1 Simple model

In this section we describe the impact of a merger in a single oligopolistic labor market. Consider a market *j* with M_j firms and productivities $\mathbf{z}_j = (z_{1j}, \ldots, z_{ij}, \ldots, z_{M_jj})$. In order to hire n_{ij} workers, given employment at its competitors (n_{-ij}) , the wage that firm *i* must pay is given by its inverse labor supply curve:

$$w_{ij} = \left(\frac{n_{ij}}{N_j}\right)^{\frac{1}{\eta}} \left(\frac{N_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} , \qquad N_j = \left[n_{1j}^{\eta+1} + \dots + n_{M_jj}^{\eta+1}\right]^{\frac{1}{\eta+1}}.$$

The aggregate *N* is exogenous with respect to labor market *j*.

For simplicity, we begin by assuming that firms one and two merge. Under a merger, the two physical plants remain in operation, however they are now both operated by the same firm. The firm therefore chooses employment at both plants in order to maximize total profit. The manager solves the following labor demand problem where $f_{ij}(n_{ij})$ is the production function which may be plant-market dependent:

$$\max_{n_{1j},n_{2j}} \underbrace{z_{1j}f_{1j}(n_{1j}) - w_{1j}n_{1j}}_{\pi_{1j}} + \underbrace{z_{2j}f_{2j}(n_{2j}) - w_{2j}n_{2j}}_{\pi_{2j}}.$$

When choosing n_{1j} , the firm understands how increasing n_{1j} increases N_j , which will make each unit of labor at plant two more expensive: $\partial w_{ij}/\partial N_j > 0$. The first order condition for plant one labor is

$$MRPL_{1j} - w_{1j} - \frac{\partial w_{1j}}{\partial n_{1j}}n_{1j} - \frac{\partial w_{2j}}{\partial n_{1j}}n_{2j} = 0.$$

Under the above supply system, this delivers:

$$MRPL_{1j} - w_{1j} - \left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) \left\langle \frac{\partial N_j}{\partial n_{1j}} \frac{n_{1j}}{N_j} \right\rangle \right) w_{1j} - \left(\frac{1}{\theta} - \frac{1}{\eta}\right) \left\langle \frac{\partial N_j}{\partial n_{1j}} \frac{n_{1j}}{N_j} \right\rangle \frac{n_{2j}}{n_{1j}} w_{2j} = 0$$

Using the fact (i) the elasticity of N_j with respect to n_{ij} is equal to the payroll share s_{ij} of plant *i*, (ii) by the definition of the payroll share $s_{1j}/s_{2j} = w_{1j}n_{1j}/w_{2j}n_{2j}$, we can write this as

$$MRPL_{1j} - w_{1j} - \left(\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)s_{1j}\right)w_{1j} - \left(\frac{1}{\theta} - \frac{1}{\eta}\right)s_{2j}w_{1j} = 0.$$

Since a symmetric condition holds for plant 2, we can express the optimal wage for each plant:

$$w_{ij} = \mu(s_{1j} + s_{2j})MRPL_{ij}$$
, $\mu(s) = \frac{\varepsilon(s)}{\varepsilon(s) + 1}$, $\varepsilon(s) = \left[s\frac{1}{\theta} + (1 - s)\frac{1}{\eta}\right]^{-1}$

The markdown μ_{ij} is the same at both plants and determined by the equilibrium combined payroll share of both plants.

It is straight-forward to show that this is more general and can be applied to a firm that owns any number of plants within a market. Let *K* be the set of plants owned by a firm, then the equilibrium wage at each plant is

$$w_{ij} = \mu\left(\sum_{i \in K} s_{ij}\right) MRPL_{ij}.$$

A natural implication of this is that if $K = \{1, ..., M_j\}$ such that one firm operates all plants, then the markdown at each firm is $\mu(1) = \theta/(\theta + 1)$. This is the same markdown as that of a single firm operating a single plant in a market in which it is the only firm.

