (0.019,0.010)	0.041 (0.015,0.068)	0.052 (0.020,0.080)	0.037 (0.018,0.054)
Private sector separation rate	$\sigma^{lp}$	IGamma (0.05,0.15)	0.053 (0.031,0.077)	0.055 (0.046,0.062)	0.049 (0.042,0.055)	0.050 (0.043,0.057)
Public sector separation rate	$\sigma^{lg}$	IGamma (0.05,0.15)	0.056 (0.015,0.103)	0.288 (0.103,0.474)	0.196 (0.089,0.319)	0.145 (0.078,0.214)
Bargaining power	$\sigma^b$	IGamma (0.05,0.15)	0.027 (0.019,0.036)	0.023 (0.015,0.031)	0.018 (0.012,0.024)	0.021 (0.014,0.029)