Complementarities between Training and Employment Policies

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Frank Oskamp[a] and Dennis J. Snower[a,b]

[a]*Kiel Institute for the World Economy* [b]*Christian-Albrechts-University, Kiel*

Abstract

This paper evaluates the joint influence of hiring subsidies and training subsidies which address a serious labor market problem: high unemployment of low-skilled. In particular, the paper deals with the question whether there are complementarities between both policies: Is the effect of each policy on welfare and employment greater when it is implemented in conjunction with the other policy?

For this purpose, we construct a simple, dynamic model of hiring decisions, derived from microfoundations. Workers are heterogenous with respect to their productivity. The model is calibrated with German data. The simulation shows that for reasonable parameter values there are complementarities between training and hiring subsidies. However, in the presence of the government budget constraint the result changes.

Keywords: hiring subsidies, training subsidies, employment; unemployment; vocational training; complementarities;

JEL classification: J21, J23, J24, J64, J68

Address:
Kiel Institute for the World Economy
Duesternbrooker Weg 120
24105 Kiel
Germany

Telephone: +49 431 8814–272
Fax: +49 431 8814–525
E-Mail: frank.oskamp@ifw-kiel.de
dennis.snower@ifw-kiel.de

1 Introduction[1]

Most OECD countries have implemented policies to encourage both employment and training. To varying degrees, both policies serve similar purposes, namely, to improve the employment and income perspectives, particularly for low-skilled workers. This paper deals with the question whether there are complementarities between both policies: Is the effect of each policy on welfare and on employment greater when it is implemented in conjunction with the other policy? Given the existence of complementarities we answer the question on what institutional and policy feature of the economy the size of complementarities depends. We focus on specific policies: hiring subsidies[2] and training subsidies.

One possible channel whereby both policies are complementary is the following: hiring subsidies facilitate the transition from vocational training to work. They increase the probability that an apprentice continues working in the firm as a medium-skilled employee after having finished vocational training successfully. Thereby, hiring subsidies improve the effectiveness of training policies, since the higher probability will amplify the positive impact of training subsidies. Moreover, hiring subsidies indirectly increase the number of people being hired as apprentices. This broadens the target group for training subsidies. However, both policies might be substitutes as both subsidies reduce low-skilled employment and unemployment and increase medium-skilled employment. They only differ with respect to the transition they are targeted at. Whereas training subsidies aim at increasing the number of people being in vocational training (first transition), hiring subsidies aim at increasing the hiring of successful apprentices as medium-skilled employees (second transition).

Our analysis tackles these issues by presenting a model of the labor market that is rich enough to capture the various groups of workers relevant to these alternative policy options, while at the same time being simple enough to generate straight-forward, intuitively transparent, policy guidelines. This paper presents a macro model that allows us to identify and qualify each effect being associated with the subsidies. To make our analysis expressly relevant to the decisions that policy makers commonly face in practice, we do not follow the mainstream practice of deriving policies as first-best responses to labor market failures. Instead, the model takes a variety of common labor market imperfections – such as wage bargaining, hiring and separation costs as well as imperfections related to the tax and transfer system – as given. Based on this framework, we examine the second-best response. Thus, we assume, as policy makers often do, that the institutions underlying these imperfections can be changed only gradually and with considerable delay; therefore we argue that it is useful to examine the effectiveness of the different policies while these institutions are in place.

We calibrate the model for the German labor market, which is especially characterized by a high unemployment of the low-skilled people. Moreover, the transition to training and from training to work has become more difficult in recent years.

This paper is organized as follows. Section 2 provides the underlying ideas and the relation to the literature. Section 3 presents the theoretical model of the labor market. In section 4, we solve a simplified model analytically in order to give the underlying intuition. In section 5 we calibrate the model and discuss the numerical results, we start with the simplified version of the

[1] We thank Alessio Brown, Wolfgang Lechthaler and Christian Merkl for valuable comments. Besides, we are very grateful to Markus Hummel (IAB) and Jessica Erbe (BIBB) for providing us data and valuable information.

[2] Sometimes, "hiring subsidies" are also called "employment subsidies" or "employment vouchers" and are implemented through a wide variety of policy instruments, such as tax breaks, grants, and so on. As they all - given that they are awarded only for a limited period of time - have analogous effects on labour market activities and government budgetary outlays, this paper groups them together under the broad heading of "hiring subsidies".

model and enlarge it gradually. Finally, section 6 concludes. The main message of the paper is that for reasonable parameter values there are complementarities between training subsidies and hiring subsidies. However, in the presence of the government budget constraint this is with respect to welfare no longer the case.

2 Underlying ideas and relation to the literature

Our study is related to a variety of previous studies analyzing the impact and optimal design of employment subsidies, both, theoretically and empirically. The initial work was done by Pigou (1933) and Kaldor (1936).[3] Frequently, the Mortensen's and Pissarides' (1994) search and matching framework has been used to analyze connections between the labor market and the education sector (see e.g. Charlot et al. (2005)) or the effect of employment subsidies (see e.g. Boone and van Ours (2004), Bovenberg et al. (2000), Cardullo and van der Linden (2006), Mortensen and Pissarides (2003), Pierrard (2005), and Vereshchagina (2002)). However, the matching technology - describing the relation between the input and output of the matching process - is assumed to be stable through time. This assumption is admissible provided that the matching technology can be considered independent of the input and output of the matching process. However, very often a negative time trend is found when estimating search and matching functions (see Blanchard and Diamond (1989) for the United States, and Fahr and Sunde (2001, 2004) for Germany), thus casting doubt on the stability through time.[4] It is admissible to use the matching function to analyze labor market policies, provided that these policies have no significant influence on the matching process itself. However, we do not see a rationale why active labor market policies should not affect the matching process. To prevent running afoul of the Lucas Critique, we do not take the aforementioned shortcut of assuming a policy-invariant matching function. Instead, we derive the policy effects in a microfounded way from the intertemporal maximization of economic agents and we model their incentives explicitly. We give special emphasis to the firm side in our model since labor demand, especially with respect to the low-skilled labor force, is the short side of the market in economies with high unemployment. The household side comes into play through the wage formation.

Many theoretical analyses are static and thus suffer from the serious drawback that they capture only short-run impact of employment policy.[5] There are, however, good theoretical and empirical reasons to believe that longer-run effects are important, often more important than the short-run effects.[6] In this context, our study differs from the literature, as we explicitly capture the dynamic effects of subsidies by specifying the transition rates between employment, unemployment and training as a function of the hiring incentives of the firm

The existing dynamic frameworks for evaluating subsidies are mainly deterministic and not well suited to analyze the impact of the policy. Mortensen and Pissarides (2003) explore the effects of

[3] For a survey of the empirical literature, see for example Katz (1998). For US evidence, see Woodbury and Spiegelman (1987) and O'Leary et al. (2006). For international evidence, see for example N.E.R.A. (1995), and for British evidence, see Bell et al. (1999). As follows, we focus on theoretical papers and the calibration thereof.

[4] Besides, many empirical studies reject the hypothesis of constant returns to scale (e.g. Warren (1996) for the United States, Fahr and Sunde (2001) for Germany). The number of matches (M) is a function of unemployment and vacancies ($M = f(U, V)$), typically specified in Cobb Douglas form ($M = U^{\alpha} V^{\beta}$). If $\alpha + \beta$ do not sum up to 1, the results are input dependent.

[5] See, e.g., Layard et al. (1991), pp. 490-492, and Snower (1994).

[6] Orszag and Snower (2000) show that the dynamic, long-run effects of employment subsidies, once the associated lagged adjustment processes have worked themselves out, differ from what may be expected in the short run.

taxes and subsidies on job creation, job destruction, employment and wages in a search and matching equilibrium model. However, in their model, like in the model of Albrecht and Vroman (2002), migration between skill groups, which is an essential component in our model, is not possible.[7] In this context, we contribute to the existing literature by explicitly allowing for migration from the low-skilled to the medium-skilled labor force. This detailed grid allows us to analyze and contrast the effects of training and hiring subsidies, explicitly taking the budgetary effects into account[8], in a common framework. As several authors have stressed the importance of training programmes aimed at fighting unemployment, training subsidies to support human capital accumulation are supposed to play a crucial role. According to Kluve and Schmidt (2002) training subsidies can combat European unemployment: as "educational credentials matter considerably"; they come to the result, that programmes improve the labor market prospects of economically disadvantaged individuals by facilitating the transition out of unemployment.

Another, huge literature analyzes possible complementarities between labour market institutions and policies (see e.g. Belot and van Ours (2001). Theoretical analyses of complementarities can be found in Coe and Snower (1997), Orszag and Snower (1999), L'Harriodon (2001) as well as Burda and Weder (2002). However, except in L'Harridon (2001), the impact on welfare is not analyzed. We now proceed to present the underlying model.

3 The model

We construct a Markov model of the labor market in which the dynamics between the different labor market states are governed by transition probabilities. The transition probabilities are derived from optimization principles of the individuals.

The model is meant to provide an analytical framework for analyzing the effectiveness of hiring and training subsidies. As noted, our model is meant to be both rich enough to capture the heterogeneity of the labor market which is relevant but it also aims to be simple enough to generate straight-forward, intuitively transparent, policy guidelines. Thus, the model involves some judicious compromises between analytical simplicity and the depiction of heterogeneous labor market behaviors. Specifically, the labor force is differentiated according its different skill levels, which are defined by the level of educational attainment. We assume, that each skill level corresponds to a certain productivity level. Total population is divided into eight groups (see table 1): people leaving school or the Newborn (B),[9] people joining vocational training (apprentices, T) and those being either employed (N) or unemployed (U). Each of the two groups are divided into three subgroups according to the skill level: low-skilled, medium-skilled and high-skilled.

Vocational training takes p periods, so that there are p cohorts. In each period, a fraction $\rho+\theta$ leaves the vocational training where ρ is the mortality rate and θ is the break off rate of training. So, given the inflow into vocational training, A_t, the outflow in period $t+p$ is given by $A_t(1-\rho-\theta)^p$. The stock of people being in vocational training is given by $T = A\sum_{c=1}^{p}(1-\rho-\theta)^{c-1}$.[10]

[7] Also in the model of Cardullo and van der Linden (2006), there is no migration between skill groups.

[8] Orszag and Snower (2003a and 2003b) pointed out the fact that the literature disregarded the complete impact of employment subsidies on the government budget constraint by requiring that aggregate payroll taxes finance aggregate employment subsidies and thereby ignoring the reduction of unemployment benefit payments, which result from reduced employment. In this paper we follow their line of reasoning.

[9] In this analysis, both interpretations of the variable B are possible: the essential point is that people in this state are available for vocational training.