Example. How does a merger affect wages and employment at both the merging firms and their competitors? We can establish general results in a simple example. Consider three firms with initial shares s_1 , s_2 and s_3 . Proceeding by contradiction, first assume that $s'_1 = s_1$ and $s'_2 = s_2$. In which case $\mu'_1 = \mu(s'_1 + s'_2) < \mu(s_1) = \mu_1$, and the same for plant 2. Therefore wages fall at both of the merging plants. The only way that the merging plants' shares remain constant will therefore be if wages fell sufficiently at plant 3 such that W_j remained constant. This is not the case (why?).

Given this then it must be the case that $s'_1 < s_1$ and $s'_2 < s_2$. But in that case $s'_3 > s_3$, which implies that $\mu'_3 < \mu_3$ so wages at plant 3 fall. Profits at plant 3 must increase, since with lower wages at their competitors shifts out the profit function of plant 3. The effect on employment at plant 3 is ambiguous since both their own and competitor's wages all decrease. The result, then, is that the merger reduces wages at *all* firms in the market. The exact effect on employment is ambiguous.

Figures 1 through 2 provide examples of the outcomes of mergers in equilibrium. In each case we consider a market with three firms where $z_1 < z_2 < z_3$. The figures consider mergers of different combinations of firms.

In Figure 1 plants 1 and 2 are merged. As markdowns are set according to the combined share, wages at both plants fall. This leads plant 3 to also cut its wage as its market power increases (s_3 goes up). The implications for employment is lower employment at plants 1 and 2

and a small increase at plant 3.

In Figure 2 it is now the largest firm that takes over the smallest firm. Relative to the case where plant 1 is taken over by plant 2, the combined share is much higher, leading to a larger cut in wages at plant 1, and a corresponding larger decline in employment.

In Figure 3 we merge the two most productive plants. Again, both firms' markdowns decrease as $s'_2 + s'_3 > \max\{s_2, s_3\}$. Both firms cut their wages, and leading to an increase in market power at plant 1 and a corresponding decrease in w_1 . The large wage cut at plant 2 leads to a steep decline in employment at the least productive of the two merged plants. The effect on employment is then to increase employment at both plant 1 and plant 3.

Market level effects. At the market level, employment falls in all three cases. The average wage actually increases in the case where the largest firm takes over the smallest. This may seem confusing: all firms' wages fall, yet the average wage increases. Employment has decreased by 4.4 percent, but the allocation of what remains is now tilted further toward plants 2 and 3 which have higher productivity and pay lower wages. Overall these changes cause output to fall in all cases. Interestingly, labor productivity of the sector measured naively by output per worker increases. Whenever a merger occurs the markdown falls most at the least productive of the two plants, reallocating labor away from unproductive plants and to productive plants.

The negative effects of the merger are highest when the two productive firms merge. The declining markup and wage at the second most productive firm reduces employment substantially while little is reallocated to the most productive firm. The result is a substantial decline at the extensive margin (market employment falls) and intensive margin (the labor share also falls).

		Merged firms		
		1-2	1-3	2-3
Total employment (percent change)	$N = \sum_i n_i$	-2.8	-4.4	-13.0
Total payroll(percent change)	$WN = \sum_i w_i n_i$	-3.0	-3.9	-22.0
Average wage (percent change)	W = WN/N	-0.3	0.5	-10.4
Total output (percent change)	$Y = \sum_i z_i n_i^{\alpha}$	-0.8	-1.3	-9.3
Labor productivity (percent change)	A = Y/N	2.1	3.3	4.3
Labor share (percentage point change)	LS = WN/Y	-1.2	-1.4	-7.3
Total profits (percent change)	$\Pi = \sum_i \pi_{ij}$	1.7	1.7	4.6

Table 1: Sector level effects of mergers



Figure 1: Oligopsonistic equilibrium under merger of firms 1 and 2



Figure 2: Oligopsonistic equilibrium under merger of firms 1 and 3



Figure 3: Oligopsonistic equilibrium under merger of firms 2 and 3