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Elasticities of Labor Supply and Labor Force Participation Flows*

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Abstract

Using a representative-household search and matching model with endogenous labor force participation, we study the interactions between extensive-margin labor supply elasticities and the cyclicality of labor force participation flows. Our model successfully replicates salient business-cycle features of all transition rates between three labor market states, the unemployment rate, and the labor force participation rate, while using values of elasticities consistent with micro evidence. Our results underscore the importance of the procyclical opportunity cost of employment, together with wage rigidity, in understanding the cyclicality of labor market flows and stocks.

Key Words: Labor force participation, labor market transitions, labor supply elasticity, unemployment

JEL Codes: E24, J64

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1 Introduction

In the traditional representative-agent real business cycle models with endogenous labor supply, the Frisch elasticity of labor supply is one of the key parameters. A longstanding puzzle in macroeconomics is that the value of the Frisch elasticity required in those models to replicate aggregate labor-market fluctuations is measurably larger than the values suggested by the micro-level evidence (see, for example, Chetty et al. (2011)). An alternative approach to modeling the aggregate labor market is to introduce search and matching frictions pioneered by the work of Mortensen and Pissarides (1994). Nonetheless, this literature has been mostly silent about the divergence between macro and micro labor supply elasticities and, in general, the labor supply. Instead, particularly after the influential work by Shimer (2005), a large part of this literature has focused on studying the sources of labor demand fluctuations.[1] Importantly, understanding the labor force participation decision is still very much relevant for macroeconomic policy, as indicated by numerous recent speeches by policymakers devoted to developments of the labor force participation rate (LFPR).[2]

In this paper, we extend a canonical search and matching model by adding an extensive margin labor supply decision and evaluate its qualitative and quantitative properties. Our aim is to develop a tractable framework that allows us to analyze the fluctuations of the unemployment rate and the LFPR as a result of the equilibrium responses of both the job-creation (labor demand) margin and the labor force participation (labor supply) margin. Our model can successfully replicate salient business-cycle features of all transition rates between three labor market states, the unemployment rate, and the LFPR.

This paper provides two important contributions to the existing literature. First, we study the cyclical movements of labor force inflow and outflow rates and relate them to elasticities of extensive-margin labor supply and substitution between home production and market-goods consumption. We do so within a tractable representative-household setup. Importantly, we use values of elasticities that are in line with micro evidence such as those reviewed by Chetty et al. (2011, 2013) for the labor supply elasticity and by Aguiar et al. (2013) for the elasticity of substitution between home production and market-goods consumption. Second, our quantitative results underscore the importance of equilibrium wage rigidity (Hall (2005), Shimer (2005)) and the procyclicality of the opportunity cost of employ-

[1] This trend is perhaps reflected in "the consensus view point that shifts in labor demand account for most of the cyclical variation in labor input" (Hall, 2008).

[2] These speeches are often motivated by questions such as whether the unemployment rate is a sufficient measure of labor market slack or some information from the LFPR should also be considered. See, for example, Bernanke (2012), Bullard (2014), Plosser (2014), Yellen (2014), Williams (2017), and Kashkari (2017).

ment (Chodorow-Reich and Karabarbounis (2016)) in understanding the cyclical movements in labor market flows and stocks. We show that these two elements play key roles in governing the returns and costs of participating in the labor market. In the existing literature, both wage rigidity and the opportunity cost of employment are studied with respect to Shimer's unemployment volatility puzzle, but our analysis reinforces the importance of these two elements in understanding cyclical responses of the labor supply margin.

Our theoretical framework is motivated by the empirical evidence on the cyclical movement of labor market transition rates between three labor market states. We summarize this evidence by estimating sign-restriction vector autoregressions (VARs). In addition to the well-known cyclical pattern in transition rates between employment and unemployment, we find that the transition rate at which nonparticipants join the unemployment pool is countercyclical, while the exit rate from unemployment to nonparticipation is procyclical. Our VAR also reveals that transition rates between employment and nonparticipation are both procyclical. These results are consistent with the existing literature analyzing unconditional second moments (e.g., Elsby et al. (2015), Krusell et al. (2017)), but provide a more complete and nuanced description of the data. In particular, our approach further allows us to coherently analyze the link between movements in transition rates and labor market stocks in a unified framework. For example, our framework reconciles small procyclical variations in the LFPR with offsetting movements in transition rates into and out of nonparticipation that exhibit much greater volatility and clearer cyclicality.

To grasp economic forces behind the cyclical movements in labor market transition rates, we develop a representative-household model with search frictions and endogenous labor force participation. In our model, nonemployed household members differ with respect to their nonmarket productivity, based on which the household optimally allocates them to either active job search (unemployment) or nonparticipation (home production). We characterize the model's equilibrium as an intersection between the job-creation (labor demand) condition and the labor force participation (labor supply) condition. The equilibrium determines the two key endogenous variables, labor market tightness (as in the standard two-state model) and the participation margin. The latter is represented by the threshold value of nonmarket productivity, above which a nonemployed household member stays out of the labor force instead of joining the unemployment pool.

When looking at the empirical evidence through the lens of the model, we find that *the participation margin threshold must be strongly countercyclical.* This result means that the household is less willing to send an additional worker to the labor force in expansions than in recessions. The direct indication of this is, for example, the strong countercyclicality of the rate at which nonparticipants join the unemployment pool in the data. This implication might seem counterintuitive given that the LFPR is weakly procyclical in the data. However, our analysis implies that the LFPR is procyclical because of large increases in participation through employment (procyclical NE transition rate) despite declines in participation through unemployment (countercyclical NU and EU transition rates). This result highlights the importance of studying labor market flows as the primitive drivers of the labor market stocks. The strong countercyclicality in the participation margin in turn implies that the opportunity cost of employment in the labor market needs to be significantly more procyclical than the returns to market work.

To be more specific, consider the representative household's decision as to whether to send an additional member to the unemployment pool or to keep the member as a nonparticipant. In the model, there are two channels through which the participation margin is affected. The first is through returns to market work (the substitution effect). As in the standard two-state model, higher market tightness (and thus the higher job-finding rate) and higher wages during expansions motivate the household to send more workers into the unemployment pool. The second channel is through the cyclical fluctuations of the opportunity cost of employment, which includes the forgone values of leisure and home production. Under our standard preference specification, a higher employment share in the household results in higher marginal values of leisure and home production with respect to market-goods consumption, keeping the household from sending more members to the labor force. The empirical evidence on the cyclicality of labor force participation flows suggests that the second channel not only dominates the first channel but also does so by a large margin, so that the equilibrium participation margin threshold falls significantly in expansions.