[10] In the initial steady state, A is exogenously given. In the presence of subsidies, A is determined endogenously.

state	variable
low-skilled employment	N_l
low-skilled unemployment	U_l
medium-skilled employment	N_m
medium-skilled unemployment	U_m
high-skilled employment	N_h
high-skilled unemployment	U_h
vocational training	T
newborn	B

Table 1: The Labor Market States.

The individual person is either unemployed or employed (respectively engaged in vocational training). For simplicity, there is no capital. Moreover, we assume constant returns to labor. Let a_i be the productivity of an employee with a skill level $i = l, m, h$. (Here, as well as for other variables below, the subscript l stands for "low-skilled"; the subscript m for "medium-skilled", the subscript h for "high-skilled"). The firms face a random operation cost $\epsilon_{i,t}$ which is iid across workers and time within the skill group i. This cost may be interpreted as an operating cost or a productivity shock. With respect to all employees, its mean of future values is normalized to zero and its cumulative distribution $\Gamma_i(\epsilon_i)$ is time invariant.

In the model, training takes place within the "dual system of vocational training", the dominant form in Germany.[11] This is a combination of vocational training provided by a private employer and theoretical education in vocational schools. With respect to the latter, the associated costs (e.g. for school buildings, salaries of the teachers) are distributed among the population. We assume that these costs have a highly fix character, so that a change in the number of apprentices does not influence the level. For the sake of simplicity, they are ignored in the remainder. The costs which are important in the model are the costs for the employer which are caused by the provision of vocational training (e.g. wages of additional employees being in charge of the instruction of apprentices within the firm). With respect to the distribution of these costs, it is necessary to distinguish two types of training: general training and specific training.[12] With respect to the German system of vocational training, one can argue, that training has a mostly general character: Due to detailed plans determining the content of on-the-job training and central examinations, vocational training within the dual system is highly standardized on a national level.[13] According to the original theory by Becker (1962, 1964), in the presence of competitive markets, the employee receives all the returns from general training and thus also has to pay the costs. However, German employers also pay a substantial amount of money for the training of apprentices [14] and thus - from the theoretical point of view - also pay for general training. It can be shown, that firms - in contrast to the original theory - have an incentive to invest in general vocational training, given that there are imperfections in the labor market.[15] In addition, the hold-up problem (see Williamson (1985) and Grout (1984)) does not occur if the investor (i.e. the firm paying for general education) receives the

[11] For a survey of the German dual system of vocational training, see Franz and Soskice (1995).

[12] See Becker (1964). For a recent survey of the literature on private sector training see Leuven (2005).

[13] See for further details Harhoff and Kane (1993: 11) and Lindner (1998: 413).

[14] According to Beicht and Walden (2002), an employer being engaged in vocational training faces a net cost of 8,705 EUR per trainee in 2000.

[15] See e.g. Harhoff and Kane (1993) and Acemoglu and Pischke (1999). Appendix A1 contains an analysis showing that firms have an incentive to invest in general vocational training.

full marginal return on investment and therefore will not under-invest. However, a hold-up problem may arise if the costs for general education arise.[16] Without a shift of financial burden away from the firm sector, the fraction of firms providing vocational training would decrease. The firms would under-invest.

Agents in our model pursue the following sequence of decisions. First, the government sets the income tax rate to ensure that its tax receipts are equal to its expenditures. Second, wages are determined through bargaining. Third, the random operating costs are revealed and finally, employment decisions are made.

We now continue by presenting the dynamic structure.

3.1 The dynamic structure

The transition among the labor market states are summarized in Figure (1). In all states besides B, people face a probability ρ of dying. With respect to the newborn, a fraction $\eta_{B,A}$ is hired as apprentice. The residual part $(1-\eta_{B,A})B$ tries to get a job as a low-skilled employee; only a fraction η_l is hired. A low-skilled worker faces a probability φ_l of being fired and a probability $\eta_{N,A}$ of being hired as apprentice, a low-skilled unemployed faces a probability $\eta_{U,A}$ of being hired as apprentice and a probability $(1-\eta_{U,A})\eta_l$ of being hired as low-skilled employee. With a probability θ per period, an apprentice breaks off training, however, we assume that an apprentice is not fired. An apprentice who successfully finishes training, is hired as a medium-skilled employee with a probability η_{A,N_m}. Here, we assume that the hiring rate is equivalent to the take over rate. A medium-skilled employee faces a probability φ_m of being fired. An unemployed medium-skilled is hired with a probability η_m. In order to keep the model simple, we treat the high-skilled labor force $(N_h + U_h)$ as a quasi-fix factor, i.e. we assume, that these states are not affected by the introduction of subsidies. However, given the government budget constraint, it is necessary to take these states into account, because N_h also carries a part of the fiscal burden and U_h is responsible for a part of the fiscal burden. Ignoring it would bias the amount of the fiscal burden which has to be carried by the low-skilled and medium-skilled employees. Given this, the labor market system can be described as follows:

$$S_{t+1} = M_{t+1}\ S_t \tag{1}$$

where S_t is a vector of the different states:

$$S_t = (N_{m,t}, U_{m,t}, N_{l,t}, U_{l,t}, A_t, B_t) \tag{2}$$

and M_t is a Markov matrix of transition probabilities: (3)

$$M_t = \begin{pmatrix} (1-\phi_m-\rho) & \eta_m & 0 & 0 & \eta_{A,N_m}(1-\rho-\theta)^p & 0 \\ \phi_m & (1-\eta_m-\rho) & 0 & 0 & (1-\eta_{A,N_m})(1-\rho-\theta)^p & 0 \\ 0 & 0 & (1-\phi_l-\eta_{N,A}-\rho) & (1-\eta_{U,A})\eta_l & 0 & (1-\eta_{B,A})\eta_l \\ 0 & 0 & \phi_l & (1-(1-\eta_{U,A})\eta_l-\eta_{U,A}-\rho) & 0 & (1-\eta_{B,A})(1-\eta_l) \\ 0 & 0 & \eta_{N,A} & \eta_{U,A} & 0 & \eta_{B,A} \\ \rho & \rho & \rho & \rho & 1-(1-\rho-\theta)^p & 0 \end{pmatrix}$$

[16] The costs of education could mainly rise for two reason: A shift of time from productive workplace activities to education and a shift from firm-specific to more general skills. Both factors create an additional financial burden which causes firms to reduce vocational training. See for an analysis of the changing environment and its consequences for vocational training e.g. Büchel (2002) and Wößmann (2004, 2006).

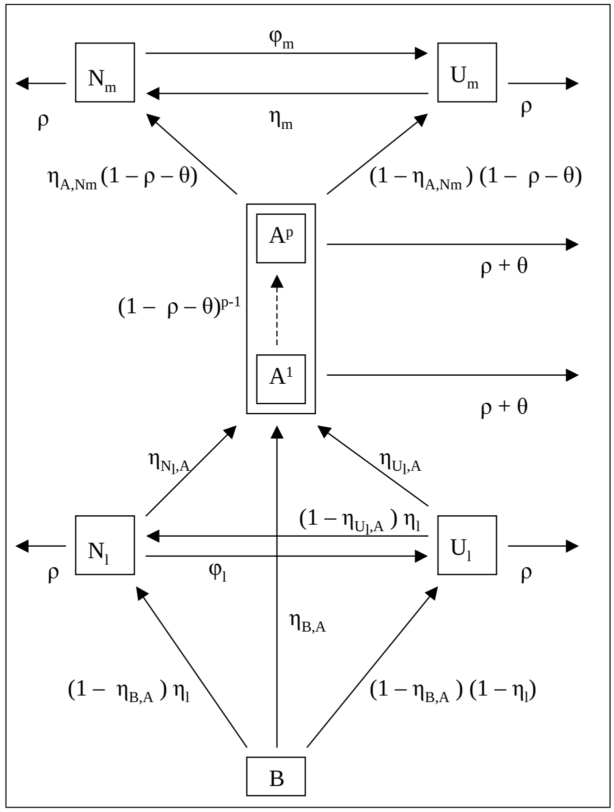

Figure 1: The dynamic structure of the model.

3.2 Government Budget Constraint

For simplicity, our model considers only four instruments of government policy: (i) a payroll tax with a tax rate τ_i, (ii) an unemployment benefit f_i, (iii) a hiring subsidy, $\sigma^{\eta_{A,N_m}}$, targeted at the employers who hire successful apprentices as medium-skilled employees and (iv) a training subsidy targeted at the employers for hiring people as apprentices. Our model has three levels of payroll tax rates, in order to match the German progressive tax system $\tau_h > \tau_m > \tau_l$. The ratios are assumed to be exogenous, whereas the levels are set so that the tax receipts are equal to the government's spending on unemployment benefits and subsidies. We assume that people being engaged in vocational training do not pay taxes. Given the presence of the subsidies above, the government budget constraint is expressed as follows:

$$\begin{aligned}\textstyle\sum_{i=l,m,h}\tau_i w_i N_i = & \textstyle\sum_{i=l,m,h}\beta_i w_i(1-\tau_i)U_i \\ & + \sigma^{k_v}\ A\textstyle\sum_{c=1}^{p}(1-\rho-\theta)^{c-1} \\ & + \sigma^{\eta_{A,N_m}}\ \eta_{A,N_m}\ A\ (1-\rho-\theta)^{p}\end{aligned} \tag{4}$$

where the left-hand side stands for the tax receipts, to be paid by the employees of different skill groups. The term on the right-hand side represents the unemployment benefits, with the net replacement rate β_i, the training subsidies σ^{k_v} paid to firms for all their apprentices and finally the hiring subsidies, $\sigma^{\eta_{A,Nm}}$., paid to firms for hiring successful apprentices as medium-skilled employees.[17]

3.3 Wage Determination

For simplicity, let the wage $w_{i,t}$ for each skill level i be the outcome of a Nash bargain between the median employee of that skill level and the firm. The median insider faces no risk of dismissal at the negotiated wage. The wage is renegotiated in each period. We start by calculating the relevant surplus for both, the employee and the firm.

