

Stochastic labour market shocks and Active labour market policies: a theoretical and empirical analysis

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Abstract

This paper develops a life-cycle model of labour supply with human capital formation that captures key aspects of labour market dynamics. The model determines both unobserved human capital that is different from experience and, in the event of unemployment, the rate at which human capital depreciates in the absence of Active Labour Market Policies (ALMP). Allowing for agent's heterogeneity, the model implies endogenous human capital formation (growth or depreciation) with respect to individual's characteristics and time-independent idiosyncratic labour market shocks. Whereas these shocks imply transitory monetary returns, the effects on human capital are long-lasting within skill class. Using several waves of the Swiss Labour Force Survey (1991 – 2003), the paper presents estimates of the dynamic process on human capital formation that allow a more complete understanding of the overall impact of labour market policies. The empirical findings show that relative to lower skill formations semi-skill workers are more efficient at increasing productivity at any level of human capital. On the other hand, the long term unemployed with medium/low skill levels experience depreciation of human capital relative to the higher skill classes. This latter do not necessarily experience depreciation rates over a spell of long-term unemployment.

Keywords: Human capital formation, life-cycle labour supply models, active labour market policies, search activities, productivity shocks, unemployment.

JEL – Classification: D31, D91, J23, J24

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1 Introduction

During the 1990s many continental European countries introduced wide-ranging active labour market policies (ALMP) in order to combat the then rising levels of unemployment. Switzerland was no exception at experiencing continuous increases in unemployment throughout the decade of the 1990s, thus in 1997 it expanded its ALMP interventions as well as prescribing new regulations for the provision of unemployment insurance. Following on the footsteps of program evaluation in North America (see for example Ashenfelter and Card (1985), Angrist and Krueger (1999) or the survey by Heckman, LaLonde and Smith (1999)) and following the widespread introduction of ALMP (both in Switzerland and other continental European countries) there has been a surge of literature that aims at evaluating the effectiveness of such labour market policies in Europe. Specifically in Switzerland, studies by Gerfin and Lechner (2000, 2002) or Gerfin, Lechner and Steiger (2001) have focused on evaluating the direct effect on employment of specific policies, for example, *Temporary Wage Subsidies*, *Sheltered Employment Programs* and/or *Training Courses*. In all these examples of program evaluation the key identification strategy lays on the assumption that labour market outcomes and the selection process into the program are independent events conditional on observed heterogeneity. The outcomes of such evaluations are the direct effects of the policies on the program participant assuming that the labour market position for the average non-participant is unaffected by the existence of the policies. The structural framework employed in these studies is that of a static partial equilibrium framework and does not usually focus on the effect that ALMP (or their absence) might have on both the short and the long run accumulation of human capital. Yet, it is stock of human capital at each particular point in time that determines individual's chances of employment assuming, of course, an appropriate vacancy flow within the individual's skill class. In the present paper the aim is to develop a life-cycle model of labour supply and human capital formation allowing for the model to capture the dynamics that characterize the labour market in Switzerland. Our structural model draws from Magnac and Rubin (1991, 1996) to define an optimization problem where optimizing individuals chose among mutually exclusive types of labour supply. At the same time we extend the framework in Costa-Dias (2002, Chapter 4) to allow for depreciating human

capital in the absence of active and passive labour market programmes. The model suggests a framework for the separate identification of the rate of human capital accumulation (for those in an employment spell) and human capital depreciation (for those in spells of long term unemployment) allowing for these rates to differ by skill class. The parameters identified by the model allow for the estimation of human capital returns from investing in labour market activities. The same parameters provide an estimate of the effects of active and passive labour market policies at maintaining pre-unemployment stocks of accumulated human capital.

In order to introduce the rationale behind the structural model (Section 2) we can illustrate the differential effects of labour market policies by comparing the effect of a wage subsidy scheme to that of other labour market policies that are more directly designed to help individuals to keep up with skill-specific knowledge such as active programs. Gerfin and Lechner (2002) studied various types of active labour market programs in terms of their relative effectiveness at promoting employment chances. Among other things, their findings suggest that one year ahead of having participated in at least one program, the average participant in a temporarily subsidized placement (*TEMP*) has 20% more chances to be employed than the average participant in other traditional labour market programs (e.g., simulated employment workshops, basic training courses, etc.). At the same time, when comparing traditional programs to a *TEMP*, the estimates show that such traditional programs can reduce the chances of employment for the average participant by as much as 15% (also estimated one year after finishing the program). A *TEMP* type of program acts very much as a wage subsidy scheme in the open market (rather than a traditional program where the unemployed will follow a particular training while receiving unemployment benefits). The results in the Gerfin and Lechner (2002) can be thought of as picking up the permanent positive effects on human capital formation resulting from a transitory labour market shock (i.e., the wage subsidy). However, their study regards only the effect of the policies on observed labour market outcomes. Even if other labour market programs (e.g., employment programs in sheltered (simulated) workshops) are not directly successful at promoting employment (relative to other programs), they might still help the program participant to maintain his or her stock of human capital from depreciating. Thus, in estimating

human capital formation (appreciation and depreciation) our study aims at providing a life cycle interpretation of the effect of active labour market programs.

We assume agents enter the labour market with a level of start up education that determines each individual's skill type from which they will not move until the age of retirement. Individual's skill type is assumed exogenous to the model. Once they enter, and at each point in time, agents make choices with respect to their labour market behaviour. The choices are either to work in return for earnings and enhanced human capital or remain unemployed. In this latter case individuals can choose to search in the open market while participating in programs that help them sustain basic skill-specific knowledge, or remain passively searching without program participation. However, receiving benefits from unemployment are often conditional on showing a level of labour market search and program participation. On the other hand, no search activity or elapse of the benefit period considerably reduces the ability to benefit from various active labour market programs as well as reduce monetary benefit. Eventually, if the spell of unemployment is prolonged for sufficiently long periods, the rights to program participation might be altogether eliminated. These implications mimic the dynamics of the unemployment system in Switzerland where individuals who become unemployed are immediately placed under the guidance of a 'caseworker' that aims at reducing the search cost for the unemployed individual and/or guide the individual towards participation in adequate active labour program. The benefits of the system (both in terms of program participation and unemployment insurance) are limited to a maximum of two years and conditional on pre-unemployment contributions to social insurance funds (see Gerfin, Lechner and Steiger (2002) for a more detailed description of the unemployment system in Switzerland).

We argue that receiving passive unemployment related benefits combined with active program participation implies the maintenance of pre-unemployment level of human capital stocks. On the other hand, becoming an outsider to the benefit system (or restricted access to it) implies entering a period of human capital depreciation that will last for as long as the individual remains unemployed. Our dynamic assumptions are necessary to capture the effect of distinct labour market regimes (by skill class) on the stock of human capital (e.g., the effect of long term unemployment on human capital versus the effect on

human capital to the new arrivals into the pool of unemployed). We reason that although both types of unemployed might have similar pre-unemployment experience within skill class, compared to the long term unemployed, new arrivals are closer to fulfilling the skill-specific knowledge required by employers, i.e., being a long term unemployed implies losing touch with contemporaneous knowledge requirement to cover existing vacancies. Our aim would imply quantifying the loss in terms of human capital stock.

The choice of labour market regime, however, is not deterministic. That is, we follow closely the ideas in Huggett (1997) and Huggett and Ventura (1999) where it is assumed that agents receive idiosyncratic *labour shocks* that determine individual's state of nature at each point in time. We assume these shocks to be time independent stochastic shocks that affect individual's contemporaneous opportunity cost of participating (or not) in paid labour market activities, i.e., at each point in time the valuation of alternative labour market regimes strongly affect the individual's decision. As in the framework by Huggett (1997) it is assumed that wages and interest rates are deterministic so that the income fluctuation problem is as result of the stochastic labour shock that directly determine individual's capital holding over time.

A difference between Huggett (1997) and our model is that we allow for alternative labour market regimes when setting up the individual's decision problem. This follows closely to Magnac and Rubin (1996) where the representative agent can choose between alternative working modes (wage work or self-employment). We assume individuals face a choice between three alternative labour market regimes: paid work, unemployment with active labour market programs participation (ALMP) and unemployment without active labour programs (N-ALMP). At any point in time the representative agent has some 'latent' or hidden valuation with regards to each of the three labour market regimes thus reflecting the agent's perceived cost of active participation. These valuations depend on the agent's state of nature which changes at each point in time as result of the time independent stochastic labour shock. Before the shock is realized the agent is uncertain about the state of the world (i.e., about his or her labour endowments and total asset holdings). Once the shock is realized the state of the world is known (i.e., capital assets and human capital are determined) and consequently the agent chooses an optimal labour

market regime. The arguments are similar to those in Kihlstrom and Laffont (1979) and Magnac and Robin (1991) where it is also assumed that individual's uncertainty on future labour market returns can be explained by attitudes towards risk, while the level of risk aversion with respect to labour market choices depends on personal characteristics and past labour market history, that is, on the stock of human capital.¹ Therefore, although unobserved, idiosyncratic taste for risk might be the most important factor that determines the choice of regime in the labour market. In our model we think of 'risk' between alternatives as the opportunity cost implied by the choice between mutually exclusive alternative, with individual's measuring the opportunity cost taking into account personal characteristics and the state of nature.

For a risk-averse individual with low levels of productivity, becoming employed implies a risky option relative to the riskless option of remaining on unemployment benefits (or social assistance in general). This is because any return from active employment might be equal or less than the benefits from unemployment and, at the same time, working implies exercising an effort. In a learning-by-doing-framework (see Cossa, Heckman and Lochner (1999)), allowing these individuals to receive a positive labour market shock that drives their gains above their own productivity (e.g., through a wage subsidy scheme) will induce participation and thus built up stocks of human capital. Since productivity level depends on the stock of human capital, an increase in human capital '*reduces*' the risk of participation in the future so that employment becomes a more likely choice in periods ahead. Likewise, if long periods of unemployment lead to human capital deterioration this increases the relative cost of employment both at present and in periods ahead so that with time the opportunity cost of 'employment' increases. For example, relative to new arrivals into the pool of unemployed, the long-term unemployed loose touch with new technologies at the work place and might have limited information to labour market programs. Clearly, the short run decision to remain unemployed might lead to long run consequences because the initial decision might trigger a period of human capital depreciation that translate into future depletion of productive capacity for periods immediately ahead. This, in turn, '*increases*' the cost (or risk) of the

¹ Because our analysis aims at explaining the labour market behaviour of low and medium skill individuals, allowing for labour market decisions to depend on savings is not as crucial an assumption as allowing for these choices to

employment option thus reducing its chance. The possibility to participate in various active programs offered by the system (e.g., short run courses, help in terms of search, etc.) might help individuals ‘maintain’ their pre-unemployment human capital level, thus creating a period where the risk attached to the choice employment is ‘*non-increasing*’ relative to the perceived risk with which they started their unemployment spell. This argument implies that ALMP can be seen not just as instruments to make the unemployed more marketable but also as a mean to help them keep their human capital (relative to their most recent human capital stock) while searching for a suitable vacancy. Taking all the above arguments into account we think of accumulated human capital as providing an insurance against risk (i.e., it lowers the opportunity cost of employment) while each individual’s taste for risk depends on individual characteristics and past labour market history. Within this framework, being subject to a positive but transitory labour market shock (e.g., a wage subsidy) may reduce the cost of participation and have a permanent effect in the form of increased human capital. Likewise, participating in active labour market programs (e.g., training courses, employment programs, etc.) can also be thought as receiving a transitory labour market shock that is neutral in terms of human capital formation relative to the pre-unemployment stock of human capital. Finally, the absence of active labour market policies or adverse labour market shocks can be thought as emulating a period with negative but permanent effects on human capital so that for as long as the unemployed remains in such labour market regime, human capital depreciates.

The above arguments imply that evaluating the impact of a policy intervention such as a wage subsidy – existing alongside other active labour policies – requires the evaluation of both short and long run effects for participants and. To this aim the starting point is to abstract away from actual interventions and to examine the dynamics of the labour market in the economy with both ‘earnings’ and ‘benefits’ as best signals in terms of disentangling the labour supply behaviour of the active population. This aim requires the definition of a life cycle model of labour-supply, human capital formation, earnings and unemployment insurance whose structural form is based on the specific observed characteristics of the economy under study. In our case this is the Swiss economy so it is fundamental that such model

depend on human capital formation.

integrates the three mentioned types of labour market regimes. Ultimately we want to identify the effect of benefits, earnings, and labour supply on human capital formation (growth and depreciation) for individuals that react differently to idiosyncratic ‘labour market shocks’. Following our previous arguments we define labour market shocks as individual specific innovations with transitory effects to monetary gains (different according to labour market regimes) but with the potential to permanently affect the productivity level of individuals within skill type (either accumulating, maintaining or depreciating human capital).

We consider *start-up* education as the first level of heterogeneity: individuals are assumed to enter the labour market with a level of start up education that determines their skill type once and for all and up to the point of retirement. Thus, human capital accumulation allows for enhanced productivity within skill type but does not allow individuals to jump to higher or lower skill types. Once individuals enter the labour market they face an idiosyncratic labour market shock at each point in time assumed to be transitory in nature. Following the arguments in Heckman and Smith (1999), the shock can be thought as determinant of labour market related activities conditional on the labour market regime dictated by the shock. A positive and sufficiently large shock implies a working decision and the shock determines the level of human capital accumulated while working. If the shock is not sufficiently large to imply a working regime the individual will choose unemployment. In the event that the shock is ‘sufficiently bad’ it will place the individual in a regime of no work and no program participation while the shock might determine the potential of searching for work in the open market, including the possibility of no search. Thus, whereas the permanent component of the transitory shock also differs by labour market regime, in all three cases the effects are with regards to human capital formation. Allowing for a third regime implies an extension to the modelling strategy in Costa-Dias (2002). In this latter, all unemployed are assumed to search at zero cost with human capital that never depreciates relative to the last employment spell. This means that as long as individuals belong to the same skill class, the long term unemployed and new arrivals to the unemployment pool are perfect substitutes in terms of productivity level. In our peruse to distinguish between active versus non active program participation as difference within the pool of

unemployed, our modelling strategy allows for endogenously determined human capital depreciation that evolves as a function of human capital skill h , conditional on skill class and pre-unemployment experience, as well as being subject to time independent idiosyncratic labour market shocks. The structural model endogenously determines human capital formation thus making a distinction between accumulated human capital and observed labour market experience.

Our results suggest that, in the event of working, those at the upper bound of the semi-skill distribution are more efficient at accumulating human capital and transforming such capital into productive capacity. For any stock of human capital, Skill type 3 can have a growth rate differential between 2% and 15% relative to lower skill classes, and still keep on showing a positive growth rate when the lower skills have already reached the maximum possible human capital change. In the event of unemployment without benefit participation (i.e., what the paper defines as long-term unemployment), the estimated rates of human capital depreciation are informative but with caution: low sample size in the formation of the data set implies that the estimates might not be very informative. However, for those in skill class 2 the estimates are meaningful and show that once in long-term unemployment individuals will experience depreciation starting from an initial 4% drop, thereafter increasing significantly slowly. The estimates for the higher skill class seem to suggest that those in Skill class 3 do not experience human capital depreciation over a long unemployment spell. These estimates provide an approximation of the benefits (or disadvantages) of the existing active labour market policies.

The paper is organized as follows. Section 2 presents the structural model as a dynamic model of labour supply with endogenous human capital formation and determines the necessary conditions to identify the parameters of interest. These conditions place restrictions on the behavioural aspects of individuals in each of the labour market regimes. Section 3 describes the estimation procedure to go from the structural model to the econometric specification. Section 4 describes the benefits and limitation in using the Swiss Labour Force Survey and provides the main estimation results. Section 5 concludes the paper. Further technical material and other data issues are relegated to an appendix.