We show that in a setup where wages are strongly procyclical, the first channel tends to dominate even with procyclical movements in the opportunity cost of employment, predicting a counterfactual behavior of participation flows.[3] We find that a powerful way of mitigating this substitution effect is to introduce equilibrium wage rigidity as proposed by Hall (2005). Thus, wage rigidity together with the procyclical opportunity cost of employment enables us to replicate the key cyclical features of labor market transition rates and stocks. In models with a frictionless labor market, the lack of movements in wages and small labor supply elasticities (as suggested by micro-level studies) would imply the lack of employment volatility. In models with search frictions, however, movements in the household's labor supply margin can be decoupled from labor demand (job-creation) fluctuations, which

[3] Tripier (2003) and Veracierto (2008) report that in their models, the unemployment rate tends to be procyclical once the participation margin is endogenized. Their models are different from ours, but the issue appears to originate from the same underlying mechanism that the substitution effect tends to dominate.

are strongly magnified by wage rigidity. Moreover, in such an environment, the lower the extensive-margin elasticity is and the larger the complementarity between home production and market-goods consumption is, the more countercyclical the participation margin threshold becomes, without compromising variability of employment, through the procyclicality of the opportunity cost of employment. The presence of wage rigidities and a procyclical opportunity cost of employment thus provides a coherent framework to explain both labor demand and labor supply fluctuations.

We convey the key economic intuitions in a model that is deliberately simplified. For our quantitative analysis, we introduce several simple extensions to the model, such as changes in timing (to deal with time aggregation in the data) and the specification of job-creation costs (to smooth out the variation in the job-creation margin). Our estimated quantitative model matches the cyclicality of *all* transition rates across the three labor market states and the behavior of labor market stocks (the unemployment rate, the employment-to-population ratio, and the LFPR). Several notable results are as follows. First, our model replicates the cyclical movements in transition rates between unemployment and nonparticipation (the NU and UN rates). As discussed above, the former is countercyclical, while the latter is procyclical. These cyclical patterns imply that both of these rates (even in the absence of movements in transition rates between employment and unemployment) contribute to countercyclical movements in the unemployment rate. Second, our model reproduces the observation that the LFPR exhibits a very small variation over the cycle and increases with a significant lag after an expansionary shock. Third, our model matches the pattern that separation rates from employment into unemployment (EU) and into nonparticipation (EN) move in the opposite direction. Although our model assumes that separations from employment occur at a constant rate, the share of separations flowing into the unemployment pool increases in downturns (and thus the share of the other flow falls). In other words, the separation rate into unemployment is strongly countercyclical, thus contributing significantly to unemployment fluctuations. This is notable because, in two-state models, a constant separation rate implies no contributions of the EU rate to unemployment fluctuations, contrary to the data. A related achievement is that our model maintains the strong negative correlation between unemployment and vacancies, known as the Beveridge curve.[4] It is worth noting that we achieve all results within a representative-household search and matching model with endogenous participation.

[4] This is also notable because it is known in the literature that two-state models with endogenous separation fail to replicate this robust empirical regularity. See Fujita and Ramey (2012) for details.

Relation to the literature. Earlier attempts that incorporate the extensive-margin labor supply decision into search models include Tripier (2003) and Veracierto (2008). These two papers emphasize the difficulty of obtaining countercyclical unemployment, once the participation decision is endogenized. More recently, Shimer (2012) studies the properties of a model similar to Tripier's, without relating his findings to the elasticities of labor supply and pays close attention to the role of wage rigidity as we do in the current paper. In his baseline model, the split between unemployment and nonparticipants is perfectly elastic and thus rigid wage itself does not help mitigate the substitution effect. Haefke and Reiter (2011) develop a search model with heterogeneous workers and endogenous participation decisions and evaluate its quantitative performance in light of micro evidence on labor supply elasticities. Based on a steady-state analysis, they also find that small elasticity values are consistent with fluctuations in the labor market stocks rate under empirically plausible degrees of wage rigidity. Galí (2010), Galí et al. (2012), and Campolmi and Gnocchi (2016) study a New Keynesian model with search frictions and endogenous participation.[5] Their models consider a richer environment with more frictions and shocks. We study a simpler environment to emphasize key economic mechanisms. Importantly, none of the papers cited so far try to match the cyclicality of labor force transition rates. We tackle the challenging task of matching the cyclicality of transition rates as well as stocks, as it enables us to intuitively connect the micro-level household decisions to aggregate labor market fluctuations. Ferraro and Fiori (2019) study asymmetric business cycles in a heterogeneous agent model with endogenous participation in which labor supply is perfectly elastic. Their model matches the volatility and cyclicality (measured as the correlation with output) of labor market transition rates by exogenously making the opportunity cost of employment strongly procyclical. Last but not least, Krusell et al. (2017) develop a heterogeneous-agent search model with endogenous participation and look explicitly at transition rates, especially those between unemployment and nonparticipation. They emphasize the role of wealth heterogeneity and associated composition effects in explaining the cyclicality of these rates. Because we study a representative-agent environment, we necessarily abstract away from such composition effects, but provide complementary channels. Furthermore, we study the link between the cyclicality of labor force participation flows and extensive-margin labor supply elasticities.

As emphasized, an important element of our model is the endogenous procyclicality of the opportunity cost of employment. In the context of a two-state model, Chodorow-Reich and Karabarbounis (2016) emphasize the empirical relevance of the procyclical opportunity cost and its implications on the resolution of the unemployment volatility puzzle (Shimer,

[5] Erceg and Levin (2014) also develop a New Keynesian model in which three labor market states can be defined without introducing search frictions.

2005, Costain and Reiter, 2008, Hagedorn and Manovskii, 2008). Our paper shows that this issue is even more relevant in models with endogenous labor force participation.

This paper is organized as follows. Section 2 presents the empirical evidence. By estimating the sign restriction VAR, we characterize the full dynamics of labor market transition rates, labor market stocks, vacancies, and real wage. Section 3 develops our baseline model and the properties of the model are extensively analyzed. This section is devoted to conveying the key economic intuitions. Section 4 extends our baseline model, making it more quantitatively attractive, while keeping the same underlying economic mechanisms. We estimate the model and show that the model replicates dynamics of the labor market well along with the steady-state levels of transition rates. Section 5 concludes the paper and offers potential future extensions.