3.3.1 Surplus of the employee

Each person has the following utility function:[18]

$$u_{i,t}(C) = \frac{1}{1-\gamma}[C_{i,t}]^{1-\gamma} \text{ with } i = l, m, h \tag{5}$$

where γ is the coefficient of relative risk aversion. Utility depends positively on consumption $C_{i,t}$.[19] Under bargaining agreement, the employee receives the net wage $w_i(1-\tau_i)$ in each period, where τ_i is the payroll tax rate being associated with a skill level i (with $i = l, m, h$). The expected present value of the employee´s utility, $V_{i,t}^N$, is:

$$V_{i,t}^N = \frac{1}{1-\gamma}[w_{i,t}(1-\tau_{i,t})]^{1-\gamma} + V_{i,t}^F \tag{6}$$

where V_i^F is the present value of utility resulting from the expected future life time income. (Here, as well as for other variables below, the superscript N stands for "employed"; the superscript U for "unemployed"). Under disagreement, the employee´s fallback position is $f_{i,t}$, assumed to be equal to the unemployment benefit. Assuming that disagreement in the current period does not affect the expected future life time income $V_{i,t}^F$, the corresponding expected present value of utility is:

$$V_{i,t}^{d,N} = \frac{1}{1-\gamma}[f_{i,t}]^{1-\gamma} + V_{i,t}^F \tag{7}$$

Given the expected present values in the cases of agreement and disagreement, we can calculate the bargaining surplus for the employee ($S_{i,t}^E = V_{i,t}^N - V_{i,t}^{d,N}$):

$$S_{i,t}^E = \frac{1}{1-\gamma}[(w_{i,t}(1-\tau_{i,t})]^{1-\gamma} - \frac{1}{1-\gamma}[f_{i,t}]^{1-\gamma} \tag{8}$$

3.3.2 Surplus of the firm

Under bargaining agreement, the firm receives the expected profit $(a_{i,t} - w_{i,t})$ in each period t. The expected present value of the profit π with respect to a medium-skilled employee is

$$\pi_{m,t} = (a_{m,t} - w_{m,t}) + \delta((1 - \varphi_{m,t+1} - \rho)\pi_{m,t+1} + \varphi_{m,t+1}(-\varsigma_{m,t+1})) \tag{9}$$

[17] Recall, that A is the inflow into training and $(1-\rho-\theta)^p A$ is the outflow from training.

[18] In what follows, only those variables have time subscripts that, for given parameter values, vary through time.

[19] In this model, for simplicity, workers consume all their income, i.e. either the net wage in the case of employment or the unemployment benefit in the case of unemployment.

where ς_m are the firing costs and δ is the discount factor. The firm's fallback profit is $-\varsigma_m$, i.e. during disagreement the employee imposes the maximal cost on the firm (e.g. through strike, work-to-rule, sabotage) short of inducing dismissal. We assume that disagreement in the current period does not affect future returns. Thus, the present value of profit under disagreement is

$$\pi^d_{m,t} = -\varsigma_{m,t} + \delta((1 - \varphi_{m,t+1} - \rho)\pi_{m,t+1} + \varphi_{m,t+1}(-\varsigma_{m,t+1})) \tag{10}$$

With respect to a low-skilled worker ($i = l$) and the future profit generated by the low-skilled worker, there is the additional probability ($\eta_{A,Nm}$) that the worker is hired as apprentice by another firm. This is based on the assumption that there are two types of firms: (1.) firm who only employ low-skilled and (2.) firms who are engaged in vocational training and who employ only medium-skilled. However, this possibility affects the expected profit of the firm in the case of agreement as well as in the case of disagreement. Thus, when calculating the surplus, this component cancels out. Finally, we can calculate the bargaining surplus for the firm ($S^F_{i,t} = \pi_{i,t} - \pi^d_{i,t}$) independent of the skill level i

$$S^F_{i,t} = (a_{i,t} - w_{i,t}) \ + \varsigma_{i,t} \tag{11}$$

3.3.3 Bargaining

To determine the wage, we use the Nash bargaining solution. The bargaining power of the employee is denoted by $\mu \ \epsilon \ \langle 0, 1\rangle$, and the bargaining power of the firm by $1 - \mu$. Then the wage w_i solves:

$$\max_{w_i} \ (S^E_i)^\mu \ (S^F_i)^{1-\mu} \tag{12}$$

We assume that the unemployment benefit level $f_{i,t} = \beta_i \ w_{i,t} \ (1-\tau_i)$ is defined on the economywide average net wage $w_{i,t} \ (1 - \tau_i)$ for each skill group. The negotiated wage is (with $\xi = 1 - \gamma$):

$$w_i = \frac{- \ a_i \ \mu \ \xi}{-1 - \beta_i^\xi(-1 + \mu) + \mu(1 + (-1 + c_\phi)\xi)} \quad \text{with } i = l, m, h \tag{13}$$

3.4 Hiring Rates

With respect to the analysis of hiring and training subsidies, two hiring decisions are of importance as they are influenced by the subsidies: (i) the hiring of newborn [20] and (ii) the hiring of successful apprentices as medium-skilled employees.

First, we consider the second hiring decision and the corresponding hiring rate η_{A,N_m} which depends on the expected profit. Recall that, a worker of type i produces a_i of output per period, the firm has to pay a wage w_i. The expected present value of profit generated by the employee, after the random cost $\epsilon_{\eta_{A,N_m}}$ is observed is

$$\pi_{A,N_m,t} = -\epsilon_{\eta_{A,N_m}} + \sigma^{\eta_{A,N_m}} + (a_m - w_m)\sum_{i=0}^{\infty}\delta^i(1 - \varphi_m - \rho)^i - \delta \ \varphi_m \ \varsigma_m \sum_{i=0}^{\infty}\delta^i(1 - \varphi_m - \rho)^i \tag{14}$$

[20] We assume that training subsidies are not paid for hiring low-skilled employees (N_l) or low-skilled unemployed (U_l) as apprentices. The corresponding transition rates, which would be affected ($\eta_{N_l,A}$) and ($\eta_{U_l,A}$) are very low, in particular in relation to ($\eta_{B_l,A}$). Thus, the quantiative impact of subsidies would be low. However, the complexity of the model would increase signficantly.

with the hiring subsidy $\sigma^{\eta_{A,N_m}}$ and the discount factor δ.[21] As the person is already in the firm, the hiring costs are zero. Given this, a person is hired as medium-skilled, if the expected present value of profit is positive: $\pi_{A,N_m,t} > 0$, thus

$$\epsilon_{\eta_{A,N_m}} < \sigma^{\eta_{A,N_m}} + \frac{a_m - w_m - \delta\ \varphi_m \varsigma_m}{1 - \delta(1 - \varphi_m - \rho_m)} \tag{15}$$

With respect to $\epsilon_{\eta_{A,N_m}}$, we assume a uniform distribution, i.e. $\epsilon_{\eta_{A,N_m}}$ is uniformly distributed between 0 and $\epsilon^{+}_{\eta_{A,N_m}}$. Given all this, the corresponding hiring rate is[22]

$$\eta_{A,N_m} = \frac{(\sigma^{\eta_{A,N_m}} + \frac{a_m - w_m - \delta\ \varphi_m\ \varsigma_m}{1 - \delta(1 - \varphi_m - \rho_m)})}{\epsilon^{+}_{\eta_{A,N_m}}} \tag{16}$$

Second, we consider the hiring of newborn (B) as apprentices. In the remainder we call the transition rate describing the hiring of newborn, respectively, as apprentices "training rate" and not "hiring rate" in order to avoid confusion with the hiring of successful apprentices as medium-skilled employees. If the training rate would only depend on the profit in the phase of vocational training, the hiring rate would be zero, because the profit in this phase is negative. The output a_v generated by the apprentice is smaller than the sum of the wage, w_v, and the additional costs of vocational training, k_v, the firms faces. However, the firm regards the costs of vocational training as of investment, which causes a profit (π_{A,N_m}, see eq. (14)), once the successful apprentice continues working in the firm as a medium-skilled employee with a certain probability $\eta_{A,N_m}(1-\rho-\theta)$. Therefore, also the latter phase has to be taken into account, when determining the training rate. The expected present value of profit generated by an apprentice is

$$\begin{aligned} \pi_{B,A,t} = {} & -\epsilon_{\eta_{B,A}} + (a_v - w_v - k_v + \sigma^{k_v}) \sum_{i=0}^{p-1} \delta^i (1 - \rho - \theta)^i \\ & + \delta^p (1 - \rho - \theta)^p\ \eta_{A,N_m}\ E(\pi_{A,N_m}) \end{aligned} \tag{17}$$

where σ^{k_v} is the training subsidy paid to the firm to increase the incentive of hiring a newborn as medium-skilled employee.[23] Given the firing costs $\chi_{K,A}$, a newborn or a low-skilled is hired as apprentice if $\pi_{B,A} > \chi_{B,A}$, thus

$$\begin{aligned} \epsilon_{\eta_{B,A}} < {} & -\chi_{B,A} + (a_v - w_v - k_v + \sigma^{k_v}) \sum_{i=0}^{p-1} \delta^i (1 - \rho - \theta)^i \\ & + \delta^p (1 - \rho - \theta)^p\ \eta_{A,N_m}\ E(\pi_{A,N_m}) \end{aligned} \tag{18}$$

Again, we assume a uniform distribution, i.e. $\epsilon_{\eta_{B,A}}$ is uniformly distributed between 0 and $\epsilon^{+}_{\eta_{B,A}}$. The corresponding hiring rate is

$$\eta_{B,A,t} = \frac{-\chi_{B,A} + (a_v - w_v - k_v + \sigma^{k_v}) \sum_{i=0}^{p-1} \delta^i (1 - \rho - \theta)^i + \delta^p (1 - \rho - \theta)^p\ \eta_{A,N_m}\ E(\pi_{A,N_m})}{\epsilon^{+}_{\eta_{B,A}}} \tag{19}$$

[21] Recall that eq. (14) is equal to eq. (9) for $-\epsilon_{\eta_{A,N_m}} = 0$ and $\sigma^{\eta_{A,N_m}} = 0$.
[22] For a derivation of the hiring rates, see Appendix A2.
[23] Note that $E(\pi_{A,N_m,t})$ is given by equation (14) with $-\ \epsilon_{\eta_{A,N_m}} = 0$.

All other transition rates, especially the hiring and firing rates of the low-skilled and the medium skilled, respectively, (η_i and ϕ_i with $i = l, m$) are not affected by the implementation of subsidies. They are treated as constant factors, so that a microfoundation is not necessary.

3.5 Labor Market Equilibrium

The labor market equilibrium is the solution of the system comprising:

- the six labor market dynamic equations given by the transition matrix (13)
- the government budget constraint, equation (4),[24] and finally
- the equations determining the hiring rates, equations (16) and (19).

Now, we first illustrate the characteristics of the subsidies. Then, we continue by presenting an analytical solution of a simplified version of the model. Then, we proceed to calibrate the full model. We continue by solving the model numerically and start again with the simplified version. The model is enlarged gradually. In each step, we calculate the impact of different combinations of subsidies on the welfare and employment, examine potential complementarities and provide an intuitive analysis of the results.