2 A model of labour supply with stochastic labour shocks

The fundamental problem for the representative individual is to maximise utility subject to the evolution of assets and human capital where the latter is endogenously determined, different from (years of) experience and subject to time independent idiosyncratic labour market shocks. The stochastic shock and start up education (skill class) are the only two exogenous components of the state space. We do not model price formation (e.g., wages, benefits, tax policy, etc.) within the framework thus assuming these is information known to the individual at each point over the planning horizon. The other component in the state space is the skill class of the individual which is fixed forever at $t = 0$. Expression (1) describes the full problem faced by the i th representative agent:

$$\max_{\{c_{i,t}\}_{t=0}^T} E_t \sum_{t=0}^T \left[\beta^t u_t(c_{i,t}) \mid (\tilde{X}_{i,t}, \psi_t)_{t=0}^T \right];$$

where,

$$\tilde{X}_{i,t} = (s_i, a_{i,t}, h_{i,t}, \pi_{i,t});$$

$$\psi_t := (r_t, W_{1t}, \dots, W_{st}, \dots, W_{St}, B_{1t}, \dots, B_{st}, \dots, B_{St}, P_{1t}, \dots, P_{st}, \dots, P_{St}, \tau_t);$$

s.t

$$\text{Assets: } a_{i,t+1} = (1 + r_t) a_{i,t} + \sum_{s=1}^S \mathbf{1}(s = s_i) \left\{ I_{i,t}^w W_{st} (1 - \tau_t) h_{i,t} \pi_{i,t} + I_{i,t}^q (B_{st} - P_{st}) h_{i,t} + I_{i,t}^n B_{st} h_{i,t} \right\} - c_{i,t};$$

Human Capital:

$$\begin{aligned} h_{i,t+1} &= h_{i,t} \cdot \exp\left\{ \nu(s_i, h_{i,t}) \pi_{i,t} \right\}, & \text{if } I_{i,t}^w &= 1; \\ h_{i,t+1} &= h_{i,t}, & \text{if } I_{i,t}^q &= 1; \\ h_{i,t+1} &= h_{i,t} \cdot \exp\left\{ -\sigma(s_i, h_{i,t}) \pi_{i,t} \right\}, & \text{if } I_{i,t}^n &= 1. \end{aligned} \quad (1)$$

The i th individual enters the market at time $t = 0$ and retires at T . Optimal allocation of lifetime resources implies maximizing expected discounted utility over the entire working horizon as expressed by the weighted sum from $t = 0$ to $t = T$, where β is the discount factor. Future realisations of the shocks are unknown (only the distribution is assumed known). This means that at each point in time the individual evaluates her options and takes actions accounting for the remaining life and subject to the contemporaneous state of nature that has all past realisations of nature in it. The suffix t explains

contemporaneous time and the suffix s stands for the exogenously determined skill type: the problem refers to a given individual who will belong to a unique skill class over his working lifetime, i.e., whereas labour market history might explain his or her ‘skill-specific knowledge’, skill class is unchanged throughout the individual’s working life. The objective of the representative individual is to maximize discounted utility over his lifetime. This objective is represented with some time separable utility function $u_t = u_t(c_{it})$. We assume this function to be time-variant and to depend on a single argument (some bundle of consumption goods c_{it} for the representative agent at time t). The vector \tilde{X}_{it} (together with the price vector ψ_t) defines *the state of nature* faced by representative agent at time t . The vector indicates that the state of the world is a function of his skill class (s_i), his accumulated returns up to that point in the form of assets and human capital (a_{it}, h_{it}) and the time-independent idiosyncratic labour market shock that the agent receives at t (π_{it}). Following from \tilde{X}_{it} , we also define the subset $X_{it} = (s_i, a_{it}, h_{it})$, that is, the state of nature for agent i at time t with respect to the endogenously determined state variables and assuming a particular time invariant skill class. Part of the state of nature faced by the agent is the set of prices that affects consumption and labour decisions. These are given by the vector ψ_t describing the wage rate for any s skill class at time t (W_{st}), the unemployment benefits also by skill class and at a given time (B_{st}), earnings tax (τ_t) at t and the rate of return from asset investments at t (r_t). Furthermore, we assume that participants in active labour market programs face a cost (monetary or otherwise) of participation in such policies described by the term P_{st} ; as with wages, this cost is also time varying and skill class dependent. The vector ψ_t has only contemporaneous effects on the state of nature. The indicator $I_{it}^j = 1, j = w, n, q$ explains the labour market choice at each point in time for the representative individual. The problem in (1) shows that together with consumption, labour market choice is the only other choice variable. This choice variable is discrete. The implication is that the problem solves for a single optimal path (consumption path) for each of the finite discrete alternatives. This also implies that each labour market regime has a unique solution to the problem (i.e., solving for a given labour market choice). The indicator $\underline{1}(s = s_i)$ clarifies that earnings, unemployment benefits, and active policy costs are skill specific and will

vary over time for each individual but within skill class. For example, the term W_{st} is a vector of prices with dimension equal to that of the number of skills in the population. Allowing for $\underline{1}(s = s_t)$ implies that for a given individual, W_{st} becomes $W_{s(i)t}$. The same applies to $B_{st}(B_{s(i)t})$ and $P_{st}(P_{s(i)t})$

Expression (1) shows a dynamic problem subject to stochastic shocks where these determine the choice of labour markets according to the combined effect these shocks have on physical and human capital. For example, lets assume that at time t the stochastic shock π_{it} is perceived as sufficiently high (say, relative to some individual dependent latent reservation policy valuation) so that relative to all other possible labour market regimes the choice is $I_{it}^w = 1$. This means that relative to receiving either the combination $\{I_{it}^q(B_{st} - P_{st}); h_{it}\}$ or $\{I_{it}^n(B_{st}); h_{it} \cdot \exp\{-\sigma(s_t, h_{it})\pi_{it}\}\}$, the (perceived) individual specific high shock implies a preference for the combined receipts of $\{I_{it}^w W_{st}(1 - \tau_t)h_{it}\pi_{it}; h_{it} \cdot \exp\{\nu(s_t, h_{it})\pi_{it}\}\}$, thus accumulating physical assets ($a_{i,t+1}$) but also human capital ($h_{i,t+1}$); this latter implies permanent effects of increasing productive capacity, thus reducing the risk associated with the option ‘work’ in future periods (as compared to alternative (unemployment) labour market regimes). Alternatively, individuals might receive a labour market shock π_{it} perceived as relatively low so that working might imply receiving benefits such that $\{I_{it}^w W_{st}(1 - \tau_t)h_{it}\pi_{it}; h_{it} \cdot \exp\{\nu(s_t, h_{it})\pi_{it}\}\} < \{B_{st} |_{I(i,t,q)=0}, h_{it} |_{I(i,t,q)=0}\}$. If so, for either of the two unemployment alternatives (i.e., $I_{it}^q = 1$ or $I_{it}^n = 1$), the benefits received imply a lower risk than working for wages. But moreover, the choice of unemployment implies a spell where productive capacity either remains constant or depreciates, thus making working in the future a more risky option (i.e., unemployment lower productive capacity h_{it} , but also lowers total net gains from working in the future for any given random shock (i.e. $I_{it}^w W_{st}(1 - \tau_t)h_{it}\pi_{it}$) so that working in the future becomes also relatively more risky through the long run effect that of contemporaneous shocks. Notice that for the unemployment alternatives we model the effect of the shock so that it only directly affects human capital formation; more specifically, we are modelling labour market dynamics so that the shock leads to human

capital depreciation if the individual perceives that the cost of ALMP participation (i.e., P_{st}) is a high burden to tolerate relative to the cost implied by future human capital depreciation. In sum, the shock determines the choice between working and unemployment. Assuming that the individual takes unemployment as the optimal choice, the shock determines the benefit of seen h_{it} decrease relative to that associated with the cost implied by ALMP, that is, P_{st} . But this relative evaluation between the two unemployment alternatives depends on the individual's characteristics along side past labour market history. It might also depend on institutional considerations. For example, starting a period of unemployment might not carry compulsory participation in ALMP but later over an unemployment spell the receipt of benefits might be subject to active program participation. Another example is that of individuals who initially participate in ALMP but unemployment spells expand to periods where individuals lose the right to further participation. These (and combinations of these) alternatives (given stochastic labour market shocks) are possible within the dynamic framework in (1).

2.1 The Bellman Representation

The dynamic problem in (1) is explained in terms of multiplicative stochastic shocks $\{\pi_{it}\}$ and two endogenous state variables (a_{it}, h_{it}) . The solution to the problem is a sequence $\{c_t\}_{t=0}^T$ among all admissible sequences for each of the labour market regimes, conditional on initial and final conditions that pin down this set of admissible sequences (see assumptions below for initial and final conditions). We choose to characterise the problem with recursive methods in terms of a Value Function. Looking at expression (1) we see labour market shocks that are time independent with the permanent effects of these shocks are picked up by the endogenous variables, the only components that carry information from today to the future. Thus, as function of these two variables the set up in (1) provides the classic set up so to summarize the problem using the Bellman representation that relates current value functions $V_t(a, h, \pi)$ – i.e., value of maximised problem given all possible paths at t – to expectations of future value function

$V_{t+1}(a, h, \pi)$, assuming knowledge of the shocks up to period t and discounted back to contemporaneous values:

$$V_{it}^s(a_{it}, h_{it}, \pi_{it}; \psi_t) = \max_{\{c, l\}_{t=0}^T} \left\{ u(c_{it}) + \beta \cdot E_{\pi} \left[V_{i,t+1}^s(a_{i,t+1}, h_{i,t+1}, \pi_{i,t+1}; \psi_{t+1}) \right] \right\} \quad (2)$$

The value function in (2) summarizes the skill-specific individual's problem representing current and future values of the optimal consumption choice that changes as the state variables change over the planning horizon. However, a unique solution characterizing the individual's optimal choice is only possible if the value function in (2) is well behaved, that is, if expression (2) complies with a set of regularity conditions that imply a unique solution for the individual's optimal consumption path for each of the discrete labour market choices. We now put forwards a set of assumptions to provide the necessary conditions to derive a set of premises that proof that the problem defined by (2) is well behaved, as needed.

2.2 Assumptions

First we state a set of assumptions to provide necessary conditions so that a set of lemmas and respective corollaries characterize the unique solution to the problem in (2).

Assumption 1 (uncertainty): Stochastic labour market shocks π are assumed to be *iid* independent across time and individuals with known and continuously (at least once) differentiable distribution function on a bounded non-negative support $[\underline{\pi}, \bar{\pi}]$.

Assumption 2 (utility function): Let $u_t = u_t(c_{it})$ depend on consumption only and be a strictly increasing, twice differentiable, concave function of its argument.

Assumption 3 (state space): Both space vectors spanned by the state variables $\tilde{X}_{it} = (a_{it}, h_{it}, \pi_{it})$ or $X_{it} = (a_{it}, h_{it})$ are assumed to be continuous, bounded and convex. Skill type (s_i) is also part of the individual's state space but we assume it to be exogenous and constant

throughout the planning horizon.

Assumption 4 (initial and final conditions): Initially, $a_{i_0} = 0$ and $h_{i_0} = \underline{h}^{s(i)} > 0$. Terminal conditions are assumed to be such that $a_{i,T} \geq 0$ and $h_{i,T} > 0$.

Assumption 5 (non-crossing): The Value Function is assumed to have a derivative in the neighbourhood of zero that tends towards $-\infty$ from the right hand side.

Assumption 6 (absolute risk aversion): Individuals display decreasing absolute risk aversion, with risk attitudes towards labour market choices that change in the opposite direction of assets, but with changes that are never far from zero in magnitude. Technically this translates into degrees of risk aversion such that $(\partial \pi_a^R / \partial a) \leq 0$ and $(\partial \pi_b^R / \partial a) \leq 0$, where π_a^R, π_b^R stand for the reservation policy levels in entering different regimes in the labour market, and 'a' stands for 'capital assets'.

Assumption 7 (human capital growth and depreciation): $\nu(\cdot) \geq 0$ and $\sigma(\cdot) > 0$, where the parameter $\nu(\cdot)$ stands for the human capital growth rate and $\sigma(\cdot)$ stands for human capital depreciation rate.

Assumption 8 (prices): $B_{st} > 0, W_{st} > 0, P_{st} > 0$ at any point in time.

Assumption 9 (uniqueness): The identification of consumption path that uniquely characterises the solution in (2) is only possible if both consumption and savings are normal goods.

2.3 Comments

At any time t , the only source of uncertainty allowed in our model is that of next period's stochastic labour market shocks (Assumption 1). Nature draws at each point in time and this draw determines the state of the world, including labour market conditions. Once the uncertainty is revealed, the agent compares the outcome to his own valuation of alternative choices taking into account his taste for risk (own reservation policy valuation of each relative labour market alternative), allowing for risk aversion to define the behaviour of agents faced with a risky choice (Assumption 6). A choice of labour

market and consumption bundle are made assuming that rational agents maximize an objective function conditional on a dynamic state space: regular classic assumptions define both the objective function and the continuous direction of the state variables (Assumptions 2 and 3); in the case of the objective function the exclusion of leisure simplifies matters because this excludes possible wealth effects (backward bending labour supply functions). Lifetime constraints in assets (Assumption 4) allow pinning down a feasible set of consumption paths from which to choose the optimal one. Initially physical assets are zero for any skill type (this also implies zero pre-entry cost of achieving a particular skill class). Individuals are allowed to borrow over their finite lifetime (no liquidity constraints) but they are bounded to choose the optimal consumption path among those such that at the point of retirement no debt is allowed (that is, $a_{iT} \geq 0$). Human capital is positive at the point of entry into the market (at $t = 0$) but differs between skill types: the lower bar in $\underline{h}^{s(i)}$ implies that at entry, human capital is at its lowest. When retiring, individual's productive capacity does not die away, while over the planning horizon this capacity can never drop to negative values (irrespective of how adverse the shocks might be, individuals always keep some minimum capacity to produce). Finally, a concave function that goes through the origin allows for monotonic changes to the unique solution if exogenous parameters shift the function in particular directions (Assumption 5), whereas positive prices and positive human capital parameters also define monotonic conditions for the dynamics in (1) and (2) (Assumptions 7 and 8).

2.4 Lemmas

Lemma 1 (choice of labour market states): Allow for Assumptions in 3.1. Given π , an optimal choice of labour market regime is characterized by a monotonic labour market reservation policy that is determined conditional on each individual's characteristics at any t such that,

(a) An agent prefers $I_{it}^w = 1$ to either $I_{it}^q = 1$ or $I_{it}^n = 1$ at t if

$$\pi_{it} > \pi_{it(b)}^R \left(X_{it} \mid (\psi_t)_t^T \right),$$

(b) and prefers $I_{it}^q = 1$ to either $I_{it}^w = 1$ or $I_{it}^n = 1$ at t if

$$\pi_{it(a)}^R \left(X_{it} \mid (\psi_t)_t^T \right) < \pi_{it} \leq \pi_{it(b)}^R \left(X_{it} \mid (\psi_t)_t^T \right) \quad (4)$$

(b) and prefers $I_{it}^n = 1$ to either $I_{it}^q = 1$ or $I_{it}^w = 1$ at t if

$$\pi_{it} \leq \pi_{it(a)}^R \left(X_{it} \mid (\psi_t)_t^T \right)$$

where $\pi_{it(a)}^R < \pi_{it(b)}^R$.

Proof: See Appendix 1

Lemma 2 (properties of the value function): Allow for Assumptions in 3.1. Then, the expected value function $E_\pi V^s(a, h, \pi)$ is strictly increasing, twice differentiable and a concave function of a (assets).

Proof: See Appendix 1

Lemma 3 (Identification of the optimal consumption path). Allow for Assumptions in 2.1. Then, the Euler Equation in (3) is not sufficient to identify an optimal consumption path, since identification further requires that both consumption and savings are normal goods for fixed labour market decisions.