2 Empirical Evidence

In this section, we summarize the cyclical behavior of transition rates across three labor market states and labor market stocks by estimating sign-restriction VARs. In contrast to the literature that focuses on the unconditional moments, we study full dynamic characteristics of the data. Our analysis extends the sign-restriction VAR analysis by Fujita (2011) to the three-state environment that also incorporates transitions into and out of the labor force.[6]

2.1 Data

We construct worker transition rates between employment (E), unemployment (U), and not-in-the-labor force (N), using the Current Population Survey (CPS) matched records. The literature has proposed various corrections to the data to deal with the margin errors, classification errors, and time aggregation bias.[7] Our main empirical results are based on flow series adjusted only for margin errors. We do not undertake any adjustments with respect to the other errors. Regarding classification errors, Elsby et al. (2015) propose an adjustment that they call "DeNUNifying." However, a recent paper by Kudlyak and Lange (2017) argues against the adjustment. We do not take a stand on this issue and instead confirm that our

[6] See, for example, Rubio-Ramirez et al. (2010), Fry and Pagan (2011), and Inoue and Killian (2013) for general discussions and reviews of the sign-restriction VARs.

[7] Margin errors arise due to nonrandom attrition of survey participants, resulting in inconsistency between worker flow data and stock data. See Abowd and Zellner (1985), Fujita and Ramey (2006), and Frazis et al. (2005) for earlier attempts to make corrections. However, the cyclicality of the data is not significantly affected by this correction (Fujita and Ramey (2006)). Gross flows are also subject to classification and time-aggregation errors (see, for example, Elsby et al. (2015)).

results are robust with respect to this and the other data adjustments (see Appendix A.1). Regarding the time aggregation bias, we measure transition rates in our model in a way that is consistent with the measurement practice used in the empirical analysis (see discussions in Section 4).

Our VAR analysis includes six transition rates: (i) EU, (ii) EN, (iii) UE, (iv) UN, (v) NE, and (vi) NU rates. The first letter represents the originating labor market state and the second letter the terminal state between the two adjacent months. The VAR also includes real wage and job vacancies.[8] The sample period for our analysis is constrained by the availability of CPS microdata and spans between 1976Q1 and 2016Q4. We convert the monthly series into quarterly series by time averaging to smooth out high frequency variations of the data. All series are seasonally adjusted, logged, and then HP-filtered with a smoothing parameter equal to 10^5. We detrend all series because the data exhibit low-frequency variations that are difficult to endogenously analyze in our stationary models. We confirm the robustness of our results with respect to various alternative detrending methods. We present those results in Appendix A.1.2. We set the lag length at two quarters.[9]

Note that the VAR does not explicitly include labor market stocks because the impulse response functions of transition rates allow us to fully characterize the dynamics of the stock variables. That is, once we know the paths of transition rates, we can use the laws of motion for the stocks to trace their paths (and thus the paths of any functions of these stocks such as the unemployment rate), conditional on initial (steady-state) values of stocks and transition rates.

2.2 Identifying Assumptions

Our sign restrictions are meant to identify the impulse that drives business cycle fluctuations of the U.S. labor market. We identify what we call the "aggregate profitability shock" (henceforth, called the aggregate shock for brevity) by imposing restrictions on the signs of responses of transition rates between unemployment and employment. Specifically, we assume that in response to the positive (negative) aggregate shock, the EU rate falls (in-

[8] Real wage is measured by compensation per hour from the BLS's productivity and cost program, deflated by the headline PCE price index. For the job vacancy series, we splice the help-wanted advertising index by the Conference Board and the job openings series reported in the Bureau of Labor Statistics's Job Openings and Labor Turnover Survey (JOLTS) program. The JOLTS vacancy series is available only after December 2000. Thus, we multiplicatively adjust the level of the Conference Board's series prior to that point to the level that matches the level of the JOLTS series.

[9] We also compute information criteria to set the lag length. The Akaike information criterion (AIC) suggests two quarters, while the Schwarz information criterion (SIC) suggests one quarter. The results are again robust with respect to the lag length around two quarters.

creases) while the UE rate increases (falls). We also assume that the shock leads to increases (declines) in vacancies and employment growth. These sign restrictions are assumed to hold for two quarters. The cyclical patterns of the EU and UE transition rates are well-established in the context of a two-state model of the labor market.[10]

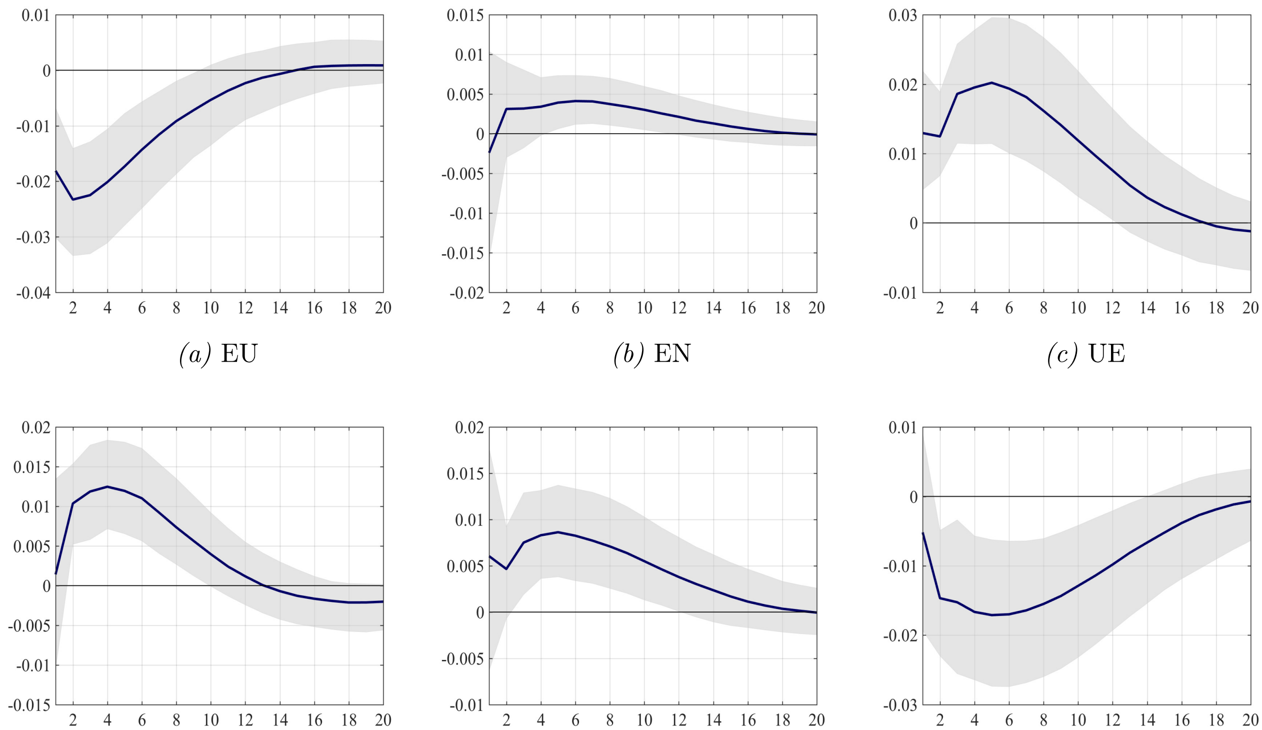
Since our main interest is to characterize the cyclicality of transition rates into and out of nonparticipation, imposing these restrictions on the directions of transition rates between employment and unemployment is sensible.

