Task Prices, Skill Selection, and Wage Inequality

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Current draft: February 2019

Abstract

This paper develops a new method to estimate changing task prices per unit of skill from a flexible economic model of endogenous sector choice. Applying our method to German administrative panel data with high-quality occupation information, we can establish important links between surging wage inequality and occupational trends over time: (1) in contrast to average wages and consistent with shocks to demand, changes of task prices are positively correlated with occupational employment growth; (2) entrants or leavers are substantially less skilled than incumbents in every occupation (and increasingly so the more it grows). Aggregating these differences with total net entry accounts for most of the declining skills of growing occupations implied by the estimation; (3) task prices explain a sizable share of rising log wage dispersion and our full empirical model, which includes workers' occupation-specific skill accumulation, generates the complete increase of upper as well as lower-half wage inequality during 1984–2010.

Keywords: Roy Model; Task prices; Skill Selection; Wage Inequality; German Administrative Panel Data

JEL codes: J21, J24, J31

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1 Introduction

In this paper we ask whether changing demand for occupations was driving the large increase of wage inequality in Germany over the last decades. As in other countries, German employment has reallocated profoundly across occupations since the 1980s, with some occupations having more than doubled in size and others halving or almost disappearing. Consistent with routine-biased technical change (e.g., Acemoglu and Autor, 2011), the employment structure overall has also polarized as high- and low-paying occupations tend to have grown compared to middle-paying occupations.

Yet not just for Germany has it been difficult to link occupational growth to changes in wages. In our data at hand, also average wages of occupations have changed but there is no systematic relationship to employment growth, especially at the lower tail of the wage distribution (Dustmann et al., 2009). This makes it hard to judge whether demand or supply were the dominant force for the occupational reallocation; and whether this has been an important driver of the increasing wage inequality during the past decades, which occurred in Germany and in other developed economies.

One reason why the link between employment growth and average wage growth of occupations is at best tenuous may be selection. If growing occupations attract on the margin less skilled entrants than the incumbent workers, and if shrinking occupations shed relatively less skilled marginal workers, average skills in the former will be declining compared to the latter. This effect may be so strong that, despite rising relative demand, average wages of growing occupations are even falling compared to shrinking occupations.

This paper uses administrative panel data with high-quality occupation information and a new estimation methodology derived from a flexible economic model to address this question. Our starting point is the Roy (1951) model with a large number of occupations and without a specific assumption for the joint distribution of skills. We show that nonetheless it is possible to identify changes of task prices, which are occupational wages in efficiency units and thus corrected for selection, by exploiting the relationship of workers' wage growth between two periods with their average occupation choices in those periods. Intuitively, if a worker chooses an occupation for which task prices rise over time, he will have higher wage growth than a worker who chooses a declining occupation even if one or both workers (endogenously) switch occupations. This is because, from their revealed choices, we can infer that the former worker possesses skills that make him benefit from the rising prices in his occupation.[1]

Workers' skills are constantly changing over their careers. In our longitudinal data, life-cycle wage growth is for example much faster among young workers and in high-skill occupations. We flexibly model this process as skill accumulation that is heterogeneous by demographic groups as well as by workers' origin and destination occupations between two periods. The identifying variation for an occupation's task price change in this respect is therefore wage growth compared to what is implied by the skill accumulation.[2] Using theory and Monte Carlo evidence we show that, conditional on the accumulation, idiosyncratic skill shocks for specific workers do not materially confound our method's estimates.

The empirical results reveal that selection indeed played a key role for the facts described above. We find that task prices in growing occupations did increase and decidedly so. This is the case across broad occupation groups, and hence consistent with a common driver for job polarization and shifting occupational wages such as routine-biased technical change (RBTC). But it also holds for individual occupations within more homogenous groups and thus indicates that demand shocks are the main source of occupational reallocation more generally.[3] The skill accumulation estimates reflect a strong concavity and heterogeneity of life-cycle wage profiles across occupations and partly large cross-accumulation parameters underscore that controlling for (self-)selection in occupation switches is indeed important to estimate the correct task prices.

The flipside of the increasing task prices are deteriorating skills in growing occupa-

[1]There is a small approximation error using the average occupation choice compared to the explicit joint distributional assumption of skills (shocks). We show that this is economically negligible.

[2]Any panel identification needs to disentangle workers' skill dynamics over the career from changing prices per unit of skill. We assume that the function of skill accumulation does not change over time and use a base period (1975–1984), in which task prices are constant, to identify its parameters.

[3]Note that this paper does not measure occupational demand or supply shocks directly. We instead infer from the co-movements of quantities and prices that these are *consistent with* demand shocks. Forces of occupational demand may include RBTC and related technical changes, international trade and offshoring (e.g., Goos et al., 2014), transformation of the industry structure (Bárány and Siegel, 2018), changes in consumption patterns (Mazzolari and Ragusa, 2013), and others.

tions. In addition to many robustness checks regarding the sample and estimation, we uncover one particularly compelling empirical fact in support of this result: we find that, in every occupation between two periods, entrants or leavers ('marginal' workers) earn substantially less than incumbents or stayers ('infra-marginal' workers), and that the wage gap increases with the occupation's growth rate. Since it arises at the same point in time and in the same occupation, this gap is independent of our estimated task prices. It therefore implies that marginal workers are substantially less skilled in each occupation than infra-marginal workers, much of which we show is due to differences in skill accumulation, and that occupations which grow fast need to particularly go down in the (occupation-specific) skill distribution in order to hire additional workers. Now, aggregating these wage differences with total net entry during the analysis period on average accounts for eighty percent of each occupation's skill selection change implied by the task price estimates.[4]

Finally, we use our economic model and estimates to assess the role of occupations in the increase of German wage inequality. We find that rising skill accumulation at the median due to workforce aging explains much of the rise in the 50–15 log wage differential, whereas task prices account for almost three quarters of the increasing 85–50 differential. The full model of changing wages at labor market entry, occupation-specific skill accumulation, task prices, and switching generates the complete rise of upper and lower half inequality as well as overall log wage dispersion. Rising task prices in high-wage occupations alone explain half of the rise in wage dispersion but this is only partly reflected in the dispersion of average occupational wages because of the above-discussed selection effects.

This paper is related to an active recent literature which estimates task prices. The explicit equation linking changing task prices to workers' (endogenous) occupation choices was first derived in Böhm (2018). Whereas Böhm employs observable

[4] These findings are consistent with some of the evidence reported in McLaughlin and Bils (2001) or Cavaglia and Etheridge (2017), and more generally with the prediction of the Roy model that marginal workers will be less skilled than infra-marginal workers if the correlation between skills across sectors is less than perfect (e.g., Young, 2014). However, we do believe we are the first to systematically document this fact, aggregate it, and compare it to task price changes which are estimated using a different type of variation in the data. We also show that the majority of the differences between marginal and infra-marginal workers are due to differences in skill accumulation and idiosyncratic shocks during their tenure in the occupation, the careful modeling and estimation of which is unique to our framework.

characteristics that predict task choices in repeated cross-section data, we use high-quality longitudinal information in order to directly implement this relationship for individual workers. Cortes (2016) proposes an alternative estimation strategy in panel data, interpreting changes of occupation-specific fixed effects over time as task prices. We conduct extensive Monte Carlo simulations and find that, while both approaches are generally robust and identify the task prices well, our method with its explicit modeling of switches performs better than occupation fixed effects in situations where the variance of idiosyncratic skill shocks is large. We also show how to use a base period and flexibly model heterogeneous skill accumulation, which otherwise confounds the task prices in any panel data estimation, and how to extend the framework so that it allows for non-pecuniary valuations and forward-looking behavior.[5]

Ours is also related to papers that have examined the implications of skill selection for the observed productivity or wage changes across industry sectors. Relative to Heckman and Sedlacek (1985) and Young (2014), but also the task price literature, we implement a new estimation method for detailed occupations so that we can generally examine the relationship between employment growth, wages, and sectoral demand. Also compared to those papers and McLaughlin and Bils (2001), we then link our estimation results to the change of the overall wage distribution. At this point, a large literature has examined occupations or tasks-based explanations for rising inequality, with more (e.g., Acemoglu and Autor, 2011) and less (Dustmann et al., 2009) affirming results. Other influential papers have analyzed the wage premia of many individual firms or workplaces (see Card et al., 2013, for Germany). Our study provides evidence that, once selection is successfully removed in high-quality data, a substantial share of the increase of wage inequality can be explained by task prices and, considering the changes of employment, it is consistent with being markedly influenced by demand shocks for the (specific) human capital that is embodied in occupations.[6]

The next section presents the economic model and derives our new estimation

[5]Reassuringly, our method and occupation fixed effects deliver similar task price changes in the actual data once rich skill accumulation is accounted for. Other papers estimating task prices include Firpo et al. (2013), Gottschalk et al. (2015), Cavaglia and Etheridge (2017), and Yamaguchi (2012, 2016). For a detailed discussion of their results and methods, see Böhm (2018).

[6]Böhm (2018) finds that task prices for broad occupation groups also have raised upper-half wage inequality among young males in the U.S..

method. Section 3 introduces the data and the key stylized facts. We then report the task price estimates and analyze the implied selection effects into occupations. In Section 5 we use our empirical results to assess their contribution to the increase of overall wage inequality. The last section concludes and the appendices provide additional details and robustness checks.

2 A Model for Estimating Task Prices and Skill Selection

In this section we develop an estimation method for changing task prices, which are the prices paid for an efficiency unit of labor in a given occupation. Task prices thus enable us to distinguish price from selection effects when occupational wages change over time. The method flexibly accounts for workers' skill accumulation over the life-cycle and endogenous occupation switches, which are critical features of individual level panel data, and it is straightforward to estimate even for the 120 occupations of our empirical implementation. Finally, we evaluate the method using Monte Carlo experiments.

There are $k = 1, \ldots, K$ distinct occupations and at time t a worker i is endowed with a K-vector of skills $S_{i,t} = \begin{pmatrix} S_{1,i,t} & S_{2,i,t} & \ldots & S_{K,i,t} \end{pmatrix}$. His potential wages obtain as the product of these skills and of the occupation-specific task prices per unit of skill that prevail in the economy, $\Pi_t = \begin{pmatrix} \Pi_{1,t} & \Pi_{2,t} & \ldots & \Pi_{K,t} \end{pmatrix}$.[7] Letting lowercase characters denote the logarithm of a variable, the worker's potential log wages in occupation k become:

$$w_{k,i,t} = \pi_{k,t} + s_{k,i,t} \; \forall \, k \in \{1, \ldots, K\}. \tag{1}$$

A key objective of this paper is to estimate the evolution of $\pi_{k,t}$ over a period of three decades. This will then also allow us to identify the changes in $s_{k,i,t}$ across occupations and their constituting components.

[7] We refrain from modeling the equilibrium but this relationship may, for example, derive from a competitive labor market where $S_{k,i,t}$ units of skill produce a proportional amount of output in occupation k and workers are paid their marginal value product. We provide evidence below that the additive separability into prices and skills is a plausible assumption in our data. The K different skills could each be an occupation-specific function of (a potentially lower dimension of) workers' fundamental productive characteristics. Our model of skill accumulation, which allows for flexible skill acquisition across occupations, will be consistent with this as well as with other notions of how skills develop.

We assume that workers maximize their incomes by choosing the occupation in which they earn the highest wage:

$$w_{i,t} = \max\{w_{1,i,t}, \dots, w_{K,i,t}\} \tag{2}$$

In general, it is difficult—even with rich panel data—to recover changes in task prices from changes in observed wages. Arguably, the most important reasons for this difficulty are endogenous sector (i.e., occupation) choices and simultaneous changes in workers' skills. We address these in turn.

2.1 A Tractable Model of Sector Choice

Consider a marginal change of task prices or skills over time. Since choices are the solution to an optimization problem, by the envelope theorem, this will only have a direct effect because workers' realized wage changes are not affected by a potential reaction in their occupation choices. Therefore, the marginal change in worker i's realized wage at time t is:

$$dw_{i,t} = \begin{cases} dw_{1,i,t} = d(\pi_{1,t} + s_{1,i,t}) & \text{if } I_{1,i,t} = 1 \\ \qquad \vdots & \\ dw_{K,i,t} = d(\pi_{K,t} + s_{K,i,t}) & \text{if } I_{K,i,t} = 1, \end{cases}$$

where $I_{k,i,t} \equiv \mathbb{1}[\max_{j=1,\dots,K}\{w_{j,i,t}\} = w_{k,i,t}] = \mathbb{1}[w_{k,i,t} \geq w_{j,i,t} \forall j \neq k]$ is a choice indicator for occupation k. We can rewrite this as:

$$dw_{i,t} = I_{1,i,t}dw_{1,i,t} + \ldots + I_{K,i,t}dw_{K,i,t} = \sum_{k=1}^{K} I_{k,i,t}dw_{k,i,t}. \tag{3}$$

Equation (3) states that a worker's observed wage grows by the same amount as the potential wage in his chosen occupation for marginal shifts in potential wages. These shifts could be due to changes in task prices or skills.

In order to arrive at an expression in discrete time, we still need to take the wage effect of an endogenous occupation switch into account. We therefore integrate over

Equation (3) from potential wages $\{w_{1,i,t-1}, \ldots, w_{K,i,t-1}\}$ to $\{w_{1,i,t}, \ldots, w_{K,i,t}\}$. With a slight abuse of notation for briefness we obtain:[8]

$$\Delta w_{i,t} = \sum_{k=1}^{K} \int_{w_{k,i,t-1}}^{w_{k,i,t}} I_{k,i,\tau} dw_{k,i,\tau}. \tag{4}$$

This result is rather intuitive: if a worker stays in his occupation k between two points in time ($I_{k,i,t-1} = I_{k,i,t} = 1$), his observed wage change is equal to the change in his potential wage in the chosen occupation (i.e., $\Delta w_{i,t} = \Delta w_{k,i,t}$). If the worker switches from some other occupation k' to k, ($I_{k',i,t-1} = 1$,$I_{k,i,t} = 1$), he obtains part of the origin occupation's wage gain (or loss) as well as part of the destination occupation's wage gain with the relative size of these parts determined by the point of indifference (i.e., the exact potential wages $w_{k',i,\tau} = w_{k,i,\tau}$ so that $\Delta w_{i,t} = (w_{k',i,\tau} - w_{k',i,t-1}) + (w_{k,i,t} - w_{k,i,\tau})$). This is also intuitive, as the worker has comparative advantage both in his origin and in his destination occupation.

In empirical analyses, Equation (4) is directly observable for occupation stayers. For switchers, we need to approximate it because we cannot observe their point of indifference. Therefore, we linearly interpolate the choice indicators for $\tau \in (t-1, t)$:

$$I_{k,i,\tau} \approx I_{k,i,t-1} + \frac{I_{k,i,t} - I_{k,i,t-1}}{w_{k,i,t} - w_{k,i,t-1}}(w_{k,i,\tau} - w_{k,i,t-1}). \tag{5}$$

Defining $\bar{I}_{k,i,t} \equiv \frac{1}{2}(I_{k,i,t} + I_{k,i,t-1})$ and combining Equations (4) and (5), we obtain:

$$\Delta w_{i,t} = \sum_{k=1}^{K} \bar{I}_{k,i,t} \Delta w_{k,i,t} = \sum_{k=1}^{K} \bar{I}_{k,i,t} \left(\Delta \pi_{k,t} + \Delta s_{k,i,t}\right) \tag{6}$$

A detailed derivation is in Appendix A.1.2. The intuition of Equation (6) after the approximation is the same as before: if a worker stays in his occupation, his wage gain is the change of his potential wage in that occupation. If the worker switches, he obtains part of the origin occupation's wage gain (or loss) as well as part of the destination occupation's wage gain, set to exactly half-half by the interpolation (i.e.,

[8]Appendix A.1.1 derives the precise formula as $\Delta w_{i,t} = \sum_{k=1}^{K} \int_{w_{k,i,t-1}}^{w_{k,i,t}} I_k(w_{1,i,t}, ..., w_{k,i,\tau}, ..., w_{K,i,t-1}) dw_{k,i,\tau}$, i.e., $I_{k,i,\tau}$ in Equation (4) is a shorthand for $I_k(w_{1,i,t}, ..., w_{k,i,\tau}, ..., w_{K,i,t-1})$. Böhm (2018) provides a derivation that does not invoke the envelope theorem for a simplified model with two tasks.

$\triangle w_{i,t} = \frac{1}{2}\triangle w_{k',i,t} + \frac{1}{2}\triangle w_{k,i,t}$). The strength of this result is that it accommodates endogenous switches that are due to changes in task prices or skills in principle for many occupations. Provided that skill changes are controlled for in an adequate manner, the first term of (6) states that task price changes can be recovered from a regression of first-differenced wages on "average" occupation choices $\{\bar{I}_{k,i,t}\}_{k=1}^{K}$, which are easily constructed in panel data with occupation identifiers. The identifying variation for this are therefore differences in workers' wage growth over time depending on their occupation choices. The two remaining questions are whether the approximation (5) is a good one and how to model the changes of workers' skills.

The first point to note is that (5) is not an approximation at all for workers who stay in their occupation and, in the case of workers who switch from occupation k' to k, for all other occupations $\neq k, k'$. This also holds true for those k' to k switchers whose wages in $t-1$ and t are equally far (symmetric) from indifference in both periods. In particular, the following holds exactly for such workers:

$$\begin{aligned} w_{k',i,t-1} - w_{k,i,t-1} &= w_{k,i,t} - w_{k',i,t} \\ w_{k,i,t} - w_{k',i,t-1} &= w_{k',i,t} - w_{k,i,t-1} \\ &= \frac{1}{2}(\Delta w_{k',i,t} + \Delta w_{k,i,t}), \end{aligned}$$

The first line is the definition of symmetry, the second and third lines reformulate this. The third line is exactly the same as Equation (6) for such workers. Our method's regression equation below exploits the mean wage changes conditional on switches k' to k and taking the expectation across all individuals preserves the relationship under the approximation. In particular, individual switchers may well have different gains from switching as long as $E\left[(w_{k,i,t} - w_{k',i,t-1}) - (w_{k',i,t} - w_{k,i,t-1}) \,\middle|\, I_{k',i,t-1} = 1, I_{k,i,t} = 1\right] = 0$. Finkelstein et al. (2015) make a related approximation in a completely different context; they nicely illustrate the argument in their Figure 1. Within our model, there is no reason to expect a skewed distribution. Also the wage gain (and thus the approximation error) is bounded by the wages changes of a switching worker's two choices. That is, if he moves from k' to k, we have $\triangle w_{i,t}\epsilon[\triangle w_{k',i,t}, \triangle w_{k,i,t}]$. Consistent with this, the Monte Carlo simulations in the appendix indicate that the approximation (5) is largely immaterial for our estimates and that the correct task prices are precisely identified.

2.2 Modeling Skill Accumulation

The previous section showed that task prices can be identified from the relationship between workers' wage growth and their average occupation choices using longitudinal data. To do this, we need to separate out skill accumulation over time, which is a first-order feature of individuals' life-cycle wage growth and would occur even in the absence of any changes in task prices. The following provides a rich model of the skill accumulation function, allowing for flexible differences within occupations, by worker demographics, and across all combinations of occupation switches.

The maintained assumption in this paper is that the task prices are changing over time but the way occupational output gets produced is not.[9] This assumption yields an empirically tractable separation of log wage growth into changes of task prices and changes of skills, and it is made throughout the task price and RBTC literature (Firpo et al., 2013; Acemoglu and Autor, 2011; Autor and Dorn, 2013, and others) as well as in earlier papers on the Roy model (e.g., Heckman and Sedlacek, 1985). In particular, we will assume that the function which governs workers' skill accumulation between periods is time-constant (skill levels at labor market entry may however change), which again parallels other papers using panel data to estimate task prices (Cortes, 2016; Cavaglia and Etheridge, 2017) or industry wage rates (McLaughlin and Bils, 2001).

This time-constancy assumption is less restrictive than appears at first glance because *overall* accelerations of skill accumulation in specific occupations can for the most part be re-interpreted as changing task prices in the theoretical model and in the empirical results. Therefore, the critical restriction amounts to the fact that *relative* accumulation between different worker groups (e.g., by age) within an occupation is assumed time-constant. We will return to this discussion below as well as provide evidence.

[9]In terms of the model sketched in footnote 7 above, the changing task prices reflect shifting marginal products of occupations' output over time. This may be the case because productivity of that output in aggregate production or consumption (demand shock) changes, or because of changing labor supply to that occupation (supply shock).

To be concrete, we flexibly model skill accumulation as learning-by-doing on the job. That is, a worker's skill s in occupation k changes depending on his occupation choice k' in the previous period:

$$\Delta s_{k,i,t} = \sum_{k'=1}^{K} I_{k',i,t-1} \cdot X'_{i,t-1}\Gamma_{k',k} + u_{k,i,t}, \tag{7}$$

where the vector $X_{i,t-1}$ contains a constant and observable variables controlling the speed of skill acquisition or depreciation via the vector $\Gamma_{k',k}$. The arguably most important variable we use are different age groups. In robustness specifications we further include education and we have also used direct measures of job-related tasks, similar in spirit to Gathmann and Schönberg (2010) or Yamaguchi (2012). Note that this formulation contains a full set of interactions of the accumulation coefficients $\Gamma_{k',k}$ with the covariates $X_{i,t-1}$. The summation term in (7) thus maps the previous occupation choice k' interacted with $X_{i,t-1}$ into skills in the current occupation k. The random component $u_{k,i,t}$ is a mean zero idiosyncratic innovation and discussed in detail below.

Substituting (7) into a worker's wage growth (6) yields our baseline estimation equation:

$$\Delta w_{i,t} = \sum_{k=1}^{K} \bar{I}_{k,i,t} \left(\Delta\pi_{k,t} + \sum_{k'=1}^{K} I_{k',i,t-1} \cdot X'_{i,t-1}\Gamma_{k',k} + u_{k,i,t} \right). \tag{8}$$

Our goal is to estimate the parameters in $\Delta\pi_{k,t}$ and $\Gamma_{k',k}$ for all $k, k' \in K$. As it stands, they are not separately identified from each other because of the intercept in $\Gamma_{k',k}$, which represents a level shifter for the speed of skill accumulation in each occupation. As indicated above, it is immaterial for workers' decisions in the model whether *overall* wage growth in an occupation stems from task prices or skill accumulation, and it is for the most part observationally equivalent in the data.

For the mechanics of our estimation, we nonetheless need to introduce a base period that separately identifies the skill accumulation parameters $\Gamma_{k',k}$ from the changing task prices $\Delta\pi_{k,t}$. In particular, we identify changes in task prices relative to the 'base period' from $t = 0$ to $t = T_{\text{base}}$ (the 'analysis period' is $t = T_{\text{base}}, \ldots, T$), estimating (8) and setting $\Delta\pi_{k,t} = 0$ for all k in $t = 1, \ldots, T_{\text{base}}$. The simplest interpretation of the resulting parameters obtains when task prices during the base period are indeed constant (i.e., $\Delta\pi_{k,t} = 0$ for $t = 1, \ldots T_{\text{base}}$ holds). In that case, all $\Gamma_{k',k}$ will be identified from the base period and changes of task prices in all occupations are identified for $t > T_{\text{base}}$. In contrast, if actual task prices are changing between $t = 1$ and $t = T_{\text{base}}$ already, the parameters have to be interpreted as accelerations or decelerations compared to the base period. That is, the skill accumulation estimates identify $\hat{\Gamma}_{k',k} = \Gamma_{k',k} + \frac{1}{2}\overline{\Delta\pi_{k,base}} + \frac{1}{2}\overline{\Delta\pi_{k',base}}$ and accordingly the task price estimates identify $\Delta\hat{\pi}_{k,t} = \Delta\pi_{k,t} - \overline{\Delta\pi_{k,base}}$.

In our discussion we mainly stick with the easier literal interpretation of the parameter estimates but we will point out instances where the more general acceleration versus deceleration interpretation does make a difference. We also provide robustness checks by estimating (8) in different base periods (though then also the analysis period may need to change). Notice again that the key restriction in both interpretations is that we can additively separate $\hat{\Gamma}_{k',k}$ and $\Delta\hat{\pi}_{k,t}$ and thus that, apart from level shifters, the relative coefficients within $\Gamma_{k',k}$ do not change over time. Accordingly, we examine below whether skill accumulation for different age groups within occupations has stayed constant over time and we estimate our model for different age groups in robustness checks.

Finally, consistent with the constancy of skill accumulation, we assume that $u_{k,i,t}$ is independently and identically distributed over time and across individuals. That is, $u_{k,i,t}$ is an innovation with respect to the previous period in the sense that its expectation conditional on all predetermined variables is zero:

$$E\left[u_{k,i,t} \,\middle|\, I_{k',i,t-1}, X_{k,t-1}\right] = 0 \;\forall\; k', k \in K.$$

Other than that, we allow for any joint distribution function $F(u_{1,i,t}, ..., u_{K,i,t})$ of the unobservables so that, for example, idiosyncratic skill shocks can be correlated among similar occupations in an unrestricted way. In robustness specifications we vary the period length to check the sensitivity of the independence over time. The most general interpretation of $u_{k,i,t}$—when we do not maintain the strict structural interpretation of the estimated $\hat{\Gamma}_{k',k}$ below—is simply as an orthogonal regression error in (7).

Bringing Equation (8) into the usual regression form by writing out the summations and considering only the observed sector choice trajectory from $t-1$ to t, the error term becomes $v_{i,t} \equiv \sum_{k=1}^{K} \bar{I}_{k,i,t} u_{k,i,t}$. This structure of shocks introduces a correlation between the error and the regressors, since a large innovation in a particular sector makes it more likely that choosing this sector happens to be optimal. First we show that a basic OLS regression of the model discussed so far is often robust to this effect. We then outline an instrumental variables strategy.

The regression (8) is a saturated skill model including all combinations of occupation choices $I_{k',i,t-1}$ and $I_{k,i,t}$. In the base period, the regression gives:

$$E\left[\Delta w_{i,t} \middle| \{I_{k,i,t}, I_{k,i,t-1}\}_{k=1}^{K}, X_{i,t-1}\right] =$$

$$E\left[\sum_{k=1}^{K} \bar{I}_{k,i,t} \left(\sum_{k'=1}^{K} I_{k',i,t-1} \cdot X_{i,t-1} \cdot \Gamma_{k',k} + u_{k,i,t}\right) \middle| \{I_{k,i,t}, I_{k,i,t-1}\}_{k=1}^{K}, X_{i,t-1}\right] \quad t = 1, \ldots, T_{\text{base}}$$

The fully interacted base period regression identifies this conditional expectation function and therefore yields expected skill accumulation $E\left[\Delta s_{k,i,t} \middle| \{I_{k,i,t}, I_{k,i,t-1}\}_{k=1}^{K}, X_{i,t-1}\right]$. Defining $\nu_{i,t} \equiv \sum_{k=1}^{K} \bar{I}_{k,i,t} \left[\Delta s_{k,i,t} - E\left[\Delta s_{k,i,t} \middle| \{I_{k,i,t}, I_{k,i,t-1}\}_{k=1}^{K}, X_{i,t-1}\right]\right]$, the regression equation in the analysis period can be re-written as:

$$\Delta w_{i,t} = \sum_{k=1}^{K} \bar{I}_{k,i,t} \Delta \pi_{k,t} + \sum_{k=1}^{K} \bar{I}_{k,i,t} E\left[\Delta s_{k,i,t} \middle| \{I_{k,i,t}, I_{k,i,t-1}\}_{k=1}^{K}, X_{i,t-1}\right] + \nu_{i,t} \tag{9}$$

Conditional on $X_{i,t-1}$ and any combination of $I_{k',i,t-1}$ and $I_{k,i,t}$, the expectation of $E\left[\Delta s_{k,i,t} - E\left[\Delta s_{k,i,t} \middle| \{I_{k,i,t}, I_{k,i,t-1}\}_{k=1}^{K}, X_{i,t-1}\right]\right]$ is zero by construction. The point here is that in the base period we already estimate the wage changes of occupation switchers, including the skill accumulation as well as idiosyncratic skill shocks. Therefore, if $E\left[\Delta s_{k,i,t} \middle| \{I_{k,i,t}, I_{k,i,t-1}\}_{k=1}^{K}, X_{i,t-1}\right]$ is consistently estimated in the base period, the error term in regression (9) is uncorrelated with the regressors $\bar{I}_{k,i,t}$ and the correct changes in task prices are identified even under general idiosyncratic skill shocks.

An alternative approach to remove the bias in Equation (8) is by instrumenting the regressors $\left\{\bar{I}_{k,i,t}\right\}_{k=1}^{K}$ with their predetermined components $\{I_{k',i,t-1}\}_{k'=1}^{K}$, which are not a function of $u_{k,i,t}$. This leads to an estimation very similar to the dynamic panel data models pioneered by Arellano and Bond (1991), the only difference being the construction of regressors $\left\{\bar{I}_{k,i,t}\right\}_{k=1}^{K}$. Further discussions of potential biases in the OLS and the IV approaches as well as Monte Carlo simulations are in the appendix.

Finally, notice that in both approaches, the estimates $\hat{\Gamma}_{k',k}$ do not correspond to the structural skill accumulation parameters in Equation (8). The reason in the IV is that if $\bar{I}_{k,i,t}$ are instrumented with lagged choices $I_{k',i,t-1}$, only one set of coefficients $\hat{\Gamma}_{k'}$ for each past sector can be identified. In the case of an age dummy in $X_{i,t-1}$, this coefficient provides the average wage growth due to skill accumulation (including due to switching) for an individual of that age working in occupation k' in the previous period. In the OLS model, the $\hat{\Gamma}_{k',k}$ are the averages of skill changes, whether due to systematic accumulation or idiosyncratic shocks, of $k' \neq k$ switchers or $k' = k$ stayers.

2.3 Model Performance

We evaluate the performance of our estimation method analytically and in Monte Carlo experiments, also exploring extensions of the underlying economic model and comparing it to an alternative approach that uses occupation-specific fixed effects. What follows is an intuitive summary while details are in the appendix.

In the Monte Carlo simulations, we generate a sample that resembles the actual SIAB data employed below by drawing from that data for occupations and wages at labor market entry, and then using our estimated task price and skill accumulation parameters to construct the workers' careers (details in Appendix B). When we estimate our main OLS specification (Equ.10 below) on this basic sample it perfectly recovers the task price changes and structural skill accumulation parameters that were fed into it. This implies that the estimation method without idiosyncratic skill shocks works as intended and that, for example, the linear approximation (5) of the indifference point is irrelevant for the results at least in this basic case.

We then add extreme value distributed idiosyncratic skill shocks to the sample. When these shocks are plausibly-sized, the estimation method still performs well and the task price as well as skill accumulation estimates are only slightly downward-biased compared to their true values.[10] The $\hat{\Gamma}_{k',k}$ estimates do not identify the struc-

[10]We set the standard deviation of shocks equal to the dispersion of period-to-period wage changes in the SIAB, which is probably an upper bound of the true idiosyncratic shock dispersion. We have tried

tural skill accumulation parameters anymore, as discussed above. When the shocks are very large, however, the bias using the main estimation Equation (10) does become substantial. But even in that case, the alternative instrumental variables estimation retains rather accurate task price estimates. We conclude from this that using the IV estimation as an empirical robustness check to our estimates is advisable and we do this below.

We also extend and generalize the economic model that underlies our estimation method in Appendix A. First, what we have termed idiosyncratic skill shocks in the previous section is observationally equivalent in our analysis to a basic model of employer learning about workers' skills (e.g., as in Altonji and Pierret, 2001; Gibbons et al., 2005). This is due to the fact that log-linearity allows us to write the model in terms of expected skills, which can evolve because of changes in actual skills (our formulation above) or because employers changing their expectations about individuals' skills over time. The two interpretations are not mutually exclusive, of course.

In our view, a key extension is the generalized and dynamic Roy model. In fact, many studies include non-pecuniary amenities of occupations in the worker's decision problem (e.g., Lee and Wolpin, 2006). Moreover, forward-looking considerations (i.e., continuation values) of choosing jobs seem particularly important in longitudinal data like ours when individuals are followed over their careers.[11] We show in the appendix that both these aspects enter similarly into the decision problem and that workers who move to an occupation with lower amenity/dynamic value will exhibit accordingly higher contemporaneous wage growth, ceteris paribus, as otherwise they would not have moved in the first place. If the amenity/dynamic values are time-constant, the skill accumulation parameter $\hat{\Gamma}_{k',k}$ in our main specification will absorb them. If they are time-changing, the estimation Equation (10) has to be augmented to include regressors for occupation switches ($\Delta I_{k,i,t}$) on top of average occupation choices ($\bar{I}_{k,i,t}$) to control for (and estimate) 'wage compensation' when moving into a low amenity/dynamic value occupation, and vice versa. We do this in empirical robustness

different distribution functions than extreme value. We have also done the Monte Carlos with non-zero task price trends in the base period and results are indeed according to the acceleration/deceleration interpretation above.

[11] For example, a young worker may prefer an occupation where he contemporaneously earns less but learns more (or has a more interesting life-style) than in some alternative occupation.

checks and we estimate the model in specific demographics (i.e., age and gender) for whom forward-looking considerations should be differentially important.

Fixed costs of switching occupations are another aspect that previous literature has emphasized. Whether they appear as wage (e.g., Gathmann and Schönberg, 2010) or as non-wage costs (Cortes and Gallipoli, 2017), we show that additive switching costs are related to (but not the same as) the above amenity/dynamic values. Therefore, the skill accumulation parameters $\hat{\Gamma}_{k',k}$ in (10) and the augmented regression including $\Delta I_{k,i,t}$ (partially) control for time-constant and time-changing switching costs, respectively. In separate Monte Carlo simulations we alternatively allow for switching costs as a fraction of the worker's current wage and find that these also do not make a material difference to the parameter identification. In empirical robustness checks we also extend the period length from one to four years because switching costs should be less important relative to task price and skill changes over a longer time horizon.

We also compare our estimation method to an alternative approach proposed by Cortes (2016) who uses occupation-specific fixed effects to estimate the changing task prices.[12] First, we show in Appendix A.3 that such an approach needs to employ separate fixed effects for each stint in an occupation and in addition control for occupation as well as age-specific skill accumulation within each stint in order to capture the main stylized facts of workers' life-cycle wage growth. When we introduce moderate idiosyncratic skill shocks in the Monte Carlos, this augmented occupation-stint-specific fixed effects approach still performs well. But with the large shocks discussed above, the task price estimates depart far from the truth and the instrumental variables version of our method remains the only reasonably accurate estimator.[13] We nevertheless employ this useful alternative approach in empirical robustness checks below.

Finally, another strand of estimation approaches in panel data employs fully spe-

[12] McLaughlin and Bils (2001) use a related approach when estimating industry-specific wage rates and Cavaglia and Etheridge (2017) directly apply Cortes' approach.

[13] It is maybe not surprising that our method, which is derived from an explicit economic model of occupation choice, performs comparatively well in situations where there is indeed a lot of endogenous switching. Paralleling our method, when implemented correctly, the occupation-(stint-)specific fixed effects can only estimate changes in task prices compared to a base period, which is needed to identify a sufficiently rich skill accumulation function. Finally, in our method it is clear how to account for amenity/dynamic values because occupation switches have to be optimal by revealed preference. In contrast, how to do this in the occupation-(stint-)specific fixed effects remains a topic for future research.

cified structural models. Lee and Wolpin (2006) is an important example as well as Yamaguchi (2012) for continuous task measures. Refer to Böhm (2018, Section 3.4) for a more detailed discussion of these papers as well as alternative task price estimation approaches in (repeated) cross-section data.

3 Data and Stylized Facts

3.1 Data

We use the Sample of Integrated Labor Market Biographies (SIAB) provided by the IAB institute at the German Federal Employment Agency. The SIAB is a 2% random sample of administrative social security records from 1975 to 2014. It is representative of 80% of the German workforce and includes employees covered by social security, marginal part-time employment, benefit receipts, officially registered as job-seeking or participating in programs of active labor market policy. The SIAB excludes the self-employed, civil servants, and individuals performing military service. Most notably, it contains an individual's full employment history, the occupation, industry, wage, and important socio-demographics such as age, gender, and level of education. The data is exact to the day as employers need to notify the employment agency if the employment relationship changes.

