Estimating insurance and incentive effects of labour market reforms

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We formulate a general equilibrium matching model with spell-dependent unemployment benefits and endogenous search effort. The model gives rise to an endogenous distribution of unemployment duration characterized by a time-varying hazard function. Using methods from the literature on Semi-Markov processes, we obtain an expression for the aggregate unemployment rate under heterogeneous search effort. We perform structural estimation of the model using a German micro-data set (GSOEP) and discuss the effects of the recent unemployment benefit reform (Hartz IV). Our results show that although the reform and economic growth have contributed to the reduction of the aggregate unemployment rate, aggregate welfare has gone down.

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1 Introduction

Continental European unemployment is notorious for its persistence. France, Italy and Germany have had rising unemployment rates from the 1960s up to 2000 and even onward. There seems to be a consensus now that a combination of shocks and institutional arrangements lies at the origin of these high unemployment rates (Ljungqvist and Sargent, 1998, 2007a, b; Mortensen and Pissarides, 1999; Blanchard and Wolfers, 2000). Neither institutions nor shocks alone explain the rise in unemployment: institutions have always been there but unemployment has not (at least not at this level) and shocks have hit many countries but not all countries have high unemployment rates. The step from this shock-institutions insight towards finding a solution to the European unemployment problem seems to be short: As

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shocks will not go, we need to address the institutions. A common suggestion is to reduce long and generous unemployment benefits.

Is this advisable? Should one reduce the length and level of unemployment benefits in order to reduce unemployment? There is a classic efficiency-equity trade-off to be faced. While reducing unemployment per se is beneficial, income of the unemployed and the insurance mechanism implicit in unemployment benefits should not be neglected.

We examine qualitatively and quantitatively the employment and welfare effects of a policy reform which reduces the length and level of unemployment benefits. We use Germany as an example of a continental European country, as the so-called "Hartz IV reform" implemented in January 2005 comprises both the reduction of benefits and the cut of the duration of entitlement and because the German unemployment benefit system has a two-tier structure typical to many other OECD countries. Unemployment insurance (UI) payments before the reform were paid for a period of 6 to 32 months followed by unemployment assistance (UA) payments, the latter potentially lasting up to infinity. The experience of Germany with rising unemployment rates over decades is shared by many other countries and, similarly to Germany in 2005, many countries did reduce length and level of unemployment payments in order to address the issue of high and persistent unemployment (OECD, 2004).

The Hartz IV reform has introduced two main modifications. First, the UA payments, formerly proportional to net earnings before the job loss, were replaced by a uniform benefit level. The effect of this new rule on the income of long-term unemployed workers was ambiguous. There were unemployed whose benefit payments were lower before 2005 than after the reform, mainly unemployed workers from the low wage sector. Those were the "winners" of the reform (47 percent of long-term unemployed). On the other hand, there were also long-term unemployed with relatively high wages before entering unemployment. These were affected negatively by the new law and their income has dropped (53 percent of long-term unemployed). Despite the fraction of "winners" and "losers" is roughly equal, the gain of the winners has turned out to be lower than the loss of the losers leading to a loss of the average worker of a bit more than 7% due to Hartz IV (Blos and Rudolph, 2005; OECD, 2007). Second, for workers who entered unemployment from February 2006 onward, the maximum duration of entitlement to unemployment insurance payments was reduced to 12 months (formerly, 15 months was the average).

At first sight, the reforms seem to have worked. The reported unemployment rate dropped between January 2005 and January 2007 from 12.3 to 10.2 percent. On the other hand, growth rates in Germany were (for German standards) fairly high. While the German economy shrank in 2003, it has recovered since then and probably also created new jobs. The real GDP grew by 0.8 percent in 2005 and by 2.9 percent in 2006. Given this background, we are left with at least three questions: Did the reform reduce unemployment and increase output? (Yes.) Did it increase welfare of the unemployed and/or employed workers? (No.) Does it increase social welfare or expected utility? (No.) In short, our findings suggest that post-reform unemployment benefits reduced the insurance mechanism of UA payments too much and overemphasized the incentive effect.[2]

We reach our conclusions by using a model which combines various strands of the litera-

[2]This does not mean that labour market institutions should not be reformed. It rather means that one should look for Pareto-improving reforms (e.g. introducing progressive social security payments).

ture and adds some new and essential features. We employ a general equilibrium matching framework and extend the standard text-book model for time-dependent unemployment benefits, endogenous effort, risk-averse households, an exogenous “residual search productivity effect”, applying Semi-Markov tools. Each of these extensions is crucial. Unemployment benefits in our model need to depend on the length of the unemployment spell as this is a feature of basically all OECD unemployment benefit systems. Letting agents optimally choose their effort to find a job, we can analyze the incentive effects of (reforms of) the unemployment benefit system on the search intensity. Risk-averse households are required as we also want to evaluate insurance effects. The residual effect allows us to capture all effects, other than the incentive effect, that influence exit rates into employment. Finally, tools from the Semi-Markov literature are required as this allows us to deduce the aggregate unemployment rate from individual search. We can thereby compute macro efficiency effects resulting from micro incentives.

We solve this model numerically by looking at Bellman equations as differential equations. This gives us solutions which are as accurate as numerical precision and which do not require us to approximate the model in any way. Optimal behaviour implies an exit rate into employment which is a function of the time spent in unemployment. We thereby obtain a flexible enough endogenous distribution of unemployment duration which we employ for structural estimation of model parameters. Estimation by maximum likelihood is then (relatively) straightforward.

The main theoretical contribution of our analysis is the explicit treatment of the Semi-Markov nature of optimal individual behaviour due to the presence of spell-dependent unemployment benefits: Optimal exit rates not only depend on whether the individual is unemployed (the current state of the worker) but also on how long an individual has been unemployed. While this Semi-Markov aspect has been known for a while, it has not been fully exploited so far in the search literature. Using results from the applied mathematics literature, we obtain analytic expressions for individual employment probabilities contingent on current employment status and duration of unemployment. They allow us to compute aggregate unemployment rates using a law of large numbers in our pure idiosyncratic risk economy. Given this link from optimal individual behaviour to aggregate outcomes, we can analyze the distribution and efficiency effects of changes in level and length of unemployment benefits.

The main empirical contribution is the careful modelling of exit rates into employment. Individual incentives due to falling unemployment benefits imply more search effort and therefore higher exit rates over time. Empirical evidence shows, however, that exit rates tend to fall - at least after some initial increase over the first 3-4 months of unemployment. We therefore combine individual incentive effects with an exogenous time-decreasing “residual search productivity effect” which allows us to obtain - from a theoretical perspective - rising, falling and hump-shaped exit rates. Structural estimation then establishes that the model can replicate empirical stylized facts of first rising and then falling exit rates.

The main policy contribution is our emphasis and structural estimation of the trade-off between insurance and incentive effects of labour market policies. The degree of risk-aversion - crucial for understanding the insurance effect - is jointly estimated with exit rates and search productivity (and other model parameters). A comparative static analysis using the estimated version of the theoretical model then allows us to derive precise predictions about the employment and distribution effects of changes in the length and level of unemployment benefits.

Our paper is related to various strands in the literature. From a theoretical perspective, we build on the search and matching framework of Diamond (1982), Mortensen (1982) and Pissarides (1985), recently surveyed by Rogerson et al. (2005). Time-dependent unemployment benefits and endogenous effort have been originally analyzed by Mortensen (1977) in a one-sided job search model. Equilibrium search and matching models include Cahuc and Lehman (2000), Fredriksson and Holmlund (2001) and Albrecht and Vroman (2005).[3] These models, however, are less powerful than our model in explaining the anticipation effect of the reduction in benefits, as exit rates within each benefit regime are constant. There also exists a substantial literature that studies optimal insurance allowing for an arbitrary time path of unemployment benefit payments (Shavell and Weiss, 1979; Hopenhayn and Nicolini, 1997; Shimer and Werning, 2007). Our focus is more of a positive nature trying to understand the welfare effects of existing systems which have a simpler benefit structure than the ones resulting from an optimization approach. We also allow for an unlimited number of transitions between employment and unemployment and undertake a general equilibrium analysis as in Moscarini (2005).[4]

From an empirical perspective, we estimate a parametric duration model (Lancaster, 1990) in which time dependence of the hazard function due to time-dependent benefits is fully described by the equilibrium solution of our theoretical model. Econometric models with time-dependent benefits were originally estimated by van den Berg (1990) and Ferrall (1997).[5] Van den Berg et al. (2004) and Abbring et al. (2005) extend the setting by introducing time dependence due to monitoring and sanctions. In contrast to our model, this literature deals with one-sided job search, which makes application of its estimates in a general equilibrium analysis rather difficult. In addition to that, focus on the incentive effect in is only partial (van den Berg et al., 2004; Abbring et al., 2005) and insurance effect remains largely unaddressed. There also exists a larger empirical equilibrium search literature that deals with unemployment benefit heterogeneity (Bontemps et al., 1999), heterogeneity in workers abilities (Postel-Vinay and Robin, 2002) and heterogeneity in workers value of nonparticipation (Flinn, 2006). Unlike in our model, however, neither of these contributions views heterogeneity as being a result of time-dependence.

Finally, Semi-Markov methods are taken from the applied mathematical literature, see e.g. Kulkarni (1995) or Corradi et al. (2004).

The structure of our paper is as follows. In section 2 we present the theoretical model, institutional setting, behaviour of supply and demand sides and the combination of both in economic welfare. Section 3 describes the equilibrium properties of the model. Section 4 illustrates the structural estimation and the underlying data. The simulation results and the evaluation of the institutions reforms are presented in section 5. Section 6 concludes.

[3] Coles and Masters (2007) also have time-dependent unemployment insurance but they do not analyze the implications for individual effort. Their focus is on a model with aggregate uncertainty implying stochastic separation rates.

[4] Acemoglu and Shimer (1999) do consider a general equilibrium model, but their setting is restricted to time-invariant benefits only.

[5] See also Eckstein and van den Berg (2007) for literature review on nonstationary empirical models.