As is clear from our VAR setup, we do not attempt to identify various forms of more structural shocks. The spirit of our approach is similar to the one taken by highly influential papers in the literature such as Blanchard and Diamond (1990) and Haltiwanger and Davis (1999). These papers also use simple sign restrictions in a parsimonious VAR to identify the shock that has a similar interpretation and also name it the aggregate shock. More recently, Fujita (2011) also follows a similar approach within the two-state model, and our analysis can be viewed as an extension of it to the three-state environment. Importantly, Fujita (2011) finds in his robustness checks that labor market variables respond very similarly to more fundamental shocks within his two-state framework. To further check the robustness of our results, we present two alternative VAR estimations in Appendix A.1. First, we show the results of a larger VAR system that includes the inflation series, to distinguish between demand- and supply-side shocks. Second, we also examine the case where the labor productivity series is used to identify technology shocks. The cyclical patterns of the labor market variables are found to be very similar across all cases.

2.3 Results

Figures 1 and 2 present the impulse response functions to a positive aggregate shock. Solid lines represent the median responses and shaded areas represent the 16th and 84th percentiles of the posterior distributions. Recall that we restrict the behavior of the EU and UE transition rates (Figure 1 (a) and (c)) for the first two quarters, and these restrictions indeed imply the unemployment rate drops significantly and persistently (Figure 2 (c)). The persistent declines in the unemployment rate together with persistent increases in vacancies (Figure 2 (e)) form the well-known Beveridge curve. Because we also restrict employment growth to be positive in the first two quarters, the employment-population ratio also increases in a hump-shaped manner (Figure 2 (a) and (b)). We also find that real wage is only weakly procyclical, as has long been known in the business-cycle literature. Although

[10]Fujita (2011) does not restrict the signs of these transition rates because his main interest is on testing the cyclicality of these rates. His finding is indeed consistent our current sign restrictions.



(a) EU *(b)* EN *(c)* UE

(d) UN *(e)* NE *(f)* NU

Figure 1: Responses to an Expansionary Shock; Transition Rates. *Notes*: Expressed as log deviations from steady-state levels. Shaded areas are error bands that represent the 16th and 84th percentiles of the posterior distribution.

the median response is positive throughout the five-year horizon, the 16-84 percentile error band tends to include zeros (Figure 2 (f)).

Figure 2 (d) presents the response of the labor-force participation rate. It is known in the literature that the participation rate is only weakly procyclical.[11] But our VAR-based result provides more complete dynamic properties of the participation rate. Moreover, by relating this response to movements in transition rates, we provide a richer story that underlies the behavior of the participation rate. In Figure 2 (d), one can see that the weak procyclicality of the participation is due to the pattern that participation takes several years after the shock to rise. Observe also that its volatility is tiny: Based on the median response, the largest deviation from the steady-state level is about 0.0004 log point. In contrast, the employment-to-population ratio and the unemployment rate deviate from their steady state levels as much as 0.002 log point and 0.03 log point, respectively. The small volatility of the

[11] See, for example, Erceg and Levin (2014) and Van Zandweghe (2017).