Proof: See Appendix 1

2.5 Characterizing the optimal consumption path

The Bellman representation in (2) shows that a realization π_{it} at time t is specific to the representative individual. The shock is a fundamental determinant of labour market choice conditional on the state of nature (which includes the individual's taste for risk). However, utility comes from consumption alone with labour market choice acting as a conditional for the choice of optimal path: to obtain this latter is the unique objective of the individual. This means that a solution to the problem in (2) must be the characterization of this optimal path conditional on a particular labour market regime. This characterization comes in the form of an Euler Equation that explains the intertemporal consumption decision and, therefore, characterizes the consumption decision rule. But the Euler Equation is a valid characterization of the solution to (2) if this latter fulfils a set of regularity conditions that we determine

with Lemmas 1 to 3. Lemma 1 characterizes the working decision and allows for an interpretation of the value function with respect to the mutually exclusive labour market regimes so that these can be clearly identified with separable additive value functions within the support of the labour market shocks. Lemma 2 establishes the continuity, differentiability and concavity of the value function *in assets* alone. The regularity conditions in Lemma 2 are sufficient and necessary so that an Euler Equation defines the optimal consumption decision (intertemporal consumption relation) for fixed labour market regimes given as follows:

$$\frac{\partial u}{\partial c_t} \Big|_{I(t)} = E \left[\beta(1+r) \frac{\partial u}{\partial c_{t+1}} \Big|_{I(t)} \right] \quad (3)$$

However, the problem (2) implies a more complex set of dynamics than just assets and the Euler Equation conditional on fixed labour market regimes is not sufficient to ‘characterize’ the consumption decision rule (even if it is sufficient to establish that the Euler Equation defines the optimal consumption decision). For a given history of shocks and for a given skill class, assets move along with human capital. Lemma 3 provides an interpretation of the necessary and sufficient conditions for the Euler Equation to represent the optimal conditions for consumption given a particular level of human capital.

3 Estimation Process and Identification

This section presents the conditions for identification of the parameters of interest, relying on the structural model to provide adequate econometric specifications that are consistent with the main assumption implied in Section 2. The two parameters of interest are the growth rate $\nu(s, h)$ and the depreciation rate $\sigma(s, h)$. It is also important to understand the use of the model to provide identification strategies for the distributional characteristics of the labour market shocks. First we provide identifying conditions for the reservation policies π_a^R , π_b^R that determine the selection into one of three labour market regimes. Second, the structural form for both assets and human capital are exploited to provide the identifying conditions for the two rates of human capital formation. Finally, an account is provided to deal with the practical estimation procedure.

3.1. Earnings when employed and Gains while unemployed

The individual's problem implies that at any $t \in [0, T]$ over an *employment spell*, the individual's total assets E_{it} are expressed as $W_{st}(1 - \tau_t)h_{it}\pi_{it}$ where W_s is the average wage rate for skill class s (or $\tilde{W} = W(1 - \tau)$ in net terms) and h_t is the contribution to total assets in terms of human capital stocks for a given productivity shock π_t . We want to study growth measures so the following applies:

$$\begin{aligned}
 E_{it}^s &\equiv \tilde{W}_{s,t} h_{it} \pi_{it} \\
 \Rightarrow \\
 \ln E_{it+1} - \ln E_{it} &\equiv \left[\ln(\tilde{W}_{s,t+1}) - \ln(\tilde{W}_{st}) \right] \\
 &\quad + \left[\ln h_{it+1}(\pi_i(t)) - \ln h_{it}(\pi_i(t-1)) \right] \\
 &\quad + \left[\ln \pi_{it+1} - \ln \pi_{it} \right]
 \end{aligned} \tag{4}$$

Expression (4) shows earnings growth over two consecutive periods as the sum of three components, that is, the structural model assumes that what we observe in the data ($\Delta \ln E_{i,t+1}$) is the sum of growth due to wage growth within the individual's skill class (can be estimated), growth due to idiosyncratic change in human capital (unobserved) and growth due to idiosyncratic between periods differential in stochastic labour market shocks (unobserved) which, in the case of working individuals, is viewed as changes in productivity gains. Characterizing h_{t+j} as determined by $\pi(t+j-1)$, $j \geq 0$, indicates that up to any contemporaneous (or future) time periods, human capital depends on the immediate past (up to the previous period) labour market history. The variable $h_{t+j, j \geq 0}$ is the only components in (4) directly related to individual's characteristics via their idiosyncratic labour market histories; Lemma 1 clearly states that labour market choice at t depends on how individuals perceive the shock at t relative to some individual specific reservation level set $[\pi_a^R, \pi_b^R]$ with such set explaining past labour market history and labour market preferences: but this preference may be the result of individuals characteristics (e.g., household and living conditions, health status, age, gender, etc). Thus,

characterizing h_{t+j} by $\pi(t+j-1)$, $j \geq 0$ links the observed information in the data (say, covariates Z specific to individual's conditions) to unobserved human capital growth $h_{t+j, j \geq 0}$. We will explore this later when we deal more directly with empirical issues.

But expression (4) does not use all information conveyed by the structural model in (1) since this latter further suggest a link between human capital dynamics and assets so that (unobserved) human capital growth rate can be explained by $[\ln h_{t+1} - \ln h_t] = \nu(s, h_t)\pi_t$ for all individuals in a working spell. Using this information the following substitutes (4):

$$\Delta(\ln E_{i,t,t+1}) \equiv \Delta(\ln(\tilde{W}_{s,t,t+1})) + \nu(s, h_t)\pi_{it} + \Delta(\ln \pi_{it,t+1}) \quad (5)$$

Expression (5) explicitly shows the parameter of interest $\nu(s, h_t)$. This parameter should be viewed as the 'skill specific' ability to learn since 'learning' is the only reason for human capital to grow between periods.

We now turn to individuals observed over *unemployment spells where ALMP are not present*. This means that we center on individuals such that $I_{it}^n = 1$ is observed over periods $t \in [0, T]$. According to our model structure, these individuals are informative with respect to human capital depreciation. From the structural model in (1), the contribution to assets or gains from a labour market regime $I_{it}^n = 1$ (call these gains Γ_{it}) are defined as $B_{st}h_{it}$ where B_s is the average benefit for skill class s (assumed to vary over time but not by individual, although we can change this) and remaining human capital h_t that also contributes towards to total assets in a multiplicative manner. Comparing gains (i.e., contribution to assets) between $I_{it}^w = 1$ and $I_{it}^n = 1$ (i.e., comparing $\tilde{W}_{st}h_{it}\pi_{it}$ to $B_{st}h_{it}$) show that the main difference is the lack of a stochastic shock component π_{it} for those in unemployment. We need to assume that unemployed do receive a shock at each point in time (see the human capital growth part in (1) expression), but our structural model and assumptions imply that the unemployed do not experience stochastic shocks in terms of productive capacity (as explained by $[\ln \pi_{it+1} - \ln \pi_{it}]$ in expression (5) in

the case of those in employment). Therefore, $B_{st}h_{it}$ is well specified. As was the case with employment spells, our interest is in terms of growth measures:

$$\begin{aligned} \Gamma_{it}^s &= B_{st}h_{it} \\ \Rightarrow \\ \ln \Gamma_{it+1} - \ln \Gamma_{it} &\equiv \left[\ln(B_{s,t+1}) - \ln(B_{st}) \right] \\ &\quad + \left[\ln h_{it+1}(\pi_i(t)) - \ln h_{it}(\pi_i(t-1)) \right] \end{aligned} \quad (6)$$

Expression (6) shows gain's growth over two consecutive periods as the sum of two components; the structural model assumes that what we observe in the data ($\Delta \ln \Gamma_{i,t+1}$) is the sum of growth due to benefit's growth within the individual's skill class (can be estimated from the data or macroeconomic data) and growth due to idiosyncratic changes in human capital (unobserved). Both of these two growth components can in principle be negative so that the net contribution to assets between two time periods in unemployment (i.e., $\ln \Gamma_{it+1} - \ln \Gamma_{it}$) could be negative. As with the case of employment spells, we have also characterizing h_{t+j} as determined by $\pi(t+j-1)$, $j \geq 0$: this has the same implications as before to indicate that up to any contemporaneous (or future) time periods, human capital depends on the immediate past (up to the previous period) labour market history. Again, the variable $h_{t+j, j \geq 0}$ is the only components in (4.II) directly related to individual's characteristics via their idiosyncratic labour market histories; using similar arguments as in the case of employed individuals the characterization of h_{t+j} by $\pi(t+j-1)$, $j \geq 0$ links the observed information in the data (say, covariates Z specific to individual's conditions) to unobserved human capital growth $h_{t+j, j \geq 0}$. Expression (6) can be further refined by using all the information from the structural model in (1) since this latter further suggest a link between human capital dynamics and assets so that (unobserved) human capital depreciation can be explained by $[\ln h_{t+1} - \ln h_t] = -\sigma(s, h_t)\pi_t$ for all individuals in an unemployment spell of the type $I_{it}^n = 1$. Using this information the following substitutes (6):

$$\Delta(\ln \Gamma_{i,t,t+1}) \equiv \Delta(\ln(B_{s,t,t+1})) + \sigma(s, h_t)\pi_{it} \quad (7)$$

Expression (7) explicitly shows the parameter of interest $\sigma(s, h_t)$. This parameter should be viewed as the ‘skill specific’ loss in learning since it is this ‘loss in learning (or non-contact with working environments)’ the reason for human capital depreciation between periods. Empirically we would expect the parameter σ to be negative for all s skill types.

Expressions (5) and (7) are the two main conditions derived from the structural model (and implied assumptions) that relate assets growth (or depreciation) to the two parameters of interest (i.e., $\nu(s, h_t)$ and $\sigma(s, h_t)$) that determine individual’s stock of human capital. The aim in this paper is to estimate these two parameters using the available data. The problem with expressions (5) and (7) is that neither h_{it} (the individual specific stock of human capital at t) or π_{it} (the individual specific stochastic labour market shock) are directly observed from the data. In terms of h_{it} , the variable enters implicitly as part of the parameters that we need to estimate ($\nu(s, h_t)$ and $\sigma(s, h_t)$). We can deal with this (*a*) making assumptions as to how the variables trends with the parameters might be sufficient to allow for this unobserved variation or (*b*) a specification that allows for h_{it} to be explained explicitly as part of $\nu(s, h_t)$ or $\sigma(s, h_t)$ joint with assumptions as to how h_{it} evolves from its starting point is also a method to deal with the problem of unobserved human capital stocks h_{it} . On the other hand, the variable π_{it} (the individual specific stochastic labour market shock) does come up explicitly in both (5) and (7) and it is the variable that determines human capital growth and depreciation, therefore we need to use both the structural model and the assumptions implied by this model to come up with this variable from the data and at different points in time so that both (5) and (7) become operational. What follows explains how to elicit π_{it} from the data. Following this, we turn our attention to the issue concerning unobserved human capital

3.2 Dealing with unobserved stochastic labour market shocks (π_{it})

In Costa-Dias (2002) various assumptions underlying the structural model are used to come up with estimates of the variable π_{it} that is not directly observed in the data. The end result is a projection

$\hat{\pi}_i$ for each individual. This projection can be used in regression specifications related to (5) so that the parameter $\nu(\cdot)$ can be estimated using NLS. Likewise, the projection can be used in (7) and the parameter $\sigma(\cdot)$ can also be estimated by NLS.

First we look at Lemma 1: the lemma establishes the existence of a pair of ‘reservation policy valuations’ for each individual ($\pi_{it[a]}^R$ and $\pi_{it[b]}^R$ where $\pi_{it[a]}^R < \pi_{it[b]}^R$) that characterize individual’s entry into the three different labour market regimes. These reservation policies depend on individual’s taste for risk possibly determined by individual’s labour market histories and personal characteristics (e.g., household type, health, taste for work, etc.): we define all variables that explain these histories and characteristics in a vector Z that includes (X, ψ) . With this and Lemma 1 we can determine the following relation:

$$\begin{aligned}
 I_{it}^w = 1 & \quad \pi_{it} \geq \pi_{it[b]}^R(Z) \\
 \text{and} & \\
 I_{it}^n = 1 & \quad \pi_{it} \leq \pi_{it[a]}^R(Z)
 \end{aligned} \tag{8}$$

Since we have specified $\pi_{it[b]}^R(Z), \pi_{it[a]}^R(Z)$, it is now possible to define a transformation of the reservation policies as linear functions of Z so that $\ln \pi_{it[b]}^R = Z_{it} \gamma_b$ and $\ln \pi_{it[a]}^R = Z_{it} \gamma_a$ apply (or more general, $\ln \pi_{it}^R = Z_{it} \gamma$). Thus, although we do not observe $\pi_{it[b]}^R(Z), \pi_{it[a]}^R(Z)$ directly, we observe individual’s labour market choice (i.e., I_{it}^w, I_{it}^n) and variables Z . The natural log transformation is purely for practical purpose: Assumption 1 suggests that π_{it} are draws from a known distribution that is bounded with non-negative bounds $[\underline{\pi}, \bar{\pi}]$ in the support of π . To comply with this assumption, Costa-Dias further assume that the bounds are symmetric in logarithms (i.e., $\ln \underline{\pi} = -\ln \bar{\pi}$) and that $\ln \pi$ follows a truncated normal distribution $N(0, \sigma_\pi)$ in $[\underline{\pi}, \bar{\pi}]$. We can make the same assumption as this is still valid for our structural model. The natural log transformation applies to (8). In fact, from (8) the following implication

is straight forwards: $I_{it}^w = 1 \rightarrow \ln \pi_{it} \geq \ln \pi_{it,[b]}^R(Z) \Rightarrow I_{it}^w = I[-Z_{it}\gamma_b + \ln \pi_{it} \geq 0]$ where π_{it} is the stochastic labour market shock. Likewise, $I_{it}^n = I[-Z_{it}\gamma_a + \ln \pi_{it} \leq 0]$ applies in the case of the reservation policy for the unemployed. Further assuming that bounds $[\underline{\pi}, \bar{\pi}]$ are located at the very thin tails of the distribution is similar to suggesting that π is unbounded: this, together with the assumption $\pi_{it} \sim N(0, \sigma_\pi)$ motivates the use of a probit model to estimate γ_a and γ_b . In our case we have three labour market regimes so that we could also motivate an ordered probit (three labour market regimes) to estimate γ in general (with further having to interpret the threshold parameters). Estimates of these probit-based parameters provide estimates of the reservation policy values $\hat{\pi}_{it}^R = \exp(Z_{it}\hat{\gamma})$: these can be estimated at different stages of the life-cycle (e.g., by age grouping) and for each of the skill types (or conditioning on skills in the estimation process).

So let's assume we have estimated sequences $\hat{\pi}_t^R, \hat{\pi}_{t+1}^R, \dots, \hat{\pi}_T^R$ over the lifecycle, where each sequence $\hat{\pi}_{ime}^R$ is based on a set of individuals that share a similar age bracket. These sequences help to estimate $\hat{\pi}_{it}$ (i.e., a projection of π_{it} , the unknown labour market shock). To create a link between π_{it} and $\hat{\pi}_{ime}^R$, the argument is that *in the case of working individuals* who is observed working at t (i.e., $I_{it}^w = 1$) indicates that the individual has evaluated the stochastic shock (i.e., the conditions in the market or state of nature) and has decided to work because $\pi_{it} \geq \pi_{it,[b]}^R$. Recall the following interpretation from Lemma 1: $\pi_{it,[b]}^R = \pi_{it,[b]}^R(X_{it} | \psi_{t=0}^T)$ where $X_{it} = (s_i, a_{it}, h_{it})$ (i.e., the state of nature at t for individual i with skill class s). The structural model determines that once π_t is revealed to each individual, next period's state of nature is determined so that $X_{i,t+1} = l(X_{it}, \pi_{it} | \psi_t^T)$ where $l(\cdot)$ is some function that interprets the transformation between periods: therefore, the random variable $X_{i,t+1}$ depends only on values of itself at t and not at earlier time. This Markov process interpretation of next period's state of nature exploits an assumption embedded in the structural model: past labour market experiences are fully explained by contemporaneous values of the state variables for any given contemporaneous labour market shock.