2 The model

We use a Diamond-Mortensen-Pissarides type matching model and extend it for time-dependent unemployment benefits, endogenous effort, risk-averse households and an exogenous search productivity effect. To solve it, we use Semi-Markov tools. The separation rate for jobs is constant and there is no search on the job. We focus on steady states in our analysis.

2.1 Production, employment and labour income

The economy has a work force of exogenous constant size N. Employment is endogenous and given by L and the number of unemployed amounts to $N - L$. Firms produce under perfect competition on the goods market and each worker-firm match produces output A, which is constant. The production process of the worker and the firm can be interrupted by exogenous causes which occur according to a time-homogenous Poisson process with a constant arrival rate λ.

Unemployed workers receive UI benefits b_1 and UA benefits b_2. Benefits are modelled to reflect institutional arrangements in many European countries. One of the most important features is the dependence of UI benefits on the unemployment spell. Empirical work has repeatedly shown (Moffitt and Nicholson,1982; Blanchard and Wolfers, 2000) that the length of entitlement to unemployment insurance payments plays a crucial role in determining the unemployment rate. Workers with a spell s shorter than $\bar{s}$ (say one year) receive UI benefits b_1, afterwards, they receive b_2,

$$b(s) = \begin{cases} b_1 & 0 \leq s \leq \bar{s} \\ b_2 & \bar{s} < s \end{cases} . \tag{1}$$

We assume $b_1 > b_2 \geq 0$. Benefits can be paid either at a fixed level or proportional to previous income.

An unemployed worker finds a job according to a time-inhomogenous Poisson process with arrival rate $\mu(.)$. This rate will also be called the job-finding rate, hazard rate or exit rate (into employment). We allow this rate to depend on effort $\phi(s(t))$ an individual exerts to find a job. Effort today in t depends on the length $s(t)$ this individual has been spending in unemployment since his last job. The spell increases linearly in time and starts in t_0 where the individual has lost the job, i.e. $s(t) = t - t_0$. An individual whose duration of unemployment spell $s(t)$ exceeds the length of entitlement to UI benefits $\bar{s}$ (i.e. $s(t) \geq t_0 + \bar{s}$) will be called a long-term unemployed.

In addition to individual effort, the exit rate will also depend on aggregate labour market conditions and on the "residual search productivity effect". Labour market conditions are captured by labour market tightness θ that differs across steady states, $\theta \equiv V/U$. We assume that effort and tightness are multiplicative: no effort implies permanent unemployment and no vacancies imply that any effort is in vain. The residual effect is a time (spell s) effect and captures stigma, ranking (Blanchard and Diamond, 1994), unobserved productivity differentials, changes in preferences and attitudes (frustration in search), loss in search productivity, etc., i.e. all effects which imply that exit rates of long-term unemployed differ from exit rates of short-term unemployed. We denote this effect by $\eta(s)$. While it can go either way, empirically, we expect that productivity goes down, i.e. $\partial\eta(s)/\partial s < 0$. Hence, in full, the exit rate reads $\mu(\phi(s(t))\theta, \eta(s))$.

It is well-known (e.g. Ross, 1996, ch. 2) that the density of unemployment duration when the exit rate is time-varying is given by

$$f(s) = \mu(\phi(s)\theta, \eta(s))\, e^{-\int_0^s \mu(\phi(u)\theta, \eta(u))du}. \tag{2}$$

This density will be crucial later for various purposes including the estimation of model parameters. It is endogenous to the model, as exit rate $\mu(\phi(s(t))\theta, \eta(s))$ follows from the optimizing behaviour of workers and firms.[6]

Unemployment benefits are financed by a tax rate κ on gross wages such that the net wage is $w = (1-\kappa)\, w^{gross}$. The budget constraint of the government therefore reads

$$\kappa \frac{w}{1-\kappa} L = b_1 U_{short} + b_2 U_{long}, \tag{3}$$

where the number of short- and long-term unemployed adds to the total number of unemployed, $U_{short} + U_{long} = N - L$. The government adjusts the wage tax κ such that this holds at each point in time.

The wage is determined by bargaining to which we return below.

2.2 Optimal behaviour

- Households

Households are infinitely lived and do not save. The present value of having a job is given by $V(w)$ and depends on the current endogenous wage w only. Employed workers enjoy instantaneous utility $u(w, \psi)$ where ψ captures disutility from working.[7] The value $V(w)$ is constant in a steady state as the wage is constant, but differs across steady states. Whenever a worker loses his job, he enters the unemployment benefit system by obtaining insurance payments b_1 for the full length of $\bar{s}$. Workers are immediately granted full benefit entitlements, i.e. unemployment payments are not experience rated (see the bargaining setup for further discussion). Hence, the value of being unemployed when just having lost the job is given by $V(b_1, 0)$ where 0 stands for a spell of length zero. This leads to a Bellman equation for the employed worker of

$$\rho V(w) = u(w, \psi) + \lambda [V(b_1, 0) - V(w)]. \tag{4}$$

The Bellman equation for the unemployed worker reads

$$\rho V(b(s), s) = \max_{\phi(s)} \left\{ u(b(s), \phi(s)) + \frac{dV(b(s), s)}{ds} + \mu(\phi(s)\theta, s)[V(w) - V(b(s), s)] \right\}. \tag{5}$$

The instantaneous utility flow of being unemployed, $\rho V(b(s), s)$, is given by three components. The first component shows the instantaneous utility resulting from consumption of

[6] Also note that due to drop of benefits at $\bar{s}$, $f(s)$ will have a more general hurdle structure (see Appendix).

[7] This parameter only serves to contrast search effort of unemployed workers and plays no major role.

$b(s)$ and effort $\phi(s)$. The second component is a deterministic change of $V(b(s), s)$ as the value of being unemployed changes over time. The third component is a stochastic change that occurs at job-finding rate $\mu(\phi(s)\theta, s)$. When a job is found, an unemployed gains the difference between the value of being employed $V(w)$ and $V(b(s), s)$.

An optimal choice of effort $\phi(s)$ for (5) requires

$$u_{\phi(s)}(b(s), \phi(s)) + \mu_{\phi(s)}(\phi(s)\theta, s)[V(w) - V(b(s), s)] = 0, \tag{6}$$

where subscripts denote (partial) derivatives. It states that the expected utility loss resulting from increasing search effort must be equal to expected utility gain due to higher effort.

We require that the value of unemployment an instant before becoming a long-term unemployed is identical to the value of being long-term unemployed at $\bar{s}$, i.e.

$$V(b_1, \bar{s}) = V(b_2, \bar{s}). \tag{7}$$

- Firms

The value of a job J to a firm is given by instantaneous profits $A - w/(1-\kappa)$, which is the difference between revenue A and the gross wage $w/(1-\kappa)$, reduced by the risk of being driven out of business

$$\rho J = A - w/(1-\kappa) - \lambda J, \tag{8}$$

where ρ stands for the interest rate (being identical to the discount rate of households).

Given that individual arrival rates are a functions of the individual unemployment spell, the expected rate of exit out of unemployment is just the mean over individual arrival rates, given the endogenous distribution of the unemployment spell $f(s)$ from (2),

$$\bar{\mu} = \int_0^\infty \mu(\phi(s)\theta, \eta(s)) f(s) ds. \tag{9}$$

As a consequence, the vacancy filling rate is $\xi(t) \equiv \theta^{-1}\bar{\mu}$. The value of a vacant job is $\rho J_0 = -\gamma + \xi(t)[J - J_0]$. With free entry, the value of holding a vacancy is $J_0 = 0$, leading to

$$J = \gamma\theta/\bar{\mu}. \tag{10}$$

- Wages

We let wages be determined by Nash bargaining. We assume that the outcome of the bargaining process is such that workers receive a share β of the total surplus of a successful match $V(w) - V(b_1, 0) = \beta\left[J\left(\frac{w}{1-\kappa}\right) - J_0 + V(w) - V(b_1, 0)\right]$. The total surplus is the gain of the firm plus the gain of the worker from the match where the latter crucially on the outside option of the worker. The fact that we use $V(b_1, 0)$ as the outside option of the worker means that all workers (even if only working for an instant or, in the limit, if only bargaining) are entitled to full unemployment benefits, i.e. b_1 over the full length $\bar{s}$ and b_2 for $s > \bar{s}$. An alternative would consist in specifying $V(b(s), s)$ as the outside option: if the bargain fails, the unemployed worker remains unemployed and continues to receive benefits he received before the unsuccessful bargaining. This would be theoretically interesting as an endogenous wage distribution would arise (see e.g. Albrecht and Vroman, 2005) where the distinguishing determinant across workers is the previous unemployment spell.[8]

Following the steps as in Pissarides (1985), we end up with a generalized wage equation that reads (see app. B.6)

$$(1-\beta)\,u(w)+\beta\frac{w}{1-\kappa}=(1-\beta)\,u(b_1,\phi(0))+\beta\,[A+\theta\gamma]\,. \tag{11}$$

The left hand side corresponds to what in models with risk-neutrality and without taxation is simply the wage rate. So, with $u(w)=w$ and $\kappa=0$ we would obtain just w on the left. The tax rate, that appears as the term $w/(1-\kappa)$, results from the instantaneous profit of a firm (8) which needs to pay a gross wage of $w/(1-\kappa)$. The right hand side is a simple generalization of the standard wage equation of Pissarides (1985). Instead of benefits for the unemployed (which we would find on the right for risk-neutral households and no time-dependence of effort), we have instantaneous utility from being unemployed. The impact of the production side is unchanged when compared to the standard wage equation.

Instead of specifying the outside option differently, one could also allow for strategic bargaining. Many recent papers have used strategic bargaining either for the reason that payoff change over time implies that Nash bargaining would correspond to myopic behaviour (Coles and Wright, 1998; Coles and Muthoo, 2003), or that a careful analysis of on-the-job search makes strategic bargaining more appropriate (Cahuc, Postel-Vinay and Robin, 2006) or that unemployment does not have such a strong effect on bargaining as generally thought (Hall and Milgrom, 2008).[9] Given that we want to focus here on the direct incentive effects of non-stationary unemployment benefits on search effort, we "switch off" the strategic channel and leave this for future work.