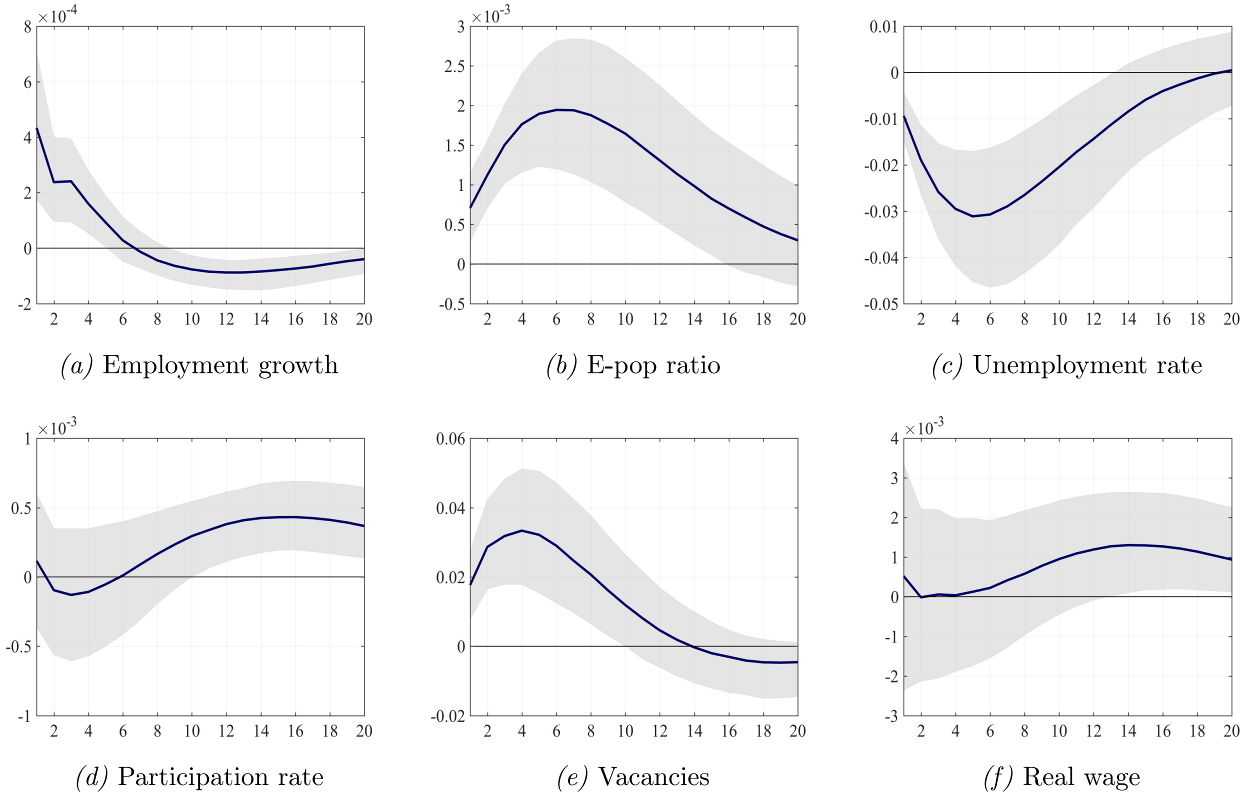


Figure 2: Responses to an Expansionary Shock; Stocks and Real Wage. *Notes*: Expressed as log deviations from steady-state levels (except for (a)). See notes to Figure 1.

participation rate is interesting, especially because volatilities of transition rates to and from nonparticipation are not particularly small compared with those of transition rates between employment and unemployment.

Let us now discuss in greater detail the cyclical patterns in transition rates. First, consider the responses of EU and UE rates (Figure 1 (a) and (c)). Our VAR restricts the direction of the initial responses of these two variables, and we see that both of these responses are highly persistent. Second, compare the responses of separation rates into unemployment (EU) and into nonparticipation (EN) ((a) and (b))). The latter rate tends to be procyclical and partially offsets the countercyclicality of the former rate, thus making the overall separation rate out of employment less cyclical, although the sum remains countercyclical.[12] Third, compare the responses of UE and NE rates, namely, job-finding rates from the two nonemployment pools ((c) and (e)). The literature has emphasized the strong procyclicality of the former, but a similar procyclicality applies to hiring from nonparticipation. The volatility of the

[12] Fujita and Ramey (2006) also note this feature of the data in their unconditional analysis.

latter is smaller given that the pool of nonparticipants is much larger and includes a large number of individuals that are dormant in terms of participation in the labor market, such as retirees. Next, compare the two transition rates that constitute inflows to the unemployment pool, namely, the EU and NU rates ((a) and (f)). Obviously, the countercyclical EU rate contributes to increasing the unemployment rate in recessions. However, the entry rate from nonparticipation increases as well, thus also contributing to a higher unemployment rate in downturns. In our model, we can quantify the latter effect. Lastly, compare the behavior of UE and UN rates ((c) and (d)). These two transition rates represent outflow rates from the unemployment pool. In contrast to the responses of inflow rates just discussed, they both move procyclically. Thus, in addition to the familiar procyclicality of the pace of hiring from the unemployment pool, the pace of exits to nonparticipation also increases in expansions.

The cyclical movements in transition rates between unemployment and nonparticipation (i.e., UN and NU rates) can be counterintuitive, when one considers statements like "workers drop out of the labor force due to discouragement after failing to find a job" or "more workers join the labor force as the labor market conditions improve." Our results are not necessarily inconsistent with these statements in that our results are based on the transition *rates* rather than worker *flows*. Even though the UN rate falls in a downturn, the number of workers making UN transitions (thus dropping out of the labor force) might increase because the unemployment pool itself increases; similarly even though the NU rate falls in an expansion, the number of workers making NU transitions (thus starting to look for jobs actively) might increase if the pool of nonparticipation falls. We view transition rates as being primitive, and worker flows are determined as a result of movements in transition rates and their interactions with stocks.

The findings regarding the cyclicality of UN and NU rates are not new. Elsby et al. (2015) point out that these patterns result from worker heterogeneity particularly with respect to the labor force attachment. In a downturn, the composition of the unemployment pool shifts more towards workers with strong attachment to the labor market (i.e., breadwinners). These attached workers are less likely to exit the labor force, thereby making the UN rate lower in a downturn. Similarly, the countercyclicality of the NU rate implies that the "need" for joining the labor force increases in a recession (for example, a spouse of the household head joining the labor force to compensate for the loss of hours or a job).[13] Krusell et al. (2017) develop a model with wealth heterogeneity and a borrowing constraint that generates these cyclical patters in UN and NU rates. Our representative-household model below also sheds light on the underlying sources of these patterns without introducing wealth heterogeneity.

[13]See Mueller (2017) for a general idea of countercyclicality of "attachment" of the pool of unemployment.

Regarding the weak procyclicality of the participation rate, notice that, out of four transition rates that directly involve nonparticipation, three of them produce countercyclical forces in the participation rate: The procyclical responses of the EN and UN rates increase flows into nonparticipation in expansions. The countercyclicality of the NU rate has the same effect. These forces are offset by the following procyclical forces. The first is the procyclicality of the NE rate. Second, even though the UN rate increases in a boom, the size of the unemployment pool falls more significantly, and thus the number of workers who move from U to N actually falls. We emphasized earlier that the participation rate varies very little over the business cycle, whereas underlying transition rates that drive the participation rate exhibit larger variations. This empirical observation arises (in an accounting sense) due to the fact that movements in transition rates to and from nonparticipation tend to have offsetting effects on the participation rate.[14]

Summary. The empirical findings so far can be summarized as follows: (i) The EN separation rate is procyclical, which partially offsets the countercyclicality of the EU separation rate, therefore making the "total separation rate" less countercyclical; (ii) the UN rate is procyclical, while the NU transition rate is countercyclical; (iii) the job-finding probability from nonparticipation (NE rate) is strongly procyclical and persistent as that from unemployment; this procyclical NE rate is an important procyclical force of the participation rate; (iv) the LFPR is, however, only mildly procyclical and varies very little over the business cycle; and (v) similarly, real wage is not very volatile and only mildly procyclical.