Recall we had determined $\pi_{it,[b]}^R = \pi_{it,[b]}^R(X_{it} | \psi_{t=0}^T)$ from Lemma 1, or what is the same, $\pi_{it+1,[b]}^R = \pi_{it+1,[b]}^R(X_{it+1} | \psi_{t+1=0}^T)$. But we have also reasoned that $X_{i,t+1} = l(X_{it}, \pi_{it} | \psi_t^T)$, therefore, $\pi_{it+1,[b]}^R = \pi_{it+1,[b]}^R(l(X_{it}, \pi_{it} | \psi_t^T))$, or more generally $\pi_{it+1,[b]}^R = \pi_{it+1,[b]}^R(X_{it}, \pi_{it} | \psi_t^T)$. This reasoning can also be applied to $\pi_{it+1,[a]}^R$, i.e., the reservation policy that determines unemployment choices. The point is that $X_{i,t+1} = l(X_{it}, \pi_{it} | \psi_t^T)$ applies to anyone whichever the choice in terms of labour market regimes, therefore we determine that the condition $\pi_{it+1,[a]}^R = \pi_{it+1,[a]}^R(X_{it}, \pi_{it} | \psi_t^T)$ also applies. So now we have established a relation between either one of the (one-period-ahead) reservation policy valuations ($\pi_{it+1,[a]}^R$ or $\pi_{it+1,[b]}^R$) and the contemporaneous but unobserved labour market shock π_{it} . Clearly, each of the two reservation policies is informative on the *size* of immediately past stochastic shock π_{it} . The following two Lemmas characterize the relation:

Lemma 4: Allow for all Assumptions 1 to 13. Then, conditional on $(X_t | \psi_{t=0}^T)$, the employment reservation policy $\pi_{[b],t+1}^R$ is a monotonic (strictly decreasing) function of the shock π_t among those in an employment spell.

Proof: Conditional on $(X_t | \psi_{t=0}^T)$, the effect to the shock π_t at time t is to increase assets and accumulate human capital at a rate $\nu(s, h)$ if the shock implies an employment spell. But as human capital accumulates (together with increasing assets), individual's level of risk aversion decrease for future periods (this has already been proven in Lemma 2 where $(\partial \pi_{[a],[b]}^R / \partial a) < 0$ as a necessary condition for uniqueness). Since the risk attached to working is reduced working becomes more likely in periods ahead so that the new taste for risk implies a lower value of $\pi_{[b],t+1}^R$. Thus, higher π_t are associated with declining $\pi_{[b],t+1}^R$. Monotonicity is proven.

Lemma 5: Allow for all Assumptions 1 to 13. Then, conditional on $(X_t | \psi_{t=0}^T)$, the non-labour activity reservation policy $\pi_{[a],t+1}^R$ is a monotonic (strictly increasing) function of the shock π_t among those in a non-working/no-program participation spell.

Proof: The proof is straight forwards following from the proof of Lemma 4. We are now dealing with the non-program participation reservation policy $\pi_{[a]}^R$ and its implications with respect to the identification of the productivity shock over a period of unemployment without program participation. Conditional on $(X_t | \psi_{t=0}^T)$, the effect to the shock π_t at time t is to reduce assets *because* human capital depreciates at a rate $\sigma(s,h)$ as the shock implies an unemployment spell. But as human capital depreciates, and even if monetary gains from unemployment remained unaffected, individuals will find it more risky to participate in employment programs and/or employment in future periods ahead, meaning that $\pi_{a,t+1}^R$ increases with the labour market shock at t . Monotonicity is proven.

Both Lemmas 4 and 5 are consistent with the set up of the structural model. Notice that by excluding leisure from the utility function we have excluded wealth effects that might have had consequences for those observed in working spells: Increases assets (including human capital) reduces the risk of working and increases the chances of participating in the future as opposed to buy out leisure time. Thus, Lemma 4 is correctly interpreting the assumptions in the model while Lemma 5 is not inconsistent with the exclusion of wealth effects (we do not interpret unemployment as leisure time, although we interpret unemployment as voluntarily based on a reservation valuation). The important issue is that both Lemma 4 and 5 provide a way of allowing for the distribution of $\pi_{t+1,b}^R$ conditional on $(X_t, I_t^w = 1)$ to be used to characterize the distribution of π_t conditional on $(X_t, \pi_t \geq \pi_{t,[b]}^R)$. Likewise, the distribution of $\pi_{t+1,a}^R$ conditional on $(X_t, I_t^n = 1)$ can be used to characterize the distribution of π_t conditional on $(X_t, \pi_t \leq \pi_{t,[a]}^R)$. First of all, we know how to estimate $\pi_{it+j,b}^R, \pi_{it+j,a}^R; j \geq 0$ from the data, so we can estimate the conditional distributions $P(\pi_{t+1,b}^R \leq \hat{\pi}_{t+1,b}^R | X_t, I_t^w = 1)$ and $P(\pi_{t+1,a}^R \leq \hat{\pi}_{t+1,a}^R | X_t, I_t^n = 1)$: in both

cases we need to observe individuals over two consecutive time periods. Thus, the monotonic relations suggested by Lemmas 4 and 5 imply the following result:

The predicted shock π_{it} at t is given as follows:

$$\hat{\pi}_{it}^R : P(\pi_{t+1,b}^R \leq \hat{\pi}_{t+1,b}^R | X_t, I_t^w = 1) = P(\pi_t \leq \hat{\pi}_t | X_t, I_t^w = 1) \quad (9)$$

and

$$\hat{\pi}_{it}^R : P(\pi_{t+1,a}^R \leq \hat{\pi}_{t+1,a}^R | X_t, I_t^n = 1) = P(\pi_t \leq \hat{\pi}_t | X_t, I_t^n = 1)$$

Expression (9) allows for the projection of the empirical distribution of the unobserved stochastic shocks π_t with information obtained from estimates $\hat{\pi}_{it+1,b}^R, \hat{\pi}_{it+1,a}^R$, and as it is stated in (9), each individual's draw π_{it} come from such distributions according to the individual's state of nature and labour market choice. Costa-Dias (2002) proceeds as follow in their case of human capital appreciation:

1. Using the full sample run a probit to come up with estimates $\hat{\pi}_{it+1,b}^R$ for those observed in employment spells. Their data set (NDSC58) is of individuals observed all at ages 23, 33 and 42, so that wave at age 33 is the one that provides the initial estimates for $\hat{\pi}_{it+1,b}^R$ (our data is different and we comment on this later). These estimates provide the empirical distribution as given in (9): they consider distributions by discrete cells defined by $(X_t, I_t^w = 1)$.
2. Using wave 23 (i.e., t), estimate $\hat{\pi}_{it,b}^R$ for each individual. We know that observing individuals working implies that $\hat{\pi}_{it,b}^R \leq \pi_{it}$ so although we do not observe π_{it} , we know that it comes from a particular distribution given in step 1 and, at the same time, for individual $i \in n$ the lower bound on the unknown labour market shock is given by $\hat{\pi}_{it,b}^R \leq \pi_{it}$.

3. The model structure requires that the distribution of π_i be bounded between $[\underline{\pi}, \bar{\pi}]$ so we make an initial ‘guess’ on $\bar{\pi}$ such that $\hat{\pi}_{i,b}^R \leq \pi_i \leq \bar{\pi}$ are not the ‘individual’s specific’ upper and lower bound. For example, we could take $\bar{\pi}$ as the highest estimated value from step 1.
4. Take estimates from step 1 and rank each individual according to place in such distribution to define a percentile location for each individual (e.g., the 12th ranking placement). Costa-Dias (2002) take the predicted value for the productivity shock $\hat{\pi}_i$ to be the corresponding percentile in the (truncated by $\hat{\pi}_{i,b}^R$ and $\bar{\pi}$) distribution of the contemporaneous distribution defined by estimates $\hat{\pi}_{i,b}^R$. The ranking is motivated by the monotonicity assumption.

In the present paper the availability of yearly allows for distinct predictions for (9) while we still have the need to estimate labour market shocks received by the unemployed (i.e., we extend the Costa-Dias structural set up to allow for human capital depreciation). Step 1 remains the same but using estimates $\hat{\pi}_{i+1,a}^R$ instead of $\hat{\pi}_{i+1,b}^R$. Step 2 is modified: observing an individual in unemployment implies that $\hat{\pi}_{i+1,a}^R \geq \pi_i$ so that the individual’s $\hat{\pi}_{i+1,a}^R$ estimate places an individual specific upper bound. We then ‘guess’ the lower bound and proceed with the same step 4 but with truncation defined by $\underline{\pi}$ and $\hat{\pi}_{i+1,a}^R$. Alternatives to step 4 (for either employed or unemployed) are random draws in the neighbourhood of the rank, for example. Once we arrive at estimates of π_i , expressions (5) and (7) are implementable if we make assumptions on the unobserved stocks of human capital.

Dealing with unobserved human capital (h_{it})

We see from expression (5) and (7) that h_{it} enters implicitly as part of the parameters that we need to estimate ($\nu(s, h_t)$ and $\sigma(s, h_t)$): Costa-Dias (2002) provides an specification to allow h_{it} to be

explained explicitly as part of $\nu(s, h_t)$ joint with assumptions as to how h_{it} evolves from its starting point as a method to deal with the problem of unobserved human capital stocks h_{it} . Expression (5) suggests that for a given skill class the parameter $\nu(s, h_t)$ is time varying since human capital h_t varies over time: the problem, of course, is that we do not directly observe h_t (past, present or future), so one way to deal with this is to assume some function where the variable h_t has an ‘explicit’ interpretation rather than the generalized implicit form in $\nu(s, h_t)$ and this is what. The trick in Costa-Dias (2002) is to characterize the parameter such that $\nu(s, h_t) = \nu(s)g(s)^{h_t^s - \underline{h}^s}$ where $\nu(s)$ is the initial rate of human capital accumulation. This characterization is not related to the structural model in any form: it is just a characterization that embeds the assumption that human capital grows at a decreasing rate (which is part of the structural model). Then, as human capital grows away from its initial levels (i.e., as $(h_t^s - \underline{h}^s) > 0 \uparrow$ over time, where \underline{h}^s is human capital at labour market entry point), the initial $\nu(s)$ adjusts at the rate $g(s)^{h_t^s - \underline{h}^s}$ where $g(s)$ is also modelled as some constant (by skill class s). The second part of the trick is to use a balance panel over three consecutive periods ($t1, t2, t3$; $t1 < t2 < t3$) of young individuals assumed to have entered the labour market at about $t1$: this assumption further justifies that $(h_{t1}^s - \underline{h}^s) = 0$. Using information from the two initial waves $t1, t2$ with $\Delta(\ln E_{i,t,t+1}) = (\ln E_{i,t2} - \ln E_{i,t1})$ implies the possibility to retrieve $\nu(s)$ from the data since the assumptions imply that $\nu(s, h_t) = \nu(s)g(s)^{h_t^s - \underline{h}^s} = \nu(s)$ from expression (5). Then, using the two consecutive waves $t2, t3$ growth in earnings are given by $\Delta(\ln E_{i,t,t+1}) = (\ln E_{i,t3} - \ln E_{i,t2})$ and this observed variable allows to estimate $\nu(s, h_t) = \nu(s)g(s)^{h_{t2}^s - \underline{h}^s} = \nu(s)g(s)^{\alpha(h_t)}$. Because it is assumed that $\nu(s)$ is constant, $g(s)^{\alpha(h_t)}$ can be retrieved mixing these two steps. A crucial problem in their approach is that $g(s)$ by itself is never identified and assumptions have to be made about the meaning of $\alpha(h_t) = (h_{t2}^s - \underline{h}^s) > 0$. The only thing that can be said is that $(h_{t2}^s - \underline{h}^s)$ represents human capital growth

over ‘some time period’ determined by the data. For example, in the distance $\alpha = (h_{t_2}^s - \underline{h}^s)$ refers to human capital growth over one year if we use annual panel data for both $\nu(\cdot)$ and $\sigma(\cdot)$. That is, first we specify similar forms for the parameters so that $\nu_t(s, h_t) = \nu(s)g(s)^{h_t - \underline{h}^s}$ and $\sigma_t(s, h_t) = \sigma(s)g(s)^{\bar{h}^s - h_t}$ apply. In terms of start up human capital the assumption ($h_{t_1}^s = \underline{h}^s$) would be needed to estimate $\nu(s)$ separately from $g(s)$ using 3 consecutive time periods. The assumption ($h_{t_1}^s = \underline{h}^s$) implies comparing homogenous individuals in terms of human capital. In the case of unemployed, the value \bar{h}^s represents the total accumulated amount of human capital before individuals enter an unemployment spell where human capital can start to depreciate. If one makes the assumption that \bar{h}^s is similar for those with similar working time experience (by skill class) then we are also grouping individuals with homogenous amounts of human capital, so that we do not group individuals by ‘physical age’, but by ‘working experience age’. Assume we also observe a balanced panel of unemployed over three consecutive years. Apply the first two years of data to (7) assuming that the following applies: $\bar{h}^s \cong h_{t_1}$, i.e., during the first period observed as unemployed the ‘homogeneous’ group in terms of work experience do not see their human capital decline. Therefore, $\sigma(s)$ (the initial rate of human capital depreciation) is identified and taken as some constant. Over time, (say, t_2, t_3), $\bar{h}^s - h_{t_2} > 0$, that is, human capital has declined between t_1 and t_2 , so that $\bar{h}^s = h_{t_1} > h_{t_2}$ and therefore $\bar{h}^s - h_{t_2} > 0$ applies and is consistent with our model. Therefore, using periods t_2, t_3 allows for the identification of $g(s)^{\bar{h}^s - h_t}$ in $\sigma_t(s, h_t) = \sigma(s)g(s)^{\bar{h}^s - h_t}$ using the first step estimation that has previously identified the parameter $\sigma(s)$.

4 Data issues (preliminary)

The Swiss Labour Force Survey (SAKE) was used to estimate the parameters on human capital formation (appreciation and depreciation rates) following the iteration procedure as define in Section 3.5. The SAKE data is the most complete longitudinal data in terms of providing labour market information – alongside other social and economic variables – representative of the active population in Switzerland. It

started in 1991 and is a rotating panel where respondents are interviewed for up to five years and on a yearly basis. For example, in 1991 a total of 16,016 individuals enter the panel. These remain in the panel up to 1995 (inclusive) or for as long as they decide to remain participants. Anyone newly interviewed in 1992 can remain in the panel up to 1996, and so on up to the most recent wave (2003 at present). In total, 69,408 unique individuals have been interviewed in the period (1991, 2003). Interviewed units are initially contacted by letter and asked to voluntarily participate in the survey irrespective of their labour market status or Swiss visa status.² The only requirement is to be registered as living in Switzerland with some degree of permanency and be at least age 17 years or older.