2.3 Welfare

When we evaluate unemployment policies, we take all agents in our economy into account. There are workers with value $V(w)$, the unemployed with value $V(b(s),s)$ depending on their spell s and firms with value J. When we want to compare one policy to another, we look at output, employment, individual effects and overall welfare. We obtain a social welfare function by aggregating - similarly to Hosios (1990) - over all these welfare levels in a standard Bentham-type utilitarian way,[10]

$$\Omega=L\,[V(w)+J]+(N-L)\left(\int_0^{\bar{s}}V(b_1,s)\,f(s)\,ds+\int_{\bar{s}}^{\infty}V(b_2,s)\,f(s)\,ds\right). \tag{12}$$

[8] Our assumption that all workers, even if they have worked only for a second, are entitled to b_1 for the full period of length $\bar{s}$ is identical to saying that benefit payments are not experience rated. While the absence of experience rating is generally distorting the firms decision to lay off workers (see e.g. Mongrain and Roberts, 2005), this does not play a role in our setup as the separation rate is exogenous. It would be interesting to study the impact of endogenous separation decisions but we leave this for future research.

[9] Coles and Masters (2004) analyse wage setting by strategic bargaining in a matching setup with non-stationary unemployment benefits. They do not consider endogenous search intensity, however.

[10] Dividing by N gives expected utility of a worker "thrown into" this economy from behind some "veil of ignorance". Higher social welfare is therefore identical to higher expected utility. One could therefore equilvalently ask in which economy such a worker would like to land.

Social welfare is given by the number L of employed workers/firms times their welfare plus the number of unemployed workers $N-L$ times the average welfare of an unemployed. This average is obtained by integrating over all spells s, where $f(s)$ is the endogenous density (2), with exit rates $\mu(\phi(s)\theta, \eta(s))$ that follow from the steady state solution of the model, and the $V(b_i, s)$ are the values of being unemployed with a spell s and benefit payments b_i from (1).

3 Equilibrium properties

3.1 Individual (un)employment probabilities

In models with constant job-finding and separation rates, the unemployment rate can easily be derived by assuming that a law of large numbers holds. Aggregate employment dynamics can then be described by $\dot{L} = \mu[N-L] - \lambda L$ which allows to compute unemployment rates. With spell-dependent effort, individual arrival rates $\mu(.)$ are heterogeneous and employment dynamics need to be derived using techniques from the literature on Semi-Markov or renewal processes, e.g. Kulkarni (1995) or Corradi et al. (2004).

The generalization of Semi-Markov processes compared to continuous time Markov chains consists in allowing the transition rate from one state to another to depend on the time an individual has spent in the current state. We apply this here and let the transition rate from unemployment to employment depend on the time s the individual has been unemployed. Hence, switching from a constant job-finding rate μ to a spell-dependent rate $\mu(s)$ implies switching from Markov to Semi-Markov processes. Processes are called "semi" as the history-dependence of the job finding rate $\mu(s)$ is not Markov. Processes are still called "Markov" as once an individual has found a job, history no longer counts. This is also why these processes are called renewal processes: whenever a transition to a new state occurs, the system starts from the scratch, it is "renewed" and history vanishes.

We start by looking at individual employment probabilities. Let $p_{ij}(\tau, s(t))$ describe the probability with which an individual, who is in state i (either e for employed or u for unemployed) today in t, will be in state $j \in \{e, u\}$ at some future point in time τ, given that his current spell is now $s(t)$. These expressions read, starting with $s(t) = 0$ and taking into account that the separation rate λ remains constant (see app. A.3),

$$p_{uu}(\tau, 0) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau e^{-\int_t^v \mu(s(y))dy} \mu(s(v))\, p_{eu}(\tau - v)\, dv, \tag{13a}$$

$$p_{eu}(\tau) = \int_t^\tau e^{-\lambda[v-t]} \lambda p_{uu}(\tau - v, 0)\, dv. \tag{13b}$$

Expressions for complementary transitions are given by $p_{ue}(\tau) = 1 - p_{uu}(\tau)$ and $p_{ee}(\tau) = 1 - p_{eu}(\tau)$, respectively.

These equations have a straightforward intuitive meaning. Consider first the case of τ being not very far in the future. Then all integrals (for $\tau = t$) are zero and the probability of being unemployed at τ is, if unemployed at t, one from (13a) and, if employed at t, zero from (13b). For a $\tau > t$, the part $e^{-\int_t^\tau \mu(s(y))dy}$ in (13a) gives the probability of remaining in unemployment for the entire period from t to τ. An individual unemployed today can also be unemployed in the future if he remains unemployed from t to v (the probability of which is $e^{-\int_t^v \mu(s(y))dy}$), loses the job in v (which requires multiplication with the exit rate $\mu(s(v))$) and then moves from employment to unemployment again over the remaining interval $\tau - v$ (for which the probability is $p_{eu}(\tau - v)$). As this path is possible for any v between t and τ, the densities for these paths are integrated. The sum of the probability of remaining unemployed all of the time and of finding a job at some v but being unemployed again at τ gives then the overall probability $p_{uu}(\tau, 0)$ of having no job in τ when having no job in t. Note that there can be an arbitrary number of transitions in and out of employment between v and τ. The interpretation of (13b) is similar. The probability of remaining employed from t to v is simpler, $e^{-\lambda[v-t]}$, as the separation rate λ is constant.

As we can see, these equations are interdependent: The equation for $p_{uu}(\tau)$ depends on $p_{eu}(\tau - v)$ and the equation for $p_{eu}(\tau)$, in turn, depends on $p_{uu}(\tau - v)$. Formally speaking, these equations are integral equations, sometimes called Volterra equations of the first type (13b) and of the second type (13a). Integral equations can sometimes be transformed into differential equations, which will simplify their solution in practice. In our case, however, no transformation into differential equations is known.

After having computed the probability of being unemployed in τ when being unemployed in t for individuals that just became unemployed in t, i.e. who have a spell of length $s(t) = 0$, we will need an expression for $p_{uu}(\tau, s(t))$. This means, we will need the transition probabilities for individuals with an arbitrary spell $s(t)$ of unemployment. Luckily, given the results from (13a and b), this probability is straightforwardly given by

$$p_{uu}(\tau, s(t)) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) \, p_{eu}(\tau - v) \, dv. \tag{14}$$

An unemployed with spell $s(t)$ in t has different exit rates $\mu(s(y))$ which, however, are known from our analysis of optimal behaviour at the individual level. Hence, only the integrals in (14) are different, the probabilities $p_{eu}(\tau - v)$ can be taken from the solution of (13a and b).

3.2 Aggregate unemployment

Given our finding in (13) and (14) on $p_{eu}(\tau)$ and $p_{uu}(\tau, s(t))$, we can now compute the expected number of unemployed for any distribution of spell $F(s)$,

$$E_t[N - L_\tau] = [N - L_t] \int_0^\infty p_{uu}(\tau, s(t)) \, dF(s(t)) + p_{eu}(\tau) \, L_t. \tag{15}$$

Starting at the end of this equation, given there are L_t employed workers in t, the expected number of unemployed workers at some future point τ out of the group of those currently employed in t is given by $p_{eu}(\tau) L_t$. Again, one should keep in mind that the probability $p_{eu}(\tau)$ allows for an arbitrary number of switches between employment and unemployment between t and τ, i.e. it takes the permanent turnover into account.

For the unemployed, we compute the mean over all probabilities of being unemployed in the future, if unemployed today, by integrating over $p_{uu}(\tau, s(t))$ given the current distribution $F(s(t))$. Multiplying this by the number of unemployed today, $N - L_t$, gives us the expected number of unemployed at τ out of the pool of unemployed in t. The sum these two expected quantities gives the expected number of unemployed at some future point τ.

The expected unemployment rate at τ is simply the expression (15) divided by N. When we focus on a steady state, we let τ approach infinity. In order to obtain a simple expression for the aggregate unemployment rate and to show the link to the textbook expression, we assume a pure idiosyncratic risk model where micro-uncertainty cancels out at the aggregate level. Hence, we assume a law of large numbers holds and the population share of unemployed workers equals the average individual probability of being unemployed. This "removes" the expectations operators, so that (15) for a steady state becomes $N - L = [N - L] \int_0^\infty p_{uu}(s(t))\, dF(s(t)) + p_{eu}L$. We have replaced $L_\tau = L_t$ by the steady state employment level L and the individual probabilities by the steady state expressions $p_{uu}(s(t))$ and p_{eu}. The probability p_{eu} is no longer a function of τ as this probability will not change in steady state, while there will always be a distribution of $p_{uu}(s)$, even in steady state.

Solving for the unemployment rates gives

$$\frac{U}{N} = \frac{p_{eu}}{p_{eu} + \left[1 - \int_0^\infty p_{uu}(s(t))\, dF(s(t))\right]} = \frac{p_{eu}}{p_{eu} + \int_0^\infty p_{ue}(s(t))\, dF(s(t))}, \tag{16}$$

where the second expression is more parsimonious. If we assumed a constant job arrival rate here, we would get $p_{eu} = p_{uu} = \lambda/(\lambda + \mu)$ and $p_{ue} = \mu/(\lambda + \mu)$. Inserting this into our steady state results would yield the standard expression for the unemployment rate, $U/N = \lambda/(\lambda + \mu)$. In our generalized setup, the long-run unemployment rate is given by the ratio of individual probability p_{eu} to be unemployed when employed today divided by this same probability plus $1 - \int_0^\infty p_{uu}(s(t))\, dF(s(t))$.

3.3 Functional forms and steady state

For estimation purposes and for the numerical solution, we need functional forms for the instantaneous utility function and for the arrival rate. We assume that the instantaneous utility function of an unemployed worker used e.g. in (5) is

$$u(b(s), \phi(s)) = \frac{b(s)^{1-\sigma}}{1-\sigma} - \phi(s). \tag{17}$$

Effort is measured in utility terms. The utility function of an employed worker has the same structure only that consumption is given by w and effort is the constant ψ.