We restrict the main sample to full-time 25 to 54 year old German men working in former West-Germany. Results for women, wider age ranges, Easterners, and foreigners are qualitatively the same and reported in the Appendix. We transform the SIAB's spell structure into a yearly panel and extrapolate daily wages to obtain equivalent wages per year. To the greatest extent, we prepare the data, including the wage variable, as in Dustmann et al. (2009) and Card et al. (2013). A full description of the dataset construction is in Appendix C.

A key strength of this data is that it contains high-quality longitudinal information on workers' occupations. We use the detailed occupations as well as more aggregated occupation groups. There are 120 different three-digit occupations consistently classified during 1975–2010 in the SIAB scientific use file and we conduct all the analyses using these detailed occupations.[14] However, in order to ease interpretation, we also follow Acemoglu and Autor (2011) and others to group the 120 occupations into broader groups based on their task content. These comprise managers, professionals, and technicians (Mgr-Prof-Tech; relatively intensive in *non-routine analytical* tasks); sales and office workers (Sales-Office; *non-routine interactive*); production, operators and craftsmen (Prod-Op-Crafts; *routine manual*); and services and care workers (Srvc-Care; *non-routine manual*).[15]

3.2 Stylized Facts

Two well-known stylized facts about the German labor market over the last decades are that its employment structure polarized, in line with other major economies (e.g. Acemoglu and Autor, 2011), and that it experienced a dramatic increase of wage inequality across-the-board (Dustmann et al., 2009; Card et al., 2013), again shared by other major economies.

Figure 7 at the end of the paper reproduces these important trends in our dataset. Panel (a) shows that the German wage distribution widened dramatically from 1984 to 2010. Both upper half (difference between 85th and 50th percentile of log wages) and lower half (50–15 difference) inequality have increased strongly during these two-and-a-half decades. At the same time, the employment share of (routine task intensive) Prod-Op-Crafts occupations declined by more than 20 log points from a baseline share of over 60 percent, especially after the early 1990s, whereas the employment share of the other occupation groups increased (Figure 7b). This trend has been termed 'job' or 'employment polarization' because Prod-Op-Crafts workers tend to be located in the middle of the occupational wage distribution.[16]

One may think that two such important trends of the labor market are interrelated and that shifting occupational employment (and wages) have at least partly driven

[14]The occupation classification changes in 2011 to an extent we cannot use the data from there on.

[15]See Appendix Table C.1 for an overview of average task intensity by occupation group and Table C.2 for the mapping of detailed occupations into these groups. The task intensities are from the Qualification and Career Surveys, which Spitz-Oener (2006) describes in detail.

[16]As we can see in Figure 7, there is even a lot more variation in the employment growth of detailed occupations and several individual occupations have bucked the respective trends of their broad groups, both in terms of employment and wages.

the trends in overall inequality. However, a closer look at the facts sheds doubt on this idea even qualitatively: first, employment growth of the very lowest-earning occupation group (Srvc-Care) has actually been strongly positive over this period. In addition, wages in middle-earning Prod-Op-Crafts occupations have actually not fallen (even risen) compared to Srvc-Care (Figure 7c). This hints against a consistent negative shock that may have lowered the demand for low- or middle-earning workers' occupations.[17]

The new empirical fact that we add to this debate zooms into the employment and wage growth of detailed occupations. Figure 1 illustrates once more that the changes in employment of the 120 occupations are very substantial. On the x-axis of Panel (a) we see that during 1984–2010 many occupations grew or declined by more than 50 log points (i.e., respectively 65 percent growth or 40 percent declines compared to their 1984 size). Yet strikingly there is no correlation detectable between occupational employment growth and wage growth on the y-axis. At first glance, this suggests that also detailed occupations' employment growth is unrelated to demand shocks (at least not dominated by them compared to supply shocks) and that occupational changes, albeit quantitatively large, may have no role in explaining the surge of wage inequality over our analysis period.

Panel (b) of Figure 1, however, shows that such a conclusion could be premature because selection effects may be shrouding the relationship between occupation demand, wages, and employment growth. The first and most important finding in Figure 1b is that on average new entrants in *all* of the 120 occupations earn substantially less than the incumbents. The difference is larger for growing sectors and smaller for shrinking ones but always larger than ten log points (10.5 percent). It also exists for leavers compared to stayers (Figure 8a).[18] It further persists, albeit smaller, when controlling for workers' age and education levels (Fig.8b). This suggests that both directly observable as well as unobservable skill differences may drive the wage gap between marginal (entrants or leavers) and infra-marginal (incumbents or stayers) workers.

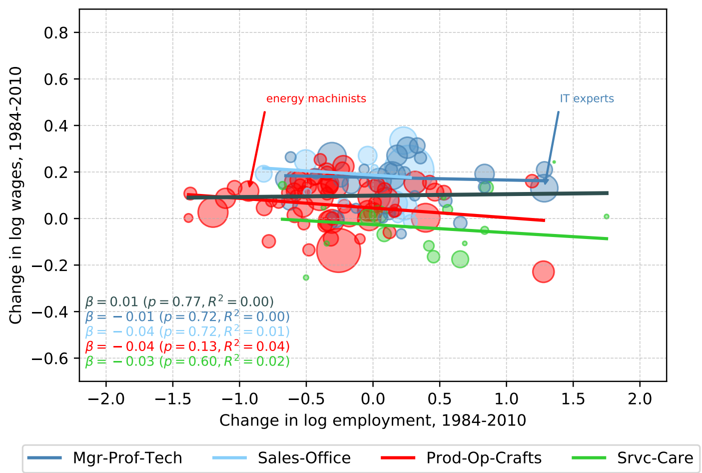
In any case, given that (strongly) growing occupations experience (a lot of) net

[17]Finding zero correlation between wage and employment growth (in the lower half) also led (Dustmann et al., 2009) conclude that the rise of lower-half inequality was not due to changes in demand.

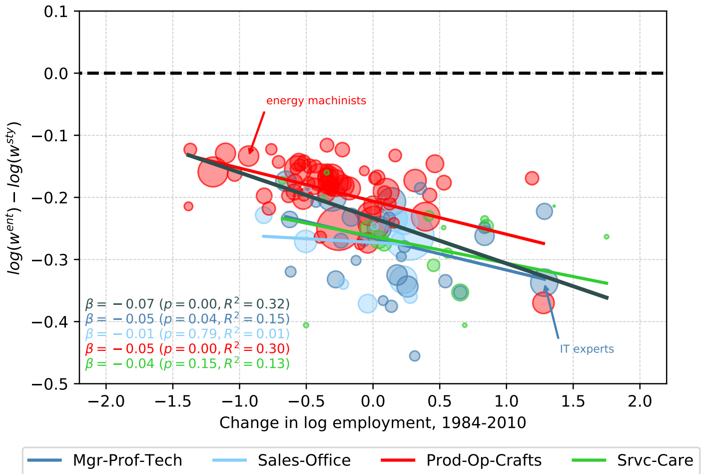
[18]McLaughlin and Bils (2001) obtain the same fact for most of the 22 U.S. industries that they analyze.

Figure 1: Correlation of wages with employment growth

(a) Wage growth vs employment growth



(b) Entrant minus incumbent wages



Notes: A bubble represents one of the detailed 120 occupations with size proportional to its average employment over all sample years. The slope coefficients β (overall and within each occupation group) were computed using weighted least squares where the weight was also equal to the average employment size.

entry of marginal workers, and (strongly) shrinking occupations (a lot of) net exit, this may hold down wages in the former and push up wages in the latter so that in Figure 1a we observe no relationship between the two trends. In the following, we will analyze this possibility, both using our new estimation method of the previous section and quantifying the more informal argument made here. The results from this analysis will also be important because the lack of a relationship between wage and employment growth (in the lower half) may have led prior literature to the conclusion that increases in (lower-half) inequality are unlikely to stem from changes in occupational demand (Dustmann et al., 2009; Mishel et al., 2013).

Finally, another finding in Figure 1b (Figure 8a) is that the wage (skill) differences between entrants (leavers) and incumbents (stayers) are larger the more an occupation grows over time, which is indicated by the negative slopes of the regression lines. This may reflect the fact that the skill pool that growing occupations can draw from narrows with the extent of their expansion (and that only the very lowest skilled workers leave growing occupations), and it may reinforce the potential negative selection effects induced by occupational growth.

4 Estimation Results

4.1 Task Price Changes and Skill Accumulation

This section presents the results from ordinary least squares estimation of

$$\Delta w_{i,t} = \sum_{k=1}^{K} \bar{I}_{k,i,t} \cdot \Delta\pi_{k,t} + \sum_{k=1}^{K}\sum_{k'=1}^{K} \bar{I}_{k,i,t} I_{k',i,t-1} \cdot X'_{i,t-1}\Gamma_{k',k} + v_{i,t} \tag{10}$$

in our sample with 1975–1984 as the base period.[19] In particular, in this main specification $X_{i,t-1}$ contains two dummies for age groups 25–34 and 35–44 in $t-1$ and an intercept representing the omitted age group 45–54. OLS is our main specification because it yields the full set of skill accumulation parameters $\Gamma_{k',k}$, which are also the structural parameters if there are no idiosyncratic skill shocks across occupations. The results using IV are very similar and in the appendix.

Figure 2a depicts the estimation results for the task prices, cumulating the yearly changes over 1984–2010. We see that although there is again substantial heterogeneity within broad groups, task prices on average strongly increased among Mgr-Prof-Tech occupations, modestly increased among Sales-Office and Srvc-Care, and decreased among Prod-Op-Crafts. Prices of the routine Prod-Op-Crafts occupations decline espe-

[19]We do not want to overemphasize this but the early 1980s were also the time when computer use in German workplaces strongly accelerated (Spitz-Oener, 2006). Note once again that, in the most general interpretation, all the task price changes are identified relative to the 1975–1984 base period.

cially after the early 1990s while prices for many Mgr-Prof-Tech occupations strongly rise. This is in contrast to the above-discussed trends between occupation groups where, critically, average wages of Prod-Op-Crafts increased compared to Srvc-Care despite falling employment. It is also broadly consistent with the hypothesized effect of routine-biased technical change (e.g., Acemoglu and Autor, 2011) on the German labor market. We further note that not only Srvc-Care prices increase compared to Prod-Op-Crafts in the figure but also relative prices of growing Mgr-Prof-Tech and Sales-Office occupations rise indeed faster than their relative average wages (most explicit in Figure 9 below).

Figure 2b graphs the estimates of the skill accumulation parameters for stayers (i.e., $\Gamma_{k,k}$) in the 120 detailed occupations as well as for the four broader groups. Skill growth in the early years of the career is steep. Then it slows down mid-career, and almost completely flattens out toward the end of prime age. This reflects the well-established concavity of life-cycle wage profiles (e.g., Lagakos et al., 2018). Another important fact to take away from this figure is that skill growth differs substantially by occupation, growth in high-earning Mgr-Prof-Tech and Sales-Office occupation groups being very steep initially and never completely ceasing whereas growth is flatter and eventually peters out in Prod-Op-Crafts and Srvc-Care.[20] This shows once again that life-cycle wage profiles are decidedly different across occupations and that it is critical in the skill accumulation estimation to control for this fact.

Table 3 at the end of the paper further reports the estimated cross-accumulation parameters for occupation group switchers.[21] Again, we can see that switching into Mgr-Prof-Tech and Sales-Office goes in hand with substantial gains. This, together with the fact that switching for younger workers is in general associated with higher gains than staying in their $t-1$ occupation,[22] suggests that the $\Gamma_{k' \neq k,k}$ estimates do not

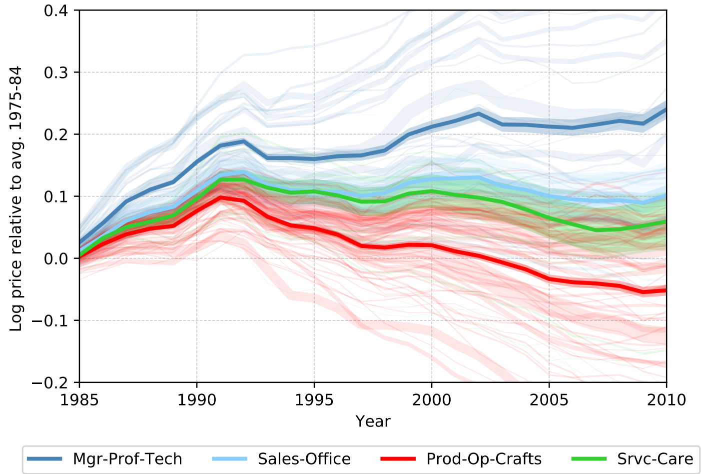
[20]One fact to note about the occupations in Germany compared to other countries is that Sales-Office is quite high-earning. Its average wages are about halfway between Mgr-Prof-Tech and Prod-Op-Crafts, employment is not declining over time, and we estimate rapid skill accumulation as well as rising task prices for this occupation group. It also scores high on non-routine interactive task content. In a related paper, using survey data, Cavaglia and Etheridge (2017) document substantially higher average wages for sales and office occupations in Germany than in the U.K..

[21]82% $(11,664)$ of the theoretically possible $120 \times 119 = 14,280$ switch combinations between the detailed $t-1$ and t occupations appear at least once in our data. Among all switches, 65% occur in the top 10% combinations. 15% are singletons (i.e., only one switch ever occurs).

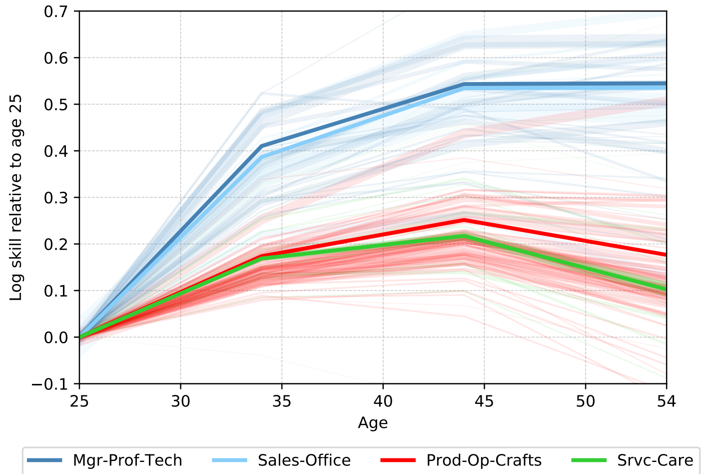
[22]In the appendix, we also present robustness estimates where we allow for intermittent unemploy-

Figure 2: Price changes and skill accumulation of occupation stayers

(a) Task prices



(b) Skill accumulation



Notes: Shaded lines in the background are the detailed 120 occupations with the thickness of the lines proportional to their average employment over all sample years. To summarize the broad trends, the solid lines in the foreground show the four aggregated occupation groups. The areas around these four lines are 95% confidence intervals.

only reflect the structural skill accumulation parameters but also some idiosyncratic skill shocks or learning about skills as discussed in Section 2. In either case, the key task at hand for the skill accumulation function is to appropriately control for any kind of wage growth that may be due to observables or unobservables changing over the career. Therefore, the detailed controls for age as well as occupation used here are

ment between jobs and the cross-accumulation parameter estimates are accordingly somewhat lower (without substantive changes to the task price estimates).

important to correctly distinguish differential life-cycle wage profiles from changes in task prices. We discuss several robustness checks using alternative specifications (including instrumental variables) and samples below.

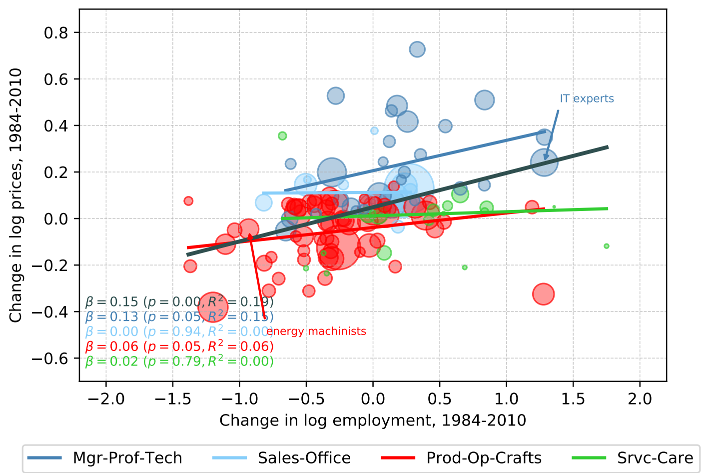
The key finding of this section is that employment growth and task price growth indeed go hand in hand. Figure 3a hones in on this result, depicting detailed occupations' log employment changes over 1984–2010 on the x-axis and cumulated task price changes on the y-axis. We see that, while there is quite some variation as discussed above, employment in Mgr-Prof-Tech, Sales-Office, and Srvc-Care occupations was growing in addition to their rising task prices whereas employment and task prices were on average falling together in Prod-Op-Crafts occupations. The upward-sloping black regression line summarizes this strong relationship across the 120 occupations, which is in marked contrast to wages that are not corrected for selection (i.e., Figure 1a). In addition, the relationship also holds within occupation groups, the respective sub-regression lines being significantly positive for the large Mgr-Prof-Tech and Prod-Op-Crafts groups.[23]

Finally, the deviations of the bubbles from the overall regression line in Figure 3a are potentially informative about elasticities of labor supply to different occupations. Since its employment grows, the bubbles for Mgr-Prof-Tech are more to the right of the graph. But the bubbles and their regression line are also located substantially above the overall regression line, which shows that wages grow relatively strongly compared to employment. This could be the case if labor supply to the Mgr-Prof-Tech occupations is rather inelastic (i.e., requires a high price change for a given employment change), which we think is plausible. The relatively steep slope of the Mgr-Prof-Tech regression line indicates that this is also the case for relative changes within this group. Accordingly, the bubbles for the (formally less educated) other occupation groups are located below Mgr-Prof-Tech and their within-group regression lines are also flatter, which suggests that labor supply to these occupations (and groups) is more elastic.

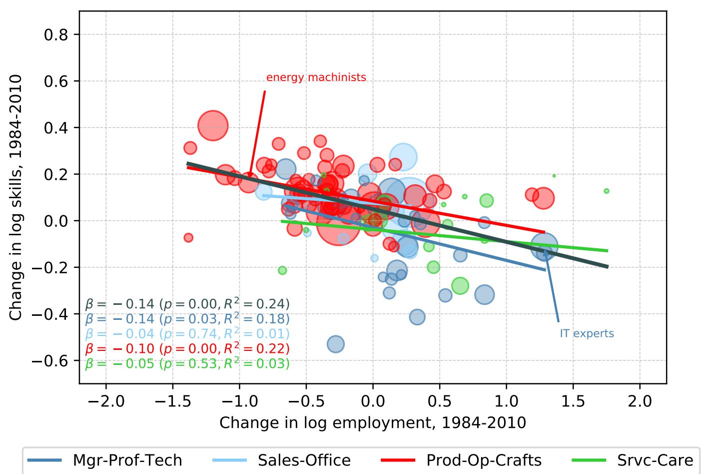
[23] Admittedly, we have not adjusted the standard errors for the sampling variation of the task price estimates because bootstrap is too demanding in this large dataset with our current computational resources. The relationship is also hardly positive or flat within the smaller Srvc-Care and Sales-Office groups, and it becomes negative for some of the robustness checks in the appendix. The interpretation of this is that, when comparing the relative employment growth of detailed occupations within Srvc-Care or Sales-Office, supply shocks have mattered, too.

Figure 3: Employment growth with estimated task prices and implied skill changes

(a) Price vs employment growth



(b) Skill vs employment growth



Notes: One bubble represents one of 120 occupations available in the SIAB data. The size of a bubble is proportional to the average size of the occupation across all sample years. The slope coefficients β were computed with weighted least squares where the weight was also equal to the average occupation size.

In order to separate task prices from life-cycle wage growth, the estimation method imposes that the skill accumulation function is time-invariant. In fact, the key assumption in our main specification is that the *relative* skill accumulation parameters between age groups within occupations are time-constant, since across-the-board changes of occupational wage growth can be re-interpreted without loss of economic insight as accelerating or decelerating task prices (see discussion in Section 2.2). Appendix D.1 provides some support that constancy of *relative* skill accumulation is a reasonable assumption by showing that differences in within-occupation wage growth between 35–44 (25–34) and 45–54 year olds have not (hardly) changed over time.[24] In Appendix D.2 we also estimate the task prices using different age ranges and find qualitatively similar results.

Appendix D conducts empirical robustness checks more generally. We estimate the task prices in different samples including women and foreigners, other base years than 1975–84, four-year instead of one-year period length, and including workers who are unemployed or out of the labor force. We also estimate the alternative specifications discussed in Section 2 by extending the skill accumulation function to include education groups, instrumenting current with last period occupation choices, and employ the alternative occupation-specific fixed effects approach. The results in these different samples and specifications indicate that, among others, our estimates are largely robust to workers' forward-looking behavior, alternative forces affecting women and minorities, choice of base period, switching costs, modeling of unemployment, and some potential endogeneity concerns (detailed discussion in Appendix D).

The task price estimation has therefore uncovered that broad as well as detailed occupations' wage and employment growth on average go hand in hand. This is consistent with the notion that occupational trends are generally (though probably not always) driven by common demand shocks. In the next sections, we will analyze the implied effects on skill selection that this finding has for occupations and in the process provide an additional independent piece of evidence supporting the results here. We will then examine the implications of occupational employment growth, skill accumulation, and task price estimates for the changes of overall German wage inequality.

4.2 Accounting for Skill Selection

Figure 3b depicts occupational employment growth against the cumulative changes of average skills that are implied by the task price estimates. For every occupation,

[24]Using just occupation stayers, we subtract one age group's wage growth from another and task prices drop out. That is, we plot a non-parametric proxy for $\gamma_{k,k,a} - \gamma_{k,k,a'}$ over time, where $\gamma_{k,k,a}$ is a scalar element of $\Gamma_{k',k}$ indicating skill accumulation for an occupation k staying worker in age group a. We do not test constancy of switchers' wage growth over time since this may also change because of changing non-pecuniary or forward-looking valuations (see Appendix A.2.2).

these are the differences between growth in task prices (Fig.3a) minus average wages (Fig.1a). Making use of log wages' additive separability into prices and skills, it is the bottom right term of

$$\underbrace{\mathrm{E}[w_{i,t}|I_{k,i,t}=1]-\mathrm{E}[w_{i,t-1}|I_{k,i,t-1}=1]}_{\text{mean wage change}} = \underbrace{\Delta\pi_{k,t}}_{\text{price change}} + \underbrace{\mathrm{E}[s_{k,i,t}|I_{k,i,t}=1]-\mathrm{E}[s_{k,i,t-1}|I_{k,i,t-1}=1]}_{\text{mean skill change}} \tag{11}$$

summed over the years $t = 1985, \ldots, 2010$. Figure 3b thus shows that implied skill changes constitute the flipside of the task price estimates in the sense that skills of growing occupations strongly (and statistically significantly) decline. The regression line in the figure for example indicates that skills of the most growing occupations declined by up to 40 log points compared to the most shrinking occupations. Figure 9 at the end of the paper plots the dynamics of the implied skill changes over time, similar to the figure for task prices above, and it shows that, for each of the growing occupation groups, skills deteriorate relative to the shrinking Prod-Op-Crafts group.

The purpose of this section is to analyze in more detail the components and the plausibility of this implied skill change. We have documented in Section 3.2 that entering (leaving) workers' skills on average appear decidedly below those of incumbents (stayers) and that faster-growing occupations draw even less skilled entrants (leavers), which is both consistent with the Roy model.[25] Given that (strongly) growing sectors by definition experience (large) positive net entry, this could substantially drag down growing occupations' wages despite rising demand and accordingly increasing task prices. Here we formalize and quantify this effect.

[25] In the terminology of McLaughlin and Bils (2001) or Young (2014) this is consistent with a Roy model where comparative advantage is aligned with absolute advantage across sectors. Notice however in this paper we document that workers' skills are substantially changing over time and therefore introduce occupation-specific skill accumulation as well as idiosyncratic shocks to our estimation method. This is a critical departure from the earlier literature with a more static view of workers' comparative advantage. Below we find that differences in skill accumulation and idiosyncratic shocks are also key to rationalize the large negative (positive) selection effects into growing (shrinking) occupations.

We decompose the change of mean skills from Equation (11) into three parts:

$$
\begin{aligned}
\mathrm{E}[s_{k,i,t}|I_{k,i,t}=1]-\mathrm{E}[s_{k,i,t-1}|I_{k,i,t-1}=1] = & \underbrace{\left(1-\frac{p^{lvr}_{k,t-1}+p^{ent}_{k,t}}{2}\right)\cdot \mathrm{E}[\Delta s^{sty}_{k,i,t}]}_{\text{1. accumulation: of stayers}} \\
& + \underbrace{\frac{p^{lvr}_{k,t-1}+p^{ent}_{k,t}}{2}\cdot\left(\mathrm{E}[s^{ent}_{k,i,t}]-\mathrm{E}[s^{lvr}_{k,i,t-1}]\right)}_{\text{2. churning: difference entrants, leavers}} \\
+ & \underbrace{\left(p^{ent}_{k,t}-p^{lvr}_{k,t-1}\right)\cdot\left(\frac{\mathrm{E}[s^{ent}_{k,i,t}]-\mathrm{E}[s^{sty}_{k,i,t}]}{2}+\frac{\mathrm{E}[s^{lvr}_{k,i,t-1}]-\mathrm{E}[s^{sty}_{k,i,t-1}]}{2}]\right)}_{\text{3. marginal selection}}
\end{aligned} \tag{12}
$$

where superscript sty indicates a occupation stayer in k from $t-1$ to t, lvr a leaver moving out of k, and ent an entrant to k. The share of last period's workers in k who left the occupation after that period is represented by $p^{lvr}_{k,t-1}$ and the share of this period's workers who entered this period by $p^{ent}_{k,t}$.[26]

The first term of Equation (12) reflects the skill accumulation of workers who stay in the occupation between the two periods. That is, if skill accumulation $\mathrm{E}[\Delta s^{sty}_{k,i,t}]$ in an occupation k is high, as in the Mgr-Prof-Tech and Sales-Office groups according to our estimates, this raises average skill growth from $t-1$ to t. But at the same time it also tends to lead to a more negative difference in skills $\left(\mathrm{E}[s^{ent}_{k,i,t}]-\mathrm{E}[s^{lvr}_{k,i,t-1}]\right)$ between entrants and leavers and the deteriorating impact of churning (second term of (12)) on skill growth will be stronger. High turnover of workers in the occupation is negative for the first as well as the second term because, ceteris paribus, it lowers the share of staying workers who can benefit from skill accumulation and it raises the churning component, respectively. There may be more entrants than leavers when the occupation is growing ($p^{ent}_{k,t} > p^{lvr}_{k,t-1}$) or vice versa. Thus $\frac{p^{lvr}_{k,t-1}+p^{ent}_{k,t}}{2}$ represents the *average*

[26] Formally, the components are defined as (re-arranging them we can obtain Equation (12)):

$$
\begin{aligned}
\mathrm{E}[s_{k,i,t}|I_{k,i,t}=1]-\mathrm{E}[s_{k,i,t-1}|I_{k,i,t-1}=1] = & \Big[\underbrace{\mathrm{E}[s_{k,i,t}|I_{k,i,t}=1,I_{k,i,t-1}=1]}_{\mathrm{E}[s^{sty}_{k,i,t}]}\underbrace{P(I_{k,i,t-1}=1|I_{k,i,t}=1)}_{1-p^{ent}_{k,t}} \\
+\underbrace{\mathrm{E}[s_{k,i,t}|I_{k,i,t}=1,I_{k,i,t-1}=0]}_{\mathrm{E}[s^{ent}_{k,i,t}]}\underbrace{P(I_{k,i,t-1}=0|I_{k,i,t}=1)}_{p^{ent}_{k,t}}\Big] & - \Big[\underbrace{\mathrm{E}[s_{k,i,t-1}|I_{k,i,t-1}=1,I_{k,i,t}=1]}_{\mathrm{E}[s^{sty}_{k,i,t-1}]} \\
\times\underbrace{P(I_{k,i,t}=1|I_{k,i,t-1}=1)}_{1-p^{lvr}_{k,t-1}}+\underbrace{\mathrm{E}[s_{k,i,t-1}|I_{k,i,t-1}=1,I_{k,i,t}=0]}_{\mathrm{E}[s^{lvr}_{k,i,t-1}]} & \underbrace{P(I_{k,i,t}=0|I_{k,i,t-1}=1)}_{p^{lvr}_{k,t-1}}\Big].
\end{aligned}
$$

turnover here.

As a first order approximation, both the accumulation and the churning effect are unrelated to an occupation's growth or decline. They should also partly offset each other because occupations that have high average skill accumulation will feature more negative differences of skills between entrants and leavers.[27] Marginal selection, the third term of (12), in contrast is directly related to employment growth because it is the product of the difference in skills between marginal and inframarginal workers in an occupation times net entry. We have documented above that both entrants' and leavers' skills are substantially lower than incumbents' and stayers' skills on average and hence both $\frac{\mathrm{E}[s^{ent}_{k,i,t}]-\mathrm{E}[s^{sty}_{k,i,t}]}{2} < 0$ and $\frac{\mathrm{E}[s^{lvr}_{k,i,t-1}]-\mathrm{E}[s^{sty}_{k,i,t-1}]}{2} < 0$. Therefore, if the occupation grows ($p^{ent}_{k,t} > p^{lvr}_{k,t-1}$), the marginal selection effect is negative, it is positive when the occupation shrinks, and it is unambiguously zero when there is no change in size ($p^{ent}_{k,t} = p^{lvr}_{k,t-1}$). Marginal selection thus formalizes and quantifies the intuition developed in Section 3.2, whereby the more an occupation grows the more net entry of less skilled workers it experiences.

Figure 4a shows the cumulated marginal selection effect on log skills (y-axis) together with occupational employment growth on the x-axis. Unsurprisingly given Equation (12), these two variables are negatively correlated. But they align almost perfectly and the relationship is similarly strong as the relationship of implied skill changes from the estimation (i.e., -0.15 regression slope versus -0.14 in Fig.3b). Figure 10 at the end of the paper shows the other two effects of Equation (12), accumulation and churning, which actually almost perfectly offset each other on average.[28]

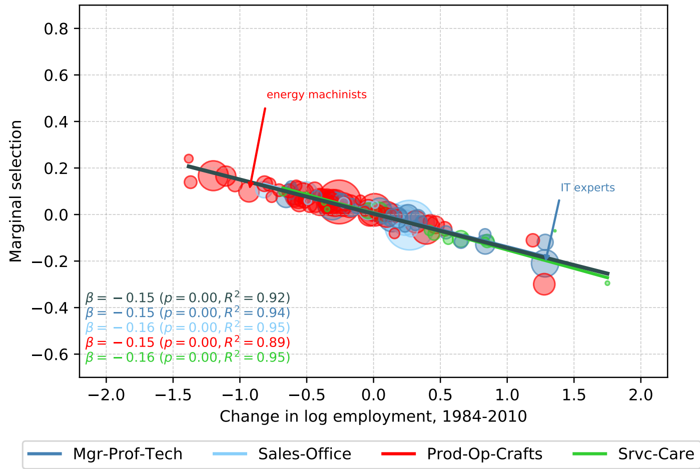
Therefore, as discussed above, the marginal selection effect should, on average, constitute a substantial share of the skill changes implied by the difference between

[27]In a 'stable occupation', in the sense that employment size and skill composition are constant, they in fact cancel each other out. Mathematically, if skills in the occupation are constant (i.e., $\mathrm{E}[s_{k,i,t}|I_{k,i,t} = 1] = \mathrm{E}[s_{k,i,t-1}|I_{k,i,t-1} = 1]$) and marginal selection is zero because of constant employment (i.e., $p^{ent}_{k,t} = p^{lvr}_{k,t-1}$), the first and second right-hand term in (12) perfectly offset each other. That is, the skill accumulation of staying workers makes up exactly the difference in skills between entrants and leavers.

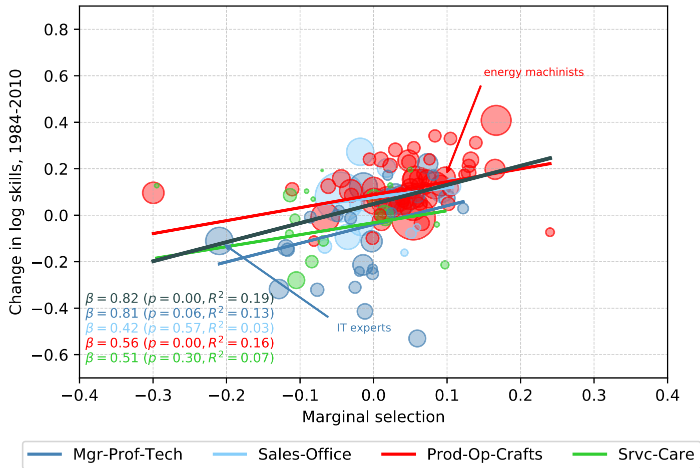
[28]That is, regression slopes of the accumulation and churning effects on employment growth are, respectively, 0.16 and -0.15. The accumulation effect rises with employment growth in Figure 10 mainly because high-accumulation Mgr-Prof-Tech and Sales-Office occupations tend to grow and the many low-accumulation occupations in Prod-Op-Crafts tend to shrink. The churning effect becomes more negative with employment growth as $\mathrm{E}[s^{ent}_{k,i,t}]$ is much lower (Fig.1b) and $\mathrm{E}[s^{lvr}_{k,i,t-1}]$ only modestly lower (Fig.8a) for faster growing occupations.

Figure 4: Skill Selection Effects

(a) Selection vs employment growth



(b) Selection vs skill growth



Notes: One bubble represents one of 120 occupations available in the SIAB data. The size of a bubble is proportional to the average size of the occupation across all sample years. The slope coefficients β were computed with weighted least squares where the weight was also equal to the average occupation size.

wages and estimated task prices. In Figure 4b, we finally depict these two variables together and indeed the bivariate regression coefficient between implied and marginal selection is 0.82. That is, a one log point higher marginal selection effect is associated with a $4/5$ log point higher skill change implied by the estimation. We see in Figure 4b that this relationship is not statistically perfect as there is variation around the regression line and quantitatively it is also not exactly 1:1. The accounting reasons for this are that accumulation and churning vary across individual occupations and that their combined effect (i.e., the sum) on average slightly increases with employment growth. Economically, this is also not surprising since marginal selection only reflects workers' skill differences exactly when they enter or leave the occupation but systematic or idiosyncratic skill accumulation may (and probably should) be different in between.[29]

Table 1 examines the contributing factors to the marginal selection effect, which as in Figure 4b is on average positive in the shrinking Prod-Op-Crafts and negative in the three growing occupation groups (top panel). We first use our estimates and the longitudinal dimension of the SIAB data to decompose workers' skill differences into "endowments" at the most recent entry into the occupation as well as systematic accumulation (i.e., due to $\Gamma_{k,k}$) and idiosyncratic skill shocks during their continuous stay in the occupation. The details are in Appendix E.1, which also shows this decomposition separately for entrants (who at that point by definition have zero accumulation and shocks) and leavers versus stayers.

The middle panel of Table 1 reports the results. It turns out that only about 25–30 percent of the skill differences with stayers in Mgr-Prof-Tech and Sales-Office are due to endowments and the majority of differences in these high-accumulation occupations are indeed due to skill accumulation. At the same time, more positive idiosyncratic skill shocks for staying workers than either entrants or leavers make up another substantial ≈20 percent of the differences, which underscores the importance of our above discussion that also staying workers are (positively) endogenously selected.[30] The differences for Prod-Op-Crafts and Srvc-Care, which are lower-accumulation occupations, are tilted toward endowments and skill shocks but qualitatively the results are the same.