In our estimation we required observing individuals for at least 3 (or 4) consecutive years. Taking only the last four available years (2000 to 2003) would seriously deteriorate the sample size in our data. Active labour market policies have been available in Switzerland since the beginning of the 1990s, thus, we use all available waves in the SAKE to create 4 artificial time periods by defining the first time period as the first year that individuals were observed, and period number 4 as the fourth period. For example, an individual observed for the first time in 1991 becomes an observation at t_1 , t_2 , t_3 , t_4 for the years 1991, 92, 93 and 94, respectively, whereas an individual observed for the first time in 1992 becomes an observation at t_1 , t_2 , t_3 , t_4 for the years 1992, 93, 94 and 95, respectively. Since we have 13 waves there are 10 possible sequences of 4 years each. Our first sample selection criteria consists on withdrawing anyone that is not continuously observed for at least these 4 consecutive periods, that is, sample attrition would imply discontinuous information on both regime and outcomes in the labour market and, therefore, attrition units are disregarded. The criterion leads to a total of 21,017 observations over the full period. Our second selection criteria selects only males between the ages of 17 and 55, either Swiss nationals or with a C visa and declaring to be active members of the labour market who are not

² Switzerland has a visa system determining the right to work and permanency for individuals with a non-Swiss nationality. Those holding C-visas have equal labour market and permanency rights as Swiss nationals. Those who hold a B-visa have equal labour market rights than Swiss nationals but for periods of time limited to 8 years. Other types of visa that are neither C nor B allow limited labour market rights with very limited time periods (e.g., seasonal work) or simply rights to remain in the country without working rights (e.g., refugees).

registered as disabled in the population.³ Thus, anyone who has not yet finishing start up education as well as early retirees are withdrawn from our data.⁴ Conditional on skill class, the selection process implies a homogenous set of individual with respect to labour market participation and labour market rights in Switzerland. Furthermore, we select only those in the population that are more likely to fulfil the conditions defining utility functions in expression (1) and the implied conditions defined in Section 2.1. Thus, high skill individuals (e.g., university, advanced vocational careers, and beyond) are withdrawn from the sample because they are more likely to either be allowed to borrow or have less constraints to choose leisure over work. Together these selection criteria reduce our sample to 4,647 individuals, and these define a balanced panel over four consecutive periods. Earnings and benefits are normalized to the base 2000. The 4,647 individuals are each assigned one of three possible skill classes. *Skill class 1* is the lowest class and corresponds to those with elementary primary school either completed or not. *Skill class 2* corresponds to having secondary education and possibly some vocational training but have not completed vocational schooling. *Skill class 3* are those who have completed vocational school after secondary school and/or those who completed up to ‘*Matura*’ but did not go to university. The skill class of an individual determines start up education. All individuals in the sample are outside the education system and full active member of the labour market in Switzerland.⁵ Appendix 3 provides a brief description of the sample by skill class and with regards to a selection of socio-economic variables.

4.1 Reservation policies and Upper bound on labour market shocks

The first set of estimates reflect the selection defined in Section 3.1 and make use of specifications (8) in the first step of the algorithm described in Section 3. A probit specification of each

³ The reason for withdrawing officially disabled is their distinct treatment with regards to various active and passive labour market policies.

⁴ The largest drop occurs due to the fact that females account for some 50% of the complete sample. This is not necessarily the corresponding labour market force percentage, but the collection system for the SAKE implies that the data is only representative after cross-sectional weights are applied in estimation.

part in (8) is applied to young individuals in the population (i.e., between 18 and 26) if observed consecutively working over three years. Similarly, a probit specification is applied to individuals of any age as long as these are observed working in the first periods and not employed in periods thereafter. Expression (8) suggest a set of variables Z that determines the selection process into alternative labour market regimes. If employed, Z includes skill class, years of experience in active employment, age, full-time/part-time dummy, household ownership, marital status, household size, dummy for cantonal language, industrial sector and dummy for ‘currently in short training courses at work’. For those in a spell of long-term unemployment the variables in Z also includes dummy variables that control for length of time in unemployment, does not include the full-Time/part-time dummy or that for ‘short training courses at work’. The probit estimates are applied to each time period (i.e., t_1, t_2 and t_3) for each set of individuals (those continuously employed, and those continuously unemployed over the three periods), separately. Due to the construction of the data and as result of the sequential needs in terms of labour market regimes, sample sizes become a problem, especially in terms of observing individuals that are such that $I_{t_1}^q = 1, I_{t_2}^n = 1, I_{t_3}^n = 1$ as would be required to estimate the parameters associated with depreciation rates. This is because the number of unemployed (registered or not) in each of the four time periods considered is relatively low (e.g., at t_1 only 254 of the 4,647 – or 5.5% – are in a non-employment regime). To maximize the sample size we allow for various alternatives taking 5 (and not four) time periods of information. These various alternatives are summarized in Table 1:

⁵ Those in apprenticeship mode are withdrawn from our sample because their human capital formation implies on-the-job-training as opposed to formation in a learning-by-doing environment as is assumed in the theoretical section.

Table 1: Defining the sample in Labour Market Regime LTU

	Period <i>t1</i>	Period <i>t2</i>	Period <i>t3</i>	Period <i>t4</i>	Period <i>t5</i>	Sample Size
Alternative (a)	Employed	<i>Declares unemployment after a period o employment</i>	<i>Unemployed, searching for work and declaring to receive no benefits</i>	<i>Unemployed, searching for work and declaring to receive no benefits</i>	--	20
Alternative (b)	<i>Unemployed but for no longer than 1 to 2 years</i>	<i>Unemployed, searching for work and declaring to receive no benefits</i>	<i>Unemployed, searching for work and declaring to receive no benefits</i>	--	--	98
Alternative (c)	--	<i>Employed</i>	<i>Declares unemployment after a period o employment</i>	<i>Unemployed, searching for work and declaring to receive no benefits</i>	<i>Unemployed, searching for work and declaring to receive no benefits</i>	29

Note 1: The employment periods in Alternatives (a) and (c) are only useful for the selecting the individuals into the sample. Sample size and estimation are always based on *t2 to t4* for Alternative (a), *t1 to t3* for Alternative (b) and *t3 to t5* for alternative (c). Numbers in brackets show successive reductions in sample size.

Thus, in our attempt to estimate depreciation rates, and given our condition of homogeneity in sample selection, those approximating the definition of ‘being observed in a spell of labour market regime similar to long-term unemployment (LTU)’ provide a sample size of 147. The ideal procedure would distribute the 147 into cells by skill class (s) and labour market experience previous to unemployment (e), and thus be able to estimate $(\sigma(s); g(s))$ for each of these cells. However, the distribution between skill classes already thins out the mass in each cell sufficiently so that we cannot consider the second level of heterogeneity. We therefore have to restrain our estimates to reflect $(\sigma(s); g(s))$. Notice that each of the three alternatives sample selections in Table 1 imply that individuals selected had been employed at least the year (and at most two year) previous to the start of the LTU spell, so that *at least* we control for ‘some’ degree of experience by skill class. It nevertheless remains an approximation and so will our estimates of the depreciation rate.

The sample size for those continuously observed as employed and, at the same time, being sufficiently young so to allow for Assumption 10 (Section 3.1) leads to a restricted size in Skill class 1, but this is a characteristic from the population that has a low percentage in the very low skill group and

relative to those in higher skill classes (see Appendix 3, Table A3.1). Table 2 shows the distribution by skill class for both the set of ‘employed consecutively over three periods’, and ‘unemployed consecutively over three periods’.

Table 2: Distribution of sample sizes over skill class

	Skill 1 (lowest)	Skill 2 (medium low skill)	Skill 3 (semi-skill)	Totals
Continuously Employed (to estimate growth in human capital by skill class)	34 (Between the ages 18 and 22)	212 (Between the ages of 20 and 24)	78 (Between the ages of 21 and 26)	<i>324</i>
Continuously not employed (to estimate depreciation rates in human capital by skill class)	51 (Any age)	57 (Any age)	39 (Any age)	<i>147</i>
Totals	<i>85</i>	<i>269</i>	<i>117</i>	<i>471</i>

Table 2 shows that the sample sizes are low, even for Skill class 2 where the frequency is higher on a yearly basis. We claim that any estimate that follows provides an approximation that best represents the state of the data and our sample selection criteria. As data becomes more available and/or other sample selection criteria are used, the sample size might become more informative (currently under further research work)

Probit models are applied to each of the two samples described in Table 2, independently at each time period. Table 3 presents the results based on period *t1* for both sets. The differences in specification reflect differences in labour market regimes (see footnotes in Table 2). In each case a set of common variables aim at capturing the fixed cost of participation.

Table 3: Results of the Probit: Dependent Variable $I_{t1}^w = I_{t2}^w = I_{t3}^w = 1$. Covariates information based on t_1
 Iteration Criteria = 10^{-6} . *Italic t-values* → significant at least at a 5% level** and at least at 10%*

Variables (at t_1)	Continuously Employed			Continuously Observed as LTU		
	Coefficients	Standard Errors	T-values	Coefficients	Standard Errors	T-values
Constant	3.133	1.024	3.061	-0.050	0.391	-0.129
Skill Class 2	-0.389	0.220	-1.764*	-0.450	0.126	-3.572**
Skill Class 3	0.454	0.250	1.815*	-0.408	0.146	-2.805**
Unemployed for less than 6 months				1.477	0.229	6.450**
Unemployed between 6 months and one year				1.924	0.300	6.413**
Unemployed between 1 year and 2 years				2.020	0.330	6.132**
Unemployed for more than 2 years				2.991	0.421	7.111**
1 to less than 2yrs w/expnce	-0.759	0.278	-2.735**	--	--	--
1 to less than 3yrs w/expnce	-0.655	0.230	-2.845**	-0.652	0.193	-3.383**
3 to less than 5 yrs w/expnce.	-1.188	0.300	-3.962**	-1.027	0.210	-4.892**
6 or more years of w/expnce	-1.038	0.301	-3.445**	-1.153	0.148	-7.777**
Age	-0.134	0.045	-2.999**	-0.029	0.008	-3.815**
Dummy=1 if fulltime	-1.289	0.225	-5.731**	N/A if LTU	--	--
Dummy=1 if owner of house	0.004	0.240	0.017	0.277	0.156	1.770*
Dummy=1 if married	-0.658	0.334	-1.971**	-0.191	0.116	-1.649*
Household Size	0.059	0.064	0.918	0.004	0.049	0.082
German speaking canton	0.049	0.186	0.262	-0.055	0.146	-0.377
French speaking canon	0.148	0.193	0.767	0.128	0.151	0.848
Manufacturing sector	0.336	0.231	1.452	0.253	0.186	1.364
Service sector	0.748	0.222	3.375**	0.026	0.184	0.140
Dummy=1 if short courses	0.212	0.151	1.403	N/A if LTU	--	--
Time dummies included	Yes	--	--	Yes	Yes	Yes
DIAGNOSTICS	$\chi_{(0.05, dem(x))}$			$\chi_{(0.05, dem(x))}$		
Value of Likelihood Function	-186.907			-366.875		
Pseudo R2	0.442			0.438		
LR Test against Mean: Reject model if LR >						
$\chi_{(0.05, dem(x))}$	296.655	0.0000		570.892	0.000	

Note 1: The exclusions for the continuously employed sample are Skill class 1, experience below 1 year (at t_1), Italian speaking cantons and primary industrial sector. Time dummies are included to control for different cohorts information since the data defines 5 artificial years from 10 cohorts. Cohorts 9 and 10 are the exclusions. Unemployment duration data is only available for the non-employed. The number continuously employed individuals in the required age interval are 324: the comparative population (alternative labour regimes over the period but of similar age) is size 719.

Note 2: The exclusion restrictions for the continuously LTU are the same as for those in continuously observed employment but adding another exclusion to identify the weight for the dummies 'unemployment duration'. This exclusion is 'if unemployed for less than 12 months at t_1 '. Furthermore the sample size of those in the continuously LTU over the three periods is 157 only and the alternative population is the remaining observations in the 4647 since we take any age into account to maximize the counts: since identification of the parameters will be limited due the small sample size the variable '1 to 2 years of experience' is further excluded from the set and iteration singularity problems in the iteration algorithm vanish. Any labour market information for the LTU refers to previous labour market experienced and from the view point of information at t_1 .

The results for the continuously employed (Columns 2 to 4 in Table 2) show that relative to Skill class 1, selection into employment is positively affected by higher levels of education (Skill class 3), but

negatively affected by experience in the labour market (relative to the lowest experience level): this result might be explain by the fact that the sample of continuously employed are those in the lower end of the age distribution for whom the dummies ‘long term experience’ will provide small (even if significant) amounts of information. As expected wealth effects (i.e., ownership o household) is not significant at explaining participation for the very young, while marital status is significant suggesting that the presence of a partner increases the chances of not being employed over long periods. Cantonal information (i.e., leaving in a German or French speaking canton, relative to an Italian one) is not significant even although unemployment rates are often higher in Non-Germanic cantons that otherwise. Thus, this would be some indication that selection into employment is not driven by regional differences (and assuming that living in a Canton is not a labour market decision). Finally working in the service sector has a positive effect into selection of continuous employment.

Column 5 to 7 in Table 7 is the selection results for individuals observed to be continuously unemployed over three consecutive periods. In this case and relative to Skill class 1, the higher the level of education the less likely it is to be observed in a long spell of unemployment. Unemployment duration is a significant factor with the weight placed in the probability of employment increasing as the unemployment spell lengthens. Likewise, shorter labour market experience increases the chances of unemployment. For all, looking at the variables that are assumed to determine the fixed cost of working, only wealth and marital status are (weakly) significant at partly explaining the selection process.

From Table 2 we conclude that education, human capital (at this point approximated by years of experience) and past labour market history (i.e., unemployment spells) are the significant variables that explain the selection process into specific labour market regimes. This suggest that the above specifications are correct at projecting the reservations policies since these are assumed to be a function of the state variables ‘skill-type’ and ‘human capital’ (approximated by experience and/or unemployment spells), among others. Thus the estimated parameters in Table 2 are applied to expression (18) to retrieve the sample distribution for the reservation policies $\hat{\pi}_{[b],t}^R$ using the sample of continuously employed over the three time periods under consideration. The three vectors of estimated reservation policies (i.e.,

$\hat{\pi}_{[b],t1}^R, \hat{\pi}_{[b],t2}^R$ and $\hat{\pi}_{[b],t3}^R$) are used to estimate some minimum value knowing that by assumption $\pi_t > \pi_t^R$ for the unknown stochastic shock. Thus, the minimum value will imply a possible upper bound as determined by Assumption 1 and 14 in Sections 2.1 and 3.4, respectively. Likewise, applying expression (18) to the continuously unemployed implies that estimation of $\hat{\pi}_{[a],t1}^R, \hat{\pi}_{[a],t2}^R$ and $\hat{\pi}_{[a],t3}^R$; following similar considerations as with the continuously unemployed, the estimates provide a second upper bound. Table 4 shows the empirical characteristics of the vectors $\hat{\pi}_{[a]}^R$ and $\hat{\pi}_{[b]}^R$:

Table 4: Characteristics of the estimated reservation policy rules (Section 3.5, expression (18))

	Sample in Employment Spell ($\hat{\pi}_{[b]}^R$)	Sample in Unemployment Spell ($\hat{\pi}_{[a]}^R$)
Mean (S.D)	0.223 (0.171)	2.317 (3.452)
Median	0.179	1.033
Range	[0.017, 1.772]	[0.026, 19.9]

For each of the two samples in Table 4 the estimates are the result of joining the three time periods, and the final estimate is consistent with the assumption that at any time period the stochastic shocks (determinant of future period's reservation policies) are draws from one unique distribution. The distribution of reservation policies for those in the employment spell imply that the employed, relative to the unemployed, have significantly low reservation values and are, on average, more likely to enter employment: this is a result that comes straight from the model assumptions imposed in the probit estimation. Recall from Step 4 in Section 3.5 that the algorithm to estimate the human capital parameters requires an initial guess on $\bar{\pi}$ that is best obtained from $\bar{\pi} = \left\{ \exp(-\ln(\min(\hat{\pi}_{[a]}^R | \hat{\pi}_{[b]}^R))) \right\}$. Table 4 shows that this minimum is 0.017 from the sample in an employment spell. Thus the first guess on $\bar{\pi}$ in the iterating process (Section 3.5) is $\bar{\pi} = 58.8$.

4.2 Growth and Depreciation Rates

The final estimates $\{(v^*, r^*), (\sigma^*, \kappa^*), \pi^*\}$ are such that $\pi^* = \pi_j, |\pi_j - \pi_{j-1}| < 0.0005$, and this is a purely arbitrary choice, but sufficiently small to justify its selection. Table 5 shows these estimates by skill class.