The arrival rate of jobs $\mu(\phi(s)\theta, \eta(s))$ is assumed to obey

$$\mu(\phi(s)\theta, \eta(s)) = \eta(s)[\phi(s)\theta]^\alpha, \quad \text{with } \eta(s) = \eta_1 e^{-\delta_j[s-\bar{s}]} + \eta_2. \tag{18}$$

If one interprets $\eta(s)$ as a productivity of search, one can look at the expression for $\mu(.)$ like at a production function. Input factors are effort $\phi(s)$ and vacancies per unemployed worker θ. With $0 < \alpha < 1$, inputs have decreasing returns. Effort $\phi(s)$ follows from behaviour of households and labour market tightness θ is the result of free entry and exit into the creation of vacancies. Search productivity is an exogenous function of the unemployment spell s. For estimation purposes below, we allow the productivity effect to change at different (depreciation) rates δ_j, depending on whether individuals receive UI payments b_1 or UA payments b_2. We also allow parameters to take any sign in the estimation procedure. From a theoretical perspective, however, we would expect all parameters η_j and δ_j to be positive.

This would imply that the productivity falls in the spell s and approaches the constant $\eta_2 > 0$. For a worker who has just lost his job, it is higher and given by $\eta(s) = \eta_1 e^{\delta_j \bar{s}} + \eta_2 > \eta_2$. Note that even if depreciation rates differ from the UI to the UA phase, search productivity is continuous at $\bar{s}$ and given by $\eta_1 + \eta_2$.

In a steady state, all aggregate variables are constant and there will be a stationary distribution for unemployment spells. The solution of the steady can most easily be found in two steps. Taking the wage w and labour market tightness θ as exogenous, one can use expressions related to the unemployed for effort, the value of being unemployed and the value of a job, $\phi(b(s), s)$, $V(b(s), s)$ and $V(w)$. Once these quantities are known, one can use the remaining equations of the model to solve for the wage rate and tightness, w and θ. In doing so, all other endogenous variables (exit rate $\mu(s)$ and the implied density $f(s)$, instantaneous utilities $u(.)$, the tax rate κ, individual employment probabilities p_{uu}, p_{eu} and the implied number of short- and long-term unemployed and the unemployment rate U/N, the number of vacancies, the value function J for the firm and social welfare Ω) are determined as well. Appendix A.2 provides an explicit presentation of all equations (which above in the model description are given implicitly) and describes the solution procedure.

4 Structural estimation

4.1 Stylized facts

The model is estimated using the data from the German Socio-Economic Panel (GSOEP).[11] Before we do so, we would like to see whether it is able to match central stylized facts. As our focus is on the exit rates, which are key to understanding incentives resulting from changes in unemployment benefits, we need to make sure that our model is flexible enough to replicate the most important features of exit behaviour observed in the data. These features are shown in Figure 2.

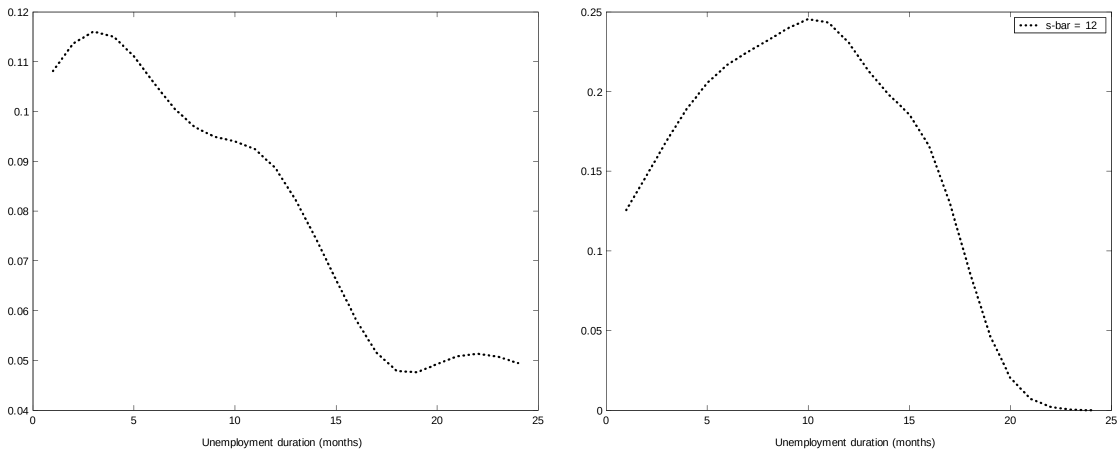

Figure 1 *Non-parametric hazard functions*

[11] GSOEP is a panel survey of individuals held on the annual basis by the Deutsche Institut für Wirtschaftsforschung (Berlin). See Appendix for more information about the data.

The left panel of Figure 2 plots a non-parametric estimate of the hazard function for the entire sample of unemployment durations.[12] This non-parametric hazard has a clear non-monotonic behaviour. Starting form the 6th month of unemployment duration changes in the speed of decline as well as local maxima are induced by multiplicity of entitlement lengths $\bar{s}$ and largely correspond to each and every value of $\bar{s}$, i.e. the time point at which benefits drop. This is further confirmed by the estimated non-parametric hazard for a subsample of individuals with $\bar{s} = 12$ (which is the case for approx. 50% of our observations) shown in the right panel.

Thus the individual exit rate derived from the theoretical model must have at least two characteristics, namely: *a*) steady increase before the expiration of entitlement (as in Mortensen, 1977), and *b*) steady decrease thereafter (as in Heckman and Borjas, 1980). Our theoretical exit rates are broadly consistent with both. When we assume that there is no effect other than that of benefits and tightness, i.e. $\eta(s)$ is constant in (18) due to $\delta_j = 0$ (no stigma), our model predicts exit rates that increase before $\bar{s}$. If δ_j is very high, exit rates fall over time. For intermediate values of δ_j, we get a non-monotonic behaviour and exit rates increase before $\bar{s}$ and fall subsequently, featuring peak at $\bar{s}$. We are therefore confident that our theoretical exit rates are sufficiently flexible for a successful estimation of the model.

4.2 Econometric model

- Specification

Estimation of model parameters uses a part of the numerical solution method for the steady state. As described in app. A.2, for a given wage w and vacancy to unemployment ratio θ, individual effort over the unemployment spell can be computed. Using individual survey data implies that the wage w for each individual is known and the corresponding θ can be taken from macro data. Individual effort can therefore be computed and model parameters can be estimated without specifying the wage setting mechanism. If we used linked employer-employee data, the model could be estimated by using the observable productivity data. This would allow us to estimate the bargaining power parameter β as well. For the rest of the parameters unrelated to wage setting mechanism, however, both approaches must be equivalent (assuming that wage setting in the second one is correctly specified). Further, computing the steady state solution suggests that estimation with given wage and tightness is faster by a factor of at least 4 or 5.

Computing individual effort simultaneously provides exit rates $\mu_1(s_i; \boldsymbol{\xi})$ and $\mu_2(s_i; \boldsymbol{\xi})$, i.e. before and after expiration of entitlement respectively. Since the data are sampled as a flow of entrants into unemployment and employment, exit rates contain all information necessary for the construction of the likelihood function. Our data consist of three types of unemployed individuals:

a) Individuals who enter unemployment with the right to claim unemployment insurance (UI) benefits and exit unemployment before the expiration of entitlement period

[12] See Tanner and Wong (1983) for the definition of the estimator and consistency proof. We use Gaussian kernel. Optimal bandwith is estimated by cross-validation discussed in Tanner and Wong (1984).

b) Individuals who enter unemployment with the right to claim UI benefits, fail to find a job before entitlement expires, transit to a lower unemployment assistance (UA) benefits and exit unemployment (or not) only after the expiration of entitlement

c) Individuals who do not have the right to claim UI benefits and enter unemployment receiving lower UA benefits from the very beginning (if at all)

Let $\boldsymbol{\xi}$ denote the vector of parameters to estimate, i.e. $\boldsymbol{\xi} = \left\{\lambda, \alpha, \sigma, \eta_j, \delta_j\right\}_{j=1,2}$.[13] Furthermore let l_i define the length of previous/current individual job. Keeping in mind that job loss is governed by a homogeneous Poisson process with rate λ, which implies exponential distribution of the length of the job spell, we get the following log-contributions for the above types of individuals

$$a) : \quad \ln \ell_i(\boldsymbol{\xi}) = d_{j,i} \ln \lambda - \lambda l_i + d_{u,i} \ln \mu_1(s_i; \boldsymbol{\xi}) - \int_0^{s_i} \mu_1(u; \boldsymbol{\xi})\, du \tag{19a}$$

$$b) : \quad \ln \ell_i(\boldsymbol{\xi}) = d_{j,i} \ln \lambda - \lambda l_i - \int_0^{\bar{s}_i} \mu_1(u; \boldsymbol{\xi})\, du + d_{u,i} \ln \mu_2(s_i; \boldsymbol{\xi}) - \int_{\bar{s}_i}^{s_i} \mu_2(u; \boldsymbol{\xi})\, du \tag{19b}$$

$$c) : \quad \ln \ell_i(\boldsymbol{\xi}) = d_{j,i} \ln \lambda - \lambda l_i + d_{u,i} \ln \mu_2(s_i; \boldsymbol{\xi}) - \int_0^{s_i} \mu_2(u; \boldsymbol{\xi})\, du, \tag{19c}$$

where $d_{u,i}$ is an indicator variable such that $d_{u,i} = 1$ if unemployment spell is uncensored and $d_{j,i}$ is an indicator variable such that $d_{j,i} = 1$ if job spell is uncensored. Finally, the log-contribution of entrants to employment is simply

$$\ln \ell_i(\boldsymbol{\xi}) = d_{j,i} \ln \lambda - \lambda l_i \tag{19d}$$

- Identification

Absence of analytical solutions for the structural exit rates $\mu_1(s_i; \boldsymbol{\xi})$ and $\mu_2(s_i; \boldsymbol{\xi})$ prevents us from obtaining any analytical result on the identifiability of a general model described by $\boldsymbol{\xi}$ as above. However, even a medium size numerical analysis shows that the model which allows both α and σ vary free is not identifiable. This holds for different shapes of the residual effect $\eta(s)$, including the constant one. Therefore we are to consider two different lines of specification,

(i): $\alpha = 1/2,\ 0 < \sigma < 1$ and
(ii): $0 < \alpha < 1,\ \sigma = 1/2,$

the rest being equal.