3 Baseline Model

In this section, we present our baseline model that is simple enough to convey key economic intuitions that help explain the cyclical behavior of labor market transition rates.[15] As shown below, this model cannot fully match the average levels of labor market flows across the three states. However, it possesses the features that allow us to replicate key cyclical properties of transition rates. Importantly, we calibrate the values of the extensive-margin labor supply elasticity and the elasticity of substitution between home and market goods to the micro-level evidence (e.g., Chetty et al. (2011, 2013) and Aguiar et al. (2013)).

[14] Note that, as first-stage effects, the movements between E and U (EU and UE) are neutral to the participation rate. However, the procyclical movement in the former and the countercyclical movement in the latter also contribute to increasing the participation rate through indirect effects. For example, when an unemployed worker moves to employment, the probability that he exits the labor force is reduced relative to the case in which he remains unemployed (i.e., EN rate is much lower than the UN rate).

[15] Details on the derivations are available in Appendix A.2.

We also show the important role that wage rigidity plays in reproducing the empirical regularities. In the following section (Section 4), we propose several simple extensions that enable us to also quantitatively match the missing aspects, while keeping the key intuitions.

3.1 Environment

We maintain the representative-household (or family) structure throughout the paper. There is a large number of identical households, each of them made up of a continuum of members represented by the unit interval. At the beginning of each period, the members can be either employed or nonemployed. If the member is employed, he is paid wage w_t and produces the consumption good with common productivity y_t, which follows an exogenous first-order autoregressive process. Job separation occurs at the end of the period with a fixed probability s, and those who separated in period t become nonemployed at the beginning of $t+1$.

If nonemployed at the beginning of t, the member i draws his productivity at home h_i from a distribution $\Phi(h_i) \in \left[0, \bar{h}\right]$. This variable h_i is assumed to be i.i.d. across members and time.[16] The household decides the activity in that period of the nonemployed members based on their productivities: either active job search (unemployed, U) or home production (not-in-the-labor force, N). Being unemployed means that the worker engages in an *active* job search in that period, receives unemployment insurance (UI) benefits (b) and finds a job at rate f_t. If assigned to home production, a worker contributes to home production according to his own productivity. However, being "not-in-the-labor force" (or being "nonparticipant") does not preclude him from receiving a job offer, albeit at a reduced rate, μf_t where $\mu < 1$.[17] New matches formed in period t become productive in period $t+1$.

Labor market matching is governed by a Cobb-Douglas matching function: $m_t = \bar{m} S_t^{\alpha} V_t^{1-\alpha}$, where m_t is the number of matches, $\bar{m}$ is the scale parameter, S_t is the effective number of job seekers, V_t are vacancies, and α is the elasticity of the matching function. Note that because we allow for passive search among nonparticipants, S_t is different from the number of the unemployed workers, i.e., $S_t = U_t + \mu N_t$. The job-finding rate per efficiency unit of

[16] Bils et al. (2012) study the environment in which workers are permanently different in terms of relative efficiency between home production and market production. That formulation is obviously more realistic and thus appealing. For our purpose, however, we like the model that is easily "aggregatable."

[17] Krusell et al. (2017) adopt the same specification and this "passive job search" makes it easier to generate a large direct flow from not-in-the-labor force to employment that is present in the data. Note also that the proportionality assumption in job-search efficiency (represented by a constant parameter μ), again adopted by Krusell et al. (2017), is empirically plausible, as shown by Hornstein et al. (2014).

search and the job-filling rate per vacancy are, respectively, written as:

$$f_t = \frac{m_t}{S_t} = \bar{m}\theta_t^{1-\alpha} \text{ and } q_t = \frac{m_t}{V_t} = \bar{m}\theta_t^{-\alpha},$$

where $\theta_t \equiv \frac{V_t}{S_t}$ is labor market tightness.

The production side is simple. The representative firm produces consumption goods Y_t using linear technology with labor being the only input. Hiring new workers is subject to search frictions with a flow vacancy posting cost κ.

3.2 Representative Household

Preferences. The momentary household-level preferences are specified as:

$$U(C_t, L_t) = \ln C_t + \omega \frac{L_t^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}}, \tag{1}$$

where L_t is aggregate leisure hours, ω is a scale parameter, ν is the *extensive-margin Frisch elasticity* of leisure, and C_t is aggregate consumption expressed as:

$$C_t = \left(\gamma C_{mt}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) C_{ht}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2}$$

Aggregate consumption consists of consumption of both market-produced goods C_{mt} and home-produced goods C_{ht}; γ is a weight parameter of the CES function; and ε is the elasticity of substitution. This specification has been used in macro models with home production (e.g., Benhabib et al. (1991)). In this environment, allocation of workers is characterized by a unique threshold value of h_i, denoted as h_t^*, above (below) which workers are allocated to home production (active job search). Given the threshold value h_t^*, aggregate home production, which is, by definition, equal to home-goods consumption C_{ht}, is written as:

$$C_{ht} = \int_{h_t^*}^{\bar{h}} h_i d\Phi(h_i)(1-E_t). \tag{3}$$

We assume that each nonparticipant has $\bar{l}$ leisure hours available, and thus the total number of leisure hours L_t is written as $L_t = \bar{l}N_t$.[18] We set $\bar{l} = 1$ without loss of generality, and thus

[18]Note that we do not explicitly model hours of market work and active job search. Thus $\bar{l}$ can be interpreted as additional hours of leisure that each nonparticipant enjoys relative to participants (including both the employed and unemployed).

note that:

$$L_t = N_t = (1 - \Phi(h_t^*))(1 - E_t). \tag{4}$$

Equation (4) implies that the parameter ν in (1) represents the extensive-margin elasticity of nonparticipation. Note that we write preferences in terms of utility of leisure (not in terms of disutility of participation). We convert the parameter ν into the elasticity of participation (i.e., the extensive-margin elasticity of labor supply) through $-\frac{N}{1-N}\nu$, where N is the steady-state nonparticipation rate.