Second, the bottom panel of Table 1 decomposes the contributions to the marginal

[29]Panels (c)–(d) of Figure 9 show the dynamics of the decomposition (12) for the other three occupation groups *relative to* Prod-Op-Crafts over time. Paralleling the implied skill changes, relative marginal selection is negative and increasingly so in these growing groups. Also, as discussed, the relative accumulation effects are strongly positive and at the same time the churning effects strongly negative in the high-accumulation Mgr-Prof-Tech and Sales-Office groups, on average largely offsetting each other.

[30]Maybe unsurprisingly, endowment and systematic accumulation differences with stayers are larger for entrants but they are also important for leavers. The leavers on average experience negative idiosyncratic skill shocks in the occupation so that this difference with stayers is even larger for them than for entrants. The relative roles of endowments versus accumulation do change when task price estimates are only accelerations or decelerations compared to the base period but the idiosyncratic skill shocks' contribution remains the same in this more general interpretation. See Appendix E.1.

Table 1: Decomposition of the Marginal Selection Effect, 1984–2010

	Mgr-Prof-Tech	Sales-Office	Prod-Op-Crafts	Srvc-Care
Marginal selection	-3.1	-2.0	5.9	-4.8
Endowments	-0.9	-0.5	2.3	-3.4
Accumulation	-1.6	-1.2	1.6	-0.7
Shocks	-0.6	-0.3	1.9	-0.7
Unemployed	-0.6	-0.3	2.9	-1.1
Out of labor force	0.0	0.1	0.9	0.3
Occupation switchers	-1.0	-0.4	0.3	-1.1
Starters or exiters	-1.6	-1.3	1.8	-2.9

Notes: Values are averages between entrants and leavers, multiplied by 100 and relative to 1984 (i.e., skill changes in log points).

selection effect by types of workers. We see that in all occupations a large part of this is due to sample starters (for entrants) and exiters (for leavers). The largest component for Prod-Op-Crafts are workers moving into and out of unemployment (especially leavers to unemployment; see Appendix E.1) while switches between occupations are also important in Mgr-Prof-Tech, Sales-Office, and Srvc-Care. Finally, switches into and out of the labor force are less important and they are actually modestly working in the opposite direction of the overall marginal selection effect for these three rising occupation groups. This might be due to entrants from abroad, self-employment, civil service, or the military who are not part of the SIAB sample (see Sect.3) and who can be rather high-skilled.

To sum up, it turns out that the marginal selection effect and the skill changes implied by the task price estimates are statistically and economically strongly related. The marginal selection effect does not depend on the estimated task prices since we empirically implement the skill differences $\mathrm{E}[s_{k,i,t}^{ent}] - \mathrm{E}[s_{k,i,t}^{sty}]$ and $\mathrm{E}[s_{k,i,t-1}^{lvr}] - \mathrm{E}[s_{k,i,t-1}^{sty}]$ by using respective contemporaneous wages. In fact, a large part of these differences are due to differences in skill accumulation and idiosyncratic shocks between entrants and leavers versus stayers, which supports our flexible modeling of these (endogenous) skill changes over time. In addition, the marginal selection effect is due to sector growth. It is exactly zero when employment in occupation k is constant, positive when employment declines, and negative when it rises because the skill difference between entrants (or leavers) and occupation incumbents is always negative in the data. Therefore, it must work in the direction of our estimated skill changes. For these reasons, we consider the results on the marginal selection effect substantive independent evidence in favor of our estimation model and for the plausibility of our task price estimates.

5 Task Prices and Wage Inequality

German wage inequality has risen dramatically over the past decades. This section analyzes, through the lens of our model, the role that occupations played in this increase. We use the estimated task price changes and life cycle skill accumulation as well as the shifting employment structure to evaluate how inequality would have changed if only these factors or subsets thereof had influenced the wage distribution.

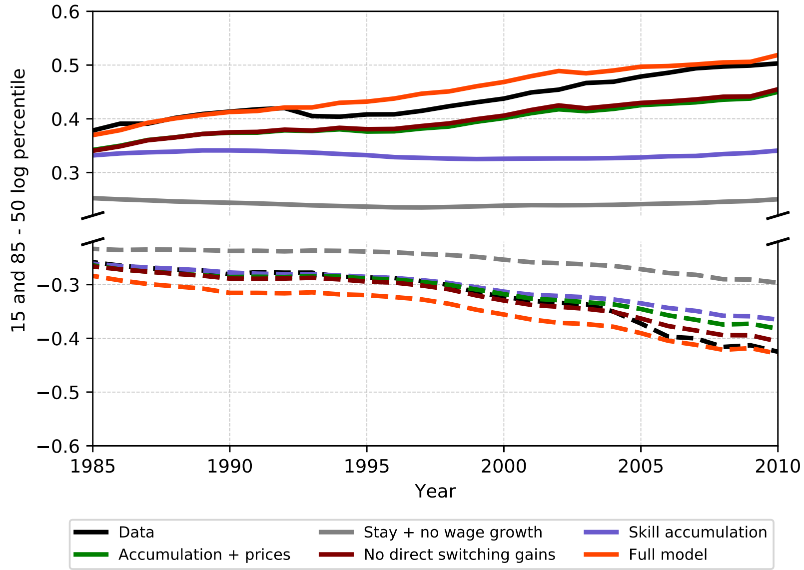
Figure 5 shows the actual and the predicted wage distributions when different contributing factors are switched on or off. The solid black line represents the difference between the 85th and 50th percentile of the actual log wage distribution (upper-half inequality) and the dashed black line the difference between the 15th and 50th percentile (lower-half inequality). As we already discussed in Section 3 (and as documented in Dustmann et al., 2009; Card et al., 2013, among others), both upper and lower half inequality have increased strongly in Germany. During the analysis period, the 85–50 differential rose by 14 log points from 38 to 52 and the 50–15 differential by 15 log points from 21 to 37 log points (detailed numbers in Table 2 below).

Within our model, inequality can increase because of changes in task prices paid for occupations or because of changes in the employment structure, which may interact with skill accumulation and the task prices. We build up the full model adding components sequentially and start by examining the role of shifting employment structure and wages at labor market entry over time. The light solid and dashed grey lines depict the trend in inequality that would prevail if only the wage distribution at age 25 (or an older age for later entrants) had shifted, with changes in task prices as well as skill accumulation or occupation-switching over the life-cycle turned off: that is, $\hat{w}^e_{i,t} = w_{i,t_{i,0}}$, where $t_{i,0} < t$ denotes the year when worker i joins the labor market.[31] We

[31] As a robustness check we have also 'deflated' later entrants' wages back to what they would have earned in 1984 using our task price estimates. This did not make a discernable difference to the light grey line in the figure. In all of the scenarios, workers over the age of 54 leave the sample with the wage

see that this explains hardly any of the increase in upper-half inequality and a modest five log points of lower-half inequality.

Figure 5: Components of wage percentiles relative to the median



Notice that in this scenario we impute the initial occupations and wages of workers who entered the labor market before the beginning of our sample, that is, who already appear in the 1975 data aged older than 25.[32] This data limitation and the impact of the imputation naturally affects the wage distribution early in the period more than later. In fact, given that workers leave the sample at age 54, after the year 2003 nobody is imputed anymore. Alternatively, we have not imputed at all and taken the first observed wage as the initial one, which actually decreased 50–15 differences in 1984 to XX log points and thus modestly raised the increase of lower-half inequality in this scenario. Conversely, we have also imputed the initial occupations and wages equally across the analysis period by pretending we do not observe workers' labor market histories more than nine years (i.e., the difference between 1984 and 1975) in the past. This reduces what the entry-only scenario can explain of the increase in lower-half inequality. In all our robustness checks regarding this data limitation, the contribution

that they have in that scenario.

[32]We do this assuming the first observed to also be the initial occupation and assign these workers an approximated entry age by calculating the average occupation-specific entry age across all years from 1976XX onward. We then impute the wage at labor market entry by subtracting the respective skill accumulation coefficients back to that entry age. More details are discussed in Appendix E.2.

of initial wages remained limited (also in the upper half of the distribution), and the contribution of changing age structure, which we discuss below, was more important in the lower-half.

The next scenario adds skill accumulation to the changing initial wages over time. In particular, the violet lines show the inequality due to changing initial occupation distribution, age structure of employment, and associated changes of skill accumulation over time:

$$\hat{w}_{i,t}^{e+a} = w_{i,t_{i,0}} + \sum_{\tau=t_{i,0}+1}^{t} \sum_{k=1}^{K} I_{k,i,t_{i,0}} I_{i,\tau-1} \cdot X'_{i,\tau-1} \hat{\Gamma}_{k,k}, \tag{13}$$

where the worker entered the labor market in time $t_{i,0} \leq t$, never switches (i.e., $I_{k,i,t_{i,0}}$ does not change with τ), and $I_{i,\tau-1}$ indicates whether the worker was employed in $\tau - 1$.[33] This scenario explains a substantial part of the level of upper-half inequality (shift from 25 to 33 log points in 1984) but again hardly any of its increase over time (one log point in Table 2). The $\hat{w}_{i,t}^{e+a}$ scenario does however generate a strong rise of inequality in the bottom segment of Figure 5. Shifting initial wages and changes in skill accumulation alone account for 12 out of the 15 log points increase in the lower half according to Table 2. This is an important finding in our analysis of the wage distribution because it has previously been hard to rationalize polarizing demand for occupations together with wage inequality that increased across the board in most countries and time periods (Goos and Manning, 2007; Mishel et al., 2013; Green and Sand, 2015; Naticchioni et al., 2014).

Figure 6 illustrates some potential drivers of the $\hat{w}_{i,t}^{e+a}$ scenario's effect on 50–15 inequality, depicting the shares of broad age and occupation groups within every quintile of the wage distribution. In 2010 (Panel b), there are more high-accumulation occupations (Mgr-Prof-Tech and Sales-Office) around the median than in the bottom quintile. There are also more middle-aged or older workers, who should have accumulated substantial amounts of skill over their labor market experience, and much less young

[33] That is, we assume that skills stagnate during non-employment spells. Also, workers who are currently unemployed or out of the labor force do not enter any of the scenarios in Figure 5. In Figure D.8 we compute the (predicted) wage distribution using the filled non-employment spells as in Appendix D.2.6 and find that the qualitative results do not change.

Table 2: Adding up the components of rising wage inequality

		Absolute 1984	Relative to 1984 1997	2010
median	$\log(p_{50})$	10.42	0.11	0.12
lower half	$\log(p_{15}) - \log(p_{50})$	-0.25	-0.04	-0.17
scenarios	stay, no wage growth		-0.01	-0.06
	+ skill accumulation		-0.03	-0.11
	+ prices		-0.04	-0.12
	+ switching		-0.04	-0.15
	+ direct gains from switching		-0.05	-0.15
upper half	$\log(p_{85}) - \log(p_{50})$	0.37	0.05	0.13
scenarios	stay, no wage growth		-0.02	-0.00
	+ skill accumulation		-0.00	0.01
	+ prices		0.05	0.12
	+ switching		0.06	0.13
	+ direct gains from switching		0.09	0.17
standard deviation	$\sigma(w_{i,t})$	0.36	0.06	0.15
scenarios	stay, no wage growth		-0.01	0.04
	+ skill accumulation		0.01	0.06
	+ prices		0.04	0.11
	+ switching		0.04	0.12
	+ direct gains from switching		0.06	0.14

(inexperienced) workers in the middle of the wage distribution. This is especially the case among the large group of Prod-Op-Crafts workers (red shades). Both of these facts largely hold up when comparing to the wage distribution in 1984 (Panel b) and therefore they may have been responsible for the stronger effect of skill accumulation on lower-half wage inequality at the end compared to the beginning of the analysis period.

Appendix E.2 studies these factors' role for rising lower-half inequality in detail. It also shows the corresponding graph to Figure 6 for only the $\hat{w}_{i,t}^{e+a}$ scenario wage and occupation distribution. We find that the increasing number of experienced workers in the middle quintile explains 4 log points of the rising median of the wage distribution due to accumulation, and that the occupation structure by itself actually plays no role. The remainder of the skill accumulation's (3 log points) effect can be accounted for by the imputation of wages or, alternatively, there is no remainder if we impute initial wages and occupations equally across the analysis period as discussed above (see Appendix E.2 for all the details). Therefore, demographic change with many baby boom generation workers, who at the time entered the labor market in Prod-Op-Crafts but still have accumulated a lot of skills (i.e., $t - t_{i,0}$ in Equation (13) is large for those workers), drove a substantial part of rising lower-half wage inequality in Germany.[34]

The next scenario includes estimated task prices. Note that from now on, results are independent of the imputation discussed above, as the sum of initial wages plus skill accumulation is invariant to these initial conditions.[35] We add the changing task prices to the model yet still do not allow for occupation switching. The full counterfactual wage when workers stay in their initial occupation throughout the life-cycle becomes:

$$\hat{w}_{i,t}^{e+a+p} = w_{i,t_{i,0}} + \sum_{\tau=t_{i,0}+1}^{t} \sum_{k=1}^{K} I_{k,i,t_{i,0}} \left(\Delta\hat{\pi}_{k,\tau} + I_{i,\tau-1} \cdot X'_{i,\tau-1} \hat{\Gamma}_{k,k} \right) \tag{14}$$

This is depicted in the green lines of Figure 5. We see that the changing task prices indeed account for the majority of the increasing inequality in the upper half of the wage distribution. According to Table 2, task prices raise the 85–50 differential by nine log points on top of changing initial wages and skill accumulation, resulting in a scenario which explains 12 out of the 14 log points increase.

Figure 6 again illustrates some of the reasons for the striking role of task prices in the upper half by showing that the vast majority (above 90 percent in 2010) of occupations in the wage distribution's top quintile are Mgr-Prof-Tech and Sales-Office.[36] The representation of these occupation groups, for which task prices strongly rise, is much

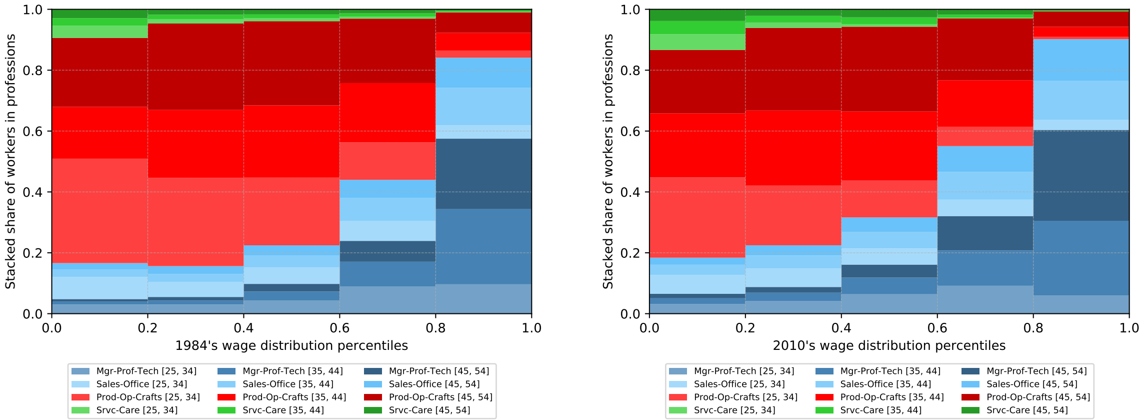
[34] In the top quintile of the wage distribution the age structure is not that different from the median and the occupation structure is already very different in 1984. Therefore, there is not much of an effect of skill accumulation on changing wage inequality in the upper half.

[35] Of course, the sequencing of scenarios matters for different factors' additional effect sizes. We start with initial wages and skill accumulation because these cannot be perfectly distinguished with the assumptions inherent in the imputation. But this is also an intuitive sequencing and rather conservative in terms of the quantitative role that task prices can play. Moreover, the dominant effect of task prices on between-occupation inequality shown in Appendix E.3 is for example independent of the sequencing.

[36] Strictly speaking, one may want to use the representation of occupations in the wage distribution in the $\hat{w}_{i,t}^{e+a+p}$ scenario at hand (not depicted for brevity) or in the $\hat{w}_{i,t}^{e+a}$ scenario before the task prices are added (shown and discussed in Appendix Figure E.2). Consistent with our argument, in both cases, the representation of 'blue high-price-increase' occupations is much larger at the top than middle wage quintile and it also strongly increases from the latter to the former. Also note that the comparison to 1984 is not important for the impact of the changing task prices. Their different representations in the 2010 distribution account for the task prices' effect.

Figure 6: Shares in the Overall Wage Distribution by Quintile

(a) 1984 (b) 2010



Notes: Each rectangle is proportional to the share of workers in the respective occupation × age bin.

lower at the median (circa 30 percent) and the representation of Prod-Op-Crafts, for which task prices fall, is much higher. Task prices are not so important at the bottom of the wage distribution, only explaining an additional two log points in Table 2, because the representation of price-increasing 'blue and green' occupation groups is roughly similar in the middle and bottom quintiles. That their contribution is not zero stems from an overtaking effect described in more detail in Böhm (2018), whereby rising task prices mainly make some low-earning Mgr-Prof-Tech and Sales-Office workers move up in the wage distribution and raise with them the percentiles around the median where they end up.[37]

Finally, we compute the full economic model, including occupation switching with associated skill and task price changes. The wage in this scenario becomes:

$$\hat{w}_{i,t}^{e+a+p+s} = w_{i,t_{i,0}} + \sum_{\tau=t_{i,0}+1}^{t} \sum_{k=1}^{K} \sum_{k'=1}^{K} \bar{I}_{k,i,\tau} \left(\Delta\hat{\pi}_{k,\tau} + I_{k',i,\tau-1} \cdot X'_{i,\tau-1} \hat{\Gamma}_{k',k} \right) \tag{15}$$

Here notably $I_{k',i,\tau-1}$ and $\bar{I}_{k,i,\tau}$ can be zero for all K occupations if the worker was unemployed or out of the labor force in the respective period.[38] The orange lines in Figure 5 indicate that the full model can account for essentially the entire increase of

[37] Accordingly, in the $\hat{w}_{i,t}^{e+a}$ scenario before task prices are added (Figure E.2), the share of 'blue' workers in the middle quintile is only 20 percent.

[38] We do assign workers who return from non-employment one $\hat{\Gamma}_{k',k}$ for their previous k' and current k occupation combination, however. This does not make a material difference for the scenario and, for ease of notation, it is not explicitly indicated in Equation (15).

85–50 as well as 50–15 differences over the period from 1984 to 2010. In particular, the whole trend of upper-half inequality is explained and an additional two of the remaining four log points increase in the lower half accounted for. A large part of this final gain from adding occupation moves arises directly at the time of switching and is thus associated with cross-accumulation parameters (see Table 3 for broad occupation groups). Therefore, one could interpret this scenario partly as adding idiosyncratic skill shocks and it is probably not surprising that such shocks during the career should have an additional impact on the overall wage distribution.

The last panel in Table 2 reports the trends of another inequality measure, the standard deviation of log wages which increased by XX points during 1984–2010. Our economic model can explain a 70pctXX of this trend (XX log points) with the skill accumulation scenario and changing task prices on top of it again playing the most important roles. Appendix E.3 shows that dispersing task prices and rising task prices in what are already high-wage occupations make up at least half of the increased standard deviation of log wages. A substantial part of this is not visible in descriptive between-versus-within wage decompositions because it is counteracted by declining skills in the rising occupations, paralleling the role of selection for masking the correlation between task prices and employment growth of Section 4.

The wage distributions of Figure 5 and Table 2 are also robust to the different interpretations of our task price estimates with respect to the base period discussed in Section 2.2.[39] In Appendix D.2.6 we show that the scenario with filled-up non-employment gives even stronger inequality trends than Figure 5. Thus, simple increases in the overall employment rate (e.g., after 2004 due to the Hartz reforms) are not the main driver of rising lower-end inequality. Finally, in unreported analyses, we have computed counterfactual wage levels and within-cohort inequality over workers'

[39]First, the wages in Equations (14) or (15) are exactly the same when they are generated by accelerating task prices $\Delta\hat{\pi}_{k,t} = \Delta\pi_{k,t} - \overline{\Delta\pi_{k,base}}$ and accordingly estimated skill accumulation coefficients $\hat{\Gamma}_{k',k} = \Gamma_{k',k} + \frac{1}{2}\overline{\Delta\pi_{k,base}} + \frac{1}{2}\overline{\Delta\pi_{k',base}}$ as when the constant task price assumption in the base period is strictly true. The more general acceleration versus deceleration interpretation of Section 2.2 only matters for wages in the counterfactual with skill accumulation but without task prices, which rises by $\sum_{\tau=t_{i,0}+1}^{t}\sum_{k=1}^{K} I_{k,i,t_{i,0}} I_{i,\tau-1}\cdot\overline{\Delta\pi_{k,base}}$ compared to the current Equation (13). Nonetheless, the currently plotted $\hat{w}_{i,t}^{e+a}$ is probably still what we would like to plot in the violet lines of Figure 5, as it is informative to see how wage inequality would have changed if skill accumulation *and* baseline task price growth across occupations had remained constant over the sample period.

careers, too. The full model for example replicates around four fifths of the life-cycle increase in wages as well as more than 85 percent of the increase of inequality within cohorts.

The overall conclusion from this section is therefore that occupations in terms of the employment distribution, skill accumulation, and task prices are remarkably important in generating the observed widening of the German wage structure. In particular, the task prices had a strong impact by themselves while the fact that they polarized is also consistent with having induced some of the changes in the employment structure that substantially affected inequality in the lower half. The occupations-based model however does not explain the full increase of wage dispersion in the bottom panel of Table 2, and a large part of the unexplained is unsurprisingly inequality within detailed occupations (Appendix E.3). For these trends other factors emphasized in the literature, such as pure firm wage premia, may have played a significant role.

6 Conclusion

In this paper we have developed a method to estimate changing task prices per efficiency unit of skill in a general number of occupations using worker longitudinal data. The method flexibly allows for endogenous occupation switches, since it is derived from a Roy (1951) model of sector choice, and it includes a rich function of skill accumulation to account for key empirical facts of workers' employment and wages over the career. We explore generalizations of the underlying economic model and resulting empirical specifications as well as provide detailed Monte Carlo evidence on the performance of the estimator.

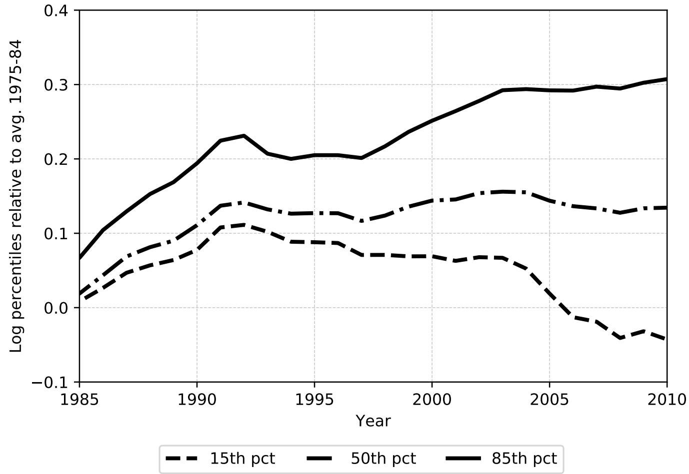
We apply our new method to German administrative data with longitudinal occupation information. The results show that, while occupational employment growth and wages are uncorrelated, task prices strongly rise in growing occupations. This is consistent with pervasive demand changes driving the occupational reallocation over time. Broader task price trends across high- and low-earning occupation groups versus middle-earning occupation groups are also consistent with routine-biased technical change and other forces that may have driven job polarization. We document that workers newly entering or leaving are on average substantially less skilled than incumbents or stayers in every occupation, with the gap being larger in faster-growing occupations and much of it due to differences in skill accumulation and idiosyncratic skill shocks. Aggregating these differences by the extent of net entry generates four fifth of the declining skills in growing occupations implied by the changing task prices, which constitutes independent evidence for the plausibility of our estimates. Finally, we use the empirical results to examine the effect of task prices, skill accumulation, and occupational reallocation on overall wage inequality. The task prices by themselves explain half of the rising dispersion of log wages during 1984–2010 and almost all of the increase in upper-half wage inequality. Our full empirical model accounts for all of rising wage dispersion and the complete increase of 85–50 and 50–15 log wage differences.

The estimation method developed in this paper could be employed in other contexts where high-quality longitudinal data are available. For example, one might study the drivers of structural transformation across detailed industries (e.g., Young, 2014, analyzes broad sectors) and re-examine the selection of skills over the business cycle (McLaughlin and Bils, 2001). It may also be possible to apply our framework, which explicitly accounts for self-selection when transitioning between jobs, to the burgeoning literature on firm-specific wage effects (Card et al., 2013; Song et al., 2018). Future methodological research could extend the approach to workers' choices along continuous job characteristics, which (as far as we are aware) have not been modeled in a comparably flexible skills and choice framework yet.

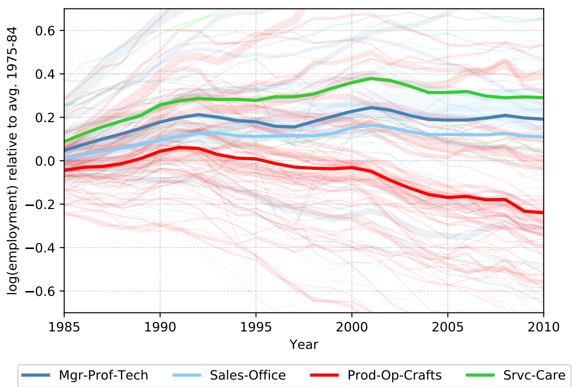
Additional Tables and Figures

Figure 7: Wage and Employment Trends

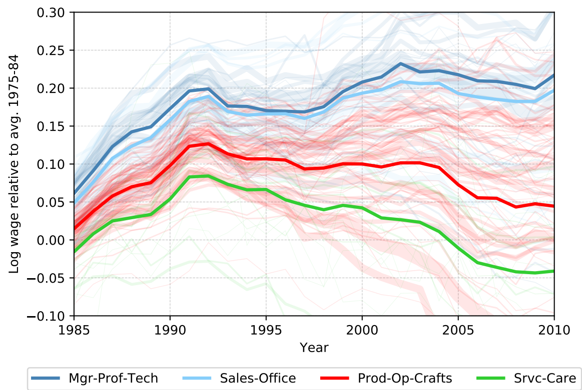
(a) Wage percentiles



(b) Occupations' employment



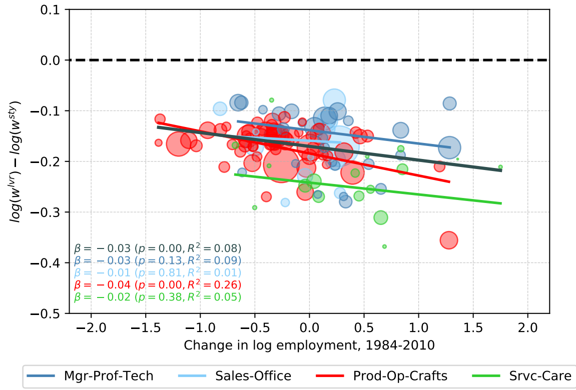
(c) Occupations' average wages



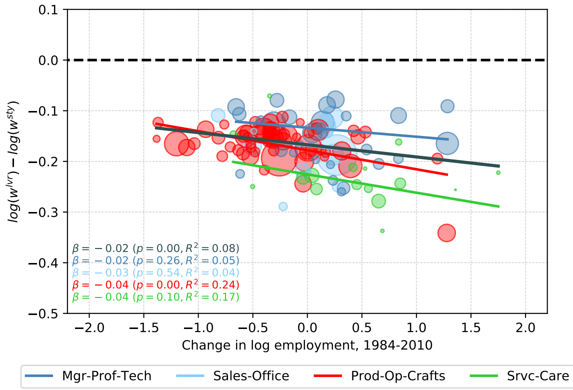
Notes: Shaded lines in the background are the detailed 120 occupations with the thickness of the lines proportional to their average employment over all sample years. To summarize the broad trends, the solid lines in the foreground show the four aggregated occupation groups. All numbers are relative to their 1975–1984 averages.

Figure 8: Wages versus employment growth

(a) Leaver minus stayer (unconditional)



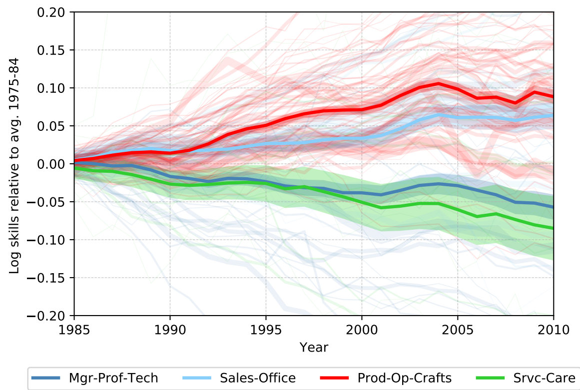
(b) Entrant minus incumbent (conditional)



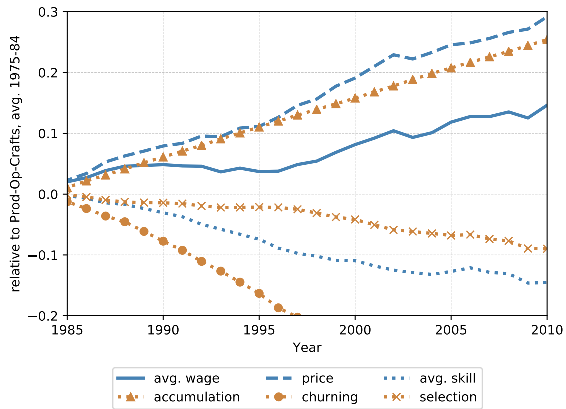
Notes: The figure computes in each occupation log average wages of (a) leavers minus stayers and (b) entrants minus incumbents, the latter controlling for age and education observables.

Figure 9: Skill changes over time in detailed occupations and broad groups

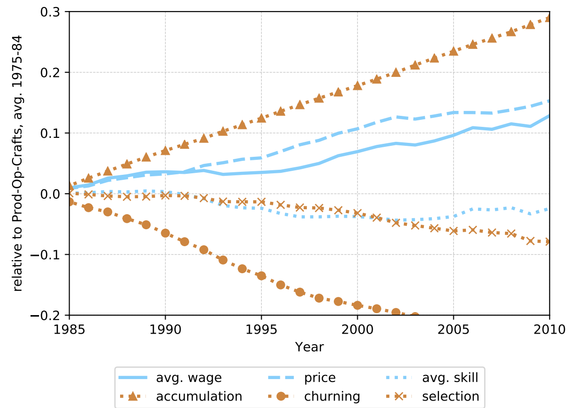
(a) Implied skills



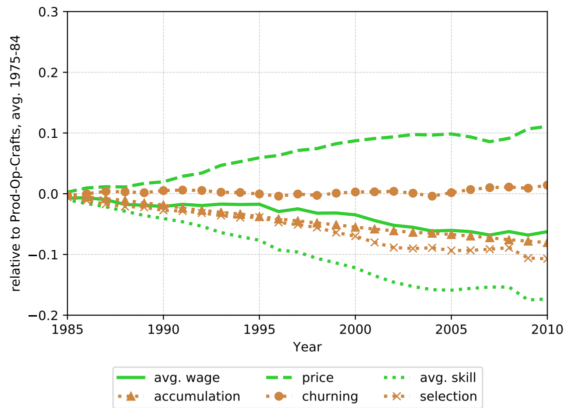
(b) Relative Mgr-Prof-Tech



(c) Relative Sales-Office



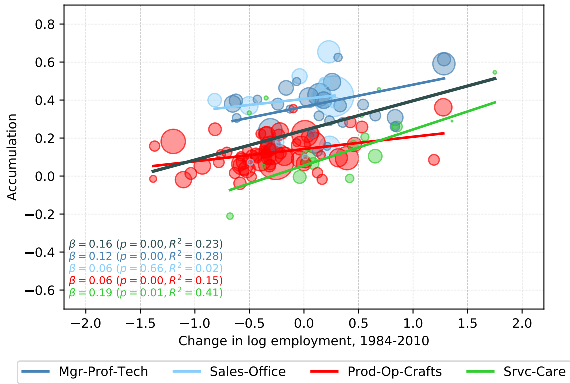
(d) Relative Srvc-Care



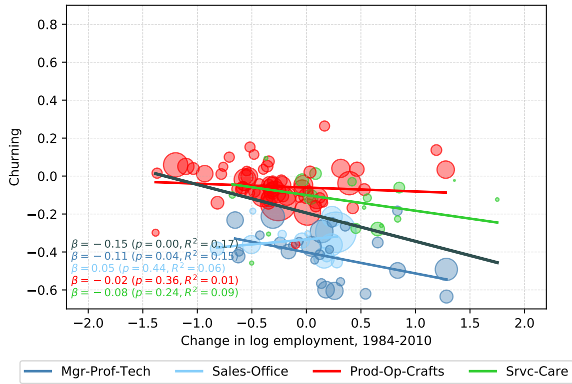
Notes: Panel shows the skill changes implied by the estimation in the 120 detailed occupations and the four broad groups. The remaining panels show the decomposition of wages and skill changes in the broad groups *relative to* Prod-Op-Crafts.

Figure 10: Accumulation and Churning

(a) Accumulation



(b) Churning



Notes: One bubble represents one of 120 occupations available in the SIAB data. The size of a bubble is proportional to the average size of the occupation across all sample years. The slope coefficients β were computed with weighted least squares where the weight was also equal to the average occupation size. Year effects were absorbed in both panels.

Table 3: Estimated skill accumulation coefficients (occupation groups)

		Age group		
Previous sector	Current sector	[25, 34]	[35, 44]	[45, 54]
Mgr-Prof-Tech	Mgr-Prof-Tech	0.046	0.013	0.000
	Sales-Office	0.140	0.012	-0.033
	Prod-Op-Crafts	0.020	-0.048	-0.068
	Srvc-Care	-0.071	-0.124	-0.016
Sales-Office	Mgr-Prof-Tech	0.218	0.063	0.024
	Sales-Office	0.043	0.015	0.000
	Prod-Op-Crafts	0.125	0.042	-0.025
	Srvc-Care	-0.014	-0.118	-0.074
Prod-Op-Crafts	Mgr-Prof-Tech	0.203	0.115	0.069
	Sales-Office	0.087	0.058	0.007
	Prod-Op-Crafts	0.019	0.008	-0.007
	Srvc-Care	-0.072	-0.051	-0.019
Srvc-Care	Mgr-Prof-Tech	0.275	0.182	0.131
	Sales-Office	0.249	0.138	0.055
	Prod-Op-Crafts	0.299	0.220	0.123
	Srvc-Care	0.019	0.005	-0.011

Notes: The table shows the estimated $\hat{\gamma}_{k',k,a}$, which is a scalar element of $\Gamma_{k',k}$ representing skill accumulation of age group a.

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Appendix

A Theory

A.1 Proofs and Derivations

A.1.1 Derivation of Equation (4)

We restate (3), explicitly indicating that $I_{k,i,t}$ is a function of potential wages in all K occupations:

$$dw_{i,t} = \sum_{k=1}^{K} I_k(w_{1,i,t}, ..., w_{k,i,t}, ..., w_{K,i,t}) dw_{k,i,t}. \tag{16}$$

To get from marginal to discrete changes, hold constant $w_{l,i,t-1} \forall\ l \geq 2$ first and integrate (16) with respect to the potential wage in occupation 1:

$$w_{i|w_{1,i,t},w_{2,i,t-1},...} - w_{i|w_{1,i,t-1},w_{2,i,t-1},...} = \int_{w_{1,i,t-1}}^{w_{1,i,t}} I_1(w_{1,i,\tau}, w_{2,i,t-1}, ...) dw_{1,i,\tau}.$$

Now, hold constant $w_{l,i,t-1} \forall\ l > k$ at $t-1$ as well as $w_{j,i,t} \forall\ j < k$ at t and integrate with respect to some $w_{k,i,t-1}$. Then, $\forall\ k \in \{1, ..., K\}$:

$$\begin{aligned} & w_{i|w_{1,i,t},...,w_{k,i,t},...,w_{K,i,t-1}} - w_{i|w_{1,i,t},...,w_{k,i,t-1},...,w_{K,i,t-1}} = \\ & = \int_{w_{k,i,t-1}}^{w_{k,i,t}} I_k(w_{1,i,t}, ..., w_{k,i,\tau}, ..., w_{K,i,t-1}) dw_{k,i,\tau}. \end{aligned} \tag{17}$$

Summing all of these elements (17) from $k = 1$ to $k = K$ we get

$$\begin{aligned} & w_{i|w_{1,i,t},...,w_{K,i,t}} - w_{i|w_{1,i,t-1},...,w_{K,i,t-1}} = w_{i,t} - w_{i,t-1} = \\ & = \Delta w_{i,t} = \sum_{k=1}^{K} \int_{w_{k,i,t-1}}^{w_{k,i,t}} I_k(w_{1,i,t}, ..., w_{k,i,\tau}, ..., w_{K,i,t-1}) dw_{k,i,\tau}. \end{aligned} \tag{18}$$

The notation of Equation (4) in the main text is therefore somewhat imprecise, as each integral with respect to $w_{k,i,\tau}$ in fact holds constant all the other wages.