As to the rest of the parameters, λ is always identified from job duration data. Given the fixed residual effect, σ in (i) and α in (ii) are always identified as unique shape parameters of the hazard function. Introduction of a time dependent residual effect implies identification bounds for σ and α which will depend on the shape of $\eta(s)$. Finally, even if identifiable under fairly general forms of $\eta(s)$, the more parsimonious is the specification of $\eta(s)$, the better is

[13] As typical for the majority of empirical search models, we impose ρ exogenously. The rate of time preference is set to 0.003, which corresponds to annual rate of 3.7%.

the numerical performance of the model. Extensive exploratory analysis of functional forms for $\eta(s)$ has led us to the following choices

$$(i) \quad : \quad \eta(s) = \eta\left[1 + e^{-\delta s}\right], \ \forall s \tag{20}$$

$$(ii) \quad : \quad \eta(s) = \begin{cases} 2\eta, & s \leq \bar{s} \\ \eta\left[1 + e^{-\delta[(s-\bar{s})]}\right], & s > \bar{s} \end{cases} \tag{21}$$

These choices insure well-behaved and at the same time best fitting models. All other extensions of (20)-(21) either converge to the above forms or fail. It is to be noted, however, that the best shape of the residual effect is a purely data-specific issue and so should differ on a case by case basis.

4.3 Estimation results

- Model selection

Given the above discussion we estimate three different models. Model 1 is the baseline model with $\alpha = 1/2$, and constant η, which assumes pure benefit/tightness effect. Model 2 is the model with unrestricted σ and the residual effect as in (20). Model 3 is a model with unrestricted α and the residual effect as in (21).

	Model 1		**Model 2**		**Model 3**	
Param.	Coeff.	p-Value	Coeff.	p-Value	Coeff.	p-Value
λ	0.0140	0.0000*	0.0141	0.0000*	0.0142	0.0000*
σ	0.1507	0.1249	0.1835	0.0774♭		
η	0.0085	0.0165*	0.0053	0.0128*	0.0422	0.0000*
δ			0.0571	0.0000*	0.0983	0.0432*
α					0.1904	0.0224*
log-L	-3187.278		-3171.318		-3162.242	

* and ♭ denote significance at 5% and 10% levels, respectively
p-Values for one-sided test $H_a : \xi_k > 0$

Specification

	α	σ	$\eta(s)$
M 1	0.5	free	η
M 2	0.5	free	$\eta\left[1+e^{-\delta s}\right]$
M 3	free	0.5	$\begin{cases} 2\eta & s \leq \bar{s} \\ \eta\left[1+e^{-\delta[s-\bar{s}]}\right] & s > \bar{s} \end{cases}$

Model selection

	Test	p-Value
M 2 vs M 1	LRT: $\chi^2_{(1)}$	0.0000
M 3 vs M 2	Vuong '89	0.0554

Table 1 *Estimation results*

Estimation results for all models are reported in Table 1. The corresponding hazard rates predicted for an individual with average sample characteristics and average duration of entitlement right are plotted in fig. 2.

The very first question we ask is that of model selection. The results of the relevant model selection tests are reported below Table 1. First of all, we recognize that Model 1 is nested in Model 2, so standard LR test applies. From the model selection summary we see that the baseline model is clearly rejected in favour of Model 2. The specifications with restricted α and σ, to the contrary, are non-nested. Looking at the values of log-likelihood, we can see that any information criterion will speak in favour of Model 3. In addition we perform a Vuong (1989) test for non-nested models. Its results show that on 10% significance level Model 3 is closer to the true model than Model 2. Taken together, these evidence make us stop our choice on Model 3.

We also see that the hazard rate implied by the best specification (solid line in fig. 2) repeats the monotonicity pattern of the nonparametric estimates in the discussion of the stylized facts.

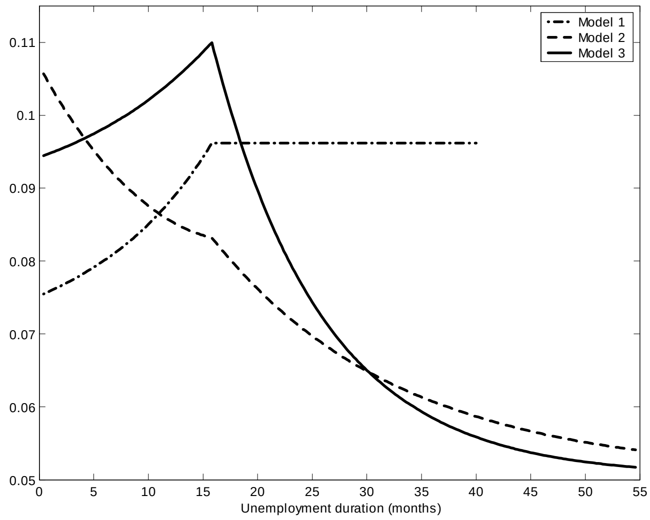

Figure 2 *Predicted hazard functions*

- Preliminary discussion

As for the estimation results alone, our main finding is the significance of α (see Table 1, Model 3; this result is true for both one-sided and two-sided alternatives). This means that changes in optimal effort in response to any unemployment benefit reform, be it the reform of b or of $\bar{s}$, will have a significant impact on the exit rate out of unemployment. This finding in particular can clarify the empirical dispute about the dependence between unemployment benefits and exit decision. Evidence in the literature are conflicting with Hujer and Schneider (1989) and Arulampalam and Stewart (1995) finding minor no negligible dependence and Carling et al. (2001) and Røed and Zhang (2003) stating the opposite. Despite we do not rule out that for certain types of heterogeneous agents the change in benefits may play no role, our above result shows that in aggregate terms there exists a positive significant relationship between the reemployment risk and any change in the level of unemployment benefit payments. Consequently, any change in the design of unemployment benefit mechanism will induce a significant response on the macro level.

- Predicting labour productivity A and vacancy cost γ

After having estimated λ, α, σ, η_i and δ_i, we are left with determining A and γ. The wage w and tightness θ were taken as exogenous in this first part of the estimation which was built on the household side of the model only. As the wage and tightness are endogenous in general equilibrium, we now take the estimated parameters $\boldsymbol{\xi}$ and compute parameters A and γ using the full general equilibrium structure of our economy in the steady state (see sect. 3.3). We compute A and γ such that the average wage and average tightness in our sample result as general equilibrium endogenous variables in our model.

5 The effects of labour market reforms

In this chapter we use the structurally estimated parameters in order to describe the steady state equilibrium of 2004 and to evaluate the reform effective as of January 2005. We base the simulations on the parameters of Model 3 where σ is set to 0.5.

5.1 The pre-reform steady state

All parameters plus some selected endogenous variables are provided in tab. 2. The rate of time preference ρ is chosen to match the annual interest rate of 3.7 percent. The bargaining power β is set equal to .5. In the pre-reform steady state, benefit payments of both short-term and long-term unemployed workers depend on the previous wage. The replacement rates are given by b_i/w and are sample means for those entitled to UI and UA payments. These ratios are pretty close to statutory replacement rates. Average sample entitlement to UI payments is 15 months, again for those entitled to UI payments. Our estimation procedure implies a degree of risk aversion σ, separation rate λ, search productivity η, depreciation rate δ and effort elasticity α. Labour productivity of A is just above the endogenous wage rate of 2250 DM (German marks), leaving the difference for firms to pay for vacancy costs γ. Comparing our tax rate κ and unemployment rate u to actual social security contribution rate (this is the only purpose of taxes in our model) of 0.06 and the actual unemployment rate of 0.12, our model fits real data very well.

For comparative statics below, we will take the exogenous parameters, the estimated and the predicted parameters as given. We will then change policy parameters to understand the effects on equilibrium values.

exogenous parameters

ρ	β	σ
.003	.5	.5

estimated and predicted parameters

λ	η	δ	α	A	γ	$\bar{\mu}$
.014	.042	.098	.190	2432	72.9	

policy parameters

$\frac{b_1}{w}$	$\frac{b_2}{w}$	$\bar{s}$
.6	.53	15

equilibrium values

w	u	θ	κ
2250 DM	11.9%	.3	7.3%

Table 2 *Parameters and selected equilibrium values (see text above for details)*

Although the economy is in the steady state, there are still dynamics on the micro level. At any point in time individuals find and lose jobs. Once unemployed, the closer expiration of the entitlement to UI benefits is, the harder they try to find a job. In addition to that, unemployed workers lose search productivity over the unemployment spell. So the exit rate that describes the steady state distribution of unemployment spells is influenced by all these factors. Figure 4 illustrates the developments on the micro level. Effort in the right upper panel, once expressed in consumption terms, is equivalent to consumption loss ranging from 12 to 21 percent.[14]

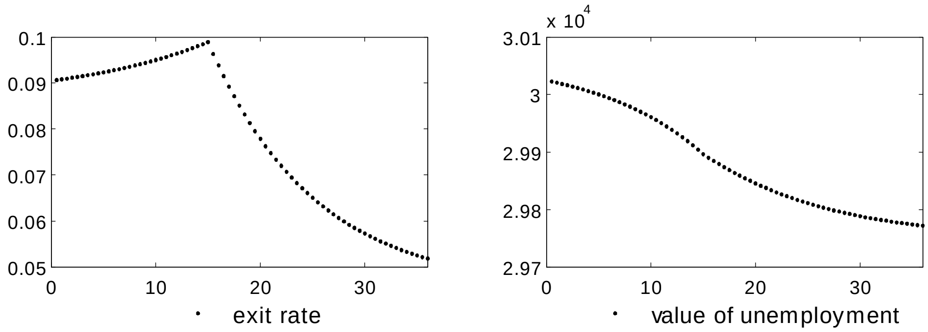

Figure 3 *Productivity, effort, exit rate and value of being unemployed over time*

5.2 Benchmarks

Before we undertake policy simulations, we need some benchmarks to evaluate whether the model does a good job at replicating data (beyond micro behaviour) and what to expect from a quantitative perspective.