Table 6: Estimates for Human Capital formation (Accumulation and depreciation)

Parameters determining Human Capital by skill class	Initial Rate of Human Capital Accumulation ($\hat{v}(s)$)		
	Skill Class 1	Skill Class 2	Skill Class 3
GROWTH RATES	0.2978 (0.2450) [0.0003; 0.9055]	<i>0.4252**</i> (0.2125) [0.0039; 0.9327]	<i>0.4237**</i> (0.1777) [0.0881; 0.7426]
	Adjustment rate of Human Capital Accumulation as function of increments of Human capital from some initial rate ($\hat{f}(s)$)		
	Skill Class 1	Skill Class 2	Skill Class 3
	<i>0.7504**</i> (0.2318) [0.0778; 0.9434]	<i>0.7159**</i> (0.2234) [0.0206; 0.9952]	<i>0.7622**</i> (0.1307) [0.2873; 0.9421]
	Initial Rate of Human Capital Depreciation ($\hat{\sigma}(s)$)		
DEPRECIATION RATES	Skill Class 1	Skill Class 2	Skill Class 3
	0.08760 (0.0951) [0.0002; 0.4030]	<i>-0.07074**</i> (0.03097) [-0.1205; -0.0300]	0.12208 (0.18471) [0.001; 0.6862]
	Adjustment rate of Human Capital Depreciation as function of remaining human stocks from a pre-employment spell ($\hat{\kappa}(s)$)		
	Skill Class 1	Skill Class 2	Skill Class 3
	0.8267 (0.3901) [0.0901; 1.0231]	0.70598 (0.4574) [0.1992; 0.7995]	0.1839 (0.2120) [0.0012; 0.7541]

Note: The first bracketed numbers show standard errors and the ranges in squared brackets are 95% confidence intervals. Both sets of figures are estimated using a naive bootstrap technique that re-samples with replacement 100 times from the original data. **Significant at a 5 % level.

The final estimate in the estimation procedure implies an optimal estimate for the upper bound on the distribution of labour market shocks. Table 6 shows this. It happens to be the upper bound obtained from skill class 2 (see expression (17)). Applying Assumption 1 and 14 and using the relation between the

log normal and normal distribution the upper bound allows for an estimate of the mean and variance in the distribution of labour market shocks:⁶

Table 6: Distributional feature of the underlying labour market shocks

	Estimated value
Estimated Upper Bound $\bar{\pi}$	1.9792** (0.31578) [1.0321; 1.9848]
Mean value for π	1.2635 (1.2851) [1.0905; 1.4365]

Note: See footnote in Table 5

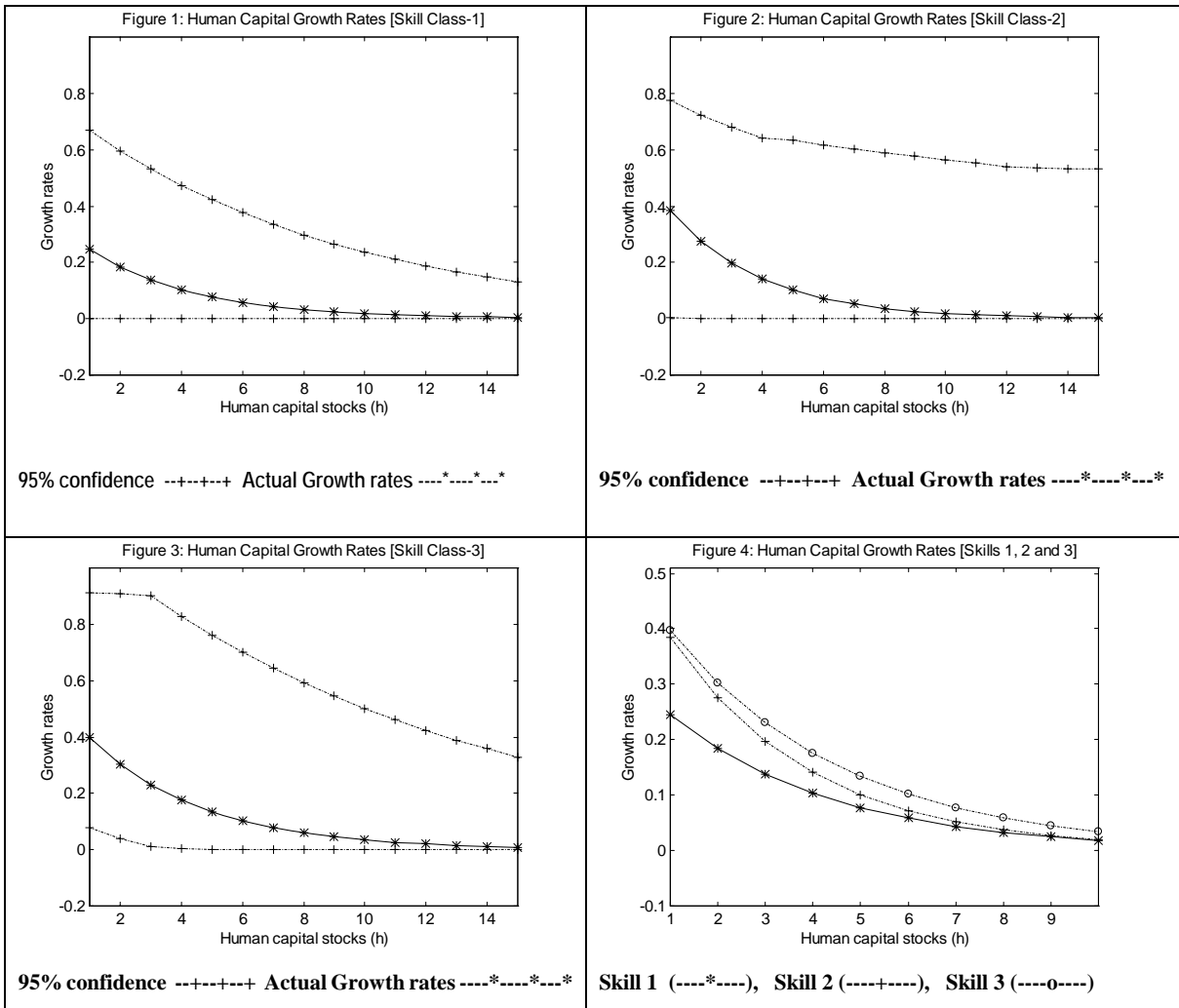
The estimates in Table 5 are based on a non-linear least square procedure applied to expression (9) for $v(s), g(s)$ and the same non-linear technique applied to (14) for $\sigma(s), \kappa(s)$. The figures show the relative difference between skill classes in terms of accumulating/depreciating human capital in reference to yearly intervals.⁷ Individuals at the very low end of the skill distribution show an initial rate of human capital growth equal to 29.8%: after an initial period, growth rates adjust over time at a basic rate of $(0.75)^{\Delta h}$ where $\Delta h > 1$ and implies cumulative stocks in human capital. That is, as human capital stocks increase there are diminishing returns in terms of growth rates. This property is found for all skill types. Compared to skill classes 2 and 3, those in skill class 1 are the least efficient in terms of human capital accumulation due to a much slower initial rate. However, beyond this initial period the resulting estimates determinant at how human capital growth adjusts as stocks increase show that the effect of the adjustment

⁶ With an upper bound $\bar{\pi} = 1.26$ and the symmetry assumption implies that the lower bound is $\underline{\pi} = 0.794$. The assumption of symmetry allows to retrieve the midpoint and approximating the variance with the range between lower and upper bound implies an approximation for the mean (μ) and variance (σ^2) of the $\ln \pi$ distribution. We then use the transformation $E(\pi) = \exp(\mu + \frac{1}{2}\sigma^2)$ and $Var(\pi) = \exp(2\mu + 2\sigma^2)$.

⁷ The data in hand implies intervals in time in terms of years. With this we assume that $(h_{t_2} - \underline{h}^s)$ equals 1, and successive increments are cumulative unit increments. Likewise, $(\underline{h}^{s,e} - h_{t_2}^{s,e})$ is also set to 1 with additional increments also implying cumulative unit increments

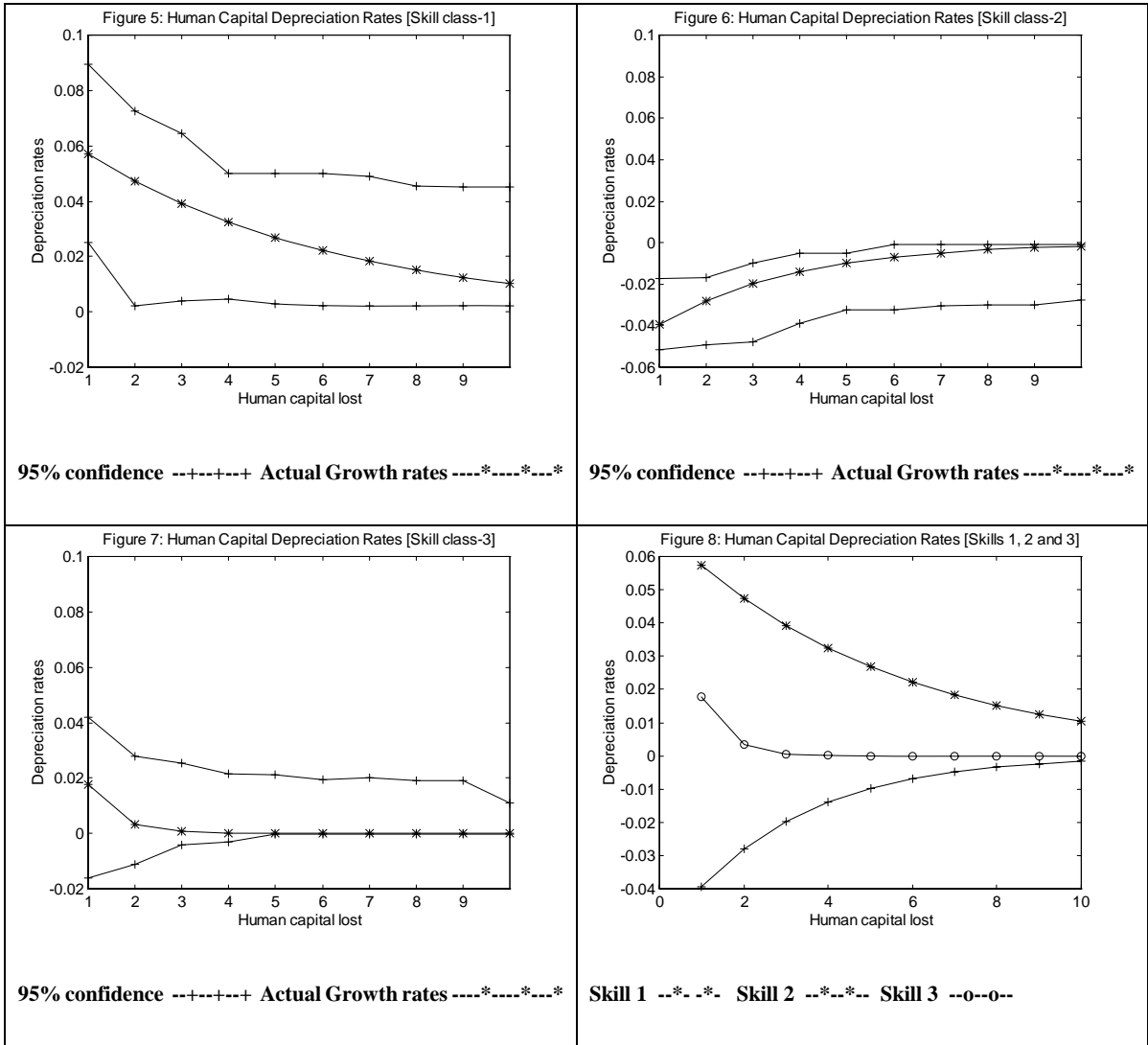
process is similar for all skill classes. Recall from our discussion in Section 3.1 that the adjustment rate $g(s)$ can be thought as a measure of how efficient agents are at converting human capital into productive capital. The estimates in Table 6 show that after the initial adjustment period the efficient rate component of human capital accumulation is almost the same for all skills. Skill class 3 shows slightly higher adjustment rates; together with the initial higher rate in growth the implication is that those in skill class 3 are the most efficient in terms of human capital accumulation reaching higher rates at each level of human capital potential, thus, becoming more productive at a faster rate than other skill types in the population. Figures 1 to 4 makes use of the estimates for $\nu(s), g(s)$ in Table 5 to plot growth in human capital as defined in expression (8). This latter expression implies that growth rates are individual and time specific because the expression depends on the stochastic draw π_t . To interpret expression (8) the plots in Figures 1 to 4 approximate the growth rates assuming that agents receive an average labour market shock equal to $E(\pi) = 1.26351$ (see Table 6). Since the data is annual, the index power in the adjustment rate accumulates in units as determined by the horizontal axis in each of the figures. The vertical axis shows growth rates. Figures 1 to 2 show lower confidence intervals for growth rates of skills 1 and 2 that are never significantly different from zero. However, the lower confidence interval in Figure 3 shows some variance over the first periods of stock accumulation. Overall, the wide confidence intervals reflect sample size problems. Figure 4 compares human capital accumulation by skill type assuming average labour market shocks. Clearly the low skills are outperformed in terms of human capital growth by the other two skill types and at any point over the horizontal range. The highest skill type (skill 3 defining semi-skilled workers) suggests that these individuals are relatively better at turning human capital into productive capital than those with slightly lower level of education (skill 2). Take, for example, a stock of human capital equal to 3 units: with such capital as stock, agents in skill class 3 experiences growth rates of 25%, compared to agents in skill 2 whose capital is growing at (approximately) 20%, and also compared to the lowest skill with human capital growth of (approximately) 12%. A positive distance is maintained over the full range of capital stocks. Our conclusions, however, cannot suggest that either of the skill types are ‘significantly’ better than their counterparts because the wide confidence intervals for

each of the first three figures implies no evidence to suggest a significant difference between the three groups.



Estimates for human capital depreciation rates are only consistent with the theoretical model for skill class 2. Skill class 1 and 3 shows that over a LTU spell, rates of ‘changes’ in human capital are positive (even if close to zero), but not negative as desired. We believe this to be the consequence of a very low sample size that implies not sufficient information to capture the true rate at which human capital depreciates: notice that for both skill classes 1 and 3 the estimates for either $\sigma(s)$ or $\kappa(s)$ are not significant. For skill class 2 the estimates for both $\sigma(s)$ and $\kappa(s)$ are consistent with the model, although the estimate for $\kappa(s)$ is not significant. Using expression (13) we plot these rates following similar

assumptions as before, that is, assuming that individuals receive an average labour market shock. We might think that those in a LTU spell might be better represented if we allow for some lower quartile of π . However, allowing for the mean value π provides a comparative ground between Figures 1-4 and Figures 5-8.



Notice that in the case of human capital depreciating the horizontal axis displays the loss in human capital assumed to start at the point 1 at some level $\underline{h}^{s,e}$ and depreciate from there after until depreciation rates reach the neighbourhood of zero: at this point our model structure would suggest that

relatively low amount of human capital remains there to depreciate. Skill type 2 suggest that initially human capital will experience a depreciation of 4%; thereafter depreciation occurs at a speed that is slower than the growth rate for the same group in the event of employment. Skill type 3 suggests that while in LTU, an initial growth rate is followed by no loss in human capital thereafter. In all cases the changes in percentage occur over a range that is always to close to zero (see the scale in the vertical axis) so that the estimates are never significantly different than zero (i.e., Figures 5 to 7 illustrate relatively large confidence intervals).