A.1.2 Derivation of Equation (6)

First, replace the indicator $I_{k,i,\tau}$ for a specific k in Equation (4) with the linear interpolation (5):

$$\begin{aligned}
\int_{w_{k,i,t-1}}^{w_{k,i,t}} I_{k,i,\tau} dw_{k,i,\tau} &= \int_{w_{k,i,t-1}}^{w_{k,i,t}} \left[I_{k,i,t-1} + \frac{I_{k,i,t} - I_{k,i,t-1}}{w_{k,i,t} - w_{k,i,t-1}} (w_{k,i,\tau} - w_{k,i,t-1}) \right] dw_{k,i,\tau} \\
&= I_{k,i,t-1} \Delta w_{k,i,t} + \frac{I_{k,i,t} - I_{k,i,t-1}}{w_{k,i,t} - w_{k,i,t-1}} \left[\frac{1}{2} w_{k,i,\tau}^2 - w_{k,i,t-1} w_{k,i,\tau} \right]_{w_{k,i,t-1}}^{w_{k,i,t}} \\
&= I_{k,i,t-1} \Delta w_{k,i,t} + \frac{1}{2} (I_{k,i,t} - I_{k,i,t-1})(w_{k,i,t} - w_{k,i,t-1}) \\
&= \bar{I}_{k,i,t} \Delta w_{k,i,t}
\end{aligned}$$

where $\bar{I}_{k,i,t} \equiv \frac{I_{k,i,t}+I_{k,i,t-1}}{2}$ is the worker's "average" occupation choice in the two periods. Then, summing up over all k gives Equation (6).

Notice that the approximated variable $I_{k,i,\tau}$ is in fact $I_k(w_{1,i,t}, ..., w_{k,i,\tau}, ..., w_{K,i,t-1})$ according to Equation (18). We use $I_{k,i,t-1} = I_k(w_{1,i,t-1}, ..., w_{k,i,t-1}, ..., w_{K,i,t-1})$ and $I_{k,i,t} = I_k(w_{1,i,t}, ..., w_{k,i,t}, ..., w_{K,i,t})$ in the empirics (and therefore in the linear interpolation) because these are observed in the data.[40] The Monte Carlo simulations in Section B and in Böhm (2018) indicate that also this approximation is innocuous for identifying the correct task prices.

A.1.3 Sign of Potential Bias for the OLS Estimation Approach

For simplicity, consider the case where task price changes in the base period are indeed zero, i.e., $\Delta\pi_{k,t'} = 0, \forall k, t' \leq T_{base}$. We will also compare price estimates for k to some other generic occupation k' as any biases tend to affect relative task price estimates (overall accelerations or decelerations of wage growth are mechanically accounted for by the estimation). We distinguish between the values entering Equation (9) in each of four cases (and ignore $X_{i,t-1}$ to save on notation):

1. if $I_{k,i,t} = I_{k,i,t-1} = 1$ and thus the worker stays in his period $t-1$ occupation, $\bar{I}_{k,i,t} E(\Delta s_{k,i,t} | I_{k,i,t} = 1, I_{k,i,t-1} = 1) = E(\Delta s_{k,i,t} | \pi_{k,t-1} + s_{k,i,t-1} + \Delta\pi_{k,t} + \Delta s_{k,i,t} > \pi_{k',t-1} + s_{k',i,t-1} + \Delta\pi_{k',t} + \Delta s_{k',i,t}, \pi_{k,t-1} + s_{k,i,t-1} > \pi_{k',t-1} + s_{k',i,t-1}, \forall k' \neq k)$. It is easier to cross the period t threshold the larger $\Delta\pi_{k,t}$, ceteris paribus. That is, $\frac{\partial E(\Delta s_{k,i,t} | I_{k,i,t}=1, I_{k,i,t-1}=1)}{\partial \Delta\pi_{k,t}} < 0$ holding the other (k') task price and skill changes constant. If in the base period the task price changes are zero but $\Delta\pi_{k,t} > \Delta\pi_{k',t}$ in the considered time t, the estimated conditional expectation $E(\Delta s_{k,i,t} | I_{k,i,t'} = 1, I_{k,i,t'-1} = 1)$ from the base period entering Equation (9) is larger than the true expectation of the error component $E(\Delta s_{k,i,t} | I_{k,i,t} = 1, I_{k,i,t-1} = 1)$, relative to the corresponding values for k'. In order to fit the wage data, this leads to a too small estimate $0 < \Delta\hat{\pi}_{k,t} < \Delta\pi_{k,t}$ (*relative* to k') and vice versa if $\Delta\pi_{k,t} < 0$.

2. if $I_{k,i,t} = 1$ but $I_{k',i,t-1} = 1$, $\bar{I}_{k,i,t} E(\Delta s_{k,i,t} | I_{k,i,t} = 1, I_{k',i,t-1} = 1) = \frac{1}{2} E(\Delta s_{k,i,t} | \Delta\pi_{k,t} +$

[40] Böhm (2018) shows that using $I_k(w_{1,i,t}, ..., w_{k,i,t-1}, ..., w_{K,i,t-1})$ and $I_k(w_{1,i,t}, ..., w_{k,i,t}, ..., w_{K,i,t-1})$ instead can in fact lead to nonsensical results.

$\Delta s_{k,i,t} - \Delta\pi_{k',t} - \Delta s_{k',i,t} > \pi_{k',t-1} + s_{k',i,t-1} - \pi_{k,t-1} - s_{k,i,t-1} > 0)$. Hence, ceteris paribus, $\frac{\partial E(\Delta s_{k,i,t}|I_{k,i,t}=1,I_{k,i,t-1}=1)}{\partial\Delta\pi_{k,t}} < 0$ and the same argument as in case 1 applies.

3. if $I_{k,i,t-1} = 1$ but $I_{k',i,t} = 1$,$\bar{I}_{k,i,t}E(\Delta s_{k,i,t}|I_{k,i,t-1} = 1, I_{k',i,t} = 1) = \frac{1}{2}E(\Delta s_{k,i,t}|\Delta\pi_{k,t} + \Delta s_{k,i,t} - \Delta\pi_{k',t} - \Delta s_{k',i,t} < \pi_{k',t-1} + s_{k',i,t-1} - \pi_{k,t-1} - s_{k,i,t-1} < 0)$. Hence, ceteris paribus, $\frac{\partial E(\Delta s_{k,i,t}|I_{k,i,t}=1,I_{k,i,t-1}=1)}{\partial\Delta\pi_{k,t}} < 0$ and again the same argument as in case 1 applies.

4. if $I_{k,i,t} = I_{k,i,t-1} = 0$,$0 \cdot E(\Delta s_{k,i,t}|I_{k,i,t} = 0, I_{k,i,t-1} = 0) = 0$ in any case and both the control term entering Equation (9) and the error component that creates the bias are zero.

Given cases 1–4, the estimation unambiguously tends to underestimate rising *relative* $\Delta\pi_{k,t}$ (or accelerating compared to base period $\Delta\pi_{k,t'}$) and to overestimate declining *relative* $\Delta\pi_{k,t}$ (or decelerating compared to base period $\Delta\pi_{k,t'}$). The baseline estimation model therefore tends to provide a lower bound in absolute value to the true changes in relative task prices. However, this need not be the case for every single one of multiple occupations as the different price and skill changes may interact (which we closed down with our rather informal "ceteris paribus" argument here). The above argument should therefore be considered together with the Monte Carlo simulations in Section B, which support it and also show that any biases tend to be small.

A.1.4 Sign of Potential Bias for the IV Estimation Approach

The clearest way to understand the instrumental variables specification is in a simplified case with only two occupations (say k and a reference occupation k') and no systematic skill accumulation ($\Delta s_{k,i,t} = u_{k,i,t}$). The wage Equation (8) becomes

$$\Delta w_{i,t} = \Delta\pi_{k',t} + \bar{I}_{k,i,t}\Delta\tilde{\pi}_{k,t} + u_{k',i,t} + \bar{I}_{k,i,t}\tilde{u}_{k,i,t}, \tag{19}$$

where $\tilde{\pi}_{k,t} \equiv \pi_{k,t} - \pi_{k',t}$ is the task price *relative* to the reference occupation and $\tilde{u}_{k,i,t} \equiv u_{k,i,t} - u_{k',i,t}$ the *relative* idiosyncratic skill shock, which affects task choices. In particular, an OLS regression here would yield the following estimate for the changing relative task price

$$\widehat{\Delta\tilde{\pi}}_{k,t} = \frac{cov(\Delta w_{i,t}, \bar{I}_{k,i,t})}{Var(\bar{I}_{k,i,t})} = \Delta\tilde{\pi}_{k,t} + \frac{cov(\bar{I}_{k,i,t}\tilde{u}_{k,i,t}, \bar{I}_{k,i,t})}{Var(\bar{I}_{k,i,t})}, \tag{20}$$

with $\bar{I}_{k,i,t} = \frac{I_{k,i,t}+I_{k,i,t-1}}{k}$ and $I_{k,i,t} = 1[\tilde{\pi}_{k,t} + \tilde{s}_{k,i,t-1} + \tilde{u}_{k,i,t} > 0]$. Hence, there is classical endogeneity bias in the second summand on the right hand side of Equation (20), which stems from the fact that $I_{k,i,t}$ is a function of $\tilde{u}_{k,i,t}$. The induced positive correlation between these two variables should work toward a too large estimate of $\widehat{\Delta\tilde{\pi}}_{k,t}$.

We now attempt to remove this bias by instrumenting the regressor $\bar{I}_{k,i,t}$ with its predetermined component $I_{k,i,t-1}$, which is not a function of $\tilde{u}_{k,i,t}$. What remains of the covariance in the numerator is then $\frac{\hat{\rho}}{2}cov((I_{k,i,t} + I_{k,i,t-1})\tilde{u}_{k,i,t}, I_{k,i,t-1})$, where $\hat{\rho}$ is the coefficient from the IV first stage ($\hat{\bar{I}}_{k,i,t} = \hat{\rho}I_{k,i,t-1}$). The second part of this covariance

$cov(I_{k,i,t-1}\tilde{u}_{k,i,t}, I_{k,i,t-1})$ is clearly zero. The first part

$$cov(I_{k,i,t}\tilde{u}_{k,i,t}, I_{k,i,t-1}) = \underbrace{E[I_{k,i,t-1}E(I_{k,i,t}\tilde{u}_{k,i,t}|I_{k,i,t-1})]}_{(1)} - \underbrace{E(I_{k,i,t}\tilde{u}_{k,i,t})E(I_{k,i,t-1})}_{(2)}, \quad (21)$$

however, is indeterminate as $E(I_{k,i,t}\tilde{u}_{k,i,t}|I_{k,i,t-1})$ may vary with $I_{k,i,t-1}$ in general. Regarding component (2), we know that $E(I_{k,i,t-1}) \equiv p\,\epsilon(0,1)$ and $E(I_{k,i,t}\tilde{u}_{k,i,t}) > 0$, since $I_{k,i,t}$ positively depends on $\tilde{u}_{k,i,t}$. Therefore, $(2) = pE(I_{k,i,t}\tilde{u}_{k,i,t}) > 0$. Regarding component (1), the outer expectation is $(1-p) \cdot 0 \cdot E(I_{k,i,t}\tilde{u}_{k,i,t}|0) + p \cdot 1 \cdot E(I_{k,i,t}\tilde{u}_{k,i,t}|1) = pE(I_{k,i,t}\tilde{u}_{k,i,t}|I_{k,i,t-1} = 1) > 0$, because also conditional on $I_{k,i,t-1} = 1$, $I_{k,i,t}$ positively depends on $\tilde{u}_{k,i,t}$. The difference

$$(1) - (2) = p[E(I_{k,i,t}\tilde{u}_{k,i,t}|I_{k,i,t-1} = 1) - E(I_{k,i,t}\tilde{u}_{k,i,t})],$$

however, is likely to be negative because $I_{k,i,t} = 1[\tilde{\pi}_{k,t-1} + \tilde{s}_{k,i,t-1} + \Delta\tilde{\pi}_{k,t} + \tilde{u}_{k,i,t} > 0]$ is likely to vary more with $\tilde{u}_{k,i,t}$ unconditionally than when conditioning on $\tilde{\pi}_{k,t-1} + \tilde{s}_{k,i,t-1} > 0$.

Therefore, as in the basic OLS regression, the estimated task price changes ($\widehat{\Delta\tilde{\pi}}_{k,t}$ here) are likely moderately downward biased in absolute value. However, this result is more complicated with $K > 2$ and, as above, it need not be the case for every single one of multiple tasks. Aside from mentioning it in the main text, we have also not formally shown how the instrumentation can in fact only estimate *average* skill accumulation per occupation and period, which may induce some further bias. Nonetheless, Section B's Monte Carlo simulations again confirm that the estimates tend to be a lower bound of the true relative task price changes and that any biases tend to be very small.

A.2 Extensions of the Model

A.2.1 Learning About Skills

In the rest of the paper we have assumed that, aside from task prices, all changes in individuals' wages over time are due to systematic skill accumulation and idiosyncratic skill shocks. In this section, we show that the model's interpretation can be widened to include imperfect information about skills and employer learning over time on top of skill accumulation.

Suppose that, as in the learning literature (e.g., Altonji and Pierret, 2001; Gibbons et al., 2005), information about skills is imperfect. Each period an additional noisy signal of the worker's productivity arrives and employers form expectations about skills based on this as well as on all past observable information. Expectations are rational, that is, employers' beliefs are correct on average. Further, information is symmetric, employers are competitive, all market participants are risk neutral, and a spot market for labor exists.

In this setup, workers' potential log wages in each occupation equal their expected productivity conditional on all available information:

$$w_{k,i,t} = \pi_{k,t} + E_t(s_{k,i,t}) \; \forall \, k \in \{1, \ldots, K\}, \quad (22)$$

where E_t indicates that we are conditioning on all the information available in t. We assume that workers maximize their log incomes by choosing the occupation in which they earn the highest wage. This yields a modified version of Equation (6) for observed wage growth over time:

$$\Delta w_{i,t} = \sum_{k=1}^{K} \bar{I}_{k,i,t} \Delta \pi_{k,t} + \sum_{k=1}^{K} \bar{I}_{k,i,t} \Delta E_t(s_{k,i,t}), \tag{23}$$

where $\Delta E_t(s_{k,i,t}) \equiv E_t(s_{k,i,t}) - E_{t-1}(s_{k,i,t-1})$ and the linearity in logs allows us to swap sum $\sum$, first difference Δ, and expectations E operators. The skill accumulation (7) with idiosyncratic updates of expected skills becomes

$$\Delta E_t(s_{k,i,t}) = \sum_{k'=1}^{K} I_{k',i,t-1} \cdot X'_{i,t-1} \Gamma_{k',k} + u_{k,i,t}, \tag{24}$$

where the first summand is the expected skill accumulation in k of workers with characteristics $X_{i,t-1}$ previously working in k'. The second summand $u_{k,i,t}$ is an expectation update about individual i's true skill (or its accumulation) in k, which may in general be differentially variable and correlated across occupations. The variance of $u_{k,i,t}$ may also decline with age as employers receive more and more precise information about workers' skills (but also the idiosyncratic skill shocks may attenuate with age in the alternative interpretation of the main text).

We have therefore shown that our setup and, by extension, the estimation method remain valid under a basic model of employer learning about skills as an alternative or in addition to systematic skill accumulation and idiosyncratic skill shocks.

A.2.2 Non-Pecuniary Benefits or Forward-Looking Behavior

In the discussion of the main text, individuals are myopic and maximizing only current wages. In this section we show how the model can be reinterpreted in light of forward-looking behavior and non-pecuniary values of the decision to enter different occupations. Suppose utility of worker i in occupation k at time t is:

$$U_{k,i,t} = w_{k,i,t} + V_{k,i,t} \text{ with} \tag{25}$$

$$V_{k,i,t} = X'_{i,t-1} \Psi_{k,t} + \varepsilon_{k,i,t}, \tag{26}$$

where $V_{k,i,t}$ can generally be occupation k's amenity value or its continuation value (i.e., the expected net present value of future income streams in a Bellman Equation) or both, and it may also be heterogeneous by worker types.[41] In particular, the vector $\Psi_{k,t}$ contains an intercept as well as occupation-specific mappings from $X_{i,t-1}$ to utility. That is, the non-pecuniary or continuation value of each occupation k will differ by workers characteristics (i.e., age) in practice. We further let idiosyncratic occupation valuations $\varepsilon_{k,i,t}$ be mean zero and independent across individuals (they may be correlated across occupations for a given individual, however). Finally, notice already that only relative values $V_{k,i,t}$, and thus the parameters $\Psi_{k,t}$ compared to a chosen reference occupation, will be identifiable from workers' observed choices and wages.

[41] For example, Lee and Wolpin (2006) is an explicit structural paper that has both of these aspects.

With Equation (25) and utility maximization at hand, we can make the derivations corresponding to the main text. In particular, the marginal relationship (3) holds for utilities and we arrive at $\Delta U_{i,t} = \sum_{k=1}^{K} \int_{U_{k,i,t-1}}^{U_{k,i,t}} I_{k,i,\tau} dU_{k,i,\tau}$. That is, workers are still exactly indifferent in utility terms at the switch point.[42] Linearly approximating the integral again, we arrive at an augmented Equation (6) for the change of utility between two periods:

$$\triangle U_{i,t} = \triangle w_{i,t} + \triangle V_{i,t} = \sum_{k=1}^{K} \bar{I}_{k,i,t} \Delta U_{k,i,t} = \sum_{k=1}^{K} \bar{I}_{k,i,t} (\Delta w_{k,i,t} + \Delta V_{k,i,t}), \tag{27}$$

where $I_{k,i,t} \equiv 1[\max_{k'=1,\ldots,K}\{U_{k',i,t}\} = U_{k,i,t}] = 1[U_{k,i,t} \geq U_{k',i,t} \; \forall \, k' \neq k]$ is a choice indicator for occupation k.

The intuition here is also parallel to Equation (6): if a worker stays in his occupation, his realized utility gain is the change of his potential utility in that occupation. That is, $\triangle U_i = \triangle U_{ki}$ if $I_{k,i,t} = I_{k,i,t-1} = 1$, which is not an approximation. If the worker switches (e.g., occupations k' to k, $I_{k,i,t-1} = 1$,$I_{k,i,t} = 1$), he obtains part of the origin occupation's utility gain (or loss) as well as part of the destination occupation's utility gain, set to exactly half-half by the approximation (i.e., $\triangle U_{i,t} = \frac{1}{2} \triangle U_{k',i,t} + \frac{1}{2} \triangle U_{k,i,t}$). The arguments of the main text why the approximation error is negligble apply.

We want to solve Equation (27) for $\triangle w_{i,t}$, which is observable in the data. Consider $V_{i,t} = \sum_k I_{k,i,t} V_{k,i,t}$ and then write

$$\begin{aligned} \triangle V_{i,t} &= \sum_k I_{k,i,t} V_{k,i,t} - \sum_k I_{k,i,t-1} V_{k,i,t-1} \\ &= \sum_k \bar{I}_{k,i,t} \Delta V_{k,i,t} + \sum_k \bar{V}_{k,i,t} \Delta I_{k,i,t}, \end{aligned}$$

with $\bar{V}_{k,i,t} \equiv \frac{1}{2}(V_{k,i,t-1} + V_{k,i,t})$ and $\Delta I_{k,i,t} \equiv I_{k,i,t} - I_{k,i,t-1}$. Inserting this into Equation (27), the realized wage growth of individual worker i in the generalized dynamic Roy model becomes:

$$\begin{aligned} \triangle w_{i,t} &= \sum_k \bar{I}_{k,i,t} \Delta w_{k,i,t} - \sum_k \bar{V}_{k,i,t} \Delta I_{k,i,t} \\ &= \sum_k \bar{I}_{k,i,t} \Delta \pi_{k,t} + \sum_k \bar{I}_{k,i,t} \Delta s_{k,i,t} - \sum_k \bar{V}_{k,i,t} \Delta I_{k,i,t}, \end{aligned} \tag{28}$$

where we have multiplied out the parentheses and substituted $\Delta w_{k,i,t} = \Delta \pi_{k,t} + \Delta s_{k,i,t}$ in the second equality. This result firstly has a purely pecuniary part as in Equation (6) of the main text: if a worker stays in his occupation, his wage gain is the potential wage change (i.e., price growth and skill accumulation) in that occupation. If the worker switches, he obtains half of the origin's as well as half of the destination's potential wage change. The strength of this result is that it accommodates endogenous switches,

[42]We cannot, however, separately write $\Delta w_{i,t} = \sum_{k=1}^{K} \int_{w_{k,i,t-1}}^{w_{k,i,t}} I_{k,i,\tau} dw_{k,i,\tau}$ (and $\Delta V_{i,t} = \sum_{k=1}^{K} \int_{V_{k,i,t-1}}^{V_{k,i,t}} I_{k,i,\tau} dV_{k,i,\tau}$) anymore, since wages may indeed discretely jump at the indifference point. This is because indifference is now determined by utility and it may be the case that, at the point of switching, wages are strictly higher and amenity/continuation values are strictly lower in the destination than in the origin occupation, or vice versa.

i.e., which are due to changes in potential wages or amenity/continuation values.

The second summand on the right of Equation (28) is then the intuitive extension of a purely pecuniary/static model: with optimal choices, a worker's observed wage growth is the change in the potential wage of his chosen occupations minus the utility gain (loss) from the behavioral response of switching occupations. That is, if a utility-optimizing worker chooses to switch occupations (e.g., from k' to k so that $\Delta I_{k,i,t} = 1$ and $\Delta I_{k',i,t} = -1$), we observe lower wage growth than the change in relevant potential wages when he gains amenities or net present value of future earnings (i.e., $\bar{V}_{k,i,t} > \bar{V}_{k',i,t}$ and thus $\sum_k \bar{V}_{k,i,t} \Delta I_{k,i,t} > 0$) via the move. Vice versa, we observe higher wage growth than the potential wage changes when he moves to a less desirable occupation in these respects ($\sum_k \bar{V}_{k,i,t} \Delta I_{k,i,t} < 0$).

Notice in Equation (28) it is the *average* amenity/continuation value over both periods $\bar{V}_{k,i,t}$ that the worker is moving into which matters for wage changes. For a switcher from k' to k,

$$\sum_k \bar{V}_{k,i,t} \Delta I_{k,i,t} = \bar{V}_{k,i,t} - \bar{V}_{k',i,t} = \frac{1}{2}(V_{k,i,t} - V_{k',i,t}) + \frac{1}{2}(V_{k,i,t-1} - V_{k',i,t-1}),$$

conditional on wage gains associated with average choices, moving into the currently high-value occupation (i.e., $V_{k,i,t} - V_{k',i,t} > 0$) is offset with lower wage growth. But also moving into a occupation that last period carried high value ($V_{k,i,t-1} - V_{k',i,t-1} > 0$) is associated with lower wage growth because it implies that the worker was compensated last period for working in the low-value occupation, which now falls away with the switch. Both of these factors enter equally into the wage equation (28). Hence one cannot distinguish them empirically and only identify the average value over the two periods. As is obvious from Equation (28), one can also not distinguish between the non-pecuniary (amenity value) and the dynamic (continuation value) considerations but only estimate a joint parameter $\bar{V}_{k,i,t}$. Finally, notice when the worker makes no switch, the value considerations do not come into play at all (i.e., $\sum_k \bar{V}_{k,i,t} \Delta I_{k,i,t} = 0$) and the changing wage is just the changing task price plus skill accumulation.

We now discuss empirical implementation for different versions of Equation (26). First, if amenity/continuation values are constant such that $\bar{V}_{k,i}$ does not carry a time index, they will be simply incorporated in the skill accumulation parameters. That is, since it $\sum_{k'=1}^{K} \bar{I}_{k,i,t} I_{k',i,t-1} \cdot X'_{i,t-1} \Gamma_{k',k}$ in Equation (10) is a fully interacted model of all task choice combinations and worker observables, it absorbs the term $\sum_k \bar{V}_{k,i} \Delta I_{k,i,t}$. Because of this, also in the time-varying case, average $\bar{V}_{k,i,t}$ parameters can only be identified relative to their base period values. Our main estimation specification therefore already controls for general time-invariant non-pecuniary values as well as forward-looking considerations of occupation choice (with the interpretation of the parameter estimates $\hat{\Gamma}_{k',k}$ adjusted accordingly).

If instead amenity/continuation values are time-varying, we first of all note again that only $\bar{V}_{k,i,t}$ relative to a reference occupation can be identified. The mechanical reason is that the $\Delta I_{k,i,t}$ sum to zero over all K, and thus one of them has to be left out of the estimation due to multicollinearity. The economic intuition is that we can use choices and wages to identify relative utilities but not their levels. Other than that, it is straightforward to introduce a full set of task choice changes into estimation Equation

(10) and also interact them with worker characteristics. That is, $\bar{\Psi}_{k,t}$ in augmented regression

$$\Delta w_{i,t} = \sum_{k=1}^{K} \bar{I}_{k,i,t} \cdot \Delta \pi_{k,t} + \sum_{k'=1}^{K} \bar{I}_{k,i,t} I_{k',i,t-1} \cdot X'_{i,t-1} \Gamma_{k',k} + \sum_{k'=1}^{K} \Delta I_{k,i,t} \cdot X'_{i,t-1} \bar{\Psi}_{k,t} + v_{i,t}, \tag{29}$$

identifies the average between time t and $t-1$ amenity/continuation values in k relative to a reference occupation and to the base period by age group.

The reason for why (29) is identified, even with time-varying valuations, is the above-discussed fact that moving into current as well as past $V_{k,i,t}$ both matter equally for wage changes. Therefore, this contribution to wage growth is only via the changing sorting $\Delta I_{k,i,t}$ into average amenity/continuation values, whereas the contribution to wage growth from changing task prices and skill accumulation is only via the average sorting $\bar{I}_{k,i,t}$. The estimation method is thus robust to both changing potential wages and amenity/continuation values over time.

Finally, conditional on specification (29), an additional (average) error term $\bar{\varepsilon}_{k,i,t}$ in (26) does not much affect the estimates, which to some extent parallels the limited confounding role of idiosyncratic skill shocks in the main text (see Böhm, 2018, for more detailed discussion of the idioyncratic non-pecuniary error term). We estimate (29) in Appendix D.3.4 and report very similar task price estimates to our main results. Broadly speaking, the amenity/continuation values of Mgr-Prof-Tech, Sales-Office, and Srvc-Care have modestly risen compared to Prod-Op-Crafts over the sample period.

A.2.3 Costs of Switching Occupations

There may also be pecuniary or non-pecuniary one-off costs of switching occupations. We can think of two types of switching costs, those that affect wages and those that are non-wage costs. The first type is due to occupation or task-specific skills and transferability (e.g., Poletaev and Robinson, 2008; Kambourov and Manovskii, 2009; Gathmann and Schönberg, 2010). We will see that this is related to (but not the same as) the skill accumulation coefficients of the main text. The other type are pecuniary and non-pecuniary non-wage costs, e.g., financial expenses and psychological stress of moving house to be close to the new job, which are potentially as important as wage costs (Dix-Carneiro, 2014; Artuç and McLaren, 2015; Cortes and Gallipoli, 2017). This is related to the amenity/continuation values from above and therefore the augmented estimation (29) accounts for (part of) it.

We use direct wage costs to illustrate the implications of switching costs. For notational brevity assume the costs are also homogenous across workers and time-invariant (though allowing for this does not create any further substantive complications). Occupation k wages of worker i in period t and $t-1$ become

$$w_{k,i,t} = \pi_{k,t} + s_{k,i,t} - \sum_{k'=1}^{K} I_{k',i,t-1} c_{k',k}$$
$$w_{k,i,t-1} = \pi_{k,t-1} + s_{k,i,t-1} - \sum_{k'=1}^{K} I_{k',i,t-2} c_{k',k},$$

where $c_{k',k} \geq 0$ is the cost of switching from occupation k' to k and we normalize the cost of a non-switcher to zero (i.e., $c_{k,k} = 0 \; \forall \, k$). This setup allows for the same derivation as in the main text. In particular, although the switching cost is discrete, workers are still indifferent at the switch point . Of course, in a generalized setup they may be indifferent in terms of utility as in the previous section (also see footnote 42 above), where a high continuation value may justify the switch and the contemporaneous wage loss that is associated with it.

We obtain

$$\begin{aligned} \triangle w_{i,t} &= \sum_{k=1}^{K} \bar{I}_{k,i,t} \Delta w_{k,i,t} = \sum_{k=1}^{K} \bar{I}_{k,i,t} (\Delta \pi_{k,t} + \Delta s_{k,i,t} - \Delta \sum_{k'=1}^{K} I_{k',i,t-1} c_{k',k}) \\ &= \sum_{k=1}^{K} \bar{I}_{k,i,t} \left(\Delta \pi_{k,t} + \sum_{k'=1}^{K} I_{k',i,t-1} \cdot (\Gamma_{k',k} - c_{k',k}) + \sum_{k'=1}^{K} I_{k',i,t-2} c_{k',k} + u_{i,t} \right), \end{aligned} \tag{30}$$

where for brevity we have also left out the dependency of skill accumulation on worker observables $X_{i,t-1}$. The contemporaneous switching cost is absorbed by the estimated skill accumulation coefficient $\Gamma_{k',k} - c_{k',k}$ and does not constitute a confounder for the task prices. However, there is also the lagged switching cost $\sum_{k'=1}^{K} I_{k',i,t-2} c_{k',k}$ in Equation (30) which affects current wages. This is because if $w_{k,i,t-1}$ were dragged down by switching costs then part of the wage growth from last period to the current period $\triangle w_{i,t}$ was not due to increasing potential wages, that is, rising task prices or skill accumulation (possibly including switching), but due to the fact that the switching cost that dragged down last period's wages fell away. This part of the wage growth is attributed to $\sum_{k=1}^{K} \bar{I}_{k,i,t} \sum_{k'=1}^{K} I_{k',i,t-2} c_{k',k}$.

The economic argument is similar when switching costs are (additionally) non-pecuniary and time-changing; we would derive an augmented Equation (30) using changes in utility as in the previous section instead of wages. Heterogeneity by worker observables can be introduced by interacting $I_{k',i,t-2}$ with $X_{i,t-1}$, and conditional on this idiosyncratic switching costs are again unlikely to have a major impact for the estimates. We could have implemented Equation (30) using second lags for occupational choice but instead vary the period length from one to four years in empirical robustness checks. This has the advantage that it checks for the importance of switching costs, since these should become (relatively) less important compared to potential wage differences over longer time horizons, but also other concerns to our estimates such as correlated-over-time skill shocks. We further run the estimation in different demographics (i.e., by age and gender) for whom switching costs may be different. Finally, the Monte Carlo simulations additionally examine the case of multiplicative switching costs whereby the worker's current (log) wage is reduced by a fraction c.

A.3 Multiple Fixed Effects as an Alternative Approach

In this section, we examine the occupation-specific fixed effects approach for estimating task prices as an alternative to our method. We show that under a flexible model of skill accumulation, this approach requires controlling for workers' whole history of occupation-specific experience or, more feasibly, extending the fixed effects to being occupation-stint specific. With idiosyncratic skill shocks, an endogeneity bias emerges that is due to the fixed effects themselves. The results from the Monte Carlo simulations in Section B support our analytical arguments.

Several papers have used fixed effects approaches in order to address worker heterogeneity when estimating task prices (Cortes, 2016; Cavaglia and Etheridge, 2017, e.g.,).[43] To be specific, consider Cortes' time-varying model for the potential wage of individual i in occupation k at time t:

$$w_{k,i,t} = \pi_{k,t} + s_{k,i,t} = \pi_{k,t} + X'_{i,t-1}\Gamma_k + \eta_{k,i}. \tag{31}$$

The changing characteristics vector $X_{i,t}$ can increase skills differentially with age or experience across occupations according to Γ_k. In addition, $\eta_{k,i}$ are occupation-specific time-invariant skill levels, which will be introduced into the regression by individual-occupation-specific fixed effects. Cortes (2016) and Cavaglia and Etheridge (2017) interchangeably call these occupation- or sector-*spell* fixed effects, which is why we instead use the term 'stint' for a worker's self-contained stay (i.e., without switches in between) in a given occupation below. Consistent with (31), Cortes' estimation equation is:[44]

$$w_{i,t} = \sum_{k=1}^{K} I_{k,i,t}\pi_{k,t} + \sum_{k=1}^{K} I_{k,i,t}\eta_{k,i} + \sum_{k=1}^{K} I_{k,i,t} \cdot X'_{i,t}\Gamma_k + \varphi_{i,t}, \tag{32}$$

A.3.1 Systematic Skill Accumulation

In the following, we examine whether estimation of Equation (32) may identify the correct task prices. First, consider the case when skill accumulation is only systematic:

$$\Delta s_{k,i,t} = \sum_{k'=1}^{K} I_{k',i,t-1} \cdot \Gamma_{k',k}, \tag{33}$$

where, compared to Equation (7) of the main text, we omit from now the $X_{i,t}$-specificity of the accumulation function to save space and thus $\Gamma_{k',k}$ is a scalar. Writing this out from when the worker joined the labor market at time $t_{i,0}$ gives

$$s_{k,i,t} = \eta_{k,i} + \sum_{k'=1}^{K} [I_{k',i,t-1} + \ldots + I_{k',i,t_{i,0}}] \cdot \Gamma_{k',k} = \eta_{k,i} + \sum_{k'=1}^{K} \sum_{\tau=t_{i,0}}^{t-1} I_{k',i,\tau} \cdot \Gamma_{k',k}, \tag{34}$$

[43] In more broadly related settings, Combes et al. (2008) estimate city wage premia, taking into account sorting across locations. Analyzing variation over the business cycle, Solon et al. (1994) account for skill selection into the labor market market, while McLaughlin and Bils (2001) examine skill selection across sectors.

[44] Similar to us, Cortes (2016) uses ten year age bins in $X_{i,t-1}$, allowing for the convexity of the life-cycle profile parallel to our Equation (7).

for $t \geq t_{i,0}$ and $\eta_{k,i}$ the initial skill endowments of i in k at when he joins the labor market. Therefore, if we are willing to assume that skill accumulation occurs similarly in each occupation of origin ($\Gamma_{k',k} = \Gamma_k, \forall k', k$), this simplifies to $s_{k,i,t} = \eta_{k,i} + (t - t_{i,0}) \cdot \Gamma_k$ and Estimation (32) identifies the correct task prices, initial endowments, and skill accumulation parameters. Notice that this specification assumes that labor market experience is not occupation-specific, just that general experience is valued differently in different occupations.