- Some facts about Germany

The central quantity against which we evaluate the macroeconomic performance of our model is the unemployment rate. From January 2005 to January 2007 it fell from 12.3% to

[14]Share of consumption x lost due to search is determined by $\left([1-x]\,b(s)\right)^{1-\sigma}/(1-\sigma) = b(s)^{1-\sigma}/(1-\sigma) - \phi(s)$.

10.2%. Can our model, taking the labour market reforms and economic growth in Germany into account, explain this drop in unemployment? When undertaking comparative static, we will also exogenously increase labour productivity A such that it reflects economic growth in Germany. Real GDP in Germany grew by 0.8 percent in 2005 and by 2.9 percent in 2006.

One can evaluate the model performance also against other time trends. For example, unemployment insurance contributions, which correspond to the tax rate in our model, were reduced from 6.5% to 4.2%. The real gross wage reduced by unemployment insurance contributions stayed basically constant over this period. The number of vacancies more than doubled from 268,296 to 593,667.[15] Evaluating the impact on the value of the firm J would also be interesting but no appropriate data has been found so far.

- Analytical benchmarks

When we ask about welfare effects of policy reforms, by which order of magnitude will welfare and values change when unemployment assistance payments b_2 reduce by approx. 7%? We use two analytical benchmarks for our numerical analysis below.

First, utility of a long-term unemployed worker who will stay unemployed forever is with (17) $\left(\frac{b_2^{1-\sigma}}{1-\sigma}-\phi_2\right)/\rho$. If we neglect effort, a reduction of b_2 by 7% implies a reduction of utility from $\frac{b_2^{1-\sigma}}{1-\sigma}/\rho$ to $\frac{(.93b_2)^{1-\sigma}}{1-\sigma}/\rho$, i.e. to $\left(\frac{(.93b_2)^{1-\sigma}}{1-\sigma}/\rho\right)/\left(\frac{b_2^{1-\sigma}}{1-\sigma}/\rho\right)=.93^{1-\sigma}$. With $\sigma=.5$, welfare in the worst case scenario reduces to 96.4%. If we reduce b_2 by 50%, we would have as upper bound for welfare effects a reduction to 70.1%.[16]

Second, when we look at welfare effects in a model with $b_1=b_2$ (or equivalently at $\bar{s}=0$), constant residual effect η and constant but endogenous effort, the value of being a long-term unemployed is $\rho V(b_2)=u(b_2,\phi^*(b_2))+\mu(\phi^*(b_2)\theta)[V(w)-V(b_2)]$ where effort is determined by $\phi^*=\{\alpha\eta\theta^\alpha[V(w)-V(b)]\}^{\frac{1}{1-\alpha}}$ and the job arrival rate is given by $\mu(\phi\theta)=\eta[\phi\theta]^\alpha$. When we decrease b_2 by 7%, $V(b_2)$ reduces to 98.79%. A 50% reduction of b_2 implies a reduction to 95.87%. In all quantitative analyses on welfare we therefore need to be prepared to "small" results.[17]

5.3 Policy modelling

Labour market reforms were characterized by a reduction in UA benefits b_2 and entitlement length $\bar{s}$. We first analyze each change individually before we combine the changes and also add an increase in labour productivity.

- A decrease of unemployment assistance benefits b_2

[15] Source: http://www.pub.arbeitsamt.de/hst/services/statistik/detail/z.html (Federal Employment Agency).

[16] A reduction by 50% is probably the strongest effect the reform can have on anybody in Germany. OECD (2007) reports that the replacement rate for a long-term unemployed with 150% of average income, single and no child, dropped from 55% to 26%.

[17] A reduction of b_2 by 7% and 50%, respectively, reduces the unemployment rate 95% and 87.7% of its original level. This shows that in principle, this model is well-suited to quantitatively analyse the effects of labour market reforms on unemployment.

As the introduction pointed out, there were winners and losers of changes in UA, ranging from -50% to actual increase. The average decrease was 7%.

Figure 4 shows the effects of a decreasing b_2 on the labour market (when reading horizontal axes from right to left). Since effort of unemployed workers increases, it becomes more likely that a job will be found faster. Consequently, as we can see in fig. 4, the long-term unemployment and also the overall unemployment rate in the economy go down. Unsurprisingly, less unemployment implies a higher vacancy-unemployment ratio θ, which means that the labour market becomes tighter for firms. This also leads to higher net wages, which becomes clear from the wage equation (11) once the positive tightness effect dominates the negative effort effect. Finally, the tax rate goes down, because reduction in b_2, and therefore in equilibrium unemployment, implies less benefits to finance (and this also by a higher number of employed workers). So without considering any welfare questions, the cut of b_2 by the Hartz IV reforms seems to be a good move against the too generous institution.

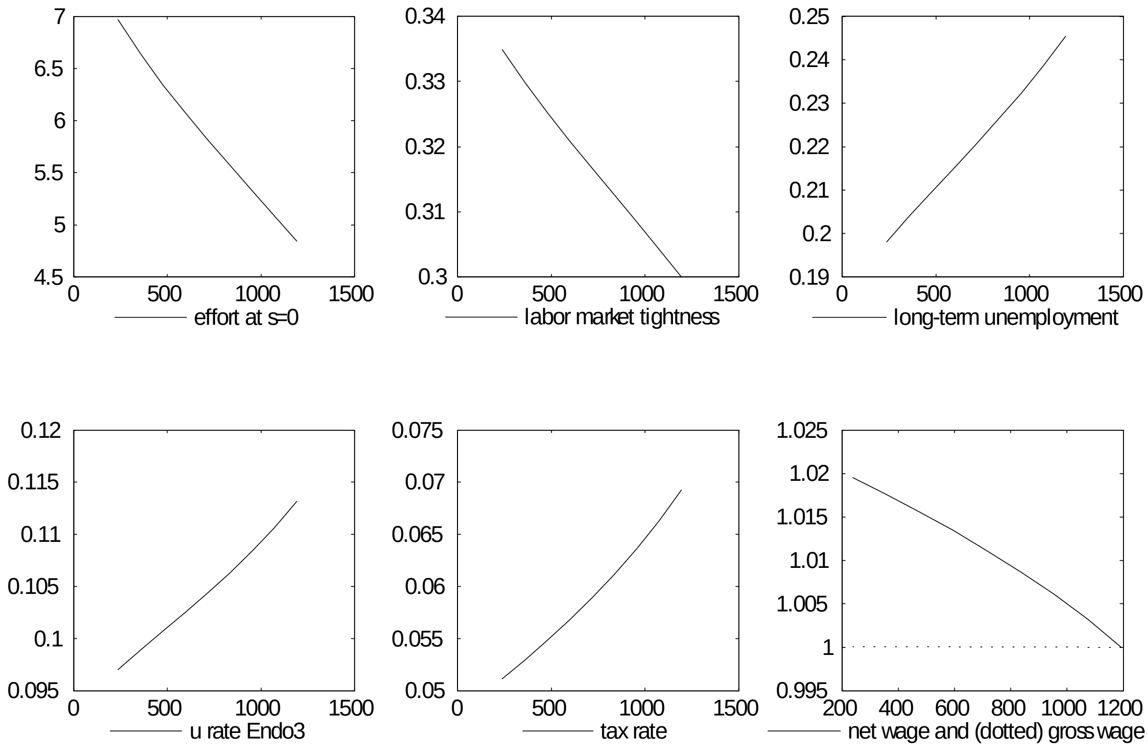

Figure 4 *(Un)employment effects of a drop in long-term benefits* b_2

When we add welfare measures in fig. 5, however, the unambiguously positive impression of the reform disappears. It makes perfect sense that both short-term and long-term unemployed are worse off in terms of their expected lifetime values. The long-term unemployed are directly hurt by the cut in b_2 and the short-term unemployed are now subject to higher search pressure, because in the nearest future their benefits may drop from b_1 to already lower level of b_2. The value of unemployment depends negatively on search effort and positively on exit rate, which can be seen from equations (5) and (17). So, as fig. 5 suggests, in both groups, the effort effect has obviously outweighed the employment effect. But it is even more remarkable that also firms and employed workers become worse off. The loss of the employed workers is slight and reflects the fact that net wage, even though increases, is not high enough to compensate for the prospective loss once becoming unemployed in the future (see eq. 4). Firms lose because the increase in net wage turns out to be higher than the decrease in the tax rate, so gross wage goes up, implying lower equilibrium profit. All in all, these results evidently explain why the total steady state welfare of the economy is expected to decrease due to Hartz IV reform. This result is very interesting because it seems generally accepted that a weakening such institutions as "benefits" is economically and socially desirable in a welfare state.