3.6 The characteristics of the subsidies

In the following we analyze two kinds of subsidies:

- Hiring subsidies ($\sigma^{\eta_{A,N_m}}$) are paid to firms which hire successful apprentices as medium-skilled employees. The subsidy will be paid during the first period of the employment spell.
- Training subsidies (σ^{k_v}) are paid to firms which hire newborn people as apprentices. The training subsidies are paid over the whole phase of vocational training. They reduce the costs of the firm being associated with vocational training.

Whereas hiring subsidies aim at improving the employment situation of the successful apprentices by increasing the hiring incentive of the firm, training subsidies aim at improving the human capital in a first step and then in a second step, the long-term employment perspective, given that the employment rate of medium-skilled labor force is higher than the employment rate of the low-skilled labor force.

Whereas training subsidies only have a direct effect - they induce an increase in the hiring rate $\eta_{B,A}$ -, hiring subsidies also have an indirect effect. Primarily, they aim at easing the transition from training to work, i.e. they increase the hiring rate $\eta_{N,A}$. Moreover they also have an indirect effect. The decision to hire an apprentice also depends on the probability, that the person continues working in the firm as medium-skilled. The higher the hiring rate, $\eta_{N,A}$, the higher is the probability and the higher is the incentive to hire a person as apprentice (see equation (19)) Thus, hiring subsidies not only increase the fraction of apprentices who are hired as medium-skilled employees, they also increase the number of apprentices.

[24] As the equation describing the government budget constraint contains three tax rates, it is necessary to introduce two additional equations in order to close the model. In the remainder this is done by introducing ratios between the tax rates, t_h, t_m and t_l.

4 A simple analytical solution

In the following analysis, welfare is calculated as the income aggregated over all labor market states[25]

$$\Phi = \sum_{i=m,l,h}(N_i\ w_i(1-t_i) + U_i\ \beta_i\ w_i(1-t_i)) + T\ w_v \tag{20}$$

4.1 The benchmark model

Before starting to solve the model numerically, we present a simple analytical solution, which helps to understand the underlying intuition. In order to get an analytically traceable solution, we make the following simplifications: (1.) there are no taxes: $t_i = 0$ with $i = l, m, h$, (2.) the individuals are risk neutral: $\gamma = 0$, (3.) training takes one period: $p = 1$ and (4.) training is not broken off: $\theta = 0$. With respect to the high-skilled labor force, all variables are treated as constant. With respect to the medium- and low-skilled labor force, the labor market states can be expressed as functions of the transition rates, i.e. in particular as functions of the hiring rates (eq. 16 and eq. 19) and thereby as a function of the hiring and training subsidies. The corresponding steady state expressions can be derived on the base of the Markov Matrix (see above). The wages are given by eq. (13).

According to Coe and Snower (1997) policies are complementary in the sense that the effect of each policy is greater when implemented in conjunction with the other policy than in isolation. To check the presence of complementarities, we calculate the cross derivative of Φ with respect to the two subsidies, σ^{k_v} and $\sigma^{\eta_{A,N_m}}$, which is

$$\begin{aligned}\frac{\partial^2\Phi}{\partial\sigma^{k_v}\ \partial\sigma^{\eta_{A,N_m}}} = \ & \underbrace{[a_m(1-\beta_m)\mu(1-\rho)\rho^2(\eta_l + \eta_{N,A}(1-\eta_l) + \rho + \phi_l)]}_{>0}\ / \\ & \underbrace{[\ \epsilon^+_{\eta_{A,N_m}}\epsilon^+_{\eta_{B,A}}(1-\beta_m(1-\mu) - c_\phi\ \mu))(1+\rho)(\eta_l(1-\eta_{U,A})(\eta_{N,A}+\rho)}_{>0} \\ & \underbrace{+\ (\eta_{U,A}+\rho)(\eta_{U,A}+\rho+\phi_l))(\eta_m+\rho+\phi_m)\ ]}_{>0}\end{aligned} \tag{21}$$

The cross derivative is unambiguously positive. Hence, the result reveals - at least for a very simple version of the model - the existence of complementarities in the sense that the effect of one subsidy is bigger in the presence of the other subsidy.

The intuition behind this is as follows: hiring subsidies facilitate the transition from vocational training to work. They increase the probability that an apprentice continues working in the firm as a medium-skilled employee after having finished vocational training successfully. Thereby, hiring subsidies improve the effectiveness of training policies, since the higher probability will amplify the positive impact of training subsidies. Moreover, hiring subsidies indirectly increase the number of people being hired as apprentices. This broadens the target group for training subsidies.

4.2 Robustness Checks

This section discusses two modifications of the simple model which may be expected to change our qualitative results (1.) we assume that the hiring rate η_{A,N_m} does not correspond to the

[25] The income in the labor market state B is 0.

take over rate, successful apprentices do face and (2.) we assume that the lower limit of the uniform distribution of ϵ is no longer 0 (see section 3.4) but $\epsilon^- > 0$. Another crucial aspect concerning complementarities is the presence of taxes (which have been ignored so far) and the government budget constraint. With the given model, a purely analytically examination is not possible. Therefore we have done the analysis with a different, 2-period-model, which is presented in appendix A3.

4.2.1 Take over rate

The critical reader might argue, that not all successful apprentices are taken over by the firm which conducted vocational training. Some of the successful apprentices will get a job in a firm which is not engaged in vocational training. So far, we have assumed that the hiring rate η_{A,N_m} was identical to the take over rate. Now, we assume, that there are two hiring rates: η^1_{A,N_m} being associated to the firm which is engaged in vocational training, and η^2_{A,N_m} being associated to the firm which is not engaged in vocational training. Now, only η^1_{A,N_m} can be interpreted as take over rate. Moreover, with respect to the dynamic equations given by the Markov Matrix, we assume that $\eta^1_{A,N_m} + \eta^2_{A,N_m} = \eta_{A,N_m}$. In the following analysis we show that the magnitude of complementarity depends on the size of the take over rate η^1_{A,N_m} for a given η_{A,N_m}. The intuition is as follows: Assume for the moment that $\eta^2_{A,N_m} = 0$ as it was the case so far. Now, total expenditures spent for hiring subsidies go to firms who conduct vocational training. Finally also the training rate $\eta_{B,A}$ increases as the expected profit generated by an apprentice increases. Thus, the hiring subsidy has the same qualitative impact as the training subsidy, so that they are substitutes to a certain extent. Now assume that the take over rate $\eta^1_{A,N_m} = 0$, i.e. all successful apprentices are hired by firms which are not engaged in vocational training. Now, again hiring subsidies are implemented. However, in contrast to the first case they are only paid to firms who are not engaged in training. Thus, now, hiring subsidies do not influence the training rate $\eta_{B,A}$. So, the qualitative impacts are different and they are expected to complement one another. The analysis requires the explicit implementation of η^2_{A,N_m} which is calculated as follows

$$\eta^2_{A,N_m} = \frac{\left(-\chi_m + \sigma^{\eta_{A,N_m}} + \frac{a_m - w_m - \delta\ \phi_m\ \varsigma_m}{1-\delta(1-\phi_m-\rho_m)}\right)}{\epsilon^+_{\eta^2_{A,N_m}}} \tag{16b}$$

In contrast to eq. (16), now, also hiring costs (χ_m) have to be taken into account, as the apprentice is hired by another firm for which the successful apprentice is an outsider. The crucial difference with respect to the previous analysis is the substitution of η_{A,N_m} in eq. (19) by η^1_{A,N_m}, i.e. hiring subsidies paid to firms which did not conduct vocational training but hire successful apprentices (with the hiring rate η^2_{A,N_m}), do not have an impact on the training rate $\eta_{B,A}$. Given the modified hiring rates, we again calculate the cross derivative of welfare (Φ^*) with respect to the two subsidies and get the following result

$$\begin{aligned}\frac{\partial^2 \Phi^*}{\partial \sigma^{k_v}\ \partial \sigma^{\eta_{A,N_m}}} = {} & [(\epsilon^+_{\eta^1_{A,N_m}} + \epsilon^+_{\eta^2_{A,N_m}}) a_m (1-\beta_m)\mu(1-\rho)\rho^2(\eta_l + \eta_{N,A}(1-\eta_l) + \rho + \phi_l)]\ / \\ & [\ \epsilon^+_{\eta^1_{A,N_m}}\, \epsilon^+_{\eta^2_{A,N_m}}\, \epsilon^+_{\eta_{B,A}} (1-\beta_m(1-\mu) - c_\phi\ \mu))(1+\rho)(\eta_l(1-\eta_{U,A})(\eta_{N,A}+\rho) \\ & +\ (\eta_{U,A}+\rho)(\eta_{U,A}+\rho+\phi_l))(\eta_m+\rho+\phi_m)\ \ ]\end{aligned} \tag{21a}$$

Calculating the ratio of the eq. (21) and (21a) reveals, that the value of the second cross derivative is higher than the first one as the values of the upper limits are all positive

$$\frac{\partial^2 \Phi^*}{\partial \sigma^{k_v}\ \partial \sigma^{\eta_{A,N_m}}} / \frac{\partial^2 \Phi}{\partial \sigma^{k_v}\ \partial \sigma^{\eta_{A,N_m}}} = \frac{(\epsilon^+_{\eta^1_{A,N_m}} + \epsilon^+_{\eta^2_{A,N_m}})}{\epsilon^+_{\eta^2_{A,N_m}}} \tag{22}$$

The result confirms our expectation: the higher the hiring rate η^2_{A,N_m}, the more complementary the subsidies are.

4.2.2 Positive lower limit of the ϵ distribution

Now, we assume that the lower limit of the uniform distribution of ϵ is $\epsilon^- > 0$. In the presence of lower limits of the distribution, the cross derivative of welfare with respect to the two subsidies is

$$\begin{aligned} \frac{\partial^2 \Phi}{\partial \sigma^{k_v}\ \partial \sigma^{\eta_{A,N_m}}} = \ & [a_m(1-\beta_m)\mu(1-\rho)\rho^2(\eta_l + \eta_{N,A}(1-\eta_l) + \rho + \phi_l)] \ / \\ & [\ (\epsilon^+_{\eta_{B,A}} - \epsilon^+_{\eta_{B,A}})(\epsilon^+_{\eta^1_{A,N_m}} - \epsilon^+_{\eta^1_{A,N_m}})\underbrace{(1-\beta_m(1-\mu) - c_\phi\ \mu)}_{>0})(1+\rho)(\eta_l(1-\eta_{U,A})(\eta_{N,A}+\rho) \\ & +\ (\eta_{U,A}+\rho)(\eta_{U,A}+\rho+\phi_l))(\eta_m+\rho+\phi_m)\ \ ] \end{aligned} \tag{21b}$$

Obviously, an increase of the lower limits ϵ^-_q with $q = (\eta_{B,A}), (\eta^1_{A,N_m})$ increases the value of the cross derivative and thereby the size of the complementarity.