5 Conclusions

The paper provides a structural framework to theoretically and empirically analyse endogenous human capital formation in the presence of three distinct labour market regimes: employment, unemployment sheltered by passive and active labour market policies and a second type of unemployment regimes where the unemployed does not participate in active labour market programs (even if they might still be entitle to some form of passive help). These three regimes characterize the actual dynamics in labour markets in Switzerland and the theoretical set up in the structural model reflects such dynamics in the evolution of assets and human capital formation. Heterogeneous agent with respect to education and taste for risk are assumed to react to a sequence of labour market shocks (e.g., wage subsidies, the chance to participate in active programs, adverse life events, etc) that determines the choice of labour market regime at each point in time. Choosing employment implies a period of human capital formation that reinforces the choice of future employment spells. This is because being employed can be thought as permanently affecting contemporaneous and future human capital formation and, consequently, productive capacity. The opposite is true in the event of unemployment, and more especially, if the right to benefit from the overall unemployment system becomes exhausted. This might trigger a period of human capital depreciation with permanent (but negative) effects in productive capacity thus further lowering the chances of labour participation if the future. An interim regime of active program participation might actually help the unemployed to maintain their stocks of human capital, thus their

productive capacity, while searching for a new employment chance. This is because actively participating in programs that target the unemployed provides a link between the unemployed and the skill specific knowledge requirements in a competitive labour market. In the absence of this interim regime of active labour market programs the link is lost and the unemployed have less contemporaneous chances to fulfil the need of new vacancy arrivals. In the long run, the unemployed might fall into a period where, relative to new arrivals in the unemployment pool, skill-specific knowledge starts to deteriorate. Thus, estimating depreciation rates implies estimating a proxy for the underlying benefits of the existence of active labour programs. The theoretical setting in this paper implies such assumptions and provides identifying conditions to retrieve growth rates and depreciation rates from the structural model. The empirical section provides estimates of these parameters for human capital formation using longitudinal data representative of the male active labour force in Switzerland. The parameters are estimated distinguishing between three skill types. Skill 1 is the lowest skill class in the population with little or no investment in education. Skill class 2 implies a minimum level of investment up to secondary schooling whereas the highest skill class, Skill class 3, represent those in the population that we often refer as with 'semi-skill formation' (e.g., vocational formation up to basic level). Anyone with a higher skill mode are not included because the aim is to find out the effect of active labour market programs on human capital formation, policies that are often not consequential to those at the upper end of the skill distribution. Our estimates of human capital growth show that for anyone skill class, human capital accumulates at a diminishing rate. However, for those at skill class 3, and within employment spells, the rate of human capital accumulation implies a higher productive capacity than any other skill class and at all levels of human capital stock. In fact, for as long as human capital keeps on accumulating those in skill class 3 accumulate capital with a growth rate that is between 2-4% higher than those in skill class 2, and between 5-15% higher than those in skill class 1. To some extent, this measures the benefit of 1 or 2 years extra of investment in education, since this is the time period that separates skill class 2 from skill class 3. In estimating human capital depreciation rates we find problems with respect to data availability both because the relative low percent of unemployed in the data and the fact that we require observing these unemployed for a sufficient number of consecutive years. Estimates of depreciation rate for the skill class 2 show some reasonable results.

They suggest that once individuals enter a period of unemployment without program participation, they will experience an initial drop in human capital of 4% assuming average type of labour market shocks. Depreciating human capital slows down as human capital erodes, and this erosion happens at a speed of 71% that changes exponentially relative to the remaining human capital stock. The rate at which human capital depreciates is much slower and starting from a much lower percentage point than human capital growth. The depreciation rates for the skill types 1 and 3 are inconclusive due to the low sample size. In the case of skill type 3 at best they indicate that depreciation does not occur for this skill type. More informative data at this point would be required to provide any real contrast between the three different skills. The fact that depreciation is captured for the medium/low skill type, and the fact that in this case the estimates are significantly different than zero implies a relative measure for the benefit of the alternative regime in the form of active labour market policies that prevent human capital deterioration.

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Appendix 1

A1.1 Proof of Lemma 1

Suppose that for a given compact space X_t for some agent i (this index will be suppressed in this section) at time t employment is the preferred labour market regime for some value $\pi = \pi'$.⁸ This particular choice of the agent implies the following:

⁸ The first part of this proof is similar to Costa-Dias (2002), but allows for a third labour market regime. The second part of the proof refers to the third regime explicitly.

$$V_{vt}^s(a_{vt}, h_{vt}, \pi'_{vt} | I^w = 1) > V_{vt}^s(a_{vt}, h_{vt}, \pi'_{vt} | I^w \neq 1) \quad (L.1)$$

Since a larger value of the shock strictly increases future human capital while working (something that does not happen in the other states) and in turn this (strictly) increases future earnings and thus future consumption possibilities, and because the period's returns from wages increase as well, for any larger value of the shock ($\pi'' \geq \pi'$), the person works as well:

$$\begin{aligned} & V_{vt}^s(a_{vt}, h_{vt}, \pi''_{vt} | I^w = 1) > V_{vt}^s(a_{vt}, h_{vt}, \pi'_{vt} | I^w = 1) > V_{vt}^s(a_{vt}, h_{vt}, \pi'_{vt} | I^w \neq 1) \\ & \Rightarrow \\ & V_{vt}^s(a_{vt}, h_{vt}, \pi''_{vt} | I^w = 1) > V_{vt}^s(a_{vt}, h_{vt}, \pi''_{vt} | I^w \neq 1) \end{aligned} \quad (L.2)$$

This establishes that there is a value of π , say π' , beyond which the agent will always choose employment (w) among all other labour market options. But then there is a range of values in the distribution of π below which contemporaneous and future earnings from employment are so low that the agent's optimal choice would be non-employment. Say this happens at $\pi = \pi^*$. Then for any lower value ($\pi^{**}, \pi^{**} < \pi^*$), the individual won't work either, because when the value of the shock declines employment becomes less attractive compared to the two non-employment options. Thus, a threshold π_b^R defined in terms of X_{vt} exists that completely characterizes the decision between choosing employment or not. The threshold π_b^R depends on assets and human capital accumulated so far as well as on state of nature (i.e. the realisations of the shock), and determines the circumstances upon which the agent is willing to work.

For the case $\pi \leq \pi_b^R$, it remains to analyse the choice between the two non-employment alternatives. From the financial capital accumulation equation we see that the shock does not influence current period physical returns from non-employment states. If there would be no effect of the shock on human capital accumulation, then individuals would all choose state $I_t^n = 1$. However, the larger shock, the less attractive alternative ' n ' becomes in terms of human capital, because the depreciation is increasing in the shock. Suppose there is a value π_a^R ($\pi_a^R \leq \pi_b^R$) for which individuals are just indifferent between q

and n . Because of Assumption 8, if π decreases below π_a^R the alternative n become more valuable since any further loss of human capital declines (i.e., below $\pi_a^R : h_{it} | I_{it}^n \rightarrow h_{it} | I_{it}^q$ & $P_{st} | I_{it}^n = 0$). If the shock increases above π_a^R , the alternative ‘ q ’ gains in value. Thus the monotone reservation policy is proved.

Proof of Lemma 2

This proof extends that in Lemma 2 Costa-Dias (2002) to cover a third labour market regime. In both cases the proof uses backward induction starting with the valued function at age T and showing similar properties for ages 0 to $T-1$ (the index i is suppressed for simplicity, so that for any i , $W_{s(i)t} = W_{st}$, etc.)

At age T the agent maximizes the contemporaneous utility only as function of consumption that equals contemporaneous assets, that is, $c_T^* = (1 + r_T)a_T + I_T^w \pi_T h_T W_{sT} + I_T^q (B_{sT} - P_{sT}) + I_T^n B_{sT}$ and the agent decides to work or not according to the realization of π_T conditional on past labour market history and characteristics. Whatever labour market regime the agent decides to select, $E_\pi V_{T+1}^s(\cdot) = 0$ and each of the (partitioned) value functions are characterized by the utility of final time period resources:

$$\begin{aligned}
V_T^s(a_T, h_T, \pi_T) &= u\left((1 + r_T)a_T + \pi_T h_T W_{sT} (1 - \tau_T)\right) & \text{if } I_T^w = 1; \\
V_T^s(a_T, h_T, \pi_T) &= u\left((1 + r_T)a_T + B_{sT} - P_{sT}\right) & \text{if } I_T^q = 1; \\
V_T^s(a_T, h_T, \pi_T) &= u\left((1 + r_T)a_T + B_{sT}\right) & \text{if } I_T^n = 1.
\end{aligned} \tag{L.4}$$

Allow for Assumption 2 at age T : the same properties for the utility function carry through for the value function for all the three labour market regimes. Allow for Assumptions in 3.1 and use the conditions in Lemma 1. Let $V_T^s(\cdot | I_T^j = 1)$ be the short hand notation of the conditional (on $j = w, n, q$) value function:

$$\begin{aligned}
E_\pi V_T^s(a_T, h_T) &= V_T^s(\cdot | I_T^w = 1)P(I_T^w = 1) + V_T^s(\cdot | I_T^q = 1)P(I_T^q = 1) + V_T^s(\cdot | I_T^n = 1)P(I_T^n = 1) = \\
&= \int_{\underline{\pi}}^{\pi_a^R} V_T^s(a_T, h_T | I_T^w = 1) f(\pi) d\pi + \int_{\pi_a^R}^{\pi_b^R} V_T^s(a_T, h_T | I_T^q = 1) f(\pi) d\pi + \int_{\pi_b^R}^{\bar{\pi}} V_T^s(a_T, h_T | I_T^n = 1) f(\pi) d\pi
\end{aligned} \tag{L.5}$$

But (L.4) implies that $V_T^s(\cdot | I_T^j = 1)$ is strictly increasing, twice differentiable and concave in assets for any j – labour market alternative, therefore, so is the expectation $E_\pi V_T^s(a_T, h_T)$; notice that this is also taking into account that at any point in the lifetime of individuals, including at T , the reservation thresholds depend on past information and not in the present levels of assets (as determined in Lemma 1).

At ages 0 to $T - 1$: The proof has four steps (following Costa-Dias (2002) and adapting Stokey and Lucas (1989) to be applicable to any number of labour market regimes)

Let $E_\pi V_{t+j}^s(a, h) = E_\pi V_T^s(a, h)$: The previous step shows that under the conditions implied by Lemma 2, the RHS is strictly increasing, twice differentiable and a concave function in assets (a).

Step 1: We show that the conditional value functions $V_t^s(\cdot | I_t^j = 1)$ are increasing, twice differentiable and concave in (physical) assets. Given that $u(c_{t+j})$ is concave (Assumption 2) and $E_\pi V_{t+j}^s(\cdot)$ are strictly increasing, concave and twice differentiable in c_{t+j} and a_{t+j} , standard recursive methods show that for bounded objective functions, $V_{t+j-1}^s(\cdot)$ has identical properties that $E_\pi V_{t+j}^s(\cdot)$. The proof can be found in Stokey and Lucas (1989), Chapter 9, page 261. Furthermore, take expectations on $V_{t+j-1}^s(\cdot)$ over the support so that we define $E_\pi V_{t+j-1}^s(\cdot)$. The latter could be represented as $E_\pi V_{t+1}^s(\cdot)$ for any t in the working life of an individual. Then, the same standard recursive methods in Stokey and Lucas (1989) imply that with $u(c_{t+1})$ and $E_\pi V_{t+1}^s(\cdot)$ strictly increasing, twice differentiable and concave in c_{t+1} and a_{t+1} , respectively, the value function $V_t^s(k, h, \pi | \cdot)$ is strictly increasing, twice differentiable and a concave function in assets (a).

Step 2: We show that the reservation value π_b^R for the labour market shock π_t is continuous in assets (a). The monotonic relation between π_a^R and π_b^R implies that both reservation values are continuous and differentiable (at least once) in assets (a). The reservation values π_a^R and π_b^R both solve the equalities between the three value-functions determined by the three labour market choices. Furthermore, *Step 1* implies the continuous differentiability (with respect to assets) of the value functions for any given labour market regime. Since assets are an increasing, continuous and differentiable function

of human capital h_v , the value functions are also strictly increasing, twice differentiable, concave functions with respect to human capital. Take, for example, the threshold π_b^R . We know from Lemma 1 that this threshold solves the equality given by $V_t^s(a, h, \pi_b^R | I_t^w = 1) = V_t^s(a, h, \pi_b^R | I_t^q = 1)$, where the latter is a function of the same arguments in the neighbourhood of π_b^R . All the above implies the following:

- (a) The partial derivatives $V_h(\cdot | I)$, $V_a(\cdot | I)$, and $V_\pi(\cdot | I)$ exist. That is, Assumption 1 and Step 1 guarantee the existence of these partial derivatives for any labour market option (notice that for $V_h(\cdot | I) = V_a \cdot (\partial a / \partial h)$ so that the existence of the partial derivative with respect to human capital is also guaranteed.)
- (b) Suppose we can define a point (a^R, h^R, π_b^R) . From Lemma 1 we know that π_b^R solves the equality $V_t^s(a, h, \pi_b^R | I_t^w = 1) = V_t^s(a, h, \pi_b^R | I_t^q = 1)$, therefore, this must also happen so that $V_t^s(a^R, h^R, \pi_b^R | I_t^w = 1) = V_t^s(a^R, h^R, \pi_b^R | I_t^q = 1)$. That is, at this point the equality is also true. Since the value function is continuous and differentiable over the support of π , and π_b^R is in the support $[\underline{\pi}, \bar{\pi}]$, then the derivative $\frac{\partial V(a^R, h^R, \pi_b^R | I)}{\partial \pi} \neq 0$ in the neighbourhood of that point.

The Implicit Function Theorem says that if a function $V(a, h, \pi): D^n \rightarrow \mathbb{R}^m, m < n$, complies with conditions (a) and (b), then, there exists a function $g(h, a)$ such that $V_t^s(a^R, h^R, g(a, h) | I_t^w = 1) = V_t^s(a^R, h^R, g(a, h) | I_t^q = 1)$ in the neighbourhood of (a^R, h^R, π_b^R) . This function has an implicit representation, say $\pi^R = g(a, h)$, satisfies $\pi_b^R = g(a^R, h^R)$, and is continuous and at least once differentiable in its arguments. Notice also that in our model $a = a(h)$, and not the other way around. Assume both (a, h) follow monotonically the same direction as is the case for fixed labour market regimes. Stokey and Lucas (1989, page 290) show that the model can be reformulated in terms of only one endogenous variable with the recursive solution applying identically to the reformulated problem. Thus, we can let $\pi_b^R = \pi_b^R(a)$. The one-to-one mapping is guaranteed.

The same argument can be applied to the reservation value π_a^R that solves for the equality between the value functions $V_t^s(a, h, \pi_a^R | I_t^q = 1) = V_t^s(a, h, \pi_a^R | I_t^n = 1)$. In both cases we have shown that Assumptions 1 and Step 1 allow for the application of the Implicit Function Theorem, and this ensures that both reservation policies are continuous differentiable functions (at least once) of assets (a). This is to be used in further steps.

Step 3: Allowing for Assumption 1 and the interpretation of the reservation policies in Lemma 1, the expected value function at time t can be written as follows:

$$\begin{aligned}
E_{\pi} V_t^s(a_t, h_t) = & \int_{\underline{\pi}}^{\pi_a^R} V_t^s(a_t, h_t | I_t^w = 1) f(\pi) d\pi + \\
& + \int_{\pi_a^R}^{\pi_b^R} V_t^s(a_t, h_t | I_t^q = 1) f(\pi) d\pi + \int_{\pi_b^R}^{\bar{\pi}} V_t^s(a_t, h_t | I_t^n = 1) f(\pi) d\pi \quad (\text{L.6})
\end{aligned}$$

Step 1 determines that $V_t^s(\cdot | I_t^j = 1)$ is strictly increasing, twice differentiable and concave in physical assets for all three labour market regimes. Step 2 determines that the reservation policies are continuous differential functions of assets, and the differentiability of the joint density function of the productivity shocks is also guaranteed in Assumption 1. Therefore, $E_{\pi} V_t^s(a_t, h_t)$ is also twice differentiable with respect to assets a_t . This is a necessary condition for Step 4 below.