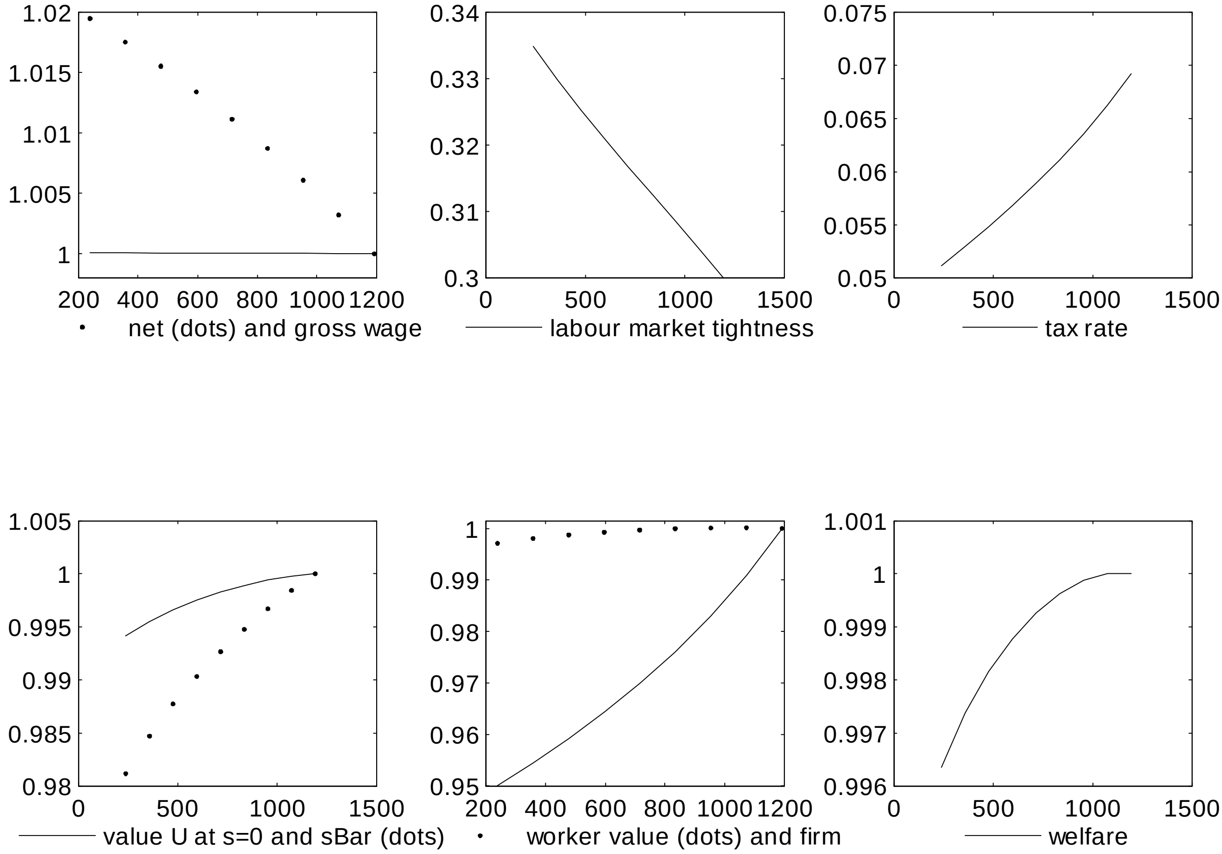

Figure 5 *Welfare effects of decreasing* b_2

- Changing the entitlement period s

- work in progress -

- Taking economic growth into account

- work in progress -

6 Conclusion

We have developed an estimable search and matching model with endogenous effort under time-dependent unemployment benefits. The main extension compared to the existing search and matching literature is the endogenous distribution of unemployment duration that arises due to individual choice of search intensity in a nonstationary environment. A link between these micro-dynamics and macro quantities like the unemployment rate is developed using tools from the literature on Semi-Markov processes.

The equilibrium distribution of unemployment duration, being a function of various parameters of the theoretical model, provides the basis for fully structural estimation via maximum likelihood. General equilibrium policy analyses are performed using the parameter estimates of the best fitting specification.

Simulations enable us to assess individual and aggregate labour market and welfare effects of changes in the length and level of unemployment benefit payments. As an example of such a reform, we evaluate the German Hartz IV reform of 2005. Total unemployment decreases due to the reform and so do government transfers to the unemployed. At the same time, despite unemployment and social security contributions do go down, the welfare changes for the economy as a whole are negative. While it is seems obvious (given the design of the reform) that the unemployed, especially the long-term unemployed, lose, employed workers and even firms lose. So when it comes to a normative evaluation, the advantages of shortened institutions are no longer unambiguous, given that efficiency effects have to be weighed against insurance effects.

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A Appendix

A.1 Data

We draw a flow sample of entrants to employment and unemployment at each month of year 1997. The choice of the year of sampling is determined by the fact that no changes to either benefit level or entitlement length were made between the 1st of January 1997 and the 1st of January 2005, when Hartz IV reform came into power. With December 2003 being the latest month of our observation period we end up with a sample that describes a stationary entitlement-benefit environment and provides a fairly reliable information on long-term unemployment (only 5.5% of unemployment durations in our sample are right-censored). For each entrant we retrieve the duration of stay in the current state since the moment of entry.

Unemployment:	Mean	Std. Dev.	**Employment** $^{a)}$:	Mean	Std. Dev.
Duration (s)	11.21	14.09	Duration (l), cens.	61.25	28.66
UI benefits (b_1)	1357.04	508.12	Duration (l), all	42.80	31.98
UA benefits (b_2)	709.24	624.17			
Entitlement ($\bar{s}$)	15.84	7.49			
Last wage (w)	2250.57	901.79			
# obs., censored	17		# obs., censored	159	
# obs., total	316		# obs., total	325	

$^{a)}$ Entrants to employment only

Table 3 *Descriptive statistics*

It is important to notice that GSOEP data do not contain information on the length of entitlement to UI benefits. There exist, however, strict and relatively simple rules that allow computing the length of entitlement once we know the length of previous job durations and the age of an individual. For this reason, for every person that enters unemployment we also have to retrieve his/her previous job history. In addition to that, previous job history provides us with the record of the latest wage earned.

Units of measurement are months for the duration data and German Marks for the wage data. Descriptive statistics can be found in Table A1.[18]

[18] $w = 2250$ Deutsche Mark is the average monthly net wage before the worker became unemployed, with job being lost during 1997. $\theta = 0.3$ is the mean of the vacancy-unemployment ratio from 1997 to 2004 in Germany.

A.2 Steady state solution

We solve for the steady state of the model by separating the model into two "blocks".

- Block 1: Household behaviour

Given the functional forms for utility and search productivity in (17) and (18), the first-order condition for effort (6) reads

$$\phi(s) = \{\alpha\eta(s)\theta^{\alpha}[V(w) - V(b(s), s)]\}^{\frac{1}{1-\alpha}}. \tag{A.1}$$

It holds for both short- and long-term unemployed. Plugging this into the Bellman equation for the unemployed (5) and expressing it as a differential equation in s gives

$$\dot{V}(b(s), s) = \rho V(b(s), s) - \frac{b(s)^{1-\sigma}}{1-\sigma} + \frac{\alpha - 1}{\alpha}[\alpha\eta(s)\theta^{\alpha}]^{\frac{1}{1-\alpha}}[V(w) - V(b(s), s)]^{\frac{1}{1-\alpha}}, \tag{A.2}$$

which is again valid for both short- and long-term unemployed. As the value of being unemployed an instant before and an instant after becoming a long-term unemployed is identical, we impose $V(b_1, \bar{s}) = V(b_2, \bar{s})$ when solving this differential equation. Finally, since for an infinite unemployment spell, search productivity in (18) becomes a constant, $\lim_{s\to\infty} \eta(s) = \eta_2$ and all other quantities are stationary as well, we get the terminal condition for (A.2) by using $\lim_{s\to\infty} \dot{V}(b_2, s) = 0$,

$$\rho V(b_2) = \frac{b_2^{1-\sigma}}{1-\sigma} - \frac{\alpha - 1}{\alpha}[\alpha\eta_2\theta^{\alpha}]^{\frac{1}{1-\alpha}}[V(w) - V(b_2)]^{\frac{1}{1-\alpha}}. \tag{A.3}$$

The Bellman equation for the employed worker (4) can be written with the explicit utility function as

$$V(w) = \frac{1}{\rho + \lambda}\left(\frac{w^{1-\sigma}}{1-\sigma} - \psi + \lambda V(b_1, 0)\right). \tag{A.4}$$

Now imagine we insert $V(w)$ from (A.4) into (A.2) and (A.3). Imagine further that we know all parameters and assume, for the time being, some values for w and θ. Then we can solve the differential equation (A.2) starting from some initial value $V(b_1, 0)$ and see whether the solution for $s \to \infty$ is identical to $V(b_2)$ from (A.3). If it does not, we need to adjust our initial guess $V(b_1, 0)$ until it does. Hence, with some exogenous w and θ, we have obtained the time path of effort over the unemployment spell, $\phi(b(s), s)$, the spell-path of the value of being unemployed, $V(b(s), s)$, and the value of a job $V(w)$.

- Block 2: Wage, tightness and vacancy filling rate

Given the equilibrium values $\{\phi(b(s), s), V(b(s), s), V(w)\}$ as a function of w and θ, we now endogenize w and θ.

The Bellman equation for the firm and the free entry result, (8) and (10), gives us

$$\frac{A - \frac{w}{1-\varkappa}}{\rho + \lambda} = \gamma\frac{\theta}{\bar{\mu}}. \tag{A.5}$$

The bargaining equation (11) reads with an explicit utility function (17)

$$\frac{w^{1-\sigma}}{1-\sigma}+\frac{\beta}{1-\beta}\frac{w}{1-\varkappa}=\left[\frac{b_1^{1-\sigma}}{1-\sigma}-\phi\left(0\right)\right]+\frac{\beta}{1-\beta}\left[A+\theta\gamma\right], \tag{A.6}$$

where $\phi(0)$ is the optimal search effort at the instant of entry into unemployment, which is given from (A.1). The above two equations require the average exit rate $\bar{\mu}$ and the tax rate $\varkappa$.

The average rate $\bar{\mu}$ is given by (9) which can easily be computed given that, after having solved block 1, the exit rates $\mu(.)$ are known from (18) and the density $f(s)$ can therefore be computed from (2).[19] The tax rate κ makes the government budget constraint (3) hold and is given by

$$\kappa=\frac{\frac{b_1U_{short}+b_2U_{long}}{wL}}{1+\frac{b_1U_{short}+b_2U_{long}}{wL}}. \tag{A.7}$$

Given the density $f(s)$, one can compute the number of short-term and long-term unemployed on the right-hand side of this expression from $U_{short}=U\int_0^{\bar{s}}f(s)\,ds$ and $U_{long}=U-U_{short}$ where U is the total number of unemployed. The number of unemployed in turn follows from (16), using (13a,b) and (14) which we can now solve, given again that exit rates are known from block 1.

Hence, we are basically left with (A.5) and (A.6) to determine the missing endogenous variables w and θ. After having solved block 1 with a guess of w and θ, we verify whether this guess fulfills (A.5) and (A.6). If not, we (Matlab) adjusts the guess until we find a solution. Appendix B.4 describes the numerical implementation in Matlab.