In the given model framework, the analysis of complementarities for a model with less restrictions can only be done numerically. This will be done in the remainder. The enlargement of the model (integration of further parameters) will be done step by step. This allows us to identify the impact of different parameters on the existence of complementarities. After the illustration of the calibration, we continue by analyzing the size of complementarity between hiring and training subsidies with respect to welfare and employment.

5 Numerical Evaluation

5.1 Calibration

We calibrate the model for Germany with a year as unit of time. As far as possible, the values of the parameters are based on observed data for the period 1997-2003. The annual interest rate, i, is set at 2.5 percent,[26] which leads to a discount rate of $\delta = 0.976$. The coefficient of relative risk aversion (CARA, γ) is set at 1.5.[27] The chosen value is relatively low. However, taking into account the whole calibration, $\gamma = 1.5$ can be justified. The period of analysis and thus the minimum duration of unemployment in the model is one year. Thus, the risk of unemployment is much bigger than in the real world, there agents could leave unemployment before the end of a year. In reality they therefore have a higher possibility to smooth income. Taking this into account, calibrating the utility function with a relatively low degree of risk aversion is justified as it compensates the higher

[26] This is the average real interest rate over the whole period, calculated as the yearly money market interest rate minus the inflation rate. Nominal values are transformed to real values by using the consumption deflator.

[27] A value of 1.5 lies within the reasonable scope. According to Rodepeter (1999:66) and Dohmen et al. (2006), the limit values for CARA are 1 and 5.

risk in the model. The number of periods, p, a person is engaged in vocational training is set at 3.[28] The apprenticeship breakup rate θ, is set at 0.035.[29] Moreover, some transition rates are taken as exogenously given. The hiring rate for the high-skilled, medium-skilled and the low-skilled are set at $\eta_m = 0.59$, $\eta_l = 0.49$ and $\eta_h = 0.55$, respectively, which correspond to the values given by Wilke´s (2005) Kaplan-Meier functions for Germany.

The labor market states are defined and quantified as follows. The low-skilled labor force (N_l+U_l) includes those with an educational attainment corresponding to less than upper-secondary education.[30] The medium-skilled labor force ($N_m + U_m$) contains all people with vocational upper secondary education. People with post-secondary and tertiary education are considered as being high-skilled ($N_h + U_h$). The corresponding values can be calculated on the basis of OECD data.[31] However, according to these data, the group of the low-skilled employed, N_l, also contains people being engaged in vocational training (apprentices). Indeed, for the purpose of this analysis, we have to distinguish explicitly between those who are only working and the apprentices. Based on administrative data,[32] we calculate the fraction of apprentices in the total labor force. Given this, we can quantify the level of apprentices and adjust the level of low-skilled employment, correspondingly. Then, we calculate simultaneously, the number of newborn, B, the firing rate of the medium-skilled employees, ϕ_m ($= 0.05$), the death rate, ρ ($= 0.023$)[33], and the number of people who enter training A.[34] After this, we normalize all relevant values (N_m, U_m, N_l, U_l, B, A), so that the sum of the relevant population[35] is 1. Table (2) shows the values for the relevant states.

	low-skilled labor	vocational training	medium-skilled labor
employment	0.15	0.06	0.65
unemployment	0.03	-	0.07
newborn (pupils)	0.04		

Table 2: Steady State values of the relevant states (before a policy exercise)

Next, we calibrate the productivities and the wages. Based on data from the German national accounts (Federal Statistical Office (2006)), the aggregate productivity, a, is set to $52,575$ EUR. According to Beicht and Walden (2002), the productivity of a person being engaged in vocational training is $a_v = 7730$ EUR (in 2000). Based on data from the German national accounts, the aggregate wage, w, which is calculated as average gross wage per employee plus social security payments, is set equal to $31,290$ EUR. In order to get the wages for different skill groups, OECD

[28] This corresponds to the length of vocational training within the dual system in Germany.

[29] The values implies (taking a death rate into account) that roughly 17 % of the apprentices do not finish vocational training sucessfully. Using data for the number of new apprentices and the number of successful apprentices which are delivered by the German Statistical Office (2006a, 2006b), this value is confirmed.

[30] This corresponds to the conventional definition which classifies people with an educational attainent corresponding to at most level 2 of the International Standard Classification of Education (ISCED) as low-skilled.

[31] See OECD (1999-2005). The underlying labor force contains people between 15 and 64.

[32] German Federal Statistical Office (2006a, 2006b).

[33] This value corresponds to an average working life time of 43.1 years, which is roughly in line with the empirical data: working life begins at the age of 17 and retirement takes places between 62 and 63 (Brussig and Wojtkowski (2006)).

[34] This is done by using the equations 1, 2 and 6 of the matrix M and the fact, that the number of people being in vocational training is given by $A\sum_{c=1}^{p}(1-\rho-\theta)^{c-1}$.

[35] As mentioned at the beginning, the high-skilled labor force has to be taken into account due to the budget constraint. However, as it is treated as a quasi fix factor, this part labor force is not relevant when analysing the transition between labor market states.

indices for the relative earnings of the population with income from employment for different skill groups are used, they yield the following ratios: $w_h/w = 1.27$, $w_m/w = 0.92$ and $w_l/w = 0.72$.[36] According to Beicht and Walden (2002), the wage of a person being engaged in vocational training is set to $w_v = 8269$ EUR, additional costs k_v are set at 5716 EUR.[37]

In order to get the consumer wages, the tax rates have to be known. In this context, we first calculate the ratios of the tax rates. This is done by using the income tax scale of the year 2002 described in Boss and Elendner (2003: 379).[38] We obtain the following ratios: $\tau_h/\tau_l = 1.437$ and $\tau_m/\tau_l = 1.178$, which describe the progressive character of the German tax system. In a next step, we define some wage-related parameter values. The net replacement rates β_i are set at 78.25 percent for the low-skilled unemployed ($i = l$), to 68.25 percent for the medium-skilled unemployed ($i = m$), and to 64.67 percent of high-skilled unemployed ($i = h$).[39] According to Chen and Funke (2005), we set the hiring costs to 10 percent of the wage and the firing costs are set to 60 percent of the wage, thus the corresponding parameters are $c_\eta = 0.1$ and $c_\phi = 0.6$. Based on data for transition rates between the education and training system on the one hand and the labor force on the other hand (Reinberg and Hummel (2006)), we calculate (i) the fraction of low-skilled people who enter vocational training: $\eta_{B,A} = 0.70$ and (ii) the ratio: $\eta_{U,A}/\eta_{N,A} = 3.6$. Moreover, based on data of the Federal Ministry for Education and Research about 25 % of the people who have successfully finished vocational training become unemployed, thus the corresponding hiring rate, η_{A,N_m} is set at 0.75. Based on these values and the given equations, the missing values characterizing the initial steady state can be calculated unambiguously. The derived values are summarized in the following table 3.[40]

Finally, we have to determine the remaining parameter for each hiring rate, i.e. the upper limit of the distribution of ϵ: $\epsilon^+_{\eta_{B,A}}$ and $\epsilon^+_{\eta_{A,N_m}}$. As all the other variables and parameters of the equations determining the hiring rates as well as the hiring rates themselves for the initial steady state are given, one can easily solve the equations determining the hiring rates, eq. (19) for $\epsilon^+_{\eta_{B,A}}$ and eq. (16) for $\epsilon^+_{\eta_{A,N_m}}$ and gets the values shown in table 4.[41] The choice of the uniform distribution also has implications for the wage elasticities of the hiring rates. We now examine, whether the implicit elasticities are in line with the empirical literature. Given the functional form of the hiring rates and the parameter values for $\epsilon^+_{\eta_{B,A}}$ and $\epsilon^+_{\eta_{A,N_m}}$, we can calculate the hiring rates for the initial values of the corresponding wage and for wages which are equal to 90 percent of the corresponding initial wage. Then, we calculate the elasticity. Given the empirical estimates, as summarized in Orszag and Snower (1999: 208) hiring elasticities range from -0.5 to -4.0. Thus, the elasticities

[36] See OECD (1999-2005). These values imply a ratio $w_m/w_u = 1.26$ which is in line with the corresponding data reported by Wienert (2006: 663).

[37] According to Beicht and Walden (2002) additional, non-wage costs are about 8166 EUR. However, according to Dohmen and Hoi (2004) the additional non-wage costs are about 30 % lower, because the costs can be deducted from the tax.

[38] Given the wages of each skill groups, it is possible to calculate the tax levels and thereby the tax rates of each skill group - in this context, we ignore that there is a difference between the labor cost of the employer (gross wage plus social security payments) and the labor income of the employee which is subject to taxation. In the remainder, we do not use the tax rates being the result of the calculation because the rates refer to a budget which contains more expenditure than unemployment benefits. In the context of this paper, only the ratio is important in order to map the tax progression in a realistic way.

[39] The values are net replacement rates (unweighted average across six family types) of workers with 67, 100 and 150 percent of average productivity. See OECD (2006b).

[40] Given all these values, it can be checked easily, that the firms have an incentive to engage in vocational training as it was assumed in section 3.2 (see Appendix A1 for a detailed calculation).

[41] The values are the result of the full calibration of the model in the absence of subsidies. In the case of the simplified version, the values are marginally different.

parameter	variable	basis of calculation	value
bargaining power	μ	eq.(13) for $i = l, m, h$	0.31
productivity (high-skilled)	a_h	and $a =$	$77,011$
productivity (medium-skilled)	a_m	$\frac{a_l N_l + a_m N_m + a_h N_h + a_v\ T}{N_l + N_m + N_h + T}$	$48,022$
productivity (low-skilled)	a_l		$25,009$
tax rate (high skilled)	τ_h	eq.(4) ($\sigma^{\eta_{A,N_m}} = 0$, $\sigma^{k_v} = 0$)	0.070
tax rate (medium-skilled)	τ_m	and the ratios of tax rates	0.057
tax rate (low-skilled)	τ_l		0.048
firing rate (low-skilled)	ϕ_l		0.103
training rate (low-skilled unempl.)	$\eta_{U,A}$	equations (3) and (4) of	0.041
training rate rate (low-skilled empl.)	$\eta_{N,A}$	Matrix M and ratio of hiring rates	0.011

Table 3: Derived parameter values for the initial steady state

are in line with the literature.

parameter	$\epsilon^{+}_{\eta_{B,A}}$	$\frac{\partial \eta_{B,A}}{\eta_{B,A}} / \frac{\partial w_v}{w_v}$	$\epsilon^{+}_{\eta_{A,N_m}}$	$\frac{\partial \eta_{A,N}}{\eta_{A,N}} / \frac{\partial w_m}{w_m}$
value	$131,680$	-1.8651	$252,757$	-1.55883

Table 4: Derived parameter values for the uniform distribution

5.2 Simulation Results

In the following calculations, we assume that the training subsidy, σ^{k_v}, and the hiring subsidy, $\sigma^{\eta_{A,N_m}}$, respectively can take two values: 0 or 5000 EUR. The subsidies are paid per person and period. We then calculate the impact on welfare for three alternatives: either only one subsidy is implemented (σ^{k_v} or $\sigma^{\eta_{A,N_m}}$) or both subsidies are implemented simultaneously. The size of complementarity is calculated as follows: First, we calculate the absolute increase in welfare for the two cases, in which only one subsidy is implemented, and then calculate the sum. Second, we compute the absolute increase in welfare given that both subsidies are implemented simultaneously. Third, we compare both results, i.e. we answer the question by how much (in percent) the increase in welfare given that both subsidies are implemented simultaneously is higher than the sum of the separate effects. Moreover, we make the same calculation with respect to employment. The following results are based on the assumption that the labor force contains 44 000 000 people.