Step 4: We show that the value function $E_{\pi} V_t^s(a_t, h_t)$ is an increasing and concave function of assets a_t . Step 3 allows for the following representation for the first derivative of $E_{\pi} V_t^s(a_t, h_t)$:

$$\begin{aligned}
\frac{\partial E V_t^s(a, h)}{\partial a_t} = & \int_{\underline{\pi}}^{\pi_a^R} \frac{\partial (V_t^s(\cdot | I^n = 1))}{\partial a_t} dF(\pi) + \int_{\pi_a^R}^{\pi_b^R} \frac{\partial (V_t^s(\cdot | I^q = 1))}{\partial a_t} dF(\pi) + \int_{\pi_b^R}^{\bar{\pi}} \frac{\partial (V_t^s(\cdot | I^w = 1))}{\partial a_t} dF(\pi) \\
& + \frac{\partial \pi_b^R}{\partial a} \left\{ (V_t^s(\cdot | I^q = 1) - V_t^s(\cdot | I^w = 1)) \right\} dF(\pi_{b,t}^R) + \\
& + \frac{\partial \pi_a^R}{\partial a} \left\{ (V_t^s(\cdot | I^n = 1) - V_t^s(\cdot | I^q = 1)) \right\} dF(\pi_{a,t}^R). \quad (\text{L.7})
\end{aligned}$$

The last two terms in the RHS vanish at the reservation value in the density function of π (the value functions are identical), while the first derivatives with respect to assets are all positive since Step 1 ensures that the conditional value function is strictly increasing. Therefore $(\partial EV_t^s(a, h)/\partial a_t) > 0$. All what is needed for concavity is to show that $(\partial E^2 V_t^s(a, h)/\partial^2 a_t) < 0$. From (L.7), the second order derivative is given by:

$$\begin{aligned} \frac{\partial E^2 V_t^s(a, h)}{\partial^2 a_t} &= \int_{\bar{\pi}}^{\pi_a^R} \frac{\partial^2 (V_t^s(\cdot | I^n = 1))}{\partial^2 a_t} dF(\pi) + \int_{\pi_a^R}^{\pi_b^R} \frac{\partial^2 (V_t^s(\cdot | I^q = 1))}{\partial^2 a_t} dF(\pi) + \int_{\pi_b^R}^{\bar{\pi}} \frac{\partial^2 (V_t^s(\cdot | I^w = 1))}{\partial^2 a_t} dF(\pi) \\ &+ \frac{\partial \pi_b^R}{\partial a_t} \left\{ \left(\frac{\partial (V_t^s(\cdot | I^q = 1))}{\partial a_t} - \frac{\partial (V_t^s(\cdot | I^w = 1))}{\partial a_t} \right) \right\} dF(\pi_{b,t}^R) + \\ &+ \frac{\partial \pi_a^R}{\partial a_t} \left\{ \left(\frac{\partial (V_t^s(\cdot | I^n = 1))}{\partial a_t} - \frac{\partial (V_t^s(\cdot | I^q = 1))}{\partial a_t} \right) \right\} dF(\pi_{a,t}^R). \end{aligned} \quad (\text{L.8})$$

The first three terms in the RHS of (L.8) are negative because of the concavity of the conditional value functions. But the value of the last two terms in (L.8) depend on the relative degree of concavity between paired labour market regimes (i.e. between I_t^q and I_t^w , and between I_t^n and I_t^q), and the degree of absolute risk aversion (given by the derivatives $(\partial \pi_b^R / \partial a)$ and $(\partial \pi_a^R / \partial a)$).⁹ Assumption 4 states that individuals are risk averse in the sense that an increase in assets reduces the reservation policy (subjective

⁹ That is, as stated in the introduction, individual's hold latent valuation on each of the labour market regimes that we define as 'reservation valuation policy set'. These sets depend on individual's taste for risk possible determined by individual's history, characteristics, etc: Lemma 1 embodies this idea. Each time the agent has to evaluate the labour market conditions as the shock is realized, they compare the realized shock π_t to own reservation policy that explains individual's taste for risk (π_t^R) , and make a labour market choice. Since the risk attitude is given by the set of reservation policies (π_t^R) , risk aversion is measured by the change on this with respect to assets, where assets includes human capital as part of the individuals wealth. This justifies that the derivatives $(\partial \pi_t^R / \partial a)$ explain the concept of risk aversion (coefficient of risk aversion).

valuation of labour market choice) thus making employment more likely than non-employment in the future for any random shock. Likewise, an increase in assets as result of non-decreased in human capital (rather than depreciation) implies that program participation becomes more likely than ‘unemployment without program participation’, also for any given random productivity shock. Therefore, $\left(\frac{\partial \pi_b^R}{\partial a}\right) < 0$ and $\left(\frac{\partial \pi_a^R}{\partial a}\right) < 0$ are implied by Assumption 4 as well as being consistent with our model (see introductory notes). But, if an increase in physical assets implies reducing the respective reservation policies through an increase in the willingness to take risk the implication is that for any given assets level, a_t , comparing the value functions between labour market regimes implies that $\left(\frac{\partial V(\cdot | I_t^w = 1)}{\partial a}\right) > \left(\frac{\partial V(\cdot | I_t^q = 1)}{\partial a}\right) > \left(\frac{\partial V(\cdot | I_t^n = 1)}{\partial a}\right)$.¹⁰ Decreasing absolute risk aversion and derivatives of value functions that are increasing as taste for risk increases implies that the second and third terms in the RHS of (L.8) can be positive and overtake the negative value of the first three terms. Then, concavity of the valued function can only be guaranteed if we assume ‘constant absolute risk aversion’ in which case $\left(\frac{\partial \pi_b^R}{\partial a}\right) = \left(\frac{\partial \pi_a^R}{\partial a}\right) = 0$. This would imply that the reservation policies are not responsive to changing wealth that is neither a realistic assumption, nor is it completely consistent with our structural model. Thus, Assumption 4 is required so that ‘decreasing absolute risk aversion’, i.e., $\left(\frac{\partial \pi_b^R}{\partial a}\right) < 0$ and $\left(\frac{\partial \pi_a^R}{\partial a}\right) < 0$, but by a magnitude that is ‘not too large’ (both values are assumed to be bounded from below in the neighbourhood of zero) guarantees that the positive terms in the last two parts

¹⁰ That is, expected value of a choice is the weighted sum of the three possible choices so that expectations of the value function is $EV = V(| I = w)P(w) + V(| I = q)P(q) + V(| I = n)P(n)$, and the choice among the three alternative depends on the realization of the shock that will determine the weight (probability). But independently, each of the value functions is an increasing, twice differentiable and concave function of assets, while the value of the value function for the working choice has to be steeper than for the non-employment alternatives and in turn. At this point is when we need to apply Assumption 5 (no crossing of the value functions).

of the RHS in (L.8) never overtake the negative values of the set of second derivatives. This is the only way to guarantee concavity.

A1.3 Proof of Lemma 3

Given Lemma 2 (i.e., having established the conditions for a well behaved value function), the Euler Equation is the necessary and sufficient condition for the optimal consumption decision ‘for fixed labour market regimes’ (since it is within labour market regimes that the value function is continuous, twice differentiable and concave function of assets). Recall the Euler Equation:

$$\frac{\partial u}{\partial c_t} \Big|_{I(j,t)} = E \left[\beta(1+r) \frac{\partial u}{\partial c_{t+1}} \Big|_{I(j,t)} \right] \quad \forall j \quad (\text{L.9})$$

But (L.9) gives the optimal intertemporal relation for the choice variable assuming concavity of the value function only with respect to assets, when in reality the problem in (2) implies a more complex set of dynamics in the state space. Then, there must be as many optimal consumptions paths that are consistent with (L.9) as possible values of h_{t+1} that are consistent with the assets path (a_t) that underlines (L.9). Then identification/characterization of the optimal consumption path is only possible if we find an expression analogous to (L.9) such that the new expression implies restrictions for human capital. Recall Step 1 in the proof of Lemma2. This step states that under the regularity assumptions for $u(c)$ and $E_\pi V_{t+1}^s(\cdot)$ in c_t and a_{t+1} , standard recursive methods show that $V_t^s(\cdot | I_t^j = 1)$ has identical properties than $E_\pi V_{t+1}^s(\cdot)$. First we apply the envelope theorem to $V_t^s(\cdot | I_t^j = 1)$ so that at the optimal consumption choice and for fixed labour regime, a change in assets implies zero additions from future changes in the value function:

$$\begin{aligned} \left(\frac{\partial V_t^s(\cdot | I^j)}{\partial a} \right) &= \frac{\partial u}{\partial a} \Big|_{c^*(t), I(j,t)} + \beta E_t \frac{\partial V_{t+1}(\cdot | I^j)}{\partial a_{t+1}} \Big|_{c^*(t), I(j,t)} \\ &= \frac{\partial u}{\partial c} \cdot \frac{\partial c}{\partial a} \Big|_{c^*(t), I(j,t)} \quad \text{since } E_t \frac{\partial V_{t+1}(\cdot | I^j)}{\partial a_{t+1}} \Big|_{c^*(t), I(j,t)} = 0 \quad (\text{L.10}) \\ &= u'(c_t)(1+r_t) \end{aligned}$$

Since $(\partial V_t^s(\cdot | I^j)/\partial a_t) = (1 + r_t)u'(c_t)$ and $V_t^s(\cdot | I_t^j = 1)$ has identical properties than $E_\pi V_{t+1}^s(\cdot | \cdot)$, we take expectations so that $E_t(\partial V_{t+1}^s(\cdot | I^j)/\partial a_{t+1}) = (1 + r_{t+1})E_t u'(c_{t+1})$; the result is labour market regime and skill specific. The result is then applied to the Euler Equation in (L.9):

$$\left. \frac{\partial u(c_t)}{\partial c_t} \right|_{I(j)} = \beta \cdot E_t \left[\left. \frac{\partial V_{t+1}^s(a_{t+1}, h_{t+1})}{\partial a_{t+1}} \right|_{I(j)} \right] \quad (\text{L.11})$$

Expression (L.11) maintains the same properties as the Euler condition in (L.9) but we have now established a relation between current consumption and the other dynamic variable in the system, human capital. We are now closer to identifying the optimal condition for consumption (optimal consumption path) taking into account the full dynamic system. Notice from the dynamics in (1) that the two endogenous state variables always follow the same direction, while the value function is concave in assets. This means that the derivative in the RHS of (L.11) is positive for any value of h_{t+1} , with this latter variables also increasing as a_{t+1} increases. At the same time (L.11) explains that any marginal change in utility today has to be matched by an equal but weighted expected marginal change in tomorrow's utility establishing a precise relation between the concavity of $u(\cdot)$ and $EV(\cdot)$ with respect to the variables c_t and a_t . From the dynamics in (1) we see that this must imply that we are pinning down the optimal human capital path. That is $a_{t+1} = (1 + r)a_t + INC(h, W, \pi) |_{I(j)} - c_t$. Then, for fixed working conditions, any increase in assets has to be met by an increase in consumption so that (L.11) is satisfied, and this leaves no room for h_{t+1} to move other than whatever value satisfies (L.11). In other words, (L.11) can be re-written as:

$$\left. \frac{\partial u(c_t)}{\partial c_t} \right|_{I(j)} = \beta \cdot E_t \left[\left. \frac{\partial V_{t+1}^s(a_{t+1}, h_{t+1} | h_{t+1})}{\partial a_{t+1}} \right|_{I(j)} \right] \quad (\text{L.12})$$

Then, given the properties of the value function, the values of the state variables and for fixed skills and working decisions, the optimal condition for consumption is given by (L.12). With this (allowing for all

regularity conditions and assumptions above), the problem in (2) has a unique solution ‘for fixed labour market regimes’ and for given skill type. In the development of (L.12) we have seen that agents are restricted to be risk averse. Expression (L.12) places further restriction in the variables that determine the behaviour of individuals: consumption (c_t) and savings (a_{t+1}) must both be *normal goods* in the sense that an increase in net income must be followed by an increase in both consumption and assets for fixed labour market regimes. The reason is the following: suppose ‘total net income’ increases (for example as result of an increase in human capital, but also as result of any other change in the state space). From the law of motion in assets (see (1)), the implication is that either (a_{t+1}) or (c_t) increase. But both u and EV are concave functions, therefore, both must increase to keep the equality in (L.12) satisfied. Another way to interpret this is as follows: allowing for $EV(a, h)$ in L.11 does not pin down a specific optimal path among all possible optimal paths given all admissible h paths, so L.11 is necessary but not sufficient. Conditioning on h implies that the Euler is now based on $EV(a, h | h)$ thus restricting the relation between assets and consumption so that the marginal intertemporal gains are now fixed for given labour market conditions. This latter is what allows to identify the optimal path but at the expense of further restrictions on the type of consumption and savings that individuals are allowed to consume and hold.

Lemmas 1, 2 and 3 complete the set of regularity conditions that allow for expression (2) to represent the individual’s problem, for the problem to be well defined and for this to have a unique solution (identification of an optimal consumption path). At the same time, expression (2) is based on (1) and we have shown that the structural model as specified in (1) is well behaved. This is what allows us to use the characterization of the endogenous variables to specify the reduced form specification, and with this to estimate the parameters. In reality, what is crucial is to make sure that for fixed labour market regimes the dynamic endogenously changing variables change all monotonically in the same direction. Our specification is correct because the newly introduced labour market regime still maintains such monotonic relation. Assuming a well behaved bounded functions in a bounded support (for anyone of the three labour market regimes), the problem boils down to ‘maximising a concave function’ subject to a set

of constrain that ‘do not jump in different directions in some unspecified form’: this is also guaranteed. Because in our case these constrains also behave nicely (monotonically), the problem can be placed in the shape of a value function with behaviour that is driven by the dynamics in the model, thus the value function is also well behaved. The regularity conditions for the value function implies three constrains (risk aversion, consumption is normal and savings is also a normal good). This completes the theoretical part (the structural model and its conditions).

Appendix 3

Table A3.1 describes the sample by skill type with respect to the variables used to project the reservation policy rules (Z). Information is based on period $t1$ and takes into account all ages in the sample.

Table A3: Sample descriptive statistics (reference information T1)

Variables	Skill 1	Skill 2	Skill 3
Sample Size	498	3,015	1,134
Age	36.7 (11.4)	36.6 (9.4)	36.8 (9.1)
% in Germanic Cantons	42.4	50.2	55.4
% in French Cantons	41.2	31.4	30.2
% in Italian Cantons	16.5	18.4	14.5
% Household ownership	68.5	62.2	69.5
% Married	55.8	56.8	60.1
Net Annual Earnings in CHF	49,423 (40,896)	66,095 (42,155)	77,027 (56,564)
Experience in the Labour Market			
<i>Less than 3 months</i>	11.04	1.3	2.4
<i>Between 4 and 12 months</i>	3.41	1.4	3.44
<i>More than 1 & less than 3 years</i>	7.03	3.5	6.5
<i>More than 3 & less than 6 years</i>	2.41	6.3	6.88
<i>More than 6 years</i>	76.10	87.53	80.78
Duration in unemployment since last labour market experience			
<i>Not unemployed</i>	92.2	96.6	96.2
<i>Less than 3 months</i>	2.61	0.83	0.71
<i>Between 4 and 12 months</i>	2.8	1.19	1.59
<i>More than 1 & less than 3 years</i>	1.4	0.76	0.71
<i>More than 3 & less than 6 years</i>	0.8	0.46	0.53
<i>More than 6 years</i>	0.20	0.17	0.26