Our evidence strongly suggests that experience is occupation-specific.[45] A model that is more aligned with the evidence hence allows for this, that is, for example allows for the fact that previous managerial experience imparts more managerial skills than previous experience in production jobs. Equation (34) becomes $s_{k,i,t} = \eta_{k,i} + \sum_{k'=1}^{K} exp_{k',i,t} \cdot \Gamma_{k',k}$, where $exp_{k',i,t} \equiv \sum_{\tau=t_{i,0}}^{t-1} I_{k',i,\tau}$ is the worker's occupation k' specific experience. Running regression (32) gives an error term $\varphi_{i,t} = \sum_{k=1}^{K} I_{k,i,t}[\sum_{k'=1}^{K} exp_{k',i,t} \cdot \Gamma_{k',k} - (t - t_{i,0}) \cdot \Gamma_k]$ in that case which varies with $I_{k,i,t}$ and is thus systematically related to the regressors. This yields biased estimates even without any unobserved idiosyncratic skill shocks that lead to endogenous sector switches.

The correct fixed effects regression for task prices is instead

$$w_{i,t} = \sum_{k=1}^{K} I_{k,i,t}\pi_{k,t} + \sum_{k=1}^{K} I_{k,i,t}\eta_{k,i} + \sum_{k=1}^{K} I_{k,i,t} \sum_{k'=1}^{K} exp_{k',i,t} \cdot \Gamma_{k',k} + \varphi_{i,t}, \tag{35}$$

that is, it controls for all previous occupation-specific experience separately. While this is conceptually possible to do, its practical implementation is difficult. It introduces many parameters to be estimated (even more when we realistically allow for occupation-specific skill accumulation to vary with age; e.g., see general skill accumulation Equation (7) and evidence in Figure 2b) and it requires high-quality panel data in order to compute the full occupation- and age-specific work experience history of each individual. Cortes (2016) and Cavaglia and Etheridge (2017) account for the fact that labor market experience is occupation-specific by introducing controls for occupation- and job-specific tenure, respectively, into regression Equation (32).[46] However, in order to deal with the growth in the number of parameters and the length of the employment history that is required for this approach, both papers assume that tenure only affects the current job and that workers lose all of its effect once they switch.

The more feasible way to implement this approach is instead to use separate individual fixed effects for each occupation stint.[47] That is, to use $\eta_{\lambda(k,i)}$ which differs flexibly for each continuous period $\lambda(k,i) = 1, ..., \Lambda(k,i)$ in i's career during which he works in occupation k. Skill accumulation can then be only occupation-specific and

[45]The skill accumulation estimates in Table 3 show this but also the fact that large wage differences between entrants and incumbents persist when controlling for general age or experience (Fig.8b). In unreported analysis we also find that wages within in a given occupation at a given age significantly depend on workers' previous occupations.

[46]Of course, there may also be proper tenure effects whereby wage growth, in addition to occupation-specific experience, depends on the exact tenure in the current job. Given the rich SIAB panel data, it would be relatively straightforward to extend our skill accumulation Equation (7) and include this in the estimation.

[47]In our data, 13% (8%) of workers have multiple stints in an occupation (group) during their career.

straightforwardly interacted with observable characteristics such as age or education (again omitted for brevity):

$$w_{i,t} = \sum_{k=1}^{K} I_{k,i,t}\pi_{k,t} + \sum_{k=1}^{K} I_{k,i,t}\eta_{\lambda(k,i)} + \sum_{k=1}^{K} I_{k,i,t} \cdot exp_{\lambda(k,i),t} \cdot \Gamma_k + \varphi_{i,t}. \tag{36}$$

Here $exp_{\lambda(k,i),t}$ is the number of years the individual has spent in this occupation stint at time t, which is effectively tenure (but interacted with age, which is omitted in Equation (36)) conditional on occupation-stint-specific fixed effects. This is in our view the best specification and we use it in empirical robustness checks as well as the Monte Carlo simulations below.

Finally, notice that the identification of all parameters in the correct occupation-specific fixed effects approach also requires a base period. This is because one period is needed to absorb the fixed effects and a second period to identify the Γ_k skill accumulation coefficients in (36) (or $\Gamma_{k',k}$ in (35)) if they are allowed to flexibly vary with age and occupation. Only in a third period can then the task prices $\pi_{k,t}$ be separately identified.

A.3.2 Idiosyncratic Skill Shocks

Another substantive difference between our proposed method and the fixed effects approach arises in the presence of idiosyncratic skill shocks and endogenous choice, which are strongly suggested by the cross-accumulation parameters of switchers (Table 3) as well as higher-than-average skill shocks of occupation incumbents and stayers (Section E.1). We use a simplified analytical argument here. The Monte Carlos in the next section show that biases in our preferred version of the occupation-stint-specific fixed effects estimation become forbidding only with large skill shocks.

We use the skill shocks in the occupation-specific fixed effects estimation (35) for notational brevity but the argument is the same in our preferred occupations-stint specification (36). With idiosyncratic skill shocks, the right-hand-side of Equation (34) becomes:

$$s_{k,i,t} = \eta_{k,i} + \sum_{k'=1}^{K} \sum_{\tau=t_{i,0}}^{t-1} I_{k',i,\tau} \cdot \Gamma_{k',k} + \sum_{\tau=t_{i,0}+1}^{t} u_{k,i,\tau}. \tag{37}$$

The regression error in Equation (35), $\varphi_{i,t} \equiv \sum_{k=1}^{K} I_{k,i,t} \sum_{\tau=t_{i,0}+1}^{t} u_{k,i,\tau}$, now systematically depends on the full history of previous idiosyncratic skill shocks, which influence current choices (i.e. the regressors in Equation (35)). Therefore, we get a classical endogeneity bias. One might expect that the sector-experience-specific controls in regression (35) largely address this problem, similar to our differenced approach (8). But this is not the case.

To see the argument and the bias most clearly, as in to Section A.1.4, suppose for simplicity that all time-varying skill parameters, which are themselves demanding to model in the fixed effects approach as we discussed, are zero ($\Gamma_{k',k} = 0$). Suppose also that there are only two sectors, k and a reference occupation k', and consider first the base period where we assume that $\pi_{k,t} = \pi_{k',t} = const$ for $t = 1, \ldots, T_{\text{base}}$. In this case, simplified wage Equation (35) becomes

$$w_{i,t} = \eta_{k',i} + I_{k,i,t}\tilde{\eta}_{k,i} + u_{k',i,t} + I_{k,i,t}\tilde{u}_{k,i,t} \text{ for } t = 1, \ldots, T_{\text{base}}, \tag{38}$$

where $\tilde{\eta}_{k,i} \equiv \eta_{k,i} - \eta_{k',i}$ and $\tilde{u}_{k,i,t} \equiv u_{k,i,t} - u_{k',i,t}$ are relative skill endowments and skill shocks parallel to the notation in Section A.1.4. The regression (38) is classically endogeneity-biased because the error term $I_{k,i,t}\tilde{u}_{k,i,t}$ most likely positively correlates with the regressor $I_{k,i,t}\tilde{\eta}_{k,i}$. This will lead to an overestimation of $\tilde{\eta}_{k,i}$, for example.

If, in order to account for this correlation along the lines of the main text, we introduce saturated choice specific controls, estimation Equation (38) becomes

$$w_{i,t} = \eta_{k',i} + I_{k,i,t}\tilde{\eta}_{k,i} + I_{k,i,t}E(\tilde{u}_{k,i,t}|I_{k,i,t}, I_{k,i,t-1}) + \nu_{i,t},$$

with $\nu_{i,t} \equiv u_{k',i,t} + I_{k,i,t}[\tilde{u}_{k,i,t} - E(\tilde{u}_{k,i,t}|I_{k,i,t}, I_{k,i,t-1})]$. Since we identify from the wage growth of sector stayers (for a switcher the fixed effects together with choice specific controls are not identified), consider $E(\tilde{u}_{k,i,t}|I_{k,i,t} = I_{k,i,t-1} = 1, \tilde{\eta}_{k,i}) = E(\tilde{u}_{k,i,t}|\tilde{\eta}_{k,i} > 0, \tilde{\eta}_{k,i} + \tilde{u}_{k,i,t} > 0)$ for occupation k. So even if the correct average $E(\tilde{u}_{k,i,t}|I_{k,i,t}, I_{k,i,t-1})$ (not conditioned on $\tilde{\eta}_{k,i}$) were identified, the error term in this regression varies systematically with the fixed effect in the regressor (most likely $\frac{\partial E(\tilde{u}_{k,i,t}|\tilde{\eta}_{k,i}>0, \tilde{\eta}_{k,i}+\tilde{u}_{k,i,t}>0)}{\partial \tilde{\eta}_{k,i}} < 0$). Therefore, $\tilde{\eta}_{k,i}$ identified from occupation k stayers should be downward-biased and, of course, also the $\Gamma_{k',k}$ in the full estimation Equation (35) are not correctly estimated from the outset. With these biases already from the base period, not only will also the task price estimates in the analysis period $\pi_{k,t}, t > T_{\text{base}}$ be potentially severely biased but also will it be hard to sign the direction of these biases. This is a reflection of the fact that fixed effects estimations fundamentally require the exogenous mobility assumption. Relying in the estimation on the wage growth of stayers in an occupation does not in principle alleviate any resulting biases (see Section E.1, which finds that stayers are strongly self-selected according to idiosyncratic skill shocks).

B Monte Carlo Evidence

In this section we provide Monte Carlo evidence for the performance of our estimation method under various assumptions about the data generating process including among others the distribution of skill shocks and switching costs.

B.1 Data Generating Process

We generate panel datasets similar in structure to the actual SIAB data and apply our estimation method to those datasets to back out prices previously fed in to construct workers' careers over their life cycle. To resemble basic moments of the SIAB, we randomly draw initial observations of 15,000 individuals from the SIAB, including their initial wages, occupational choices, age (25–54) and year (1975–2010) of first observation. We then merge prices and skill accumulation parameters previously estimated for the actual SIAB data (see Figure 2) to that sample and construct the initial skill of an individual as the difference between the initial (observed) wage and the estimated price of the occupation the worker is initially observed in:

$$s_{k,i,t_{i,0}} = w_{i,t_{i,0}} - \pi_{k,t_{i,0}} \quad \text{if} \quad i \in k, \tag{39}$$

where $t_{i,0} \in \{1975, \dots, 2010\}$ is the year a worker is first observed. As the initial choice must also be optimal $w_{k,i,t_{i,0}} \geq w_{k',i,t_{i,0}} \forall k'$, we have a natural bound for the skills a worker possesses in the remaining sectors k' in line with the model and assumed prices given by:

$$s_{k',i,t_{i,0}} \leq s_{k,i,t_{i,0}} + \pi_{k,t_{i,0}} - \pi_{k',t_{i,0}} \quad \text{if} \quad i \in k. \tag{40}$$

We draw the initial skills separately and independently for every worker in the sample from a truncated normal distribution with the upper bound given by $s_{k,i,t_{i,0}} + \pi_{k,t_{i,0}} - \pi_{k',t_{i,0}}$. We set the location parameter $\mu_{k,i,t_{i,0}} = s_{k,i,t_{i,0}} + \pi_{k,t_{i,0}}$ and fix the scaling parameter $\sigma = 3$ across workers.[48] For the following years of a worker's career, we then simulate wage growth as the sum of systematic skill growth and price growth given by $\hat{\Gamma}_{k',k}, \Delta\hat{\pi}_{k,t}$. On top of that, we add idiosyncratic skill shocks depending on the specification and finally let workers choose their preferred sector based on comparative advantage (possibly including costs of switching) according to Equation (2). We repeat this until a worker's maximum age of 54 is reached or the sample period ends. We rerun the exercise for 70 Monte Carlo repetitions and estimate price and skill changes on each sample. We then compute the average price trends and skill accumulation function across repetitions.

Finally, for computational reasons we only use four occupations (i.e., the four broad occupation groups of the main text) in the following.

[48] Note that the estimation method removes all time constant variables by first differencing and so is robust to any choice of initial skills and their correlations with each other. We employed the truncated normal distribution for computational reasons only.

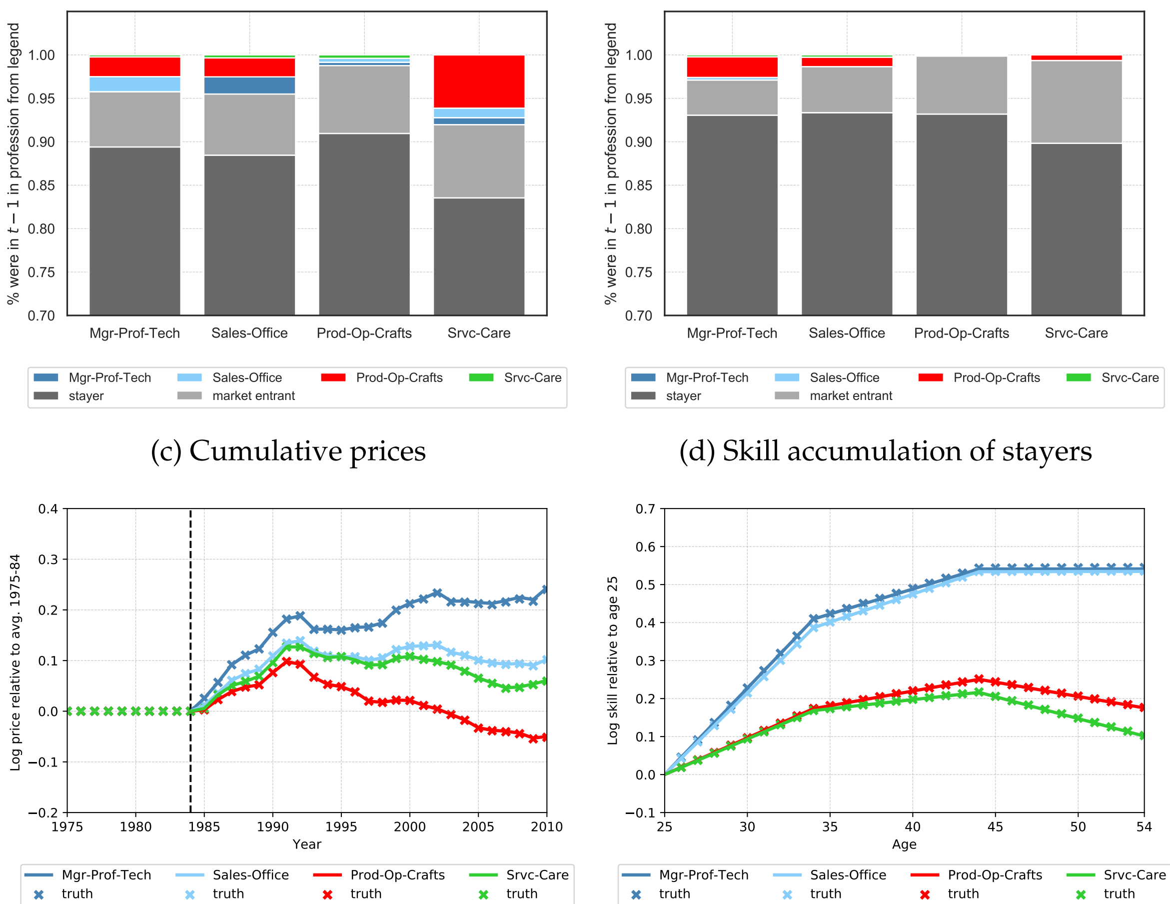
B.2 No Idiosyncratic Shocks

Figure B.1 reports the results for a Monte Carlo exercise where wage growth only stems from price growth or systematic skill growth but not from idiosyncratic shocks. Panel (b) shows that there are unsurprisingly not many occupation switchers in such a setup, with only a few of them accumulating such an amount of skills that they move into Mgr-Prof-Tech and Sales-Office. Alternatively, relative task prices can rise so much that some move into those high-paying occupations or Srvc-Care. The other three panels of Figure B.1 show the cumulative task prices and skill accumulation in the four occupations, with estimates depicted in the solid lines and true parameter values as crosses "**x**".

Figure B.1: Approximation quality without shocks

(a) Switchers in SIAB (b) Switchers in MC

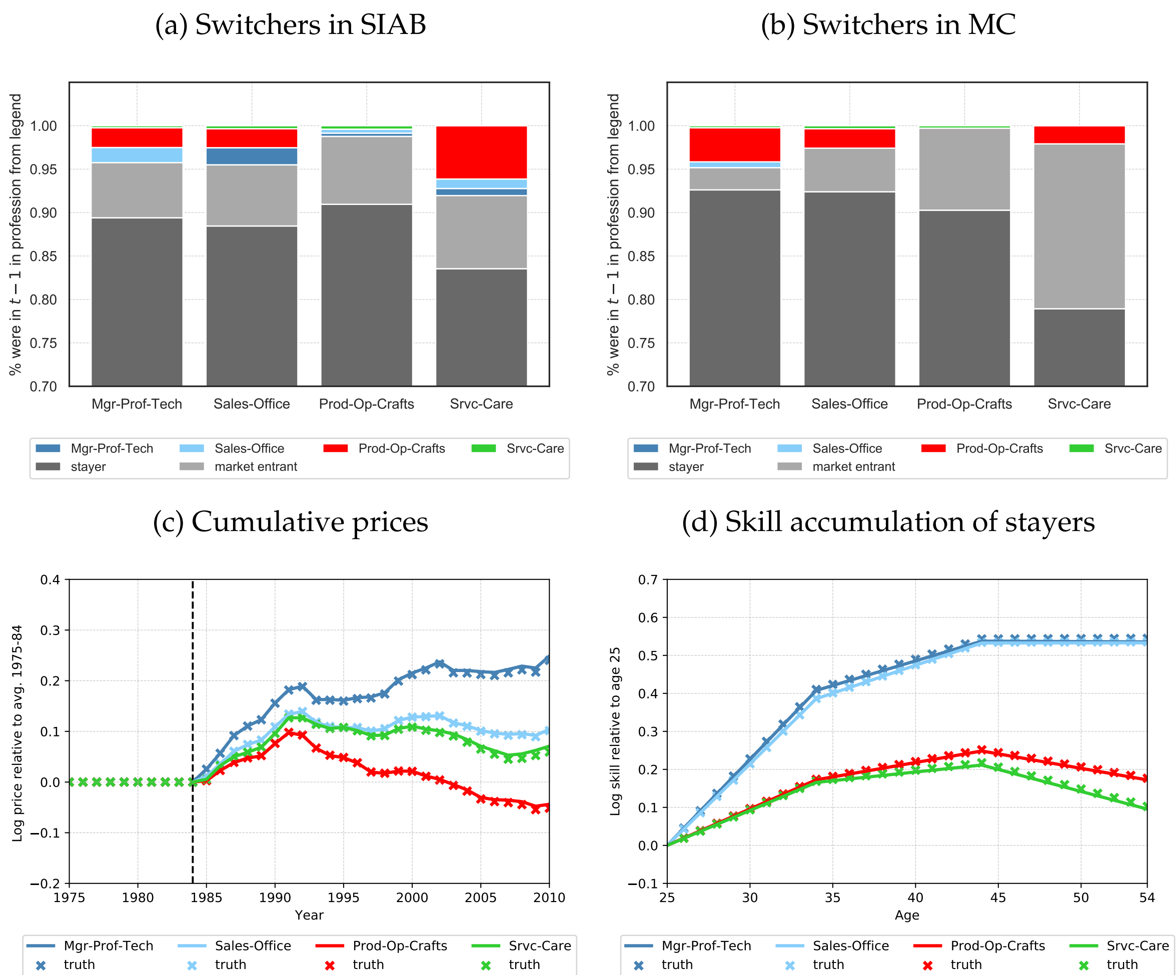
(c) Cumulative prices (d) Skill accumulation of stayers



Notes: Shimmering lines the background show the individual Monte Carlo replications.

Clearly, the proposed method is able to estimate skill accumulation and task price trends from observed wage changes when skill changes are only systematic and the function describing those changes is time invariant. To more fairly assess if the approximation (linear interpolation) of the indifference point is an issue, in the next step we decrease the standard deviation of initial skills from $\sigma = 3$ to $\sigma = 1$. This makes initial skills more alike within workers and therefore naturally increases the amount

Figure B.2: Approximation quality with more alike initial skills



Notes: Shimmering lines the background show the individual Monte Carlo replications.

of switchers as shown in Panel (b) of Figure B.2. Despite that, the price changes and skill accumulation parameters are still well identified. This supports our argument of the main text that the approximation of the indifference point (5) is not a first-order concern for the estimates.

The next four subsections present further possible challenges to our approach, evaluating the any potential biases and discussing their empirical relevance.

B.3 Idiosyncratic Skill Shocks

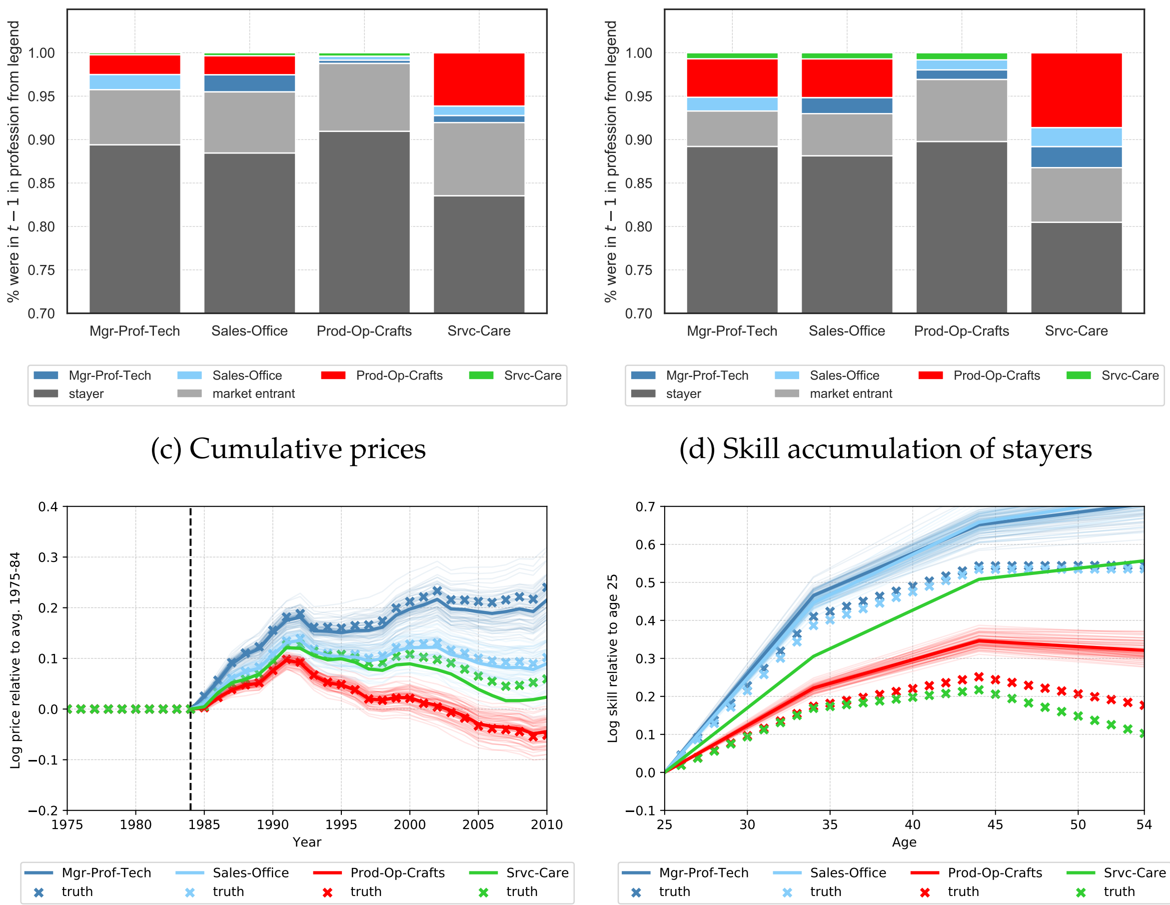
Making the idiosyncratic skill shocks $u_{k,i,t}$ occupation-specific, as in Equation (7), introduces an endogeneity bias to our estimates, which is rising with the standard deviation of the shocks. This subsection shows that the bias is small under empirically realistic assumptions for the dispersion of shocks.

Figure B.3 depicts the results when the skill shocks are drawn from a Gumbel distribution with parameters μ, β set so that the errors' standard deviation $\sigma_u = \frac{\pi}{\sqrt{6}}\beta$ is equal to the standard deviation of log wage growth across years in the SIAB data and

Figure B.3: Impact of skill shocks on baseline method, moderate shocks

(a) Switchers in SIAB (b) Switchers in MC

(c) Cumulative prices (d) Skill accumulation of stayers



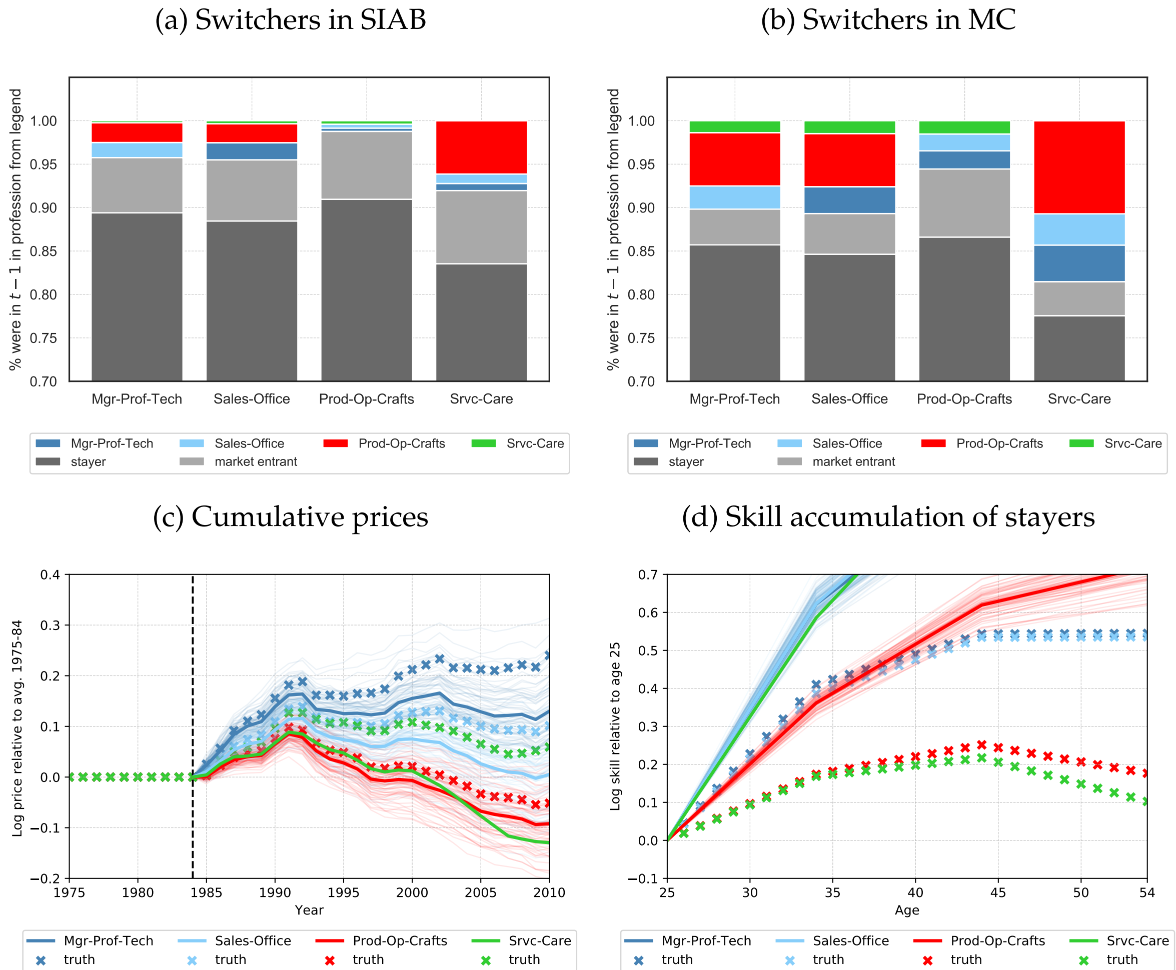
the mean $\mu+\beta\gamma$ is equal to zero.[49] Note that the standard deviation of wage growth is a complicated product but likely an upper bound for the standard deviation of idiosyncratic shocks, which our Monte Carlo analyses confirm. This is because the standard deviation of wage growth is not only a function of the dispersion of skill shocks but also of wage growth due to switching (and thereby the initial skill variation), systematic skill accumulation, and changes in task prices. The extent of switching in the Monte Carlo sample is also already larger than in the actual SIAB data (Figure B.4 Panels (a) versus (b)).

The results indicate that indeed there is a moderate bias for the task prices. Moreover, as predicted in Section A.1.3, the task price changes relative to a reference occupation are downward biased: estimates for Mgr-Prof-Tech, Sales-Office, and Srvc-Care rise less compared to Prod-Op-Crafts than the true increase of their relative prices. Measured against the assumed trend in prices, all these biases seem to be small in empirical applications though.

From now on, we increase the standard deviation of $u_{k,i,t}$ to two times the standard deviation of log wage growth observed in the SIAB data, so that the dispersion of log

[49] γ is the Euler-Mascheroni constant ≈ 0.57721.

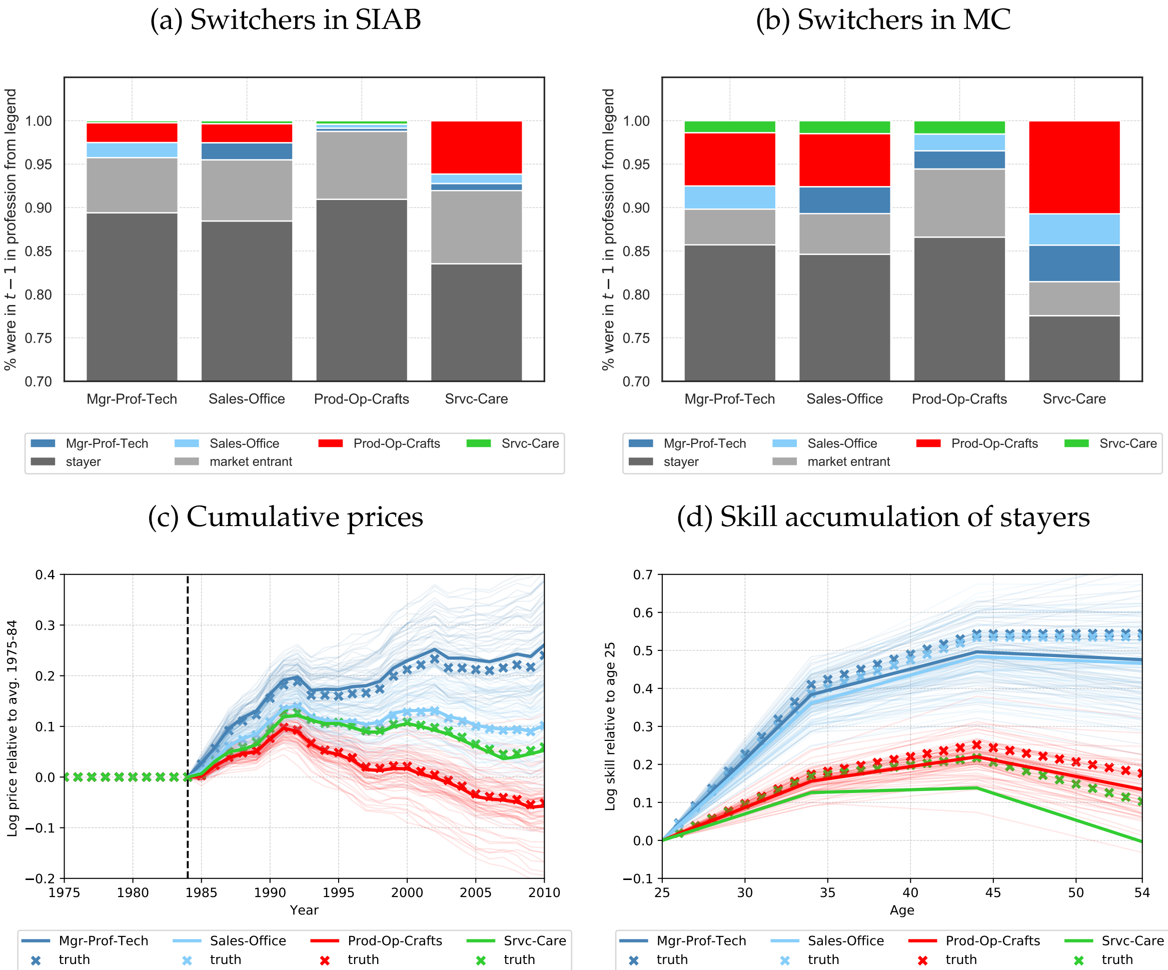
Figure B.4: Impact of skill shocks on baseline method, many shocks



wage growth in the Monte Carlo samples approximately doubles as well. The number of switches in the Monte Carlo sample increase a lot, too, and it is now much larger than in the actual SIAB data. So this is clearly an extreme setting, which we create to see whether the bias due to skill shocks can in some instances become substantial. The OLS estimates in Figure B.4 show that the bias does indeed becomes large. All the task price changes are underestimated and the skill accumulation is overestimated compared to the true data generating process. Again, the relative task price changes are downward biased.

In the next Figure B.5 we implement the instrumental variables strategy that was outlined in the main text (more detail in Section D.3.2) in order to deal with the now large bias due to idiosyncratic skill shocks. It turns out that IV does indeed largely estimate the correct task price changes even in this extreme case. Apart from skill accumulation in Srvc-Care, the task price changes and the other (average) skill accumulation parameters are very close to correctly identified (though the small bias for Mgr-Prof-Tech is now upward, contrary to the simple two-occupations example in Section A.1.4). Therefore, the instrumental variables estimation is robust even to rather extreme idiosyncratic skill shocks. It also seems that implementing the IV will be help-

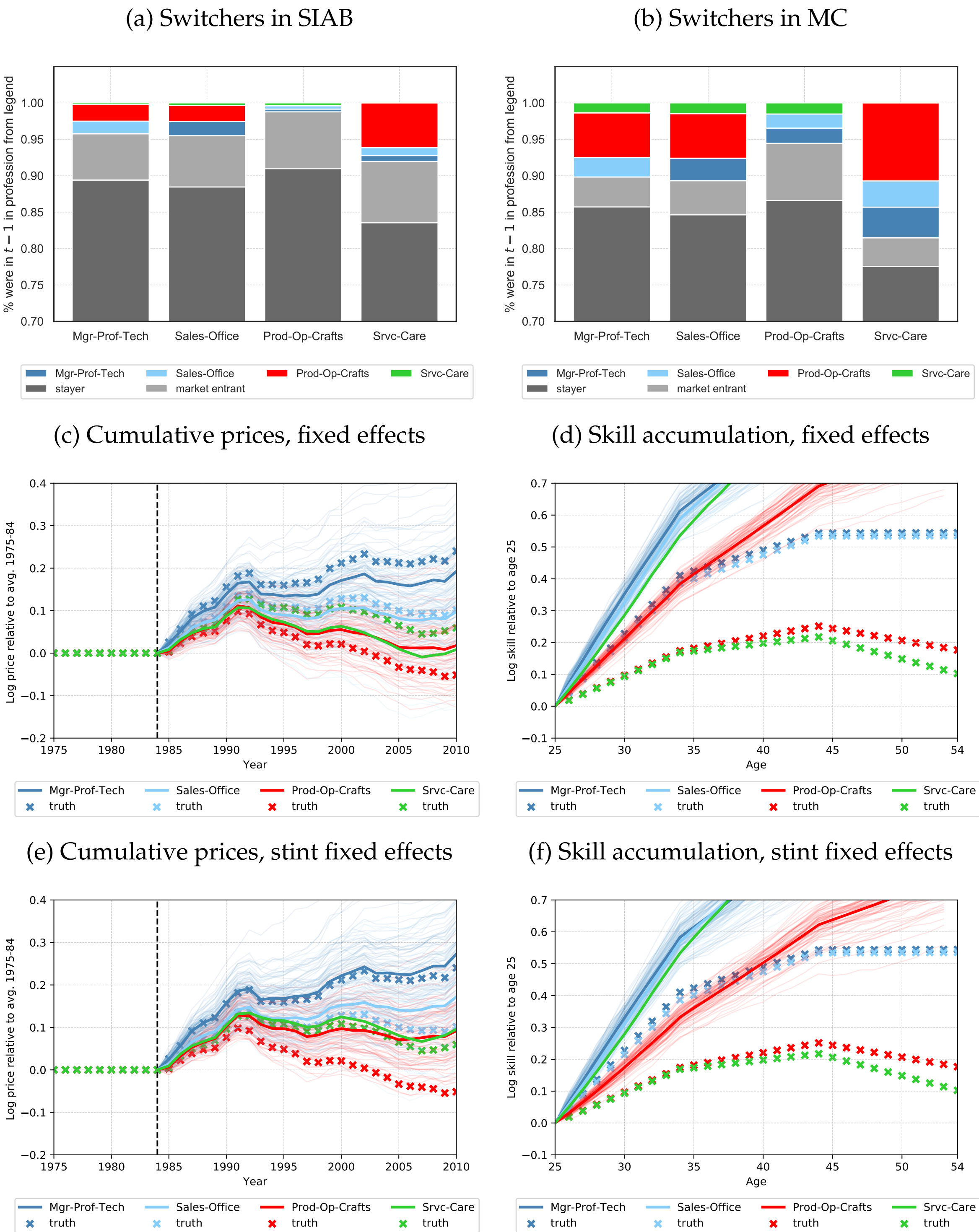
Figure B.5: Performance of IV, many shocks



ful in practice to check whether the (main) OLS estimates might be biased because of such large shocks.