5.2.1 Benchmark solution

Before analyzing the model by taking into account all parameters, we first calculate the numerical solution of the simple model given above, i.e. among other things we assume the absence of the taxes and the thus the absence of a government budget constraint. For this case, the impact of the subsidies on welfare and employment is shown in table 5. The analytical solution is confirmed by the numerical exercise. With respect to welfare, there are complementarities between the two subsidies. With respect to employment, the size of complementarity is significantly higher. An alternative way to examine complementarities with respect to welfare is to calculate the value of

policy	$\sigma^{k_v} = 5000$	$\sigma^{k_v} = 0$	$\sigma^{k_v} = 5000$	size of
	$\sigma^{\eta_{A,N_m}} = 0$	$\sigma^{\eta_{A,N_m}} = 5000$	$\sigma^{\eta_{A,N_m}} = 5000$	complementarity
increase in ...				(%)
... welfare (million EUR)	$4,843.53$	$7,497.03$	$12,364.6$	0.194515
... employment (persons)	$31,790.7$	$132,545$	$166,969$	1.60263

Table 5: The size of complementarity in the benchmark case

the cross derivative according to equation (21). For the benchmark model we get $\frac{\partial^2 \Phi}{\partial \sigma^{k_v}\, \partial \sigma^{\eta_{A,N_m}}} = 2.1822 * 10^{-8} > 0$.

5.2.2 Realistic calibration of the training phase

So far, for the sake of simplicity - especially to get a analytically traceable result -, we have assumed, that (1.) training is not broken off ($\theta = 0$), and that the duration of training is one period ($p = 1$). Now, we adjust the values to realistic ones ($\theta = 0.035$ and $p = 3$). The impact of the subsidies on welfare and employment is shown in table 6. As in the benchmark model, there are

policy	$\sigma^{k_v} = 5000$	$\sigma^{k_v} = 0$	$\sigma^{k_v} = 5000$	size of
	$\sigma^{\eta_{A,N_m}} = 0$	$\sigma^{\eta_{A,N_m}} = 5000$	$\sigma^{\eta_{A,N_m}} = 5000$	complementarity
increase in ...				(%)
... welfare (million EUR)	$12,629.8$	$5,571.82$	$18,284.5$	0.455218
... employment (persons)	$78,359$	$55,685.7$	$136,164$	1.58079

Table 6: The size of complementarity for a realistic calibration of the training phase

complementarities with respect to employment and welfare. However, the complementarity with respect to welfare is - in relative terms -, significantly higher than in the benchmark case,[42] whereas the complementarity with respect to employment is roughly the same. The result is caused by another remarkable result: Now, as the training takes $p = 3$ periods, the training subsidies which are paid per person and per period have a higher impact than in the benchmark case. However, the training subsidies mainly cause a shift of employment from the low-skilled sector to the medium-skilled sector being connected with higher wages. Thus, the impact on welfare increases, whereas the impact on employment is roughly the same.

The critical reader might argue that the fraction of apprentices which breaks off vocational training, θ, is not exogenous but has to be treated as a function of the hiring subsidy, $\sigma^{\eta_{A,N_m}}$. The reasoning is the following: a subsidy rate increase the hiring rate η_{A,N_m} and thus the probability of the apprentice to get a medium-skilled job. The higher probability of the high, medium-skilled income could be expected to reduce the incentive to break off training. Therefore θ should be treated as an endogenous variable with $\frac{\partial \theta}{\partial \sigma_{\eta_{A,N_m}}} < 0$. However, studies trying to identify reason for breaking training do not give any argument to think that financial incentives would reduce θ. One main reason to break training is a problem in the relationship between apprentice and the instructor in the firm. Many apprentices breaking off training do not intend to finally stop training

[42] Also, the value of the cross derivative is higher, it lies between $7.4 * 10^{-8}$ and $7.7 * 10^{-8}$. Here, in contrast to the benchmark solution, the cross derivative is also a function of $\sigma^{\eta_{A,N_m}}$ and $\sigma^{\eta_{A,N_m}}$, so that we get a range of values.

but they try to get a training position somewhere else. Thus, the financial incentive cannot be supposed to play a role.

5.2.3 Individuals with risk aversion

So far, we have assumed, that the individuals are risk neutral ($\gamma = 0$). However, as mentioned this does not correspond to reality. Now, we assume that $\gamma = 1.5$. The impact of the subsidies on welfare and employment is shown in table 7.

policy increase in ...	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 0$	$\sigma^{k_v} = 0$ $\sigma^{\eta_{A,N_m}} = 5000$	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 5000$	size of complementarity (%)
... welfare (million EUR)	$12,994.3$	$5,732.95$	$18,814.9$	0.468071
... employment (persons)	$80,620.6$	$57,211.7$	$140,067.$	1.62147

Table 7: The size of complementarity in the presence of risk aversion

The given degree of risk aversion does not change the results significantly. Again, there are complementarities, but with respect to the simulation before, the results are roughly the same.[43]

5.2.4 Explicit take over rate

Now, we assume that the take over rate of successful apprentices is lower than the hiring rate. According to the analysis in section 4.2.1, we assume that the take over rate is given by η^1_{A,N_m}. Additionally, successful apprentices can be hired by firms which do not conduct vocational training, the corresponding hiring rate is η^2_{A,N_m}. However, we assume, that $\eta^1_{A,N_m} + \eta^2_{A,N_m} = \eta_{A,N_m}$. The following numerical exercise confirms the result of the analytical examination: The magnitude of complementarity depends on the size of the take over rate η^1_{A,N_m} for a given η_{A,N_m}. We assume that $\eta^1_{A,N_m} = 0.52$ and $\eta^2_{A,N_m} = 0.23$.[44] The analytical solution is confirmed by the numerical exercise.[45] The impact of the subsidies on welfare and employment is shown in table 8. Again,

policy increase in ...	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 0$	$\sigma^{k_v} = 0$ $\sigma^{\eta_{A,N_m}} = 5000$	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 5000$	size of complementarity (%)
... welfare (million EUR)	$20,602.3$	$6,261.9$	$27,014.3$	0.558955
... employment (persons)	$127,823$	$60,664.1$	$192,114$	1.9242

Table 8: The size of complementarity in the case of an explicit take over rate

there are complementarities, but now, they are higher as in the previous simulation.

[43] The value of the cross derivative lies between $7.4 * 10^{-8}$ and $7.7 * 10^{-8}$. This also corresponds to the previous result.

[44] According to the Federal Ministry for Education and Research (2007), about 52 % of the successful apprentices can stay in the firm, in which they did their vocational training.

[45] Also the value of the cross derivative is higher than in the previous simulation. It lies between $1.3 * 10^{-7}$ and $1.4 * 10^{-7}$.

5.2.5 Impact of taxes

In contrast to the analysis so far, we now implement taxes. For the moment, we assume, that only the expenditures for unemployment benefits have to be financed via taxes. According to eq. (4), the tax rates are set in a way which ensures that the tax receipts of the government are equal to its expenditures for unemployment benefits. However, with respect to the government budget constraint, we ignore the presence of subsidies, so that the expenditures for subsidies do not affect the level of the tax rates. The impact of the subsidies on welfare and employment is shown in table 9. Again, there are complementarities. With respect to employment, the result is absolutely identical

policy increase in ...	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 0$	$\sigma^{k_v} = 0$ $\sigma^{\eta_{A,N_m}} = 5000$	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 5000$	size of complementarity (%)
... welfare (million EUR)	$28,113.8$	$8,921.11$	$37,286.3$	0.678773
... employment (persons)	$127,823$	$60,664.1$	$192,114$	1.9242

Table 9: The size of complementarity in the presence of taxes

to the previous result. As the introduction of taxes does not affect the producer wages and thereby the hiring rates, there is also no impact on employment. With respect to welfare, the impact of both subsidies and thus the size of complementarity are higher than in the previous simulation.[46] Now, the implementation of subsidies affects welfare over two channels. The first channel is the same as in the previous simulation: subsidies increase employment and reduce unemployment. Additionally, there is a second channel: a higher level of subsidies also reduces unemployment benefits compared to the initial steady state: By leading to a fall in the number of people requiring unemployment benefits and an increase in the number of people paying taxes, subsidies generate subsidy-induced revenue for the government. Consequently, the tax rates are reduced and the net wages increase. Due to the second channel, welfare increases finally more strongly than in the previous analysis.

5.2.6 Impact of the government budget constraint (Policy exercise 6)

In the next step, we analyze the impact of the government budget constraint, i.e. in contrast to the previous analysis, also subsidies have to be financed by taxes. According to eq. (4), the tax rates are now set in a way which ensures that the tax receipts of the government are equal to its total expenditures, i.e. the sum of unemployment benefits and subsidies. The impact of the subsidies on welfare and employment is shown in table 10. With respect to employment, the result

policy increase in ...	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 0$	$\sigma^{k_v} = 0$ $\sigma^{\eta_{A,N_m}} = 5000$	$\sigma^{k_v} = 5000$ $\sigma^{\eta_{A,N_m}} = 5000$	size of complementarity (%)
... welfare (million EUR)	$14,697.1$	$5,930.91$	$19,707.3$	-4.46334
... employment (persons)	$127,823$	$60,664.1$	$192,114$	1.9242

Table 10: The size of complementarity in the presence of the government budget constraint

is absolutely identical to the previous result for the same reason as before. But there are no longer

[46] Also the value of the cross derivative is higher than in the previous simulation. It lies between $2.2 * 10^{-7}$ and $2.3 * 10^{-7}$.

complementarities with respect to welfare. In contrast to the simulation before, in the presence of the government budget constraint, the implementation of subsidies does not necessarily cause a reduction but an increase of the tax rates. This is the case if the subsidies are not self-financing. Subsidies create additional employment. By leading to a fall in the number of people requiring unemployment benefits and an increase in the number of people paying taxes, subsidies generate subsidy-induced revenue for the government. However, the revenue is not sufficient to compensate the additional expenditures caused by the subsidies. In the given framework, hiring subsidies are almost self-financing whereas training subsidies are not self-financing. Consequently, the tax rates have to be increased in order to ensure that the receipts are equal to the expenditures.