Optimal decisions. The household maximizes the discounted present value of utility flows (1) by choosing market-goods consumption C_{mt}, the participation margin h_t^*, and employment at the beginning of the next period E_{t+1}. The stock of employment evolves according to:

$$E_{t+1} = E_t - sE_t + f_t\Phi(h_t^*)(1 - E_t) + \mu f_t(1 - \Phi(h_t^*))(1 - E_t). \tag{5}$$

Note that the mass of U_t and N_t are determined with no dependence on their past values, i.e. they are not state variables in the current model.[19] Second, although the separation rate s is a constant parameter, transition rates into unemployment and nonparticipation are both cyclical. Note also that those who separated at the end of period t (sE_t) make a participation decision at the beginning of period $t+1$: The mass $\Phi(h_{t+1}^*)sE_t$ enters the unemployment pool, while the mass $(1-\Phi(h_{t+1}^*))sE_t$ exits the labor force. Third, given that E_t is predetermined, choosing h_t^* implies choosing total home production C_{ht} and leisure L_t (see Equations (3) and (4), respectively).

The household decision is also subject to the following budget constraint:

$$A_{t+1} + C_{mt} = w_t E_t + bU_t + (1 + r_t)A_t + \Pi_t - T_t, \tag{6}$$

where A_t represents (zero net-supply) wealth yielding the real return r_t, Π_t the firm's flow profits equal to $(y_t - w_t)E_t - \kappa V_t$, and T_t the lump sum tax that is used to finance unemployment insurance benefits b. Note that each employed worker brings w_t to the household, while each unemployed worker brings b.

Market-goods consumption C_{mt} is determined by the usual Euler equation:

$$\Lambda_t^{C_m} = \beta \mathbb{E}_t \Big[\Lambda_{t+1}^{C_m}(1 + r_{t+1})\Big],$$

[19] This is not the case in our extended model.

where $\Lambda_t^{C_m} \equiv \frac{\partial U(C_t,L_t)}{\partial C_{mt}}$ is the marginal utility of market-goods consumption and $\mathbb{E}_t$ is a conditional expectation operator.

The participation condition is given by:

$$z_{ht}h_t^* + z_{lt} + \mu f_t \mathbb{E}_t \hat{\beta}_{t,t+1}\mathcal{V}_{t+1}^E = b + f_t \mathbb{E}_t \hat{\beta}_{t,t+1}\mathcal{V}_{t+1}^E, \tag{7}$$

where the future value of employment $\mathcal{V}_{t+1}^E$ is discounted by the stochastic discount factor $\hat{\beta}_{t,t+1} \equiv \beta\frac{\Lambda_{t+1}^{C_m}}{\Lambda_t^{C_m}}$. z_{ht} and z_{lt} represent marginal values of home production C_{ht} and leisure L_t, respectively, measured in C_{mt} (i.e., the marginal rate of substitution between C_{ht} and C_{mt} and between L_t and C_{mt}, respectively) and are written as:

$$z_{ht} = \frac{\Lambda_t^{C_h}}{\Lambda_t^{C_m}} = \frac{1-\gamma}{\gamma}\left(\frac{C_{mt}}{C_{ht}}\right)^{\frac{1}{\varepsilon}} \text{ and } z_{lt} = \frac{\Lambda_t^{L}}{\Lambda_t^{C_m}} = \frac{\omega L_t^{-\frac{1}{\nu}}}{\frac{\gamma}{C_t}\left(\frac{C_t}{C_{mt}}\right)^{\frac{1}{\varepsilon}}}, \tag{8}$$

where $\Lambda_t^{C_h} \equiv \frac{\partial U(C_t,L_t)}{\partial C_{ht}}$ and $\Lambda_t^{L} \equiv \frac{\partial U(C_t,L_t)}{\partial L_t}$. The participation condition (7) equalizes the marginal return of having another nonparticipant in the household (left-hand side) to the marginal return of sending another worker to the unemployment pool (right-hand side). On the right-hand side, having another active job seeker brings b to the household plus the expected value of being employed next period $\mathcal{V}_{t+1}^E$ with probability f_t. On the left-hand side, having another nonparticipant brings to the household the flow value of home production $z_{ht}h_t^*$, the flow value of leisure z_{lt}, and the expected value of employment with reduced probability of μf_t.

A few additional remarks regarding Equation (7) are in order. First, as discussed above, we introduced μ to allow for the direct hiring from the pool of nonparticipation to be consistent with the empirical evidence. However, this assumption does not change the underlying economics; the same idea applies when $\mu = 0$. Second, in a standard real business cycle model with endogenous labor supply (or leisure) (as in King et al. (1988)) but without explicit modeling of home production, the optimal condition is simply $z_{lt} = w_t$. In our setup, without home production ($\gamma = 1$), $C_t = C_{mt}$ and thus $z_{ht} = 0$ and $z_{lt} = \frac{\omega L_t^{-\frac{1}{\nu}}}{C_t^{-1}}$. Further, without search frictions, the optimal decision is to equate z_{lt} and w_t.

The value of employment (net of the value of nonemployment) is written as:

$$\mathcal{V}_t^E = w_t - g_t + (1 - s - \hat{f}_t)\mathbb{E}_t \hat{\beta}_{t,t+1}\mathcal{V}_{t+1}^E, \tag{9}$$

where $\hat{f}_t$ is the "effective" job-finding probability defined below and g_t is the flow value of nonemployment given by:

$$g_t = \Phi(h_t^*)b + (1 - \Phi(h_t^*))(\hat{h}_t z_{ht} + z_{lt}). \quad (10)$$

Importantly, the opportunity cost g_t fluctuates over the business cycle even though unemployment benefits are fixed. $\hat{h}_t$ is a conditional mean of h_i above h_t^*, i.e., $\hat{h}_t \equiv \frac{\int_{h_t^*}^{\bar{h}} h_i d\Phi(h_i)}{1-\Phi(h_t^*)}$. The effective job-finding probability is given by:

$$\hat{f}_t = \Phi(h_t^*)f_t + (1 - \Phi(h_t^*))\mu f_t = f_t\psi(h_t^*), \quad (11)$$

where $\psi(h_t^*) \equiv \Phi(h_t^*) + (1 - \Phi(h_t^*))\mu$. In a standard two-state search model (e.g., the one in Chapter 1 of Pissarides (2000)), the value of employment (net of the value of unemployment) takes exactly the same form as Equation (9) except that the outside option value consists only of unemployment benefits b and that $\mu = 0$ and thus $\hat{f}_t = f_t$. In the current model, all nonemployed workers at the beginning of t (including those who have just separated) can optimize on whether to engage in an active job search or to exit the labor force. This decision is based on the i.i.d. home production values that all nonemployed workers draw every period regardless of their previous labor force statuses. In our extended model below, we relax this assumption and thus no longer obtain a simple expression like Equation (9).