Finally, we implement the occupation-specific fixed effects, which we have also expanded to include stint fixed effects and occupation-specific skill accumulation in order to account for the important facts of workers' labor market histories as argued in Section A.3. As Figure B.6 shows, this alternative estimation strategy does not perform well in the face of potentially severe endogeneity biases due to switching, especially in terms of the skill accumulation coefficients which are completely off. This might not be surprising as it was not devised to incorporate (lack of) endogenous switches in the first place (again, see discussion in Section A.3). In less extreme circumstances (e.g., those depicted in Fig.B.3) it performs quite well (unreported for brevity), however, and therefore we do use occupation-stint fixed effects with occupation-specific skill accumulation as another robustness check in our empirical estimates.

Figure B.6: Performance of occupation-specific fixed effects, many shocks



(a) Switchers in SIAB

(b) Switchers in MC

(c) Cumulative prices, fixed effects

(d) Skill accumulation, fixed effects

(e) Cumulative prices, stint fixed effects

(f) Skill accumulation, stint fixed effects

B.4 Switching Costs

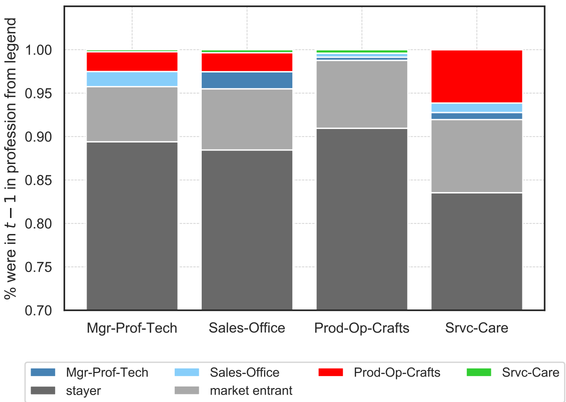
The theory appendix formally discussed additive switching costs occupations already. In an empirical robustness check we also extend the estimation period length because switching costs should be lower *relative* to task price and skill changes over a longer time horizon.

Here we examine a different specification for the switching costs, namely as a fraction of (log) wages. We assume that every worker has to pay a (psychic) utility cost when wanting to switch, which is individual-specific and equal to $c = 0.01 \cdot w_{i,t}$.[50] This cost reduces the number of switchers. We therefore still employ highly volatile skill shocks as in Figure B.4. Figure B.7 shows that also this type of switching costs does not bias our results. In fact, as they are identified well from wage growth of stayers, switching costs make any bias of the task prices less severe.

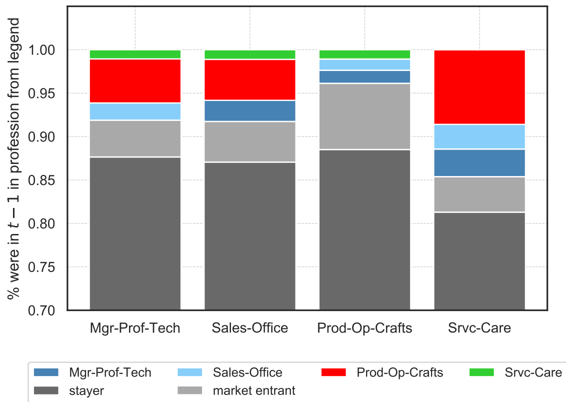
[50]The results are robust to setting the costs equal for every worker or only equal for every worker within a sector.

Figure B.7: Impact of switching costs

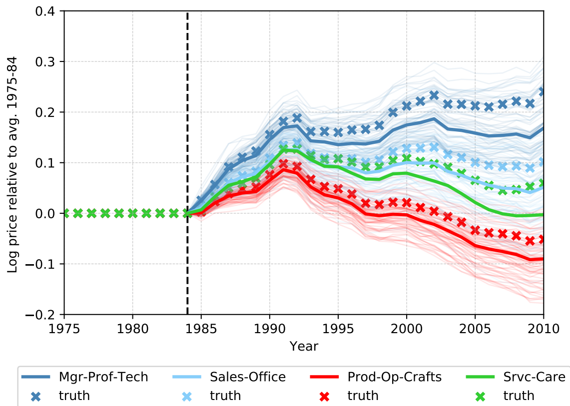
(a) Switchers in SIAB



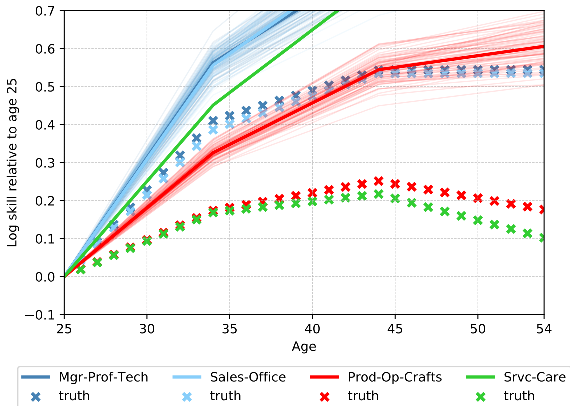
(b) Switchers in MC



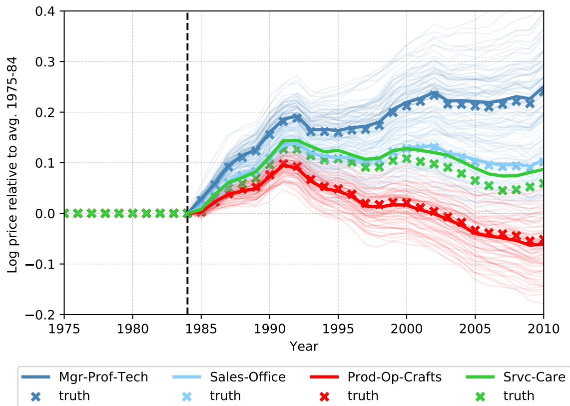
(c) Cumulative prices - OLS



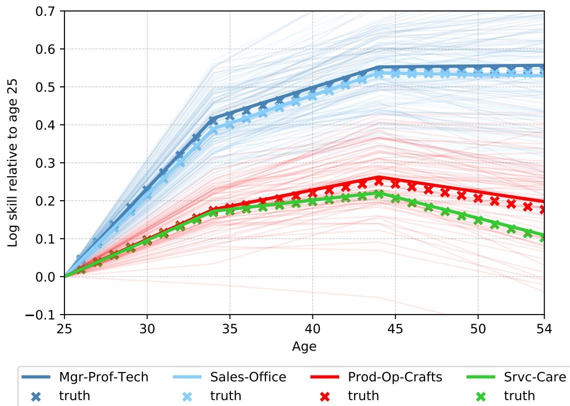
(d) Skill accumulation - OLS



(e) Cumulative prices - IV



(f) Skill accumulation - IV



C Dataset Construction

This study uses the factually anonymous Sample of Integrated Labor Market Biographies (version 7415). Data access was provided via a Scientific Use File supplied by the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB) under contract Number 101357.[51] See Ganzer et al. (2017) for an up to date overview of the data.

Structure: The SIAB is a 2% random sample of administrative social security records from 1975 to 2014. It includes employees covered by social security, marginal part-time employment, benefit receipts, officially registered as job-seeking or participating in programs of active labor market policies. It excludes the self-employed, civil servants, individuals performing military service as well persons not in the labor force. In total, it is therefore representative for 80% of the German workforce. Most notably, it contains an individual's full employment history, including a time-consistent occupational classifier, the corresponding wage, year of birth, place of work, and education. The data is exact to the day as employers need to notify the employment agency if the employment relationship changes. This means, there can be various employers for an individual worker within a year and those spells may even overlap as workers can have multiple employment contracts at a time. We transform this spell structure into a yearly panel structure by identifying the longest spell (a spell can have length of 365/366 days at most in a year) within a given year and deleting all the remaining spells. This procedure differs from the previous inequality literature employing the SIAB in the German context. For instance, Dustmann et al. (2009) aggregate all the information from various spells within a year, adding up all the earnings from multiple employment spells. Since our focus is on occupations, this is impossible to do as one cannot aggregate multiple categorical occupation information. Fortunately, the number of *full time* workers with more than one spell in a year is negligible and so of minor concern.

Occupations, education, age: The detailed 120 occupations of our main analysis can be found in Table C.2. We group these 120 occupations into 4 major groups in the spirit of Acemoglu and Autor (2011);

1. Managers-Professionals-Technicians (Mgr-Prof-Tech)
2. Sales-Office (Sales-Office)
3. Production-Operators-Craftsmen (Prod-Op-Crafts)
4. Services-Care (Srvc-Care)

[51]You can get access to a *test version* here: `http://fdz.iab.de/en/FDZ_Individual_Data/integrated_labour_market_biographies.aspx`. The full Scientific Use File can only be downloaded after having signed a contract with the FDZ. We carried out all the analyses making use of the templates provided by von Gaudecker (2014). The code is available at `https://gitlab.iame.uni-bonn.de/hmg/task-prices-de` upon request.

The grouping reflects differences in tasks performed on the job. Table C.1 shows average task intensities by occupation group obtained by aggregating answers of individual workers to questions such as: "how often do you repair something?"

Table C.1: Average tasks by occupation group

	analytical	interactive	routine	manual
Mgr-Prof-Tech	0.37	0.39	0.20	0.14
Sales-Office	0.19	0.34	0.13	0.10
Prod-Op-Crafts	0.09	0.16	0.32	0.26
Srvc-Care	0.23	0.30	0.20	0.29

Notes: Qualifications and Career Surveys (QCS), own calculations. In the QCS surveys, workers are asked what tasks they perform in their job. They provide answers on a scale: "never, sometimes, often". We assign numerical values 0, 0.3, 1 to these answers, respectively. We group all the questions into the four task categories in the top row of the table and average over occupation groups, implying that the four task categories do not need to sum to one as some occupations might overall be more intense in tasks than others. The six different QCS waves were pooled together. For more details see, for instance, Spitz-Oener (2006).

The education variable contained in the SIAB has a lot of inconsistencies and missing values as described in Fitzenberger et al. (2006). We use Fitzenberger et al.'s imputation to obtain an education variable with five distinct outcomes: missing information, without any postsecondary education, with apprenticeship training, with a high school diploma (Abitur), with a university or technical college degree. Depending on the specification we aggregate this education variable further into three groups: low (missing or without any postsecondary education), medium (apprenticeship training or high school diploma), and high (university or technical college degree).

The age bins used for estimating skill accumulation parameters are $[25, 34]$, $[35, 44]$, and $[45, 54]$.

Wage imputations: Despite being accurately measured as the employer can be punished for incorrect reporting, the contained wage variable has two major drawbacks for our analysis. First, wages are top coded, amounting to 12% censored observations for men and 4% censored observations for women on average across years. We impute the wages using a series of tobit imputations as in Card et al. (2013) or Dustmann et al. (2009), fitted separately for each year, gender and East-West region. We predict the upper tail employing controls for five age groups ($[17, 24]$, $[25, 34]$, $[35, 44]$, $[45, 54]$, $[55, 62]$) and five education groups as well as their interaction and allow the error variance to vary between age and education groups. Further, we include controls for the censored log wage on a constant, age (within age groups), the mean wage in other years, the fraction of censored wages in other years as well as a dummy if the person was only observed once in his life as in Card et al. (2013).[52] We use the predicted values $X'\hat{\beta}$ from the tobit regressions together with the estimated standard deviation $\hat{\sigma}$ to impute the censored wages y^c as follows: $y^c = X'\hat{\beta} + \hat{\sigma}\Phi^{-1}[k + u(1-k)]$, where $u \sim U[0, 1]$ and $k = \Phi[(c - X'\hat{\beta})/\hat{\sigma}]$ and c is the main censoring limit.[53] We deflate wages with respect to prices as of 2010 and smooth them using three year moving averages. Finally, we

[52] If this is the case, the mean wage in other years and the fraction of censored wages in other years is replaced by the sample mean.

[53] Accessible at `http://fdz.iab.de/en/FDZ_Overview_of_Data/working_tools.aspx`.

multiply them with a factor of 365/366 to receive yearly wages from daily wages.

Wage break 1983/1984: The second major concern with the wage variable is that the definition of a wage changed from 1983 to 1984. Prior to 1984 wages did not contain bonuses and one time payments. If one does not correct this break, it leads to a spurious increase in inequality between those years when the consistent periods 1975–1983 and 1984–2014 are not analyzed separately. We deal with this break by correcting wages prior to 1984 upwards following Fitzenberger (1999) and Dustmann et al. (2009). Their idea is that a worker's rank in the wage distribution between 1984 and 1983 should be similar. Additionally, they control for the fact that different percentiles of the wage distribution should be differently affected by the break since workers from higher percentiles are likely to receive higher bonuses. Therefore, they estimate locally weighted regressions of an individual's wage ratio in 1983/1984 and 1983/1982 on the rank of a person in the wage distribution. They then calculate a correction factor as the difference between the predicted, smoothed values from the two wage ratio regressions and multiply wages prior to the break with that factor. After this, some wages are corrected above the censoring limit. Dustmann et al. (2009) reset these wages back to the censoring limit and impute them in the same way they imputed wages which were above the limit anyway. This, however, is very problematic when analyzing wages within high skill occupations. For instance, by employing this procedure, the amount of censored wages within the Mgr-Prof-Tech group aged $[45, 54]$ increases from 40% to 80% in 1975. In contrast, there is only a rise from 38% to 50% in 1983. Therefore, the imputation now over-corrects wages the more they date back, which makes imputed and corrected wages of Mgr-Prof-Tech fall between 1975 and 1983, especially for older and more experienced workers. As this is likely to be a problem of the wage break correction approach and not a feature of the data because wages of all other occupations increased in that period, we follow a different approach by not imputing wages which were moved above the censoring limit. Instead, we do not reset wages back to the censoring limit if they were corrected above the limit.

Sample selection: The main dataset is restricted to full time working 25 to 54 year old men. Workers without information on the occupation are dropped from the analysis. Additionally, the years 2011 - 2014 are left out as the employment agency's official occupational classification changed in 2011 (from KLDB1988 to KLDB2010). A crosswalk exists in the data but is not 1:1 so that a clear break in employment and wages by occupation is observable between 2010 and 2011. Furthermore, we drop all spells of workers who ever worked in East Germany as well as permanently foreign workers.[54] After that, we are left with 721,953 persons and 8,492,131 person times year combinations. Out of this, 296,703 persons and 2,703,303 person times year combinations are women, whereas 425,250 individuals and 5,788,828 person times year combinations are men who are employed in the main estimations.

[54] That is, workers who are German at some point but foreign at another, are not dropped from the sample. In robustness checks we include include the dropped East Germans and foreigners.

Imputing unemployment and out of labor force spells: As unemployed persons receive social security benefits, we observe the universe of unemployed persons. If a worker leaves the labor force, however, we do not observe him anymore unless he returns to the labor force. Therefore, we impute out of labor force (olf) spells by filling up all missing years between two employment/unemployment spells in the data.[55] We then drop persons with olf spells longer than 10 consecutive years as prime age men in this case are more likely to have switched into the public sector than truly having left the labor force[56]. One of our key robustness checks (Section D.2.6) concerns the role of unemployment and out of labor force spells. For this, we relax the exogeneity assumption for unemployment and out of labor force by imputing the occupation where the worker "would have worked in had he not become unemployed or left the labor force". We do the imputation by comparing the (real) wage after an unemployment/olf spell with the wage before the unemployment/olf spell. We then impute the wage while in unemployment/olf as the lower of those two wages adjusted for inflation and set the occupation within this time to the occupation that corresponds to that lower wage. The rationale for this procedure is based on the idea that a worker could always choose the lower paying job but eventually decides to quit employment if he prefers becoming unemployed. As we only fill up olf spells between two employment or unemployment spells we therefore treat both unemployment and permanently leaving the labor force without returning to employment as exogenous actions. The number of men aged 25 to 54 who permanently leave the labor force is, however, extremely low (1.1%). The imputed sample therefore has no gaps for any person during his first and last observed employment spell. Using this sample, we then repeat all of the analysis but set the skill accumulation variables to zero if a person is unemployed or out of the labor force. After that, we are left with 789,413 persons and 10,639,100 person times year combinations. Out of this, 345,544 persons and 3,999,340 person times year combinations are women, whereas 443,869 individuals and 6,639,760 person times year combinations are men who are used for the main estimations.

[55]The imputation is done before the sample selection so that we also fill up with out of labor force spells at age 54 in the main specification. That is, the last employment observation can occur during ages > 54.

[56]Between 1996 and 1998, many workers in occupation 102 "Physicians until Pharmacists" leave the sample and return afterwards as mentioned by Ganzer et al. (2017). We impute those likely erroneously missing observations by setting the occupation to 102 if a worker was in 102 in 1995 and returned in 1999 or 2000 and linearly interpolate the missing wage using the observations in 1995 and 1999/2000.

Table C.2: Grouping of occupations

Group	SIAB occupation
Managers	Entrepreneurs, managing directors, divisional managers
	Management consultants, organisors until chartered accountants, tax advisers
	Members of Parliament, Ministers, elected officials until association leaders, officials
Professionals	Architects, civil engineers
	Artistic and assisting occupations (stage, video and audio) until performers, professional sportsmen, auxiliary artistic occupations
	Chemists, chemical engineers until physicists, physics engineers, mathematicians
	Data processing specialists
	Economic and social scientists, statisticians until scientists n.e.c.
	Electrical engineers
	Home wardens, social work teachers
	Journalists until librarians, archivists, museum specialists
	Mechanical, motor engineers
	Music teachers, n.e.c. until other teachers
	Musicians until scenery/sign painters
	Navigating ships officers until air transport occupations
	Physicians until Pharmacists
	Social workers, care workers until religious care helpers
	Soldiers, border guards, police officers until judicial enforcers
	Survey engineers until other engineers
	University teachers, lecturers at higher technical schools and academies until technical, vocational, factory instructors
Technicians	Biological specialists until physical and mathematical specialists
	Chemical laboratory assistants until photo laboratory assistants
	Electrical engineering technicians until building technicians
	Foremen, master mechanics
	Measurement technicians until remaining manufacturing technicians
	Mechanical engineering technicians
	Other technicians
	Technical draughtspersons
Sales	Bank specialists until building society specialists
	Commercial agents, travellers until mobile traders
	Forwarding business dealers
	Health insurance specialists (not social security) until life, property insurance specialists
	Publishing house dealers, booksellers until service-station attendants
	Salespersons
	Tourism specialists until cash collectors, cashiers, ticket sellers, inspectors
	Wholesale and retail trade buyers, buyers
Office	Cost accountants, valuers until accountants

Continued on next page

Table C.2: Grouping of occupations

Group	SIAB occupation
	Office auxiliary workers
	Office specialists
	Stenographers, shorthand-typists, typists until data typists
Production	Building labourer, general until other building labourers, building assistants, n.e.c.
	Ceramics workers until glass processors, glass finishers
	Chemical laboratory workers until vulcanisers
	Chemical plant operatives
	Drillers until borers
	Electrical appliance fitters
	Electrical appliance, electrical parts assemblers
	Engine fitters
	Farmers until animal keepers and related occupations
	Generator machinists until construction machine attendants
	Goods examiners, sorters, n.e.c.
	Goods painters, lacquerers until ceramics/glass painters
	Iron, metal producers, melters until semi-finished product fettlers and other mould casting occupations
	Locksmiths, not specified until sheet metal, plastics fitters
	Machine attendants, machinists' helpers until machine setters (no further specification)
	Metal grinders until other metal-cutting occupations
	Metal polishers until metal bonders and other metal connectors
	Metal workers (no further specification)
	Miners until shaped brick/concrete block makers
	Other assemblers
	Packagers, goods receivers, despatchers
	Paper, cellulose makers until other paper products makers
	Paviors until road makers
	Plant fitters, maintenance fitters until steel structure fitters, metal shipbuilders
	Plastics processors
	Sheet metal pressers, drawers, stampers until other metal moulders (non-cutting deformation)
	Sheet metal workers
	Special printers, screeners until printer's assistants
	Spinners, fibre preparers until skin processing operatives
	Steel smiths until pipe, tubing fitters
	Tracklayers until other civil engineering workers
	Turners
	Type setters, compositors until printers (flat, gravure)
	Welders, oxy-acetylene cutters
	Wood preparers until basket and wicker products makers

Continued on next page

Table C.2: Grouping of occupations

Group	SIAB occupation
Operators	Assistants (no further specification)
	Motor vehicle drivers
	Post masters until telephonists
	Railway engine drivers until street attendants
	Stowers, furniture packers until stores/transport workers
	Transportation equipment drivers
	Warehouse managers, warehousemen
Craftsmen	Agricultural machinery repairers until precision mechanics
	Bakery goods makers until confectioners (pastry)
	Bricklayers until concrete workers
	Butchers until fish processing operatives
	Carpenters
	Carpenters until scaffolders
	Cutters until textile finishers
	Dental technicians until doll makers, model makers, taxidermists
	Electrical fitters, mechanics
	Gardeners, garden workers until forest workers, forest cultivators
	Motor vehicle repairers
	Other mechanics until watch-, clockmakers
	Painters, lacquerers (construction)
	Plumbers
	Roofers
	Room equippers until other wood and sports equipment makers
	Stucco workers, plasterers, rough casters until insulators, proofers
	Telecommunications mechanics, craftsmen until radio, sound equipment mechanics
	Tile setters until screed, terrazzo layers
	Toolmakers until precious metal smiths
	Wine coopers until sugar, sweets, ice-cream makers
Service	Cashiers
	Cooks until ready-to-serve meals, fruit, vegetable preservers, preparers
	Doormen, caretakers until domestic and non-domestic servants
	Factory guards, detectives until watchmen, custodians
	Hairdressers until other body care occupations
	Household cleaners until glass, buildings cleaners
	Housekeeping managers until employees by household cheque procedure
	Laundry workers, pressers until textile cleaners, dyers and dry cleaners
	Others attending on guests
	Restaurant, inn, bar keepers, hotel proprietors, catering trade dealers until waiters, stewards
	Street cleaners, refuse disposers until machinery, container cleaners and related occupations

Continued on next page

Table C.2: Grouping of occupations

Group	SIAB occupation
Care	Dietary assistants, pharmaceutical assistants until medical laboratory assistants
	Medical receptionists
	Non-medical practitioners until masseurs, physiotherapists and related occupations
	Nursery teachers, child nurses
	Nurses, midwives
	Nursing assistants

D Robustness of Task Price Estimates

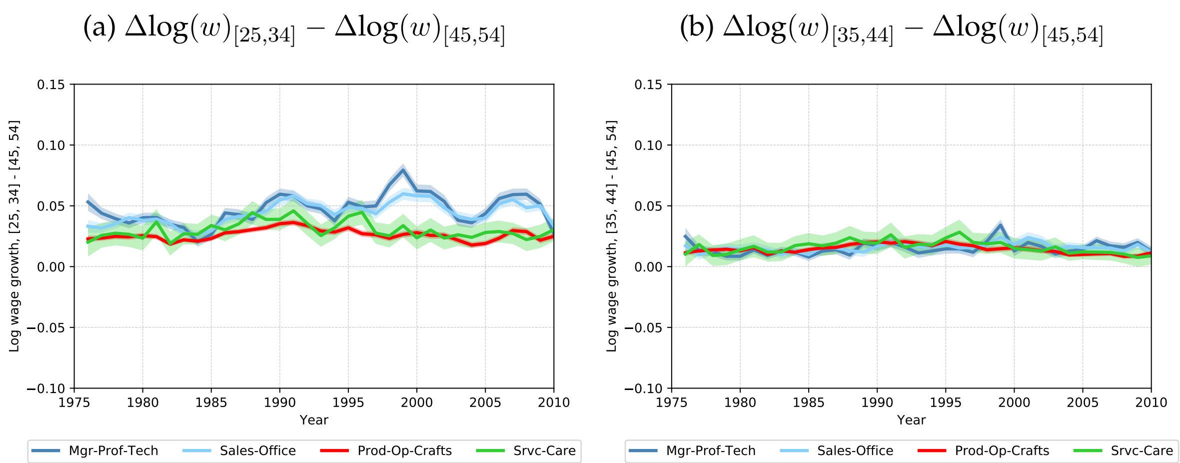
Section 4 of the main text has estimated the skill accumulation functions and changes in task prices for detailed occupations as well as broader groups. We found that task prices in fact increased with employment growth in Germany during 1984–2010, contrary to changes in average wages across occupations. This section shows that these results are robust to various alternative sample definitions and estimation specifications. We start by providing evidence regarding the identification assumption of time-invariant skill accumulation.

D.1 Constancy of Skill Accumulation

Section 2.2 showed that, if skill accumulation in occupations is time-invariant, we can consistently estimate it in the base period and control for it thereafter. Since we interpret general accelerations or decelerations in wage growth *between* occupations as changes in task prices, and changes of estimated cross-accumulation parameters may also occur because of changing non-pecuniary or forward-looking valuations (see Appendix A.2.2), this constancy of skill accumulation assumption mainly implies that the relative $\Gamma_{k',k,}$ for different types of workers *within* occupations do not change over time.

Figure D.1 summarizes evidence in this respect by plotting relative average wage growth within broad occupation groups during the sample period. We compute the yearly wage growth for the same age groups as in the price estimations separately and then deduct the mean wage growth of older workers (45–54 year olds) from the wage growth of younger workers (25–34 or 35–44 year olds). In the model, all workers should face the same task prices within a occupation independent of their age. Therefore, normalizing wage growth of younger workers with that of older workers should eliminate the changes in prices from the changes in wages, giving us the relative changes (i.e., accumulation) of skills.

Figure D.1: Wage growth differences between age groups



Notes: Shaded areas are 95% confidence intervals.

That the lines in Figure D.1 are above zero, and that they are located higher in the left than in the right panel, once again reflects the faster skill accumulation of younger compared to older workers. The relative wage growth in Prod-Op-Crafts and Srvc-Care occupations also largely stayed constant over time (i.e., the lines are essentially horizontal), which is consistent with our identification assumption of time-invariant skill accumulation. The growth rate of

25–34 year olds' wages in Mgr-Prof-Tech and Sales-Office occupations did however temporarily accelerate during the late 1990s and again after the mid-2000s, also slightly during the early 1990s. The accelerations were partly statistically significant as indicated by the confidence intervals.

We make the following observations and empirical checks in order to assess the implications of this finding for our task price estimates. First, this steepening of high-earning occupations' early career wage profiles appears temporary, reverting in the dotcom crisis of the early 2000s and in the great recession of the late 2000s, and it thus may not be a fundamental longterm change of the skill accumulation function. It does not occur for the middle age group of 35–44 year olds and hence seems relevant only early in the career. It also occurs late in the sample and therefore should not directly influence the substantial trends in task prices up until the end of the 1990s (Fig.2a). In the next section, we further restrict the sample to the older 45–54 age group and obtain qualitatively similar estimates to our main results. Overall, we therefore think that the acceleration of young Mgr-Prof-Tech and Sales-Office occupation workers' wage growth toward the end of the analysis period is not a critical concern for our task price estimates.

D.2 Alternative Samples

D.2.1 Different Age Groups

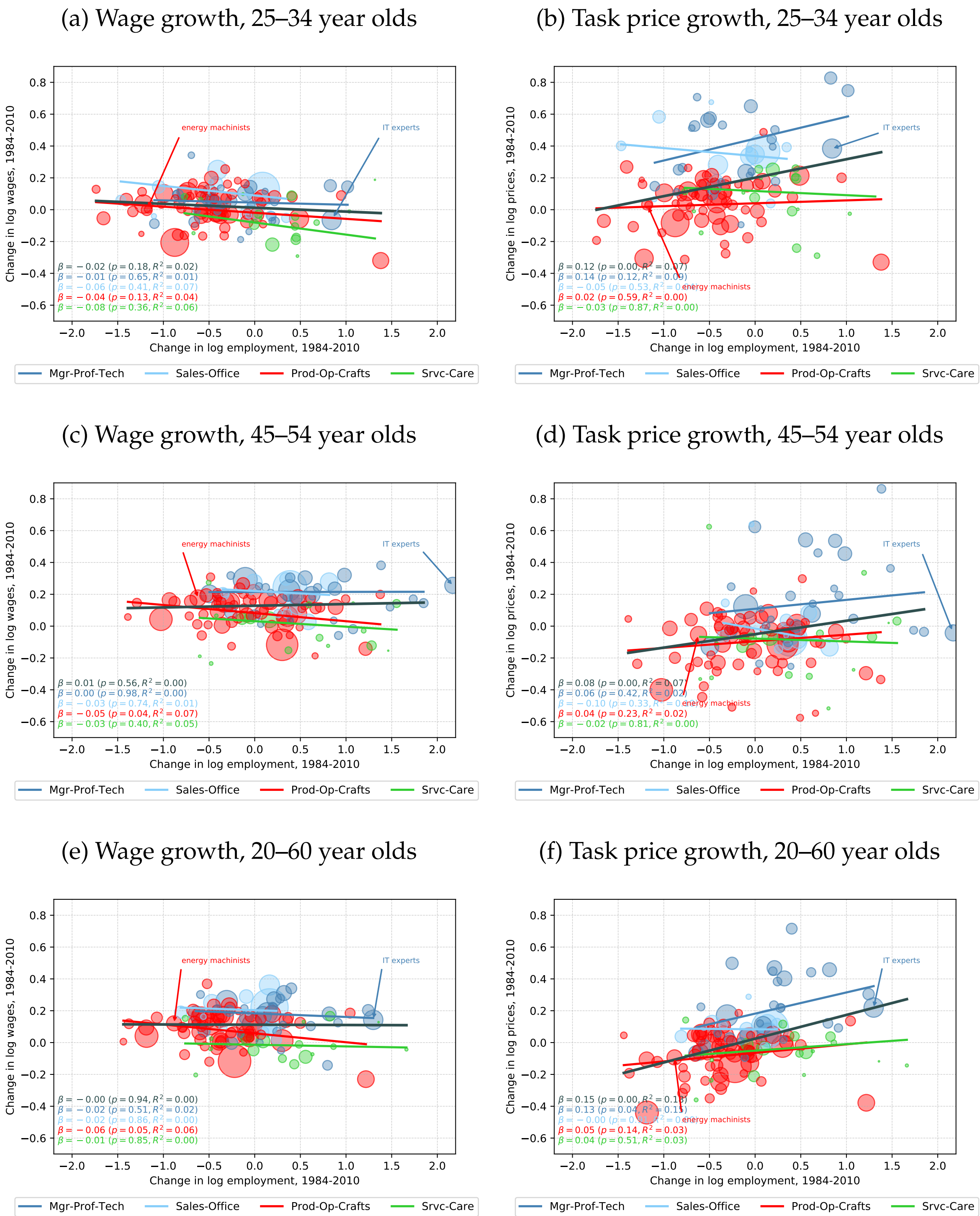
One important robustness check is to estimate the model for different age groups. This is because, as discussed in detail in Appendix A.2.2, *changing* dynamic considerations in workers' occupation choices should arguably not be as important for older as for younger workers. We also saw in the previous subsection evidence for steepening life cycle wage growth within Mgr-Prof-Tech and Sales-Office occupations, which would imply differing task price estimates for these occupations in an old versus young sample.

The top panels of Figure D.2 depict the results from estimating Equation (10) for the subsample of 45–54 year olds only. We saw in the previous section that within-occupation wage growth of this oldest age group did not change compared to 35–44 year olds whereas young (25–34) Mgr-Prof-Tech and Sales-Office occupation workers' relative wage growth exhibited phases of acceleration toward the end of the period. Therefore, it may be a valuable check whether task price estimates are very different in this sample. Given the rather flat skill accumulation estimates for 45–54 year olds (e.g., Figure 2b), estimating the task prices (while still controlling for what remains of skill accumulation in the base period) for this age group might also be considered a basic application of Heckman et al. (1998)'s flat spot identification strategy.

We see in Figure D.2a that wage growth is again uncorrelated with employment growth in this older subsample whereas in Figure D.2b task price growth once more increase with occupations' size growth. This positive relationship is somewhat weaker than in the full sample (regression slope of 0.08 compared to 0.15), but it is still in general economically and statistically significant. The estimates for 25–34 and 35–44 year olds are accordingly somewhat stronger but overall the estimation results are robust to different subsamples by age. Therefore, our task price estimates appear largely robust to the steepening life-cycle profiles in Mgr-Prof-Tech and Sales-Office occupations as well as to potentially changing dynamic considerations of workers over time. The latter conclusion is also reinforced by similar task price estimates in the sample of women, which we turn to next, as women should arguably also have more muted dynamic incentives than men when choosing occupations.

For completeness, the bottom panels of Figure D.2 show task price estimates, occupational

Figure D.2: Employment growth versus wages or prices (alternative ages)



Notes: One bubble represents one of 120 occupations available in the SIAB data. The size of a bubble is proportional to the average size of the occupation across all sample years. The slope coefficients β were computed with weighted least squares where the weight was also equal to the average occupation size.

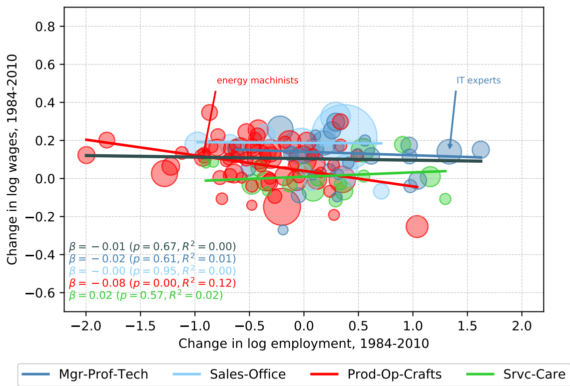
wages, and employment changes for wider age ranges (20 to 60) than in our main sample. In particular, we see that the results are essentially the same for 20–60 year olds as for prime age males.

D.2.2 East Germans, Foreigners, and Women

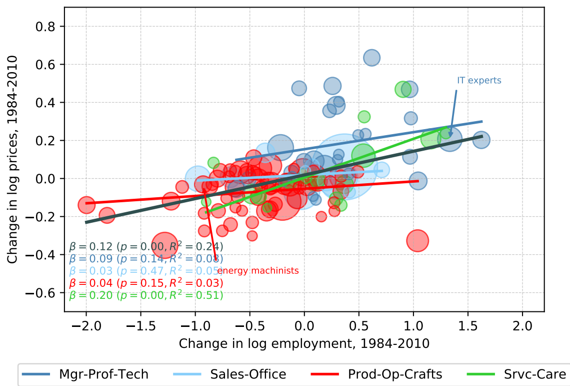
We have restricted our main sample to West German men as these can be defined consistently over the 1975–2010 period and many potentially confounding factors that may have affected women or foreigners, such as increasing labor force participation rates, declining workplace discrimination (e.g., Hsieh et al., 2013), and rapidly rising educational attainment, do not apply. Nonetheless, the entry of women and foreigners as well as the reunification with the East constituted major supply shifts affecting the German labor market during our sample period. If women or foreigners were more inclined to work in Srvc-Care, for example, rising employment and falling wages in these occupations may be due to changes in labor supply. Also, if women or foreigners tend to earn less in certain occupations, estimated task prices may be confounded by the closing of such wage gaps over time. We therefore examine whether general equilibrium and composition effects due to supply shifts are important by checking if our estimates differ when we include these groups in our sample.