The analysis of the necessary increases of the tax rates reveals a new type of complementarity. The necessary increase of the tax rates given that both subsidies are implemented simultaneously is higher than the sum of the necessary increases given that both subsidies are implemented separately. With respect to welfare this type of complementarity overcompensates the positive type of complementarity illustrated in the previous simulation, so that there is no complementarity with respect to welfare.

6 Conclusion

In many European countries, high unemployment is one of the biggest economic problems, especially, for the low-skilled people. In particular in Germany, the unemployment rates are relatively high. However, in Germany, the unemployment rate of the youth relative to those of the unemployment of the prime-age people (25 - 54 years) is the lowest in the OECD. The fact is often explained by the German dual system of vocational training. However, with respect to the vocational training, the situation has decreased in recent years. There has been an increasing gap between the declining demand for apprentices by firms and the supply of students wishing to enter the apprenticeship. Therefore it is often argued that the financial burden of firms providing vocational training should be reduced. A second problem for young people is the transition from vocational training to work. A significant fraction of apprentices who have successfully finished vocational training becomes unemployed.

In our model, we analyze the effects of subsidies which are expected to reduce the problems. Training subsidies are paid to employers in order to increase their incentive to provide vocational training. Hiring subsidies are meant to increase the transition from apprenticeship to work. They are provided for a limited period of time, in which they drive a wedge between the income, the worker receives, and the labor costs the employer is confronted with. For a certain labor income, the producer wage is reduced in the presence of an employment voucher and thus the hiring incentives are increased.

Our analysis tackles these issues by presenting a macro model of the labor market that allows us to identify and qualify each effect being associated with the subsidies. To make our analysis expressly relevant to the decisions that policy makers commonly face in practice, we do not follow the mainstream practice of deriving policies as first-best responses to labor market failures. Instead, the model takes a variety of common labor market imperfections as given.

In our simulations, we investigate whether there are complementarities between training subsidies and hiring subsidies. The simulation shows that for a reasonable parameter constellation there are complementarities. However, in the presence of the government budget constraint this is - with respect to welfare - no longer the case

7 References

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Appendix

A1 Firm´s Incentive to engage in General Vocational Training

When answering the question why firms have - in contrast to the initial theory - an interest to pay for general education, two aspect are noteworthy with respect to the German labor market. (1.) High firing costs can create an incentive to invest in general training: Since an employer can decide not to hire an apprentice at the end of the training phase, but faces considerable costs when firing a regular employee, the firm may be willing to subsidize apprenticeship training. Such a training may serve as an expensive employment test for which employers are willing to pay (Harhoff and Kane, 1993). (2.) Another reason for the willingness of the firm to pay for general training can lie in the wage structure. A compressed wage structure, caused by labor market frictions can create an incentive for the firm to invest in general training. This aspect is analyzed in detail by Acemoglu and Pischke (1999). As there is reason to believe that this is an essential aspect in Germany, the theoretical background is shown in the following: Assume that the amount of training is a continuous variable, Θ. The product of the worker $a(\Theta)$ and the wage $w(\Theta)$, are functions of the amount of training. The worker gets a wage which corresponds to the outside option $o(\Theta)$. Given that there are no frictions, $w(\Theta) = o(\Theta) = a(\Theta)$. The profit of the employer is $\pi = a(\Theta) - w(\Theta) = 0$. The employer has no incentive to invest in general human capital, a higher amount of human capital would cause an increase of the productivity and an increase of the wage in the same extent, thus the profit is not affected. Now, assume that there are labor market frictions (e.g. mobility costs). The outside option of the worker is $o(\Theta) = a(\Theta) - \Delta(\Theta)$ with $\Delta'(\Theta) > 0$. Again, the worker gets a wage corresponding to his outside option: $w(\Theta) = o(\Theta) = a(\Theta) - \Delta(\Theta)$. But now, due to $\Delta(\Theta)$, the wage structure is compressed, which is illustrated by the fact that $\frac{\partial w(\Theta)}{\partial \Theta} < \frac{\partial a(\Theta)}{\partial \Theta}$. Now, the profit of the firm is given by $\pi(\Theta) = a(\Theta) - w(\Theta) = \Delta(\Theta)$. As the profit increases with Θ, the firm has an interest to invest in general human capital.

So far, there was the implicit assumption that the wage can be set by the firm, thus the bargaining power, μ, of the employee is 0. Now, we assume $\mu > 0$. For the moment we assume, that there are no firing costs, i.e. $c_\phi = 0$. The wage is calculated as follows:

$$w(\Theta) = \frac{\mu}{1 - \beta + \beta\mu}\, a(\Theta)$$

Thus, the profit of the firm, π, can be calculated as follows:

$$\pi = a(\Theta) - w(\Theta) = a(\Theta)(1 - \frac{\mu}{1 - \beta + \beta\mu})$$

Thus, as $\frac{\partial a(\Theta)}{\partial \Theta} > 0$, an increase in the amount of training, Θ, has a positive impact on the profit of the firm. This implies that (1.) $(1 - \frac{\mu}{1-\beta+\beta\mu}) > 0 \Leftrightarrow \mu < 1$ and (2.) $(1 - \frac{\mu}{1-\beta+\beta\mu}) > 0 \Leftrightarrow \beta < 1$. Both conditions have to be satisfied: $\mu < 1$ implies that the firm must be able to capture a fraction of the profit in the bargaining process. This is also one central conclusion of the analysis by Kessler and Lülfesmann (2006). They focus on the incentive complementarity between employer sponsored general and specific training. They argue that the possibility to provide specific training leads the employer to invest in general human capital. The second condition again stresses what has already been mentioned. Here, β is the replacement rate, but in a more general interpretation it can also be seen as the ratio of the outside option relative to the wage. $\beta < 1$ implies that the skilled workers face a relatively worse outside option. This causes a compression of the wage structure, which is a necessary condition for the firm to invest in general training. Taking also firing costs into account (i.e. $c_\phi > 0$), the conditions to be fulfilled is $\frac{\mu}{1-\beta+\beta\mu-\mu c_\phi} < 1$. Given that $\mu = 0.245$, $c_\phi = 0.6$ and $\beta = 0.6825$, this condition is satisfied!

Now, we deliver a short numerical exercise showing, that firms have an incentive to invest in general human capital, i.e. they have an financial interest to conduct vocational training. During the training phase, the firm realize a loss of training: $(7730 - 8269 - 5716)\sum_{t=0}^{3-1}\delta^t(1-\rho-\theta)^t = 17,283.5$ EUR. Given that the apprentice will be employed (with a probability: 0.75 $(1-\rho-\theta) = 0.71$) and generate an expected profit of $189,568$ EUR (according to eq. (9), the overall surplus is $116,617$ EUR. Even for a probability 0.52 $(1-\rho-\theta) = 0.49$, the overall surplus is positive ($75,554$ EUR) . Thus, the firm (on average) has an incentive to engage in vocational training.

However, one might argue, that a positive overall surplus is not enough because the opportunity costs have to be taken into account. In our model, the opportunity costs are 0 but - in contrast to our model - one might argue, that firm have the decision between the employment of a low-skilled employee and the training of the person. In this case, the opportunity costs are given by the expected profit generated by a low-skilled person. In our model, the corresponding profit is 22,998 EUR, which is lower, than the expected overall surplus in the case of training.

A2 Hiring and Firing Rates

Successful apprentice

Given, that the person has successfully finished vocational training, the expected present value of the firm´s profit is calculated as follows

$$
\begin{aligned}
\pi_{A,N_m,t} = & -\epsilon_{\eta_{A,N_m}} + (a_m - w_m + \sigma^{\eta_{A,N_m}}) \\
& + \delta^1(1-\varphi_m-\rho)^1(a_m-w_m) - \delta\varphi_m\,\varsigma_m \\
& + \delta^2(1-\varphi_m-\rho)^2(a_m-w_m) - \delta\varphi_m\,\varsigma_m\delta^1(1-\varphi_m-\rho)^1 \\
& + \delta^3(1-\varphi_m-\rho)^3(a_m-w_m) - \delta\varphi_m\,\varsigma_m\delta^2(1-\varphi_m-\rho)^2 \\
& + \;...
\end{aligned}
\tag{A2.1}
$$

This can be rewritten as

$$\pi_{A,N_m,t} = -\epsilon_{\eta_{A,N_m}} + \sigma^{\eta_{A,N_m}} + (a_m - w_m)\sum_{t=0}^{\infty}\delta^t(1-\varphi_m-\rho)^t - \delta\;\varphi_m\;\varsigma_m\sum_{t=0}^{\infty}\delta^t(1-\varphi_m-\rho)^t \tag{A2.2}$$

The term on the right hand side can be simplified, so that the equation becomes

$$\pi_{A,N_m,,t} = -\epsilon_{\eta_{A,N_m}} + \sigma^{\eta,A,N_m} + \frac{a_m - w_m \;-\; \delta\varphi_m\,\varsigma_m}{1-\delta(1-\varphi_m-\rho)} \tag{A2.3}$$

A successful apprentice is hired whenever: $\pi_{A,N_m,,t} > 0$. As the person is already in the firm, there are no hiring costs. Solving for the random component $-\epsilon_{\eta_{A,N_m}}$, we obtain the following equation

$$\epsilon_{\eta_{A,N_m}} < \sigma^{\eta,A,N_m} + \frac{a_m - w_m \;-\; \delta\varphi_m\,\varsigma_m}{1-\delta(1-\varphi_m-\rho)} \tag{A2.4}$$

With respect to $\epsilon_{\eta_{A,N_m}}$, we assume a uniform distribution, i.e. is uniformly distributed between 0 and $\epsilon^{+}_{\eta_{A,N_m}}$. Given all this, the corresponding hiring rate can be expressed as follows

$$\eta_{A,N_m} = \frac{(\sigma^{\eta_{A,N_m}} + \frac{a_m - w_m - \delta\;\varphi_m\;\varsigma_m}{1-\delta(1-\varphi_m-\rho_m)}) - 0}{\epsilon^{+}_{\eta_{A,N_m}} - 0} \tag{A2.5}$$