- Exit rates (in the estimation procedure)

For any given pair $\{w,\theta\}$, both elements of which are observable in the estimation procedure, via (18) the solution of Block 1 immediately provides us with exit rate $\mu\left(\phi\left(s\left(t\right)\right)\theta,\eta\left(s\right)\right)$. However, because of the necessity to integrate over $\mu(s)$ in (19a)-(19c), in practice it is more convenient to express exit rates as differential equations given $V(w)$. Irrespective of the level of paid out benefits, $\mu_1(s)$ and $\mu_2(s)$ have identical structure

$$\begin{aligned}\dot{\mu}_j\left(s\right)=\alpha\left[\mu_j\left(s\right)\right]^2+\left(\frac{\partial\eta\left(s\right)/\partial s}{\alpha\eta\left(s\right)}+\rho\right)\frac{\alpha}{1-\alpha}\mu_j\left(s\right)\\-\frac{\alpha^2}{1-\alpha}\left[\eta\left(s\right)\theta^\alpha\right]^{\frac{1}{\alpha}}\left[\mu_j\left(s\right)\right]^{2-\frac{1}{\alpha}}\left[\rho V\left(w\right)-\frac{b_j^{1-\sigma}}{1-\sigma}\right]\end{aligned} \tag{A.8}$$

[19] Given the regime change at $\bar{s}$, the density in (2) will have a hurdle structure. Denoting the exit rate $\mu(.)$ by $\mu_1(s)$ for short-term unemployed and $\mu_2(s)$ for long-term unemployed, we get

$$f\left(s\right)=\begin{cases}\mu_1\left(s\right)e^{-\int_0^s\mu_1(u)du} & \text{for } s\leq\bar{s}\\ \frac{\exp\left\{-\int_0^{\bar{s}}\mu_1(u)du\right\}}{\exp\left\{-\int_0^{\bar{s}}\mu_2(u)du\right\}}\mu_2\left(u\right)e^{-\int_0^s\mu_2(u)du} & \text{for } s>\bar{s}\end{cases}.$$

The expression for $s>\bar{s}$ is the probability of surviving $\bar{s}$ with a high level of benefit payments times the density of unemployment duration conditional on the expiration of entitlement, i.e. on $s>\bar{s}$, and transition to a lower level of benefit payments.

with $j = 1, 2$. On the $(\bar{s}, \infty)$ interval terminal condition for (A.8) obtains from $\lim_{s \to \infty} \eta(s) = \eta_2$ and $\lim_{s \to \infty} \dot{\mu}_2(s) = 0$, giving us

$$(1-\alpha)\mu_2 - \alpha [\eta_2 \theta^\alpha]^{\frac{1}{\alpha}} [\mu_2]^{1-\frac{1}{\alpha}} \left[\rho V(w) - \frac{b_2^{1-\sigma}}{1-\sigma}\right] + \rho = 0, \tag{A.9}$$

This allows to pin down the value $\mu_2(\bar{s})$ and write down the terminal condition for $\mu_1(s)$ on the $(0, \bar{s})$ interval

$$\mu_1(\bar{s}) = \mu_2(\bar{s}). \tag{A.10}$$

A.3 A Semi-Markov process

This is a short version of a more general introduction to Semi-Markov processes. The longer version is available upon request (see app. B.3). The first subsection describes the general approach to Semi-Markov processes while the second adapts it to our question.

A.3.1 The general approach

This follows Kulkarni (1995) and Corradi et al. (2004). The original work is by Pyke (1961a,b).[20] Let Y_n denote the state of a system after the nth transition. Let this state be i. Let the point in time of the nth transition be denoted by S_n. Define the probability that the system after the next transition is in j and that this transition takes place within a period of length x or shorter, conditional on the system being in i after the nth transition, as

$$Q_{ij}(x) \equiv P\{Y_{n+1} = j, S_{n+1} - S_n \leq x | Y_n = i\}.$$

The probability that any transition takes place is then given by summing up the probabilities for each j, $Q_i(x) = \Sigma_{j \neq i} Q_{ij}(x)$, not taking into account transitions from i to i. The probability that the system will be in j in τ is given by

$$p_{ij}(\tau) = (1 - Q_i(\tau))\delta_{ij} + \Sigma_{k \neq i} \int_0^t p_{kj}(\tau - x)\, dQ_{ik}(x). \tag{A.11}$$

The interpretation of this integral equation is as follows: the first part of the right hand side gives the probability that the system, being currently in state i, never leaves state i until τ. In this case $j = i$ and $\delta_{ij} = 1$, so $1 - Q_i(\tau)$ is the survival probability in state i. If $j \neq i$, $\delta_{ij} = 0$. The second part of the right hand side collects all cases in which the transition from i to j (which includes i) occurred via another state $k \neq i$. First, we take the probability that the process stayed in state i for a period of length x and passed to state k then (captured by $Q_{ik}(x)$). Then we need the probability that the process which is in state k after x will be in state j at τ (captured by $p_{kj}(\tau - x)$). As the transition from i to k can be anywhere between 0 and τ, we have to integrate over x in order to cover all possible transitions.

[20] We are grateful to Ludwig Fahrmeir for comments on Semi-Markov processes. For an excellent introduction in German, see Fahrmeir et al. (1981).

Equation (A.11) can slightly be rewritten, provided that $Q_{ik}(x)$ is once differentiable (which holds for our case), as

$$p_{ij}(\tau) = (1 - Q_i(\tau))\,\delta_{ij} + \Sigma_{k \neq i} \int_0^t p_{kj}(\tau - x) \frac{dQ_{ik}(x)}{dx} dx. \tag{A.12}$$

The derivative $dQ_{ik}(x)/dx$ now gives the density of going from i to k after duration x. Multiplied by the probability of subsequently going from k to j gives the density of ending up in j after having gone to k after x. Integrating over all durations x gives the probability of starting in i and being in j at τ.

A.3.2 Our two-state system

We now need to adjust the notation such that it suits our purposes. We look at a worker who just moved in t (like today) into either employment e or unemployment u. Define $Q_{eu}(\tau)$ as the probability that a worker who just found a job in t "jumps" to u in a period of time shorter or equal to $\tau - t$. With a duration s dependent arrival rate $\lambda(s(v))$, this is then simply given by

$$Q_{eu}(\tau|t_e) = 1 - e^{-\int_t^\tau \lambda(s(v))dv}, \tag{A.13}$$

where $s(v) = v - t$ is the duration in his current state. In perfect analogy and using a spell-dependent arrival rate $\mu(s(v))$, we get $Q_{ue}(\tau) = 1 - e^{-\int_t^\tau \mu(s(v))dv}$. For the complementary events - remaining in a given state - the probabilities are simply $Q_{ee}(\tau) = 1 - Q_{eu}(\tau)$ and $Q_{uu}(\tau) = 1 - Q_{ue}(\tau)$. The probabilities that a transition takes place at all in this two state process are

$$Q_e(\tau) \equiv Q_{eu}(\tau),\ Q_u(\tau) \equiv Q_{ue}(\tau). \tag{A.14}$$

With two possible states, we have four transition probabilities for the future: an unemployed (employed) person can either be unemployed or employed at some future point in time τ. Two are redundant as the probability of e.g. an unemployed worker of being employed is complementary to the probability of being unemployed, $p_{ue}(\tau) = 1 - p_{uu}(\tau)$, and similarly $p_{ee}(\tau) = 1 - p_{eu}(\tau)$. Hence, we only focus on $p_{uu}(\tau)$ and $p_{eu}(\tau)$. These probabilities are, using the general equation (A.12),

$$p_{uu}(\tau) = 1 - Q_u(\tau) + \int_t^\tau p_{eu}(\tau - v) \frac{dQ_{ue}(v)}{dv} dv, \tag{A.15a}$$

$$p_{eu}(\tau) = \int_t^\tau p_{uu}(\tau - v|t_u) \frac{dQ_{eu}(v)}{dv} dv. \tag{A.15b}$$

These equations can be most easily be understood by looking at the following figure.

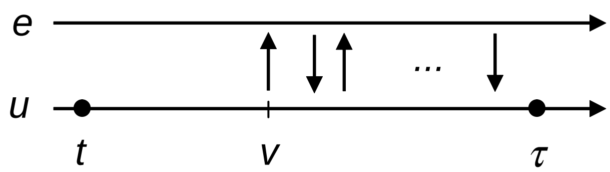

Figure 6 *Illustrating transition probabilities*

Let's consider $p_{uu}(\tau)$: An individual unemployed in t can be unemployed in τ by always remaining unemployed. This is the term $1 - Q_u(\tau)$. The individual can be unemployed in τ by remaining unemployed until v where he jumps into employment, the density for which is $dQ_{ue}(v|t_u)/dv$. After v, the probability of returning to unemployment in the remaining time span of $\tau - v$ is $p_{eu}(\tau - v)$. Note that this probability includes an arbitrary number of transitions larger than zero in this remaining period $\tau - v$. In contrast to integrating over x as in (A.12), we integrate over the point in time v here simply as this is the more intuitive way.

As a last step, we need to determine the two derivatives $dQ_{ue}(v)/dv$ and $dQ_{eu}(v)/dv$. Given duration-dependent arrival rates, the derivatives of (A.13) are,

$$\frac{dQ_{ue}(v)}{dv} = e^{-\int_t^v \mu(s(y))dy} \frac{d}{dv} \int_t^v \mu(s(y))\, dy = e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) \tag{A.16a}$$

$$\frac{dQ_{eu}(v)}{dv} = e^{-\int_t^v \lambda(s(y))dy} \frac{d}{dv} \int_t^v \lambda(s(y))\, dy = e^{-\int_t^v \lambda(s(y))dy} \lambda(s(v)) . \tag{A.16b}$$

Given (A.14) and the derivatives, the equations (A.15) become

$$p_{uu}(\tau) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau p_{eu}(\tau - v)\, e^{-\int_t^v \mu(s(y))dy} \mu(s(v))\, dv,$$

$$p_{eu}(\tau) = \int_t^\tau p_{uu}(\tau - v)\, e^{-\int_t^v \lambda(s(y))dy} \lambda(s(v))\, dv.$$

The final adjustment we need to make is to replace $\lambda(s(v))$ by λ as the separation rate is assumed to be constant. This then gives equations (13) in the main text.

B Appendix

All references to appendices starting with B are available upon request.