Figure D.3: Different samples

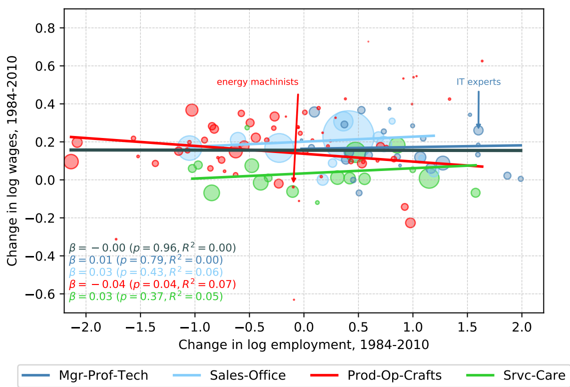
(a) Include foreigners + women, wages



(b) Include foreigners + women, prices



(c) Women only, wages



(d) Women only, prices

Notes: One bubble represents one of 120 occupations available in the SIAB data. The size of a bubble is proportional to the average size of the occupation across all sample years. The slope coefficients β were computed with weighted least squares where the weight was also equal to the average occupation size.

Figure D.3 shows the results. First, the task prices hardly change when we include males working in XX East Germany (increases sample by circa 15%) and foreign males (circa 6%) in the estimation. Still more notable, when we estimate our model for 25–54XX year old women only (circa XX percent of the male sample), the relationship between occupations' task price and employment growth is almost the same as among prime age males (i.e., 0.16 versus 0.15 slope of the regression line) and task prices similarly tend to polarize (i.e., rise for the Mgr-Prof-Tech, Sales-Office, and Srvc-Care occupation groups). It is also interesting to see that these results are the same despite a very different employment composition in general, with many more Sales-Office and Srvc-Care occupations among women than among men (e.g., compare the bubble sizes here to Figure 1 in the main text).

Overall the findings show that composition or general equilibrium effects due to changes in labor supply seem not to have substantially altered our estimates. They also indicate that the same forces that have polarized task prices for men may have been at least as strong for women, potentially dominating other drivers of wages for women in different occupations over the last decades. Finally, they again suggest that dynamic considerations may not be a first-order concern for severely confounding our conclusions, since these should be different for men and women in the labor market while the actual task price results are similar.

D.2.3 Different Base Periods

In the main estimations, our base period are the years 1975–1984. We examine whether our results are robust to different choices of base period.

XX does the downward-sloping green regression line suggest that for Srvc-Care supply shocks among men dominated after 1990 (and in Figure D.2b among older men in general)?XX

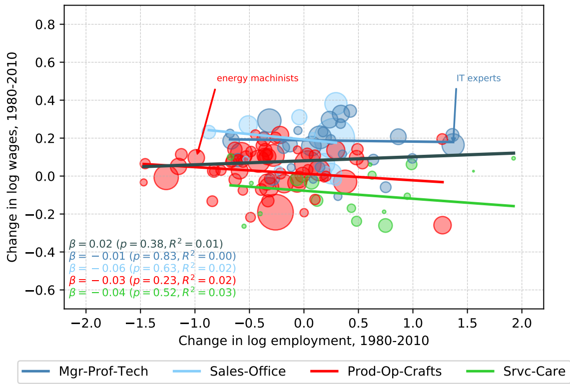
Figure D.4 reports task price estimates for different base periods, plotting them *relative* to Prod-Op-Crafts in order to remove differences in general wage growth across periods which are not our focus. We see that estimates using the period 1975–85 are very similar to our main results and that shorter initial base periods (1975-80 and 1975–83) yield more extreme relative task price changes. This is likely due to the fact that relative wage growth in Prod-Op-Crafts was strong during 1975-80. Accordingly, the long base period 1975–90 covering all of the pre-unification era yields substantially smaller relative task price changes than our main estimates. What all the different base periods have in common, however, is that task prices robustly polarize during the subsequent estimation periods. Our main result that task prices polarized over the last decades is therefore robust to the choice of the base period and the assumption of constant relative task price changes during that time.

D.2.4 No wage imputation

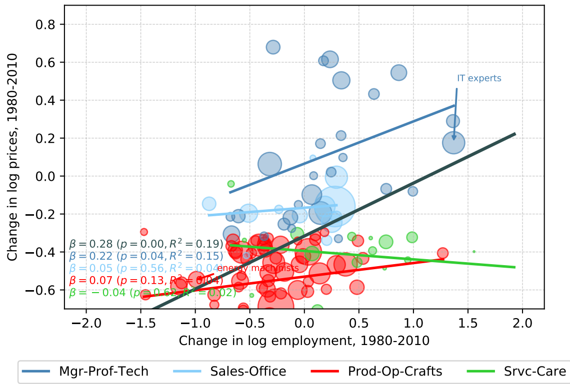
The following examines our main estimates when we do not impute top-coded wages as described in Section C. One may be concerned that our task price estimates and their relation to employment growth may unduly stem from the fact that more than XX percent of wages in high-earning Mgr-Prof-Tech and Sales-Office XX occupations are (incorrectly) imputed.

Figure D.4: Alternative Choices of the Base Period

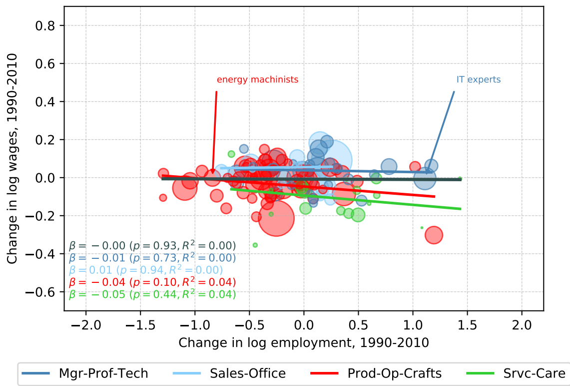
(a) Base period 1975–1980, wages



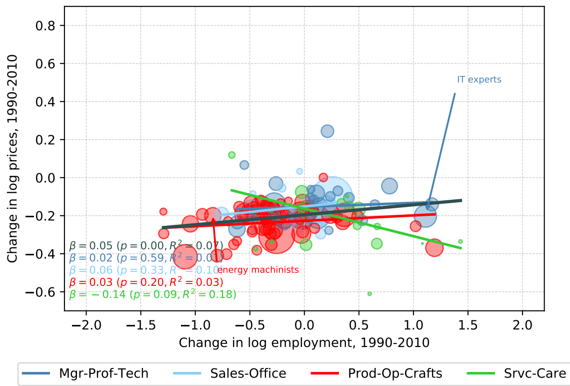
(b) Base period 1975–1980, prices



(c) Base period 1975–1990, wages



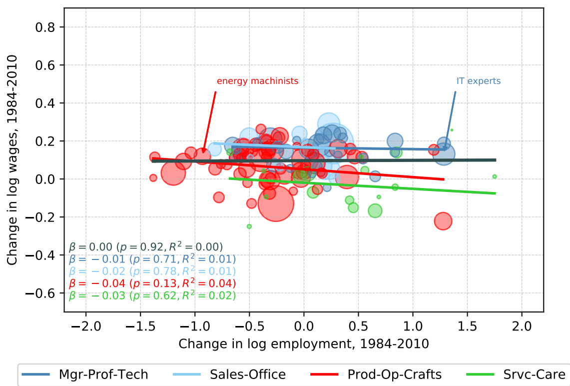
(d) Base period 1975–1990, prices



Notes: One bubble represents one of 120 occupations available in the SIAB data. The size of a bubble is proportional to the average size of the occupation across all sample years. The slope coefficients β were computed with weighted least squares where the weight was also equal to the average occupation size.

Figure D.5: No wage imputation

(a) Wages



(b) Prices

Notes: One bubble represents one of 120 occupations available in the SIAB data. The size of a bubble is proportional to the average size of the occupation across all sample years. The slope coefficients β were computed with weighted least squares where the weight was also equal to the average occupation size.

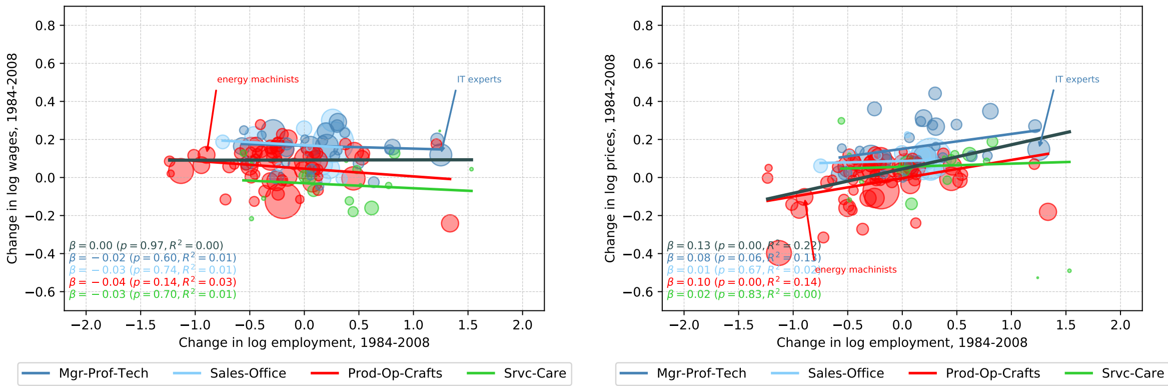
Figure D.5 shows a robustness check for this in which we do not impute at all and rather keep top-coded wages at the censoring limit XX. Despite this (probably too) conservative treatment, our main results still hold: employment growth is uncorrelated with occupational wages but it is significantly positively correlated with task price changes, albeit more modestly so. Also, task prices for Mgr-Prof-Tech and Sales-Office unsurprisingly rise less than in our main sample where we do impute. Overall, we conclude that potential mistakes in the imputation of top-coded wages do not drive our qualitative results.

D.2.5 Different Period Lengths

We also estimate the model with four- and five-year periods instead of the annual data used for the main results. Such increased period lengths should be informative about whether our findings are generally robust to exploiting relatively short-run wage changes in order to estimate task prices and skill accumulation. For example, if initial wage costs of switching occupations are large and potentially changing over time, longer periods may be able to bypass the bulk of these wage drops as argued in Appendix **??**. Also, if workers' idiosyncratic skill shocks or learning about skills (e.g., as in Appendix A.2.1) are correlated over time,[57] one-year periods may be too short for estimates not to be confounded by such autocorrelations.

Figure D.6: Different Period Lengths

(a) Wages (b) Prices



Notes: One bubble represents one of 120 occupations available in the SIAB data. The size of a bubble is proportional to the average size of the occupation across all sample years. The slope coefficients β were computed with weighted least squares where the weight was also equal to the average occupation size.

Figure D.6 graphs the results from increasing the period length to four and five years.[58] We see that, although we need to stop after 2008 as our last four-year period, the results for the former are very similar to our main estimation. We conclude that our findings are robust to longer period lengths and that concerns such as serial correlation of skill shocks or changing wage costs of switching do not appear of first-order importance for our estimates. These

[57]The evidence on the existence and importance of correlated wage shocks seems to be mixed (e.g., see Gibbons and Waldman, 1999, and the references therein).

[58]We drop all the years which are not a multiple of 4 relative to the base period 1984 and so keep 1976, 1980, 1984, 1988 and so on. Wage growth and occupation choice indicators are then calculated on the basis of these 4 year periods, i.e., the wage growth refers to the growth between 1988 and 1992, for instance. To avoid having only one point of data within the base period and therefore estimating the skill accumulation from differences between 1975 and 1980 only, we use 4 years.

results are also reassuring concerning robustness with respect to the treatment of employment and labor force status in our main estimations because longer period lengths should also include more workers who switched occupations via an intermittent unemployment spell (see discussion of $\gamma_{k',k,a}$ estimates in Table 3). We now turn to explicit modeling of unemployment and labor force exit.

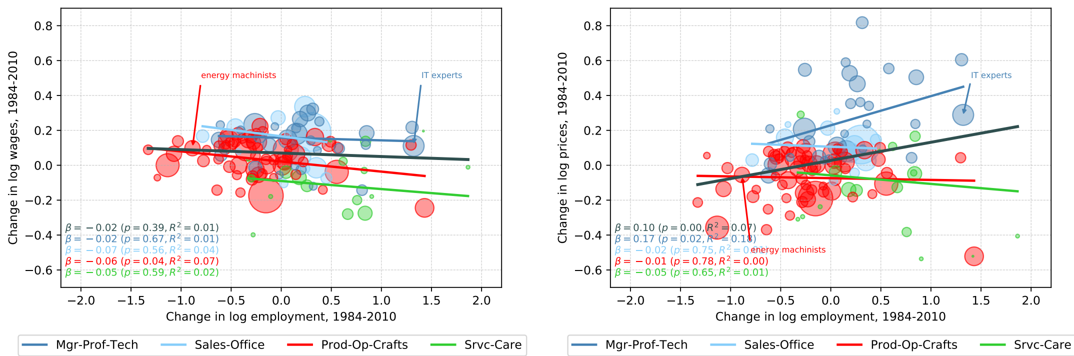
D.2.6 Unemployment and Leaving the Labor Force as a Choice

Another key robustness check is to allow for endogenous unemployment and exit from the labor force. In the main estimation we have assumed that entering and leaving the sample is exogenous. This is obvious for individuals who reach age 25 or 60 (the borders of our sample age range) but it might not be an innocuous assumption during the career. In particular, workers may *choose to* become unemployed or leave the labor force if they obtain a sufficiently bad idiosyncratic skill shock or vice versa for a sufficiently good shock, and if the (time-limited) benefits or other non-labor income they obtain are sufficiently high. This could lead to endogeneity bias in our task price estimates.

In this robustness check, we therefore assume that becoming unemployed or leaving the labor force temporarily is fully endogenous.[59] We do this by imputing workers' wages and their occupation choices if they are unemployed or out of the labor force for any number of years between two spells of employment. We impute those by comparing pre and post unemployment/olf wages and assigning them the lower of those two wages adjusted for inflation. That, is we assume that workers could well have worked within the lower paying occupation but chose to become unemployed or leave the labor force for some period of time instead (further details in Section C). On this sample, which is about 10% larger in size, we then repeat the estimation. The results, depicted in Figure **??** are again almost unchanged, which indicates that exogenous unemployment or leaving the labor force is not a critical assumption in our model.

Figure D.7: Filling Unemployment and Out of Labor Force Spells

(a) Wages (b) Prices



Notes: One bubble represents one of 120 occupations available in the SIAB data. The size of a bubble is proportional to the average size of the occupation across all sample years. The slope coefficients β were computed with weighted least squares where the weight was also equal to the average occupation size.

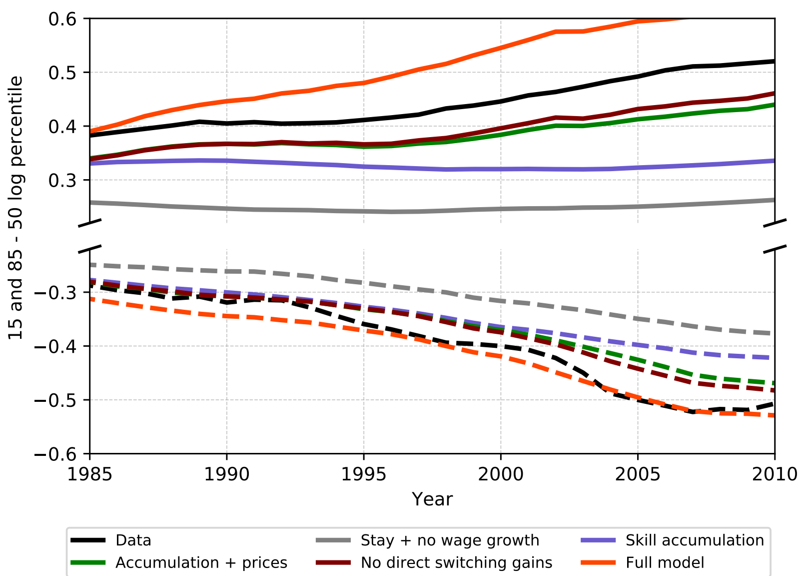
[59]The reality is likely somewhere in between these two extremes. We do maintain the assumption that permanently leaving employment is exogenous because for prime age males this is quite rare (roughly 1.1% each year as opposed to 2.3% for temporary unemployment) and likely often due to relatively exogenous factors such as illness/death, moving to East Germany or abroad, becoming self-employed or civil servant, etc..

Table D.1: Estimates for $\gamma_{k',k,a}$ with filled unemployment and olf spells

		Age group		
Previous sector	Current sector	[25, 34]	[35, 44]	[45, 54]
Mgr-Prof-Tech	Mgr-Prof-Tech	0.044	0.011	-0.002
	Sales-Office	0.174	-0.035	-0.135
	Prod-Op-Crafts	0.021	-0.160	-0.211
	Srvc-Care	-0.092	-0.391	-0.397
Sales-Office	Mgr-Prof-Tech	0.441	0.070	-0.004
	Sales-Office	0.039	0.012	-0.002
	Prod-Op-Crafts	0.103	-0.044	-0.154
	Srvc-Care	-0.098	-0.329	-0.281
Prod-Op-Crafts	Mgr-Prof-Tech	0.549	0.168	0.062
	Sales-Office	0.240	0.076	-0.029
	Prod-Op-Crafts	0.017	0.006	-0.009
	Srvc-Care	-0.121	-0.180	-0.145
Srvc-Care	Mgr-Prof-Tech	0.947	0.345	0.167
	Sales-Office	0.464	0.192	0.059
	Prod-Op-Crafts	0.348	0.193	0.110
	Srvc-Care	0.011	0.001	-0.013

Notes: The table shows the estimated $\hat{\gamma}_{k',k,a}$ for age groups a. k' is last period's occupation. k is the current occupation. Intermittent unemployment gaps were filled up as described in the text.

Figure D.8: With filled up UE/OLF Spells



D.3 Alternative Estimation Approaches

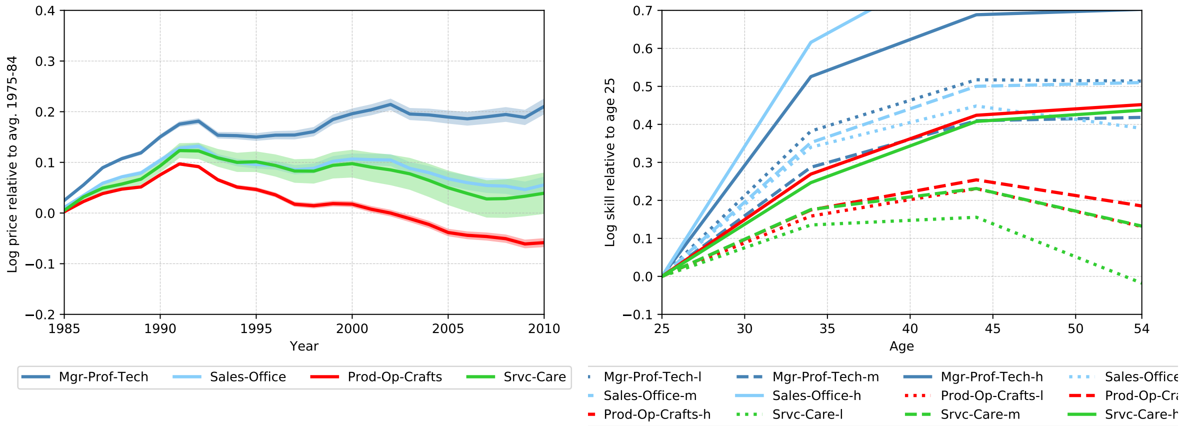
D.3.1 Education Specific Skill Accumulation

In the main estimation, skill accumulation varies by combination of current and last year's occupation as well as by age in order to account for the different life-cycle wage facts documented in Section **??**. In this robustness check we also allow for the fact that skill accumulation may additionally vary by the worker's education level on top of detailed occupation and age.

Figure D.9 depicts the results. Practically, considering skill accumulation Equation (7), we add dummies for high (university or college degree), medium (apprenticeship or Abitur), and low (without postsecondary) education level to the worker characteristics vector $X_{i,t-1}$ and the according coefficients to the $\Gamma_{k',k}$ accumulation parameter vector.

Figure D.9: Education Specific Skill Accumulation

(a) Prices (b) Skill Accumulation



Legend: -l: missing or without any post-secondary education; -m: apprenticeship training or high school diploma; -h: university or technical college degree.

D.3.2 Instrumental Variables Estimation

In Section 2.2 we argued that our main estimation using the fully saturated model for skill accumulation across occupation combinations largely accounts for endogenous switches due to idiosyncratic skill shocks. We also discussed an alternative estimation method, which instruments endogenous current period occupations using exogenous last period occupations.

Figure D.9 reports the results from this instrumental variables (IV) estimation.[60] The task prices as well as the average skill accumulation are almost identical to our main estimation.

[60]As discussed in Section 2.2, we can only estimate the skill accumulation for each of four average task choices, since we only have four instruments. That is, we reduce Equation (7) to $\Delta s_{k,i,t} = \sum_{k'=1}^{K} I_{k',i,t-1} \cdot X'_{i,t-1}\Gamma_{k'} + u_{k,i,t}$ so that the estimation Equation (8) becomes

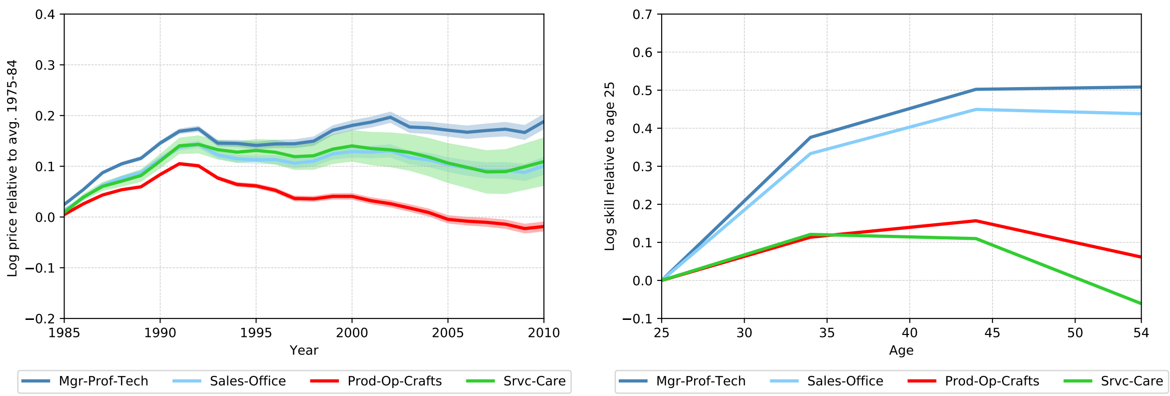
$$\Delta w_{i,t} = \sum_{k=1}^{K} \bar{I}_{k,i,t}\Delta\pi_{k,t} + \sum_{k'=1}^{K} I_{k',i,t-1} \cdot X'_{i,t-1}\Gamma_{k'} + v_{i,t},$$

with $v_{i,t} \equiv \sum_{k=1}^{K} \bar{I}_{k,i,t}u_{k,i,t}$ and $\bar{I}_{1,i,t}I_{1,i,t-1}, \bar{I}_{2,i,t}I_{2,i,t-1}, \bar{I}_{3,i,t}I_{3,i,t-1}, \bar{I}_{4,i,t}I_{4,i,t-1}$ instrumented by $I_{1,i,t-1}, I_{2,i,t-1}, I_{3,i,t-1}, I_{4,i,t-1}$, both interacted with the age dummies. Notice that under the above assumption of homogeneous skill accumulation, the middle term in the estimation equation does not depend on current period sector choice anymore.

Sections A.1 and B.3 showed formally and using Monte Carlo simulations, respectively, that both the fully saturated model as well as the IV should give us a lower bound to the absolute changes in task prices relative to the reference occupation (Prod-Op-Crafts). Therefore, these results make us confident that actual task prices polarized at least as strongly as shown in our main estimation results.

Figure D.10: Instrumental Variables Estimation

(a) Prices (b) Skill Accumulation



Notes: Instruments are one and two period lagged occupational choices as well as the interaction of one and two lag choices.

D.3.3 Fixed Effects Estimation

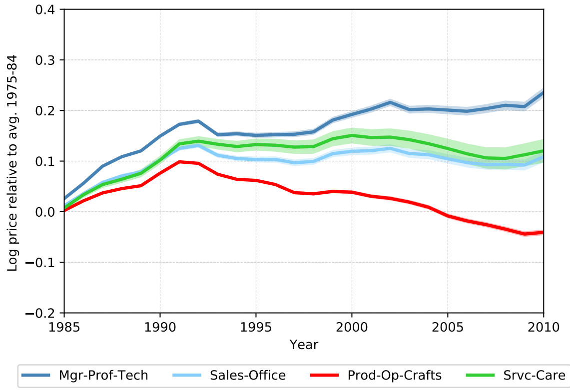
We also compare our results to the alternative estimation method using fixed effects. As discussed in Appendix A.3, even without idiosyncratic skill shocks, the fixed effects requires detailed controls of worker's entire labor market history. With idiosyncratic skill shocks, the bias is potentially large and cannot in general be signed, which is illustrated in the Monte Carlo simulations of Appendix **??**. Nonetheless, it should be supportive of our empirical results if this alternative estimation approach yielded qualitatively similar findings.

Figure D.9 graphs the results from the fixed effects estimation. The task prices widen much more strongly than in our main estimation and skill accumulation continues to increase over the whole life-cycle (it is hardly concave and almost linear, even at ages 45–60). The fact that task prices for the high skill accumulation Mgr-Prof-Tech and Sales-Office occupations rise substantially faster in the fixed effects than the main estimation method is consistent with the results from our Monte Carlo simulations, even in the absence of skill shocks (see Figure **??**). These suggested that task price growth in the fixed effects, as it is usually implemented, is confounded by skill accumulation and therefore overestimated for the high skill-growth occupations. Footnote **??** in Appendix A.3 provides a somewhat informal economic explanation. Nonetheless, the estimated task prices also polarize in the fixed effects approach, which lends further support to our main empirical results in this paper.

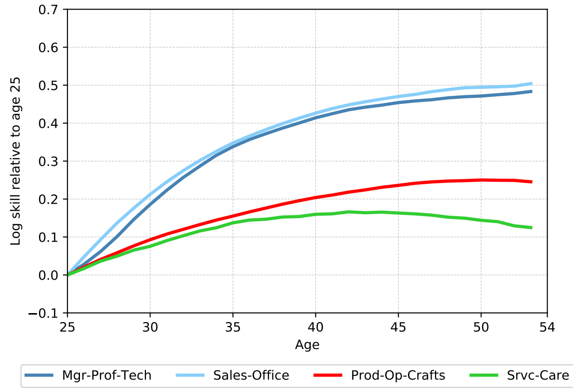
D.3.4 Accounting for Non-Pecuniary Benefits

Figure D.11: Fixed Effects Estimation

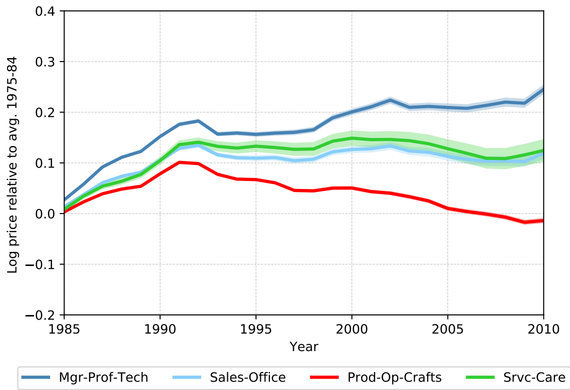
(a) Prices, fixed effects



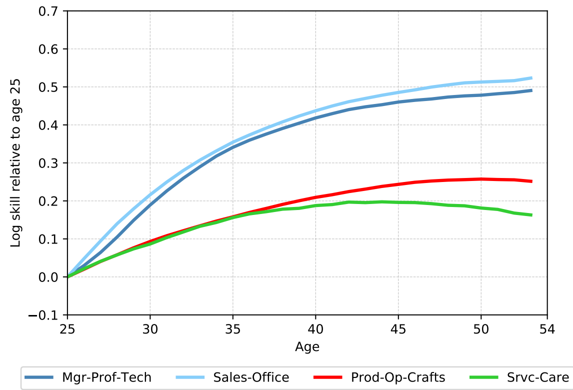
(b) Skill Accumulation, fixed effects



(c) Prices, stint fixed effects



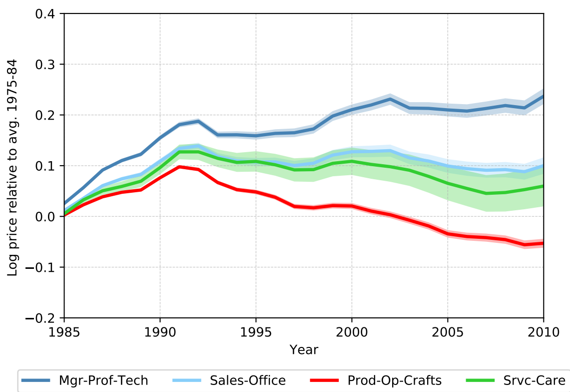
(d) Skill Accumulation, stint fixed effects



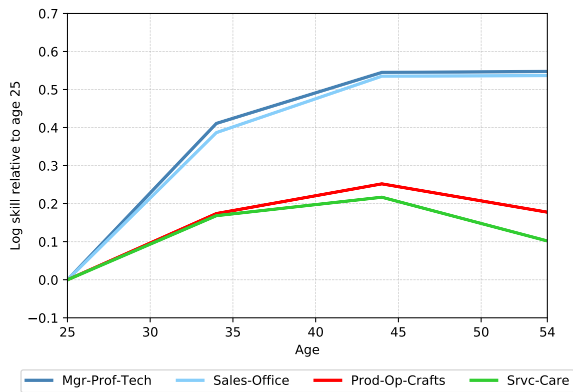
Notes: The first two panels include occupation $\times$ worker fixed effects. The last two panels include a fixed effect for every new occupation $\times$ worker stint as workers might switch back and force.

Figure D.12: Controlling for Changing Amenities

(a) Prices



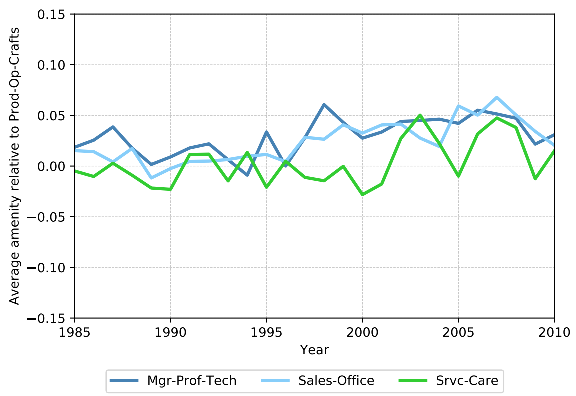
(b) Skill Accumulation



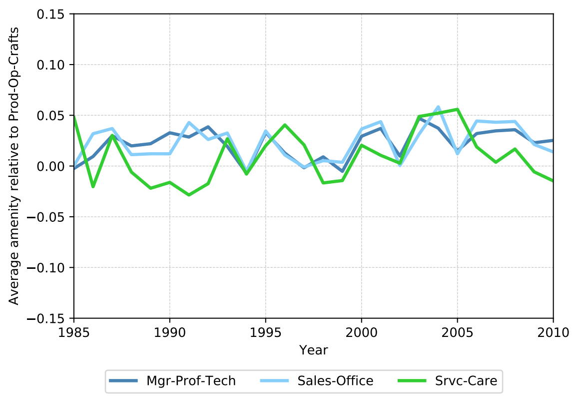
Notes: Amenity values were estimated relative to Prod-Op-Crafts.

Figure D.13: Changes in Amenities

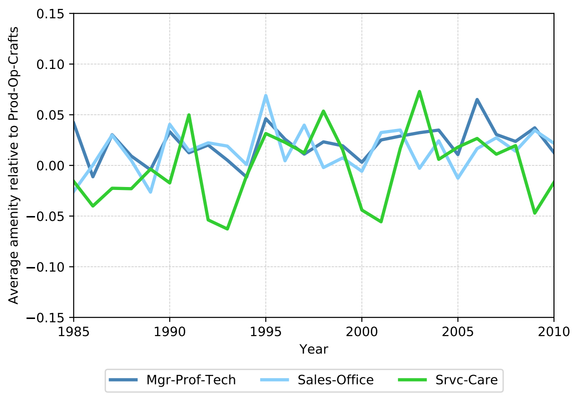
(a) 25–34 year olds



(b) 35–44 year olds



(c) 45–54 year olds



Notes: Amenity values were estimated relative to Prod-Op-Crafts.

E Detailed Decompositions

E.1 Sources of the Marginal Selection Effect

This section investigates the sources of marginal selection in more detail. The effect can either be "classic selection" due to differences in skill endowments or it can be due to (differences in) skill changes of workers after they entered the occupation. These changes may be systematic skill accumulation but they will also have an idiosyncratic component which is consistent with workers who experience positive shocks endogenously staying in their occupation. We also decompose the marginal selection effect into the contributions of occupation switchers, sample starters and exiters, and movers from or into unemployment and out of the labor force.

The main text showed the contributing factors to the average marginal selection of Equation (12). Here we do this for occupation entrants and leavers separately, and we derive the decomposition in detail. In particular, we make use of the high quality of our panel data to write the skills of a worker i in occupation k as:

$$s_{k,i,t} = s_{k,i,t_{k,i,0}} + \sum_{\tau=t_{k,i,0}}^{t-1} X'_{i,\tau}\Gamma_{k,k} + \sum_{\tau=t_{k,i,0}+1}^{t} u_{k,i,\tau}, \tag{41}$$

where the first term is the initial "endowment" when the worker entered this occupation at time $t_{k,i,0}$, the second term is systematic skill accumulation up to the current period t, and the last term are the cumulated idiosyncratic skill shocks in k since entry for this particular worker.[61]

The marginal selection *for entering workers only* can then be computed as

$$\begin{aligned}
\underbrace{\left(p^{ent}_{k,t} - p^{lvr}_{k,t-1}\right)\cdot\left(\mathrm{E}[s^{ent}_{k,i,t}] - \mathrm{E}[s^{sty}_{k,i,t}]\right)}_{\text{3. marginal selection for entrants}} &= \underbrace{\left(p^{ent}_{k,t} - p^{lvr}_{k,t-1}\right)\left(\mathrm{E}[s^{ent}_{k,i,t}] - \mathrm{E}[s^{sty}_{k,i,t_{k,i,0}}]\right)}_{\text{differences in endowments}} \\
&+ \underbrace{\left(p^{ent}_{k,t} - p^{lvr}_{k,t-1}\right)\left(\mathrm{E}[-\sum_{\tau=t_{k,i,0}}^{t-1} X'_{i,\tau}\Gamma_{k,k}|sty]\right)}_{\text{systematic skill accumulation}} \\
&+ \underbrace{\left(p^{ent}_{k,t} - p^{lvr}_{k,t-1}\right)\left(\mathrm{E}[-\sum_{\tau=t_{k,i,0}+1}^{t} u_{k,i,\tau}|sty]\right)}_{\text{idiosyncratic skill shocks}}
\end{aligned} \tag{42}$$

[61]Strictly speaking, we do not know levels of task prices and skills but we can compute i's overall accumulation $s_{k,i,t} - s_{k,i,t_{k,i,0}} = w_{k,i,t} - w_{k,i,t_{k,i,0}} - (\pi_{k,t} - \pi_{k,t_{k,i,0}})$ and thus back out $\sum_{\tau=t_{k,i,0}+1}^{t} u_{k,i,\tau}$ from (41). Then, for comparisons of entrants versus incumbents or leavers versus stayers at a given point in time, levels of task prices and thereby level shifters of skills in the population cancel out.