Hiring as apprentice

The expected present value of the firms´s profit with respect to the hiring of a newborn as apprentice is calculated as follows

$$\pi_{B,A,t} = -\epsilon_{\eta_{B,A}} + (a_v - w_v - k_v + \sigma^{k_v})\sum_{i=0}^{p-1}\delta^i(1-\rho-\theta)^i + \eta_{A,N_m}\;\delta^p(1-\rho-\theta)^p \tag{A2.6}$$

$$(\;\sigma^{\eta,A,N_m} + (a_m - w_m)\sum_{i=0}^{\infty}\delta^i(1-\phi_m-\rho)^i - \delta\;\phi_m\;\varsigma_m\sum_{i=0}^{\infty}\delta^i(1-\phi_m)^i\;\;)$$

The term on the right hand side can be simplified, so that the equation becomes:

$$\pi_{B,A,t} = -\epsilon_{\eta_{B,A}} + (a_v - w_v - k_v + \sigma^{k_v}) \sum_{i=0}^{p-1} \delta^i (1-\rho-\theta)^i + \eta_{A,N_m}\ \delta^p (1-\rho-\theta)^p \qquad \text{(A2.7)}$$
$$[\sigma^{\eta,A,N_m} + \frac{a_m - w_m\ -\ \delta\varphi_m\ \varsigma_m}{1-\delta(1-\varphi_m-\rho)}]$$

A person is hired as apprentice whenever: $\pi_{B,A} > \chi_{B,A,t}$. Solving for the random component $-\epsilon_{\eta_{B,A}}$, we obtain the following equation:

$$\epsilon_{\eta_{B,A}} < -\chi_{B,A,t} + (a_v - w_v - k_v + \sigma^{k_v}) \sum_{i=0}^{p-1} \delta^i (1-\rho-\theta)^i \qquad \text{(A2.8)}$$
$$+\ \delta^p (1-\rho-\theta)^p\ \eta_{A,N_m}\ [\sigma^{\eta,A,N_m} + \frac{a_m - w_m\ -\ \delta\varphi_m\ \varsigma_m}{1-\delta(1-\varphi_m-\rho)}]$$

Again, we assume a uniform distribution, i.e. $\epsilon_{\eta_{B,A}}$ is uniformly distributed between 0 and $\epsilon^+_{\eta_{B,A}}$. The corresponding hiring rate can be expressed as follows

$$\eta_{B,A} = \frac{(-\chi_{B,A,t} + (a_v - w_v - k_v + \sigma^{k_v}) \sum_{i=0}^{p-1} \delta^i (1-\rho-\theta)^i + \delta^p (1-\rho-\theta)^p\ \eta_{A,N_m}\ E[\pi_{A,N_m,t}]) - 0}{\epsilon^+_{\eta_{B,A}} - 0} \qquad \text{(A2.9)}$$

A3 A simple model with government budget constraint

In contrast to the main model, the following one is a 2-period model. This and some other properties allow to get an analytical solution even in the presence of the government budget constraint. The worker´s possible labor market states are illustrated in Figure (2): training T, medium-skilled employment N_m, medium-skilled unemployment U_m, low-skilled employment in period 1 and 2, N_l^1 and N_l^2, and finally low-skilled unemployment in period 1 and 2, U_l^1 and U_l^2. At the end of the second period, all people die. We assume, that the number of deaths is equal to the number of newborn, so the labor force is constant. A newborn is hired with a probability $\eta_{B,A}$ as apprentice and with a probability η_l as low-skilled employee. The rest remains unemployed. A trained person becomes a medium-skilled employee in the 2^{nd} period with a probability $\eta_{A,N}$. For the sake of simplicity, we assume that low-skilled people do not change their labor market state in the 2^{nd} period. Given all this, we get the following expressions for the steady state

$$N_l^1 = \eta_l\ (\ N_m + U_m + N_l^2 + U_l^2) \qquad \text{(A3.1)}$$

$$U_l^1 = (1 - \eta_{B,A} - \eta_l)\ (\ N_m + U_m + N_l^2 + U_l^2) \qquad \text{(A3.2)}$$

$$N_m = \eta_{A,N}\ T \qquad \text{(A3.3)}$$

$$U_m = (1 - \eta_{A,N})\ T \qquad \text{(A3.4)}$$

$$N_l^2 = N_l^1 \qquad \text{(A3.5)}$$

$$U_l^2 =\ U_l^1 \qquad \text{(A3.6)}$$

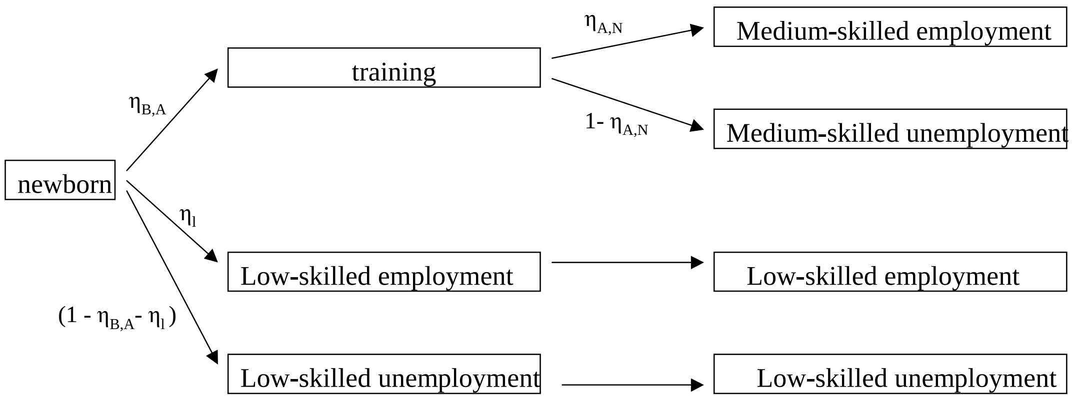

Figure 2: The dynamic structure of the simple model.

Moreover, we normalize the total labor force to 1

$$1 = T + N_m + U_m + N_l^1 + U_l^1 + N_l^2 + U_l^2 \tag{A3.7}$$

Given these equations, we can calculate the steady state expressions for each labor market state as a function of the transition rates: $N_m = 0.5\ \eta_{A,N}\ \eta_{B,A}$, $U_m = 0.5(1-\eta_{A,N})\eta_{B,A}$, $T = \frac{\eta_{B,A}}{2}$, $N_l^{1,2} = \frac{\eta_l}{2}$ and finally $U_l^{1,2} = 0.5(1 - \eta_{B,A} - \eta_l)$. The hiring rates $\eta_{A,N}$ and $\eta_{B,A}$ which are affected by the subsidies are expressed as

$$\eta_{A,N} = \frac{a_m - w_m + \sigma^{\eta,A,N_m}}{\epsilon^{+}_{\eta_{A,N_m}}} \tag{A3.8}$$

and

$$\eta_{B,A} = \frac{a_v - w_v + \ \sigma^{k_v} + \eta_{A,N}\ (a_m - w_m + \sigma^{\eta,A,N_m})}{\epsilon^{+}_{\eta_{B,A}}} \tag{A3.9}$$

where $w_m = (a_m + \sigma^{\eta,A,N_m})\mu$ which is given by eq. (13) for $\xi = 1$, $\beta = 0$ and $c_\phi = 0$. Given these expression, the labor markets states can be written as function of the subsidies. In order to determine aggregate welfare, we have to connect each labor market state with a value. In the case of training the value corresponds to the wage w_v and in the case of employment, the value corresponds to the net wage $w_i(1-t)$ with $i = N_m, N_l^1, N_l^2$. With respect to unemployment, we assume the absence of unemployment benefits, so that the value associated with unemployment is 0.

In the absence of the government budget constraint, $t = 0$, so that aggregate welfare is calculated as follows:

$$W = T\ w_v + N_m\ w_m + (N_l^1 + N_l^2)\ w_l \tag{A3.10}$$

Substituting the variables presenting the labor market stated in eq. (A3.10) by their steady state expressions gives an expression of welfare as a function of the two subsidies. Then, we can calculate the cross derivative of welfare w.r.t. to the two subsidies:

$$\frac{\partial^2 W}{\partial \sigma^{\eta,A,N_m} \partial \sigma^{k_v}} = \frac{(1-\mu)\mu(a_m + \sigma^{\eta,A,N_m})}{2\ \epsilon^{+}_{\eta_{A,N_m}} \epsilon^{+}_{\eta_{B,A}}} \tag{A3.11}$$

The two subsidies are complementary w.r.t. welfare. The cross derivative is unambiguously positive.

In the presence of the government budget constraint, $t > 0$, so that aggregate welfare is calculated as follows:

$$W = Tw_v + N_m w_m(1-t) + (N_l^1 + N_l^2)\ w_l\ (1-t) \tag{A3.12}$$

The government budget constraint is given by

$$t\ N_m w_m + t\ (N_l^1 + N_l^2)\ w_l = T\ \sigma^{k_v} + N_m \sigma^{\eta,A,N_m} \tag{A3.13}$$

Solving for t yields

$$t = (T\ \sigma^{k_v} + N_m \sigma^{\eta,A,N_m})/(N_m w_m +\ (N_l^1 + N_l^2)\ w_l) \tag{A3.13a}$$

Substituting the variables presenting the labor market stated in eq. (A3.12) and in eq. (A3.13a) by their steady state expressions and then substituting t in eq. (A3.12) by the expression in eq. (A3.13a) gives an expression of welfare as a function of the two subsidies. Then, we can calculate the cross derivative of welfare w.r.t. to the two subsidies

$$\frac{\partial^2 W}{\partial \sigma^{\eta,A,N_m} \partial \sigma^{k_v}} = \frac{(1-\mu)\mu(a_m + \sigma^{\eta,A,N_m}) - (1-\mu)(3(a_m + \sigma^{\eta,A,N_m})(1-\mu) + \sigma^{\eta,A,N_m})}{2\ \epsilon^{+}_{\eta_{A,N_m}} \epsilon^{+}_{\eta_{B,A}}} \tag{A3.14}$$

This equation shows the contrast to the cross derivative in the absence of the government budget constraint, which is given by the last term on the right-hand side. The additional term is unambiguously negative. In order to get to know the sign of the whole expression, we make some transformations which yields

$$\frac{\partial^2 W}{\partial \sigma^{\eta,A,N_m} \partial \sigma^{k_v}} = \frac{(1-\mu)\ ((4\mu - 3)a_m - (1-\mu)\ 4\ \sigma^{\eta,A,N_m})}{2\ \epsilon^{+}_{\eta_{A,N_m}} \epsilon^{+}_{\eta_{B,A}}} \tag{A3.14a}$$

Now, in the presence of the government budget constraint, there are no complementarities. The term on the right-hand side in brackets is for a plausible parameter ($\mu < 0.75$) value negative.