Notice however that the empirical implementation of (41) is not invariant to the more general acceleration/deceleration interpretation of the task price ($\Delta\hat{\pi}_{k,t} = \Delta\pi_{k,t} - \overline{\Delta\pi_{k,base}}$) and skill accumulation estimates ($\hat{\Gamma}_{k,k} = \Gamma_{k,k} + \overline{\Delta\pi_{k,base}}$ for stayers). The reason is that our calculations then give us $\hat{s}_{k,i,t} - \hat{s}_{k,i,t_{k,i,0}} = s_{k,i,t} - s_{k,i,t_{k,i,0}} + (t - t_{k,i,0})\overline{\Delta\pi_{k,base}}$ and $\sum_{\tau=t_{k,i,0}}^{t-1} X'_{i,\tau}\hat{\Gamma}_{k,k} = \sum_{\tau=t_{k,i,0}}^{t-1} X'_{i,\tau}\Gamma_{k,k} + (t - t_{k,i,0})\overline{\Delta\pi_{k,base}}$. Because the resulting $(t - t_{k,i,0})\overline{\Delta\pi_{k,base}}$ on each side of (41) cancel out, the idiosyncratic shocks term remains nonetheless unaffected.

and correspondingly *for leavers only*

$$\underbrace{\left(p^{ent}_{k,t} - p^{lvr}_{k,t-1}\right) \cdot \left(\mathrm{E}[s^{lvr}_{k,i,t-1}] - \mathrm{E}[s^{sty}_{k,i,t-1}]\right)}_{\text{3. marginal selection for leavers}} = \underbrace{\left(p^{ent}_{k,t} - p^{lvr}_{k,t-1}\right)\left(\mathrm{E}[s^{lvr}_{k,i,t_{k,i,0}}] - \mathrm{E}[s^{sty}_{k,i,t_{k,i,0}}]\right)}_{\text{differences in endowments}} \tag{43}$$

$$+ \underbrace{\left(p^{ent}_{k,t} - p^{lvr}_{k,t-1}\right)\left(\mathrm{E}[\sum_{\tau=t_{k,i,0}}^{t-2} X'_{i,\tau}\Gamma_{k,k}|lvr] - \mathrm{E}[\sum_{\tau=t_{k,i,0}}^{t-2} X'_{i,\tau}\Gamma_{k,k}|sty]\right)}_{\text{differences in systematic skill accumulation}}$$

$$+ \underbrace{\left(p^{ent}_{k,t} - p^{lvr}_{k,t-1}\right)\left(\mathrm{E}[\sum_{\tau=t_{k,i,0}+1}^{t-1} u_{k,i,\tau}|lvr] - \mathrm{E}[\sum_{\tau=t_{k,i,0}+1}^{t-1} u_{k,i,\tau}|sty]\right)}_{\text{differences in idiosyncratic skill shocks}}.$$

The average of each of the components of (42) and (43) give the summary results reported in the main text whereas the Table E.1 shows them separately for entrants (upper panel) and leavers (lower panel). Notice that the level of the marginal selection effect is somewhat higher for entrants than for leavers (Srvc-Care is an exception), so that some of what follows interprets the components in relative terms when comparing the their effects for entrants and leavers.

First, we see that differences in skill accumulation tend to be more important for entrants but that also for leavers they are not negligible compared to the overall effect, especially in high-accumulation Mgr-Prof-Tech and Sales-Office. The main reason for this is that leavers tend to stay in the occupation for a shorter time on average than stayers, which one could interpret as these workers being at some point revealed unsuitable for the job. It is also consistent with the strong negative contribution of the skill shocks for leavers shown in the table, since these shocks on (or learning about) skills may eventually make the leavers quit the occupation.

Second, initial endowments are stronger in the low-accumulation Prod-Op-Crafts and Srvc-Care occupations, both for entrants and leavers. In addition, skill accumulation differences are large for entrants into Prod-Op-Crafts whereas negative shocks are also important for leavers in this occupation group, consistent with the leaving into unemployment that we discuss below. Therefore, these results again underscore the importance of skill changes over the career. On the one hand, systematic skill accumulation is quantitatively large but, on the other hand, also stayers in occupations are clearly self-selected according to their idiosyncratic skill shocks.

We further decompose the contributions of occupation switchers, movers out of or into unemployment or the labor force during the career, and sample starters or exiters to the marginal selection effect. That is, we rewrite the average skills of entrants into occupation k as:

$$\mathrm{E}[s^{ent}_{k,i,t}] = h^{ent,swt}_{k,t}\mathrm{E}[s^{ent,swt}_{k,i,t}] + h^{ent,unem}_{k,t}\mathrm{E}[s^{ent,unem}_{k,i,t}] + h^{ent,olf}_{k,t}\mathrm{E}[s^{ent,olf}_{k,i,t}] + h^{ent,start}_{k,t}\mathrm{E}[s^{ent,start}_{k,i,t}] \tag{44}$$

where the shares of entrants who are occupation switchers $h^{ent,swt}_{k,t}$, entering from unemployment $h^{ent,unem}_{k,t}$ or out of the labor force during their careers $h^{ent,olf}_{k,t}$, and new sample starters $h^{ent,start}_{k,t}$ sum to one. Accordingly, for leavers from occupation k:

$$\mathrm{E}[s^{lvr}_{k,i,t-1}] = h^{lvr,swt}_{k,t-1}\mathrm{E}[s^{lvr,swt}_{k,i,t-1}] + h^{lvr,unem}_{k,t-1}\mathrm{E}[s^{lvr,unem}_{k,i,t-1}] + h^{lvr,olf}_{k,t-1}\mathrm{E}[s^{lvr,olf}_{k,i,t-1}] + h^{lvr,exit}_{k,t-1}\mathrm{E}[s^{lvr,exit}_{k,i,t-1}] \tag{45}$$

We then use Equations (44) and (45) to decompose the marginal selection effects of entrants and leavers separately.[62]

[62]Formally, in the case of entrants, the components are $\left(p^{ent}_{k,t} - p^{lvr}_{k,t-1}\right) h^{ent,swt}_{k,t}\left(\mathrm{E}[s^{ent,swt}_{k,i,t}] - \mathrm{E}[s^{sty}_{k,i,t}]\right)$,

Table E.1: Decomposition of the Marginal Selection Effect, 1984–2010

		Mgr-Prof-Tech	Sales-Office	Prod-Op-Crafts	Srvc-Care
Entrants	Marginal selection	-4.0	-2.6	6.8	-4.8
	Endowments	-1.3	-0.7	2.6	-3.3
	Accumulation	-2.2	-1.6	2.7	-0.7
	Shocks	-0.5	-0.3	1.5	-0.8
	Unemployed	-0.7	-0.4	2.6	-1.1
	Out of labor force	0.1	0.1	1.1	0.4
	Occupational switchers	-1.0	-0.3	0.4	-0.7
	Starters	-2.4	-1.9	2.7	-3.3
Leavers	Marginal selection	-2.3	-1.5	4.9	-4.9
	Endowments	-0.6	-0.3	2.1	-3.4
	Accumulation	-1.0	-0.8	0.5	-0.8
	Shocks	-0.7	-0.4	2.3	-0.7
	Unemployed	-0.4	-0.3	3.1	-1.1
	Out of labor force	-0.0	0.0	0.8	0.2
	Occupational switchers	-1.0	-0.5	0.2	-1.5
	Exiters	-0.8	-0.7	0.9	-2.4

Notes: Values are multiplied by 100 and relative to 1984 (i.e., skill changes in log points).

The upper panel of Table E.1 shows that for entrants a large part (between 40 and 75 percent) of the effect is due to new sample starters and substantial shares are due to entry from unemployment (16–39 percent) and occupational switchers (5–25). The latter is, perhaps unsurprisingly, largest for high-earning Mgr-Prof-Tech occupations since switchers likely need a lot of accumulation to catch up with incumbents' skills. Entrants from out of the labor force do not always contribute to the marginal selection effect (the contribution is only positive for Prod-Op-Craft), presumably because they are relatively high-skilled switchers from abroad, self-employment, military, or civil service (these individuals are not part of our dataset; Sect.C).

In the case of leavers (lower panel of Table E.1), moving into unemployment is relatively stronger and for Prod-Op-Crafts it is particularly important. It thus seems that (potentially older) Prod-Op-Crafts workers have little other attractive options in the labor market (although admittedly we would need to distinguish between the number of leavers in Prod-Op-Crafts and the extent of their skill differences with stayers to be precise). Another interesting finding is that even sample exiters contribute to the marginal selection effect of leavers, that is, they are on average less skilled than stayers.[63] Finally, occupational switchers have a quite strong effect in all three rising occupation groups, and especially so for leavers.

E.2 Effect of Skill Accumulation on Wage Percentiles

This section analyzes the reasons for skill accumulation's strong effect on the change of lower-half inequality, also in comparison to the modest effect on the upper-half.

$\left(p^{ent}_{k,t} - p^{lvr}_{k,t-1}\right) h^{ent,unem}_{k,t} \left(\mathrm{E}[s^{ent,unem}_{k,i,t}] - \mathrm{E}[s^{sty}_{k,i,t}]\right)$, $\left(p^{ent}_{k,t} - p^{lvr}_{k,t-1}\right) h^{ent,olf}_{k,t} \left(\mathrm{E}[s^{ent,olf}_{k,i,t}] - \mathrm{E}[s^{sty}_{k,i,t}]\right)$, and $\left(p^{ent}_{k,t} - p^{lvr}_{k,t-1}\right) h^{ent,start}_{k,t} \left(\mathrm{E}[s^{ent,start}_{k,i,t}] - \mathrm{E}[s^{sty}_{k,i,t}]\right)$.

[63] Part of the reason for this is that also workers who leave the sample at a younger age and do not return before age 54 are counted as exiters here. Similarly, workers who first appear at an age older than 25 nonetheless count as sample starters.

To do this, we rewrite the overall skill accumulation as a function of average accumulation within detailed worker cells times the frequency of these cells. In particular, for every percentile of the wage distribution in a given year, we compute the average skill accumulation in each worker cell defined by age and initial occupation. Then we average over these cells by their shares in the respective percentiles. That is, we compute the average skill accumulation in each percentile had workers stayed in the occupation of when they first entered the labor market as:

$$avg_t = \sum_{k=1}^{120} \sum_{a=25}^{54} P_t(a,k) \cdot \overline{acc}_{k,a,t} \text{ with} \tag{46}$$

$$\overline{acc}_{k,a,t} = \frac{1}{N_{k,a,t}} \sum_{\tau=t_{i,0}+1}^{t} \sum_{i\epsilon\{k,a,t\}} I_{k,i,t_{i,0}} I_{i,\tau-1} \cdot X'_{i,\tau-1}\hat{\Gamma}_{k,k}, \tag{47}$$

where in the second equation $\sum_{i\epsilon\{k,a,t\}}$ is a shorthand for summing over all workers of age a in year t whose initial occupation was k, and $N_{k,a,t}$ is the total number of such workers. In Equation (46) we then weigh by the relative cell sizes $P_t(a,k)$ to obtain the average skill accumulation in the respective wage percentile.[64]

The black solid line in Figure E.1 depicts this average skill accumulation across the percentiles of the 2010 wage distribution. We can see that skill accumulation's contribution to log wages is substantially higher at the median than at the bottom of the distribution, and much higher at the top of the distribution. The grey solid line depicts the corresponding skill accumulation for the year 1984, which is substantially flatter in its lower half but comparably steep as the 2010 accumulation between the 50th and the 85th percentile. The difference between the two lines is the effect of the accumulation (i.e., $\hat{w}_{i,t}^{e+a}$ compared to the scenario with only initial wages changing) reported in Table 2 of the main text (i.e., XX log points).

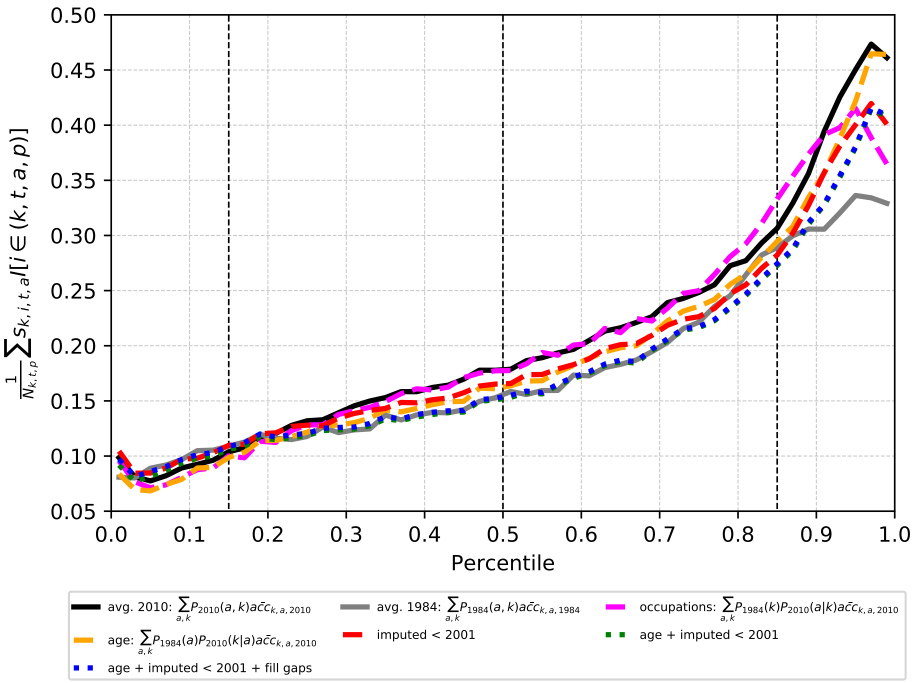
Using Equations (46)–(47), we now decompose the difference between the 2010 and 1984 skill accumulation effect into its parts with a particular focus on the lower half. One obvious component of this is supposed to be the changing occupation structure. Using Bayes' law, we compute the accumulation that would have prevailed if (conditionally) the age structure and within-cell accumulation changed over time but the (initial) occupation structure had stayed the same as in 1984:

$$avg_{2010}(occup = 1984) = \sum_{k=1}^{120} \sum_{a=25}^{54} P_{1984}(k) \cdot P_{2010}(a|k) \cdot \overline{acc}_{k,a,2010} \tag{48}$$

The pink dashed line depicts this accumulation, showing that it actually does not explain any of the increase in lower-half inequality. The reason for this rather surprising result is that the occupation structure actually did not shift decidedly toward higher-accumulation occupations at the median. In particular, Figure 6 in the main text shows that the share of high-accumulation Mgr-Prof-Tech and Sales-Office groups at the median and the 15th percentile of the actual wage distribution was not much different in 2010 than in 1984. Figure E.2 depicts the corresponding graph in the scenario at hand, i.e., with workers' initial occupations in the distribution of wages only due to entry and skill accumulation. We see that there are the same share of high-accumulation occupations at the median, and actually more at the bottom, in 2010 as in

[64] An index for the specific wage percentile is omitted, since this is always conditioned on anyway.

Figure E.1: Skill accumulation by percentile of the wage distribution



Notes:

1984.[65]

The next potential component of the skill accumulation effect is the shifting age structure in the different parts of the wage distribution. That is, we change the unconditional age distribution at each percentile to its 1984 value but hold the accumulation and the conditional occupation structure at their 2010 values:

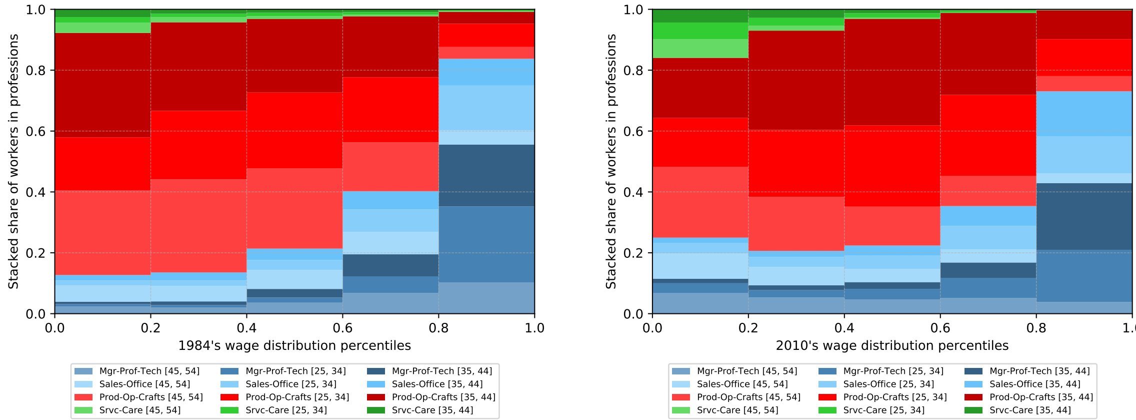
$$avg_{2010}(age = 1984) = \sum_{k=1}^{120} \sum_{a=25}^{54} P_{1984}(a) \cdot P_{2010}(k|a) \cdot \overline{acc}_{k,a,2010} \tag{49}$$

The yellow dashed line in Figure E.1 shows that this strikingly explains more than half of the accumulation effect. In particular, the many experienced workers at the 2010 median, already mentioned in the main text, have accumulated a lot of skills over their careers. Figure E.2 illustrates this even more clearly in the $\hat{w}_{i,t}^{e+a}$ scenario at hand: the share of 45–54 (but also 35–44) year old Prod-Op-Crafts workers at the median is very large in 2010 and much higher than in 1984. Now, if we down-weigh the share of these workers to 1984 and hold everything else constant, as we do in Equation (49), skill accumulation at the median is substantially lower.

[65] Admittedly, this may also partially be a reflection of data limitations here, since we need to approximate initial occupations in 1984 by their 1975 occupations for workers who entered the labor market before that (see also discussion further below and in main text). That is, some (middle-aged or older) 'blue' Mgr-Prof-Tech or 'Sales-Office' workers in 1984 may have started in the 'red' or 'green' occupations before 1975. This would also explain the higher share of 'blue' initial occupations at the top quintile of the Figure E.2 wage distribution in 1984 than in 2010 and the concurrent stronger 85th percentile increase in the pink 1984 occupation structure series of Figure E.1. Nonetheless, Figure 6 of the main text, which is independent of the imputation using the actual distribution, shows that the 'blue' occupations' shares at the mean or bottom quintile are not too different in 2010 and 1984 either.

Figure E.2: Shares in the Wage Distribution by Quintile ($\hat{w}^{e+a}_{i,t}$ Scenario)

(a) 1984 (b) 010



Notes: Each rectangle is proportional to the share of workers in the respective occupation $\times$ age bin.

Therefore, the large baby boomer birth cohorts, who still started their careers in Prod-Op-Crafts occupations,[66] are strongly increasing median wages at the end of our analysis period. Bottom wage earners in 2010 are in contrast much younger, and therefore skill accumulation due to demographic change raises lower-half wage inequality in this point in time.

The last factor that may have changed between 1984 and 2010 is the skill accumulation within worker-cells, which we analyze by computing:

$$avg_{2010}(accum = 1984) = \sum_{k=1}^{120} \sum_{a=25}^{54} P_{2010}(a,k) \cdot \overline{acc}_{k,a,1984}. \tag{50}$$

This counterfactual represents a specific version of changing worker employment biographies: since we condition on a fixed initial occupation and age, it can only differ when workers on average have more or less gap years of not working or different ages at labor market entry in 1984 than in 2010. In Equation (47) above, the former corresponds to differences in the number of $I_{i,\tau-1} = 0$ instances (e.g., due to unemployment) and the latter to differences in the total number of years $t - t_{i,0}$ over which skill accumulation is summed for a given age.

The differences in labor market entry ages are hard to measure in our data because we have to impute them for workers who entered before 1975 (see also description in the main text). We do this by computing, for every occupation, the average entry age across all years from 1976XX onward and assign this as the entry age together with their 1975 occupation to every worker who appears in the sample older than 25 in that year. This imputation, which affects our computed 1984 (but not 2010) skill accumulation, may generate a bias. We assess this possibility by pretending we only observed 2010 workers' labor market histories after 2001 (i.e., as in 1984, everything longer than nine years ago is unobserved) and then conducting the same imputation.

Since the red dashed line is below the solid black line, Figure E.1 shows that the imputation does indeed underestimate the skill accumulation in 2010, especially at and above the median.

[66]The baby boom in Germany started later than in the U.S., with cohorts comprising birth years 1955–69, i.e., 41–55 year olds in 2010. Had more of the baby boom workers started in high-accumulation Mgr-Prof-Tech or Sales-Office occupations instead of Prod-Op-Crafts, they would have ended up in the top quintile of the wage distribution, raising upper-half as much as lower-half inequality.

Therefore, part of our measured skill accumulation effect on lower-half inequality between 2010 and 1984 may be due to this data issue. Nonetheless, if we apply our calculation (49) on top of the imputed labor market biographies in 2010, age structure differences between the beginning and the end of the analysis period still have a substantial effect on 50–15 inequality. In fact, the combination of imputation and age structure (green dotted line) is almost exactly the same as the actual 1984 skill accumulation in the lower half of the wage distribution.

Finally, we examine the effect of potentially more intermittent prior labor market attachment at either end of the analysis period. That is, similar to Section XX we fill up gaps in employment biographies (e.g., due to unemployment) during the nine years leading up to 1984 and 2010, respectively. The dotted blue line shows the results and that actually this has no discernible effect on skill accumulation in addition to the imputation and the age structure effect. In unreported analyses, we found that labor market biographies were indeed somewhat more intermittent leading up to 2010 than 1984, but this only affected the very bottom of the wage distribution (below 5th percentile XX) and turns out quantitatively unimportant here.

To summarize the overall result, had we plotted

$$avg_{2010}(age = 1984, accum = 1984) = \sum_{k=1}^{120} \sum_{a=25}^{54} P_{1984}(a) \cdot P_{2010}(k|a) \cdot \overline{acc}_{k,a,1984} \tag{51}$$

into the Figure E.1, it would have almost exactly overlapped with the green dotted line. This implies that the workforce's changing age structure was the main driver of rising lower-half inequality. There is in fact no role for the occupation distribution at labor market entry *conditional on* the age structure (i.e., replacing $P_{2010}(k|a)$ with its 1984 value does not matter for the lower half). Replacing $\overline{acc}_{k,a,1984}$ with a version of $\overline{acc}_{k,a,2010}$ in which initial occupations and wages are imputed as in 1984 also gives the same result (which is the green dotted line). On top of that, filling labor market biographies during the preceding nine years also does not matter (blue dotted line). Therefore, some of the lower-half inequality effect may or may not be attributed to changing initial wages instead of skill accumulation, but the economically and quantitatively important part is accounted for by aging of the workforce.

E.3 Analysis of Between and Within Occupation Inequality

Another way to assess the importance of occupations for the trends of inequality is to compute their explanatory power in wage regressions (e.g., Card et al., 2013; Acemoglu and Autor, 2011, Figures 4 and 17, respectively). In the following we compare the results from such a decomposition to the conclusions that one would obtain from our model.

For each worker, we can write their wage as the average in the occupation that they are working in and an individual deviation from that average:

$$w_{i,t} = \bar{w}_{k,t} + \tilde{w}_{k,i,t}, \tag{52}$$

where $\bar{w}_{k,t}$ is also the coefficient on the dummy for occupation k in a wage regression and $\tilde{w}_{k,i,t}$ the orthogonal regression error.[67] The between versus within occupation decomposition then simply becomes $\sigma^2(w_{i,t}) = \sigma^2(\bar{w}_{k,t}) + \sigma^2(\tilde{w}_{k,i,t})$, with $\sigma^2(w_{i,t})$ the overall variance of log wages at time t.

[67] We refrain from controlling for other observable characteristics such as education, age, or experience here, since these have no clear interpretation in our model. However, we have done this as an (unreported) robustness check, also projecting task prices and skills from the model off these Mincer variables, and obtained comparable results.

E.3.1 Between Occupation Variance of Log Wages

In the first part of the analysis, we focus on the role of inequality between occupations over time. To do this, consider the covariances:

$$\begin{aligned}\sigma(\Delta\bar{w}_{k,t},\bar{w}_{k,1984}) &= \sigma(\Delta\pi_{k,t},\bar{w}_{k,1984})+\sigma(\Delta\bar{s}_{k,t},\bar{w}_{k,1984})\\ &= \sigma(\bar{w}_{k,t},\bar{w}_{k,1984})-\sigma^2(\bar{w}_{k,1984}) \qquad (53)\\ \sigma(\Delta\bar{w}_{k,t},\bar{w}_{k,t}) &= \sigma(\Delta\pi_{k,t},\bar{w}_{k,t})+\sigma(\Delta\bar{s}_{k,t},\bar{w}_{k,t})\\ &= \sigma^2(\bar{w}_{k,t})-\sigma(\bar{w}_{k,t},\bar{w}_{k,1984}), \qquad (54)\end{aligned}$$

where we have exploited the additive linearity of the covariance operator and inserted our economic model for the wage. $\bar{s}_{k,t}$ is the average skill in occupation k at time t.

Summing the right hand sides of (53) and (54), the change of between occupation inequality since 1984 becomes[68]

$$\begin{aligned}\Delta\sigma^2(\bar{w}_{k,t}) &= 2\cdot[\sigma(\Delta\pi_{k,t},\frac{\bar{w}_{k,t}+\bar{w}_{k,1984}}{2})+\sigma(\Delta\bar{s}_{k,t},\frac{\bar{w}_{k,t}+\bar{w}_{k,1984}}{2})]\\ &= 2\cdot\sigma(\Delta\pi_{k,t},\bar{\bar{w}}_k)+2\cdot\sigma(\Delta\bar{s}_{k,t},\bar{\bar{w}}_k). \qquad (55)\end{aligned}$$

Therefore, there are two broad factors in our model that drive between occupation wage inequality. First, inequality increases when task prices rise in high-wage occupations and vice versa for low-wage occupations, i.e., $\sigma(\Delta\pi_{k,t},\bar{\bar{w}}_k)>0$. The second factor is the covariance of changes in average skills and average wages across occupations $\sigma(\Delta\bar{s}_{k,t},\bar{\bar{w}}_k)$. Table E.2 summarizes the changes in wage inequality and the contributions of these two factors.

The top panel of Table E.2 reports that the cross-sectional variance of wages increased by 13.5 log points during 1984–2010. Close to eight log points of this was inequality within occupations and 5.7 log points between occupations. The latter are the spreading out average occupational wages of Figure 7b. Therefore, this trend by itself already had a substantial effect on the rising overall inequality.

Moreover, the second panel shows that the role of occupational task prices was even larger; almost as large as that of within-inequality, as the covariance between task prices and average wages increased by 6.7 points. Since we have seen in the main text that task prices appear mainly driven by demand shocks across occupations, one may conclude that this implies that in general the changing demand for occupations has made the wage distribution more extreme. This is also consistent with our general results in the paper whereby high-paying Mgr-Prof-Tech and Sales-Office occupations have seen their employment as well as task prices increase over time.[69]

The changes in skill selection across occupations however cushioned the impact of task prices on between occupation inequality. In particular, the covariance between occupational skills and average wages decreased by one log point. This is consistent with our findings above whereby the selection of skills into sectors which experience positive demand shocks, and thus rising employment and task prices, deteriorate. Remember the uncorrelatedness of employment changes and average occupational wages in Figure 1a. This shrouds the importance of occupational shocks to wage inequality that one can see in a simple between versus within occupation wage decomposition.[70]

[68]In this notation Δ always denotes the change from 1984 to t and $\bar{\bar{w}}_k \equiv \frac{\bar{w}_{k,t}+\bar{w}_{k,1984}}{2}$ is the mean average occupation k wage over the two periods.

[69]The increases in employment and task prices of the lowest-paying Srvc-Care occupations are somewhat counteracting one another and anyways relatively unimportant, as this is quite a small sector.

[70]In addition to counteracting the rising task prices of high-wage occupations, changing skill selec-

Table E.2: Accounting for Trends in Between and Within Occupation Inequality

		Absolute	Relative to 1984	
		1984	1997	2010
overall	$\sigma^2(w_{i,t})$	0.132	0.048	0.135
between	$\sigma^2(\bar{w}_{k,t})$	0.045	0.016	0.057
broad	$2 \cdot \sigma(\Delta\pi_{k,t}, \bar{w}_k)$		0.026	0.067
	$2 \cdot \sigma(\Delta\bar{s}_{k,t}, \bar{w}_k)$		-0.010	-0.010
detailed	$2 \cdot \sigma(\Delta\pi_{k,t}, \bar{w}_{k,1984})$		0.024	0.049
	$\sigma^2(\Delta\pi_{k,t})$		0.007	0.032
	$2 \cdot \sigma(\Delta\bar{s}_{k,t}, \bar{w}_{k,1984})$		-0.013	-0.019
	$\sigma^2(\Delta\bar{s}_{k,t})$		0.007	0.023
	$2 \cdot \sigma(\Delta\pi_{k,t}, \Delta\bar{s}_{k,t})$		-0.009	-0.028
within	$\sigma^2(w_{i,t} - \bar{w}_{k,t})$	0.086	0.032	0.078
	$\sum_k \Delta\sigma^2_{k,t}\Delta p^{>0}_{k,t}$		0.003	0.015
	$\sum_k \Delta\sigma^2_{k,t}\Delta p^{\leq 0}_{k,t}$		-0.003	-0.009
	$\sum_k \Delta p_{k,t}\sigma^2_{k,1984}$		0.001	0.005
	$\sum_k \Delta\sigma^2_{k,t} p_{k,1984}$		0.030	0.066
	$(\sum_k \Delta\sigma^2_{k,t}\frac{1}{120})$		0.024	0.062

In order to observe these factors more clearly at work, we develop a detailed version of Equation (55):

$$\Delta\sigma^2(\bar{w}_{k,t}) = 2 \cdot [\sigma(\Delta\pi_{k,t}, \bar{w}_{k,1984}) + \sigma(\Delta\bar{s}_{k,t}, \bar{w}_{k,1984})] \\ + \sigma^2(\Delta\pi_{k,t}) + \sigma^2(\Delta\bar{s}_{k,t}) + 2 \cdot \sigma(\Delta\pi_{k,t}, \Delta\bar{s}_{k,t}), \tag{56}$$

where we have factored $\sigma^2(\Delta\bar{w}_{k,t})$ out of the first row of (56) and split it up into prices and skills in the second row. Table E.2 again reports the results. Parallel to before, the task prices of occupations that were high-wage in 1984 increased, which raised wage inequality (4.9 log points), while skill selection into these occupations deteriorated, which counteracted the effect of the task prices on inequality (-1.9 log points). Also, both the variance of task price changes $\sigma^2(\Delta\pi_{k,t})$ and average skill changes $\sigma^2(\Delta\bar{s}_{k,t})$ contributed to rising inequality.

Yet the key insight from the more detailed decomposition is the covariance of changes in task prices and average skills across occupations. This covariance is negative, which we have concluded from Section 4 already, but it also is quantitatively strong in its impact on between occupation wage inequality. In particular, the deteriorating skills in occupations with rising task prices reduce the change in the between occupation variance of wages by 2.8 log points, which at an overall between inequality increase of 5.7 log points is very substantial.

Therefore, we conclude from this analysis that, through the lens of our model, a standard between occupation analysis underestimates the true role of occupations for trends in wage inequality. The skill selection effects of rising occupations that we have documented in Section 4 turn out to be important also for the overall wage inequality, as they substantially reduce the observable variance of wages across occupations.The true role of occupations in changing inequality is quite larger, as we have seen in the different model scenarios of the main text, and

tion also increased within occupation wage dispersion, as we discuss below.

as they have a role for within occupation trends too, which we see next.

E.3.2 Within Occupation Variance of Log Wages

Inequality within occupations is rising as fast as between, even when accounting for selection effects as in the previous section. However, within inequality can also be affected by selection into occupations due to changing task prices. If, for instance, rising prices attract workers of lower skill than the incumbents, inequality will increase within growing sectors. If occupations with high inequality grow, then within inequality will rise overall. Conversely, within inequality might decrease in shrinking occupations with declining task prices because their low skilled workers may leave.

In Equation (52) we defined $\tilde{w}_{k,i,t}$ as the difference between an individual's wage and the average wage within his occupation. In addition, given that task prices are the same for a given occupation in our model, the residual wage difference is the same as the residual skill difference: $\tilde{w}_{k,i,t} = \tilde{s}_{k,i,t} = s_{k,i,t} - \bar{s}_{k,t}$. Therefore, the average within occupation variance of log wages becomes:

$$\sigma^2(\tilde{w}_{k,i,t}) = \sigma^2(\tilde{s}_{k,i,t}) = \frac{1}{N_t}\sum_{i=1}^{N_t}\tilde{s}^2_{k,i,t} = \sum_{k=1}^{K}\overbrace{\frac{N_{k,t}}{N_t}}^{p_{k,t}}\cdot\overbrace{\frac{1}{N_{k,t}}\sum_{i\in k}\tilde{s}^2_{k,i,t}}^{\sigma^2_{k,t}} = \sum_{k=1}^{K}p_{k,t}\sigma^2_{k,t}, \tag{57}$$

where $p_{k,t}$ is occupation k's share of total employment at time t and $\sigma^2_{k,t}$ is the variance of wages or skills within the occupation. The change of the average within variance is:

$$\begin{aligned}\Delta\sigma^2(\tilde{w}_{k,i,t}) = \Delta\sigma^2(\tilde{s}_{k,i,t}) &= \sum_{k=1}^{K}\left(p_{k,t}\sigma^2_{k,t} - p_{k,1984}\sigma^2_{k,1984}\right)\\ &= \sum_{k=1}^{K}\Delta\sigma^2_{k,t}p_{k,1984} + \sum_{k=1}^{K}\Delta p_{k,t}\sigma^2_{k,1984} + \sum_{k=1}^{K}\Delta\sigma^2_{k,t}\Delta p_{k,t}\end{aligned} \tag{58}$$

Therefore, the rise of within occupation inequality can be decomposed into terms linked to the changing employment structure and 'pure' increases of the variance of log wages in occupations at fixed sizes. In particular, the last summand of Equation (58) is actually the covariance of changing within inequality with changing employment share. That is, how much the variance of skills in occupations rises for growing occupations, which is closely linked to the declining skills in growing occupations discussed in Section 4. This relationship generates 0.6 (i.e., 1.5-0.9) log points of the increase in within inequality in the bottom panel of Table E.2.

The other component related to the changing employment structure is the growing size of sectors with high initial within inequality. These are often relatively large occupations inside the rising Mgr-Prof-Tech, Sales-Office, and Srvc-Care groups, which is partly due to the German KLDB occupation classification as it is more detailed in production and crafts related occupations than in managerial, office, or service type occupations (see Appendix Table C.2). The effect of this is the second summand in Equation (58) and it makes up another 0.5 log points of the increase in within occupation inequality in Table E.2. Clearly the largest part of the rising within variance is the first summand in Equation (58). However, also here the employment structure played a role because larger occupations, which as we said are often in Mgr-Prof-Tech, Sales-Office, and Srvc-Care occupations, had higher increases of within inequality. To see this, compare this term in Table E.2's bottom panel with its unweighted version ($\frac{1}{120}\sum_k \Delta\sigma^2_{k,t}$).

Therefore, 1.5 of the 7.8 log point increase in within occupation inequality can be attributed to the changing employment structure. In unreported analysis we find that our model captures more than this, since the majority of the 'pure' within occupation inequality increase is due to changes of skill accumulation within occupations. Overall, the different wage distribution scenarios of Section 5 generate about sixty percent of the increase in between occupation inequality and almost eighty percent of within inequality during 1984–2010.