3.3 Representative Firm

The representative firm produces the consumption goods via the following technology:

$$Y_t = y_t E_t,$$

where y_t is exogenous stochastic labor productivity that follows:

$$\ln y_{t+1} = (1 - \rho)\ln \bar{y} + \rho \ln y_t + \epsilon_{t+1} \text{ with } \epsilon \sim \mathcal{N}(0, \sigma^2). \quad (12)$$

The firm maximizes its value by choosing the number of vacancies posted every period and thus its size (employment) in the following period (E_{t+1}). The decision is characterized by the following standard job-creation condition:

$$\frac{\kappa}{q_t} = \mathbb{E}_t \hat{\beta}_{t,t+1} \mathcal{V}_{t+1}^J, \quad (13)$$

where $\mathcal{V}_t^J$ is the value of the filled job that evolves according to:

$$\mathcal{V}_t^J = y_t - w_t + (1-s)\mathbb{E}_t\hat{\beta}_{t,t+1}\mathcal{V}_{t+1}^J. \tag{14}$$

3.4 Equilibrium

The equilibrium in the goods market and the zero net supply condition on an asset A_t imply:

$$y_t E_t = C_{mt} + \kappa V_t.$$

It remains to describe how wages are determined. To build intuition, we consider two stark cases, one in which wages are continuously renegotiated through Nash bargaining and the other in which wages are completely fixed.

Flexible wage. The surplus of the employment relationship is defined by:

$$\mathcal{S}_t = \mathcal{V}_t^J + \mathcal{V}_t^E. \tag{15}$$

We assume that surplus is shared in fixed proportions between the firm and the household with η being the household's bargaining power and $1-\eta$ being the firm's. Using Equations (9) and (14) into (15) allows us to write the evolution of surplus as:

$$\mathcal{S}_t = y_t - g_t + (1-s-\eta\hat{f}_t)\mathbb{E}_t\hat{\beta}_{t,t+1}\mathcal{S}_{t+1}. \tag{16}$$

Using the free-entry condition (13) in the above equation, we obtain:

$$\frac{\kappa}{q_t} = \mathbb{E}_t\hat{\beta}_{t,t+1}\left[(1-\eta)(y_{t+1}-g_{t+1}) + \frac{\kappa}{q_{t+1}}(1-s-\eta\hat{f}_{t+1})\right]. \tag{17}$$

We can also rewrite the participation condition (7) simply as:

$$z_{ht}h_t^* + z_{lt} = b + (1-\mu)\frac{\eta}{1-\eta}\kappa\theta_t. \tag{18}$$

Finally, using Equations (9) and (14) in $\eta\mathcal{V}_t^J = (1-\eta)\mathcal{V}_t^E$, we can explicitly solve for w_t:

$$w_t = \eta y_t + (1-\eta)g_t + \eta\kappa\theta_t\psi(h_t^*).$$

As in the standard two-state model, the last two terms capture the effect of the outside option on w_t. In the standard model, however, $g_t = b$ and $\psi(h_t^*) = 1$, while in our model both terms are time-varying and $\psi(h_t^*) < 1$.

Fixed wage. When the wage is fixed at some value $\bar{w}$ (which is assumed to be in the bargaining set in all states of the economy), we can combine Equations (13) and (14) together with $w_t = \bar{w}$ and rewrite the job-creation condition as:

$$\frac{\kappa}{q(\theta_t)} = \mathbb{E}_t \hat{\beta}_{t,t+1} \left[y_{t+1} - \bar{w} + \frac{(1-s)\kappa}{q(\theta_{t+1})} \right]. \tag{19}$$

The participation condition is obtained by combining Equations (7) and (9):

$$\frac{z_{ht} h_t^* + z_{lt} - b}{f_t} = \mathbb{E}_t \hat{\beta}_{t,t+1} \left[(1-\mu)(\bar{w} - g_{t+1}) + (1 - s - \hat{f}_{t+1}) \frac{(z_{ht+1} h_{t+1}^* + z_{lt+1} - b)}{f_{t+1}} \right]. \tag{20}$$

Except for the indirect effect through the stochastic discount factor, the fixed wage assumption essentially isolates the firm's job-creation (labor demand) decision from the household's participation (labor supply) decision. Wage rigidity also plays an important role not only for the former decision but also for the latter, because wage represents an important component of the returns to market work.

3.5 Calibration

We calibrate this model at monthly frequency. Table 1 summarizes the parameter values we use for the exercises below. In Appendix A.2.2, we describe in detail the exact procedure through which we set those values. We first set values of some of the parameters exogenously, as described in Panel A. The values of β, α, and η are all standard in the literature. For the exogenous productivity process, we first normalize its steady-state level $\bar{y}$ to 1 and select its persistence and volatility to match the cyclical properties of the quarterly U.S. labor productivity between 1976 and 2016.[20]

The elasticity of substitution between C_m and C_h, ε is set to 2.5, following Aguiar et al. (2013). As these authors explain (see footnote 13 in their paper), the values used in the literature range from somewhat less than 2 to 4. We also study two additional cases: $\varepsilon = 1$ (Cobb-Douglas preferences) and $\varepsilon = \infty$ (linear preferences). To set the nonparticipation (or leisure) elasticity ν, we follow Chetty et al. (2011, 2013) who suggest that the "micro"

[20] Following Shimer (2005), labor productivity is seasonally-adjusted real output per person in the nonfarm business sector (series PRS85006163 published by the BLS). After taking logs and deviations from an HP trend with smoothing parameter 10^5, the standard deviation of quarterly labor productivity equals 0.0176 and its quarterly autocorrelation is 0.899 in the data. With our selected values, our exogenous productivity series matches these moments.

Table 1: Parameter values

Parameter	Interpretation	Value
	Panel A: Externally calibrated	
β	Discount factor	0.995
α	Elasticity of the matching function	0.5
η	Worker's bargaining power	0.5
ϵ	Elasticity of substitution	2.5
ν	Elasticity of nonparticipation	0.47
$\bar{y}$	Aggregate productivity in steady state	1
ρ	Autoregressive parameter for log aggregate productivity	0.99
σ	Standard deviation for log aggregate productivity	0.0053
$\bar{\omega}$	Ratio between z_l and $\hat{h}z_h$ in the steady state	1
	Panel B: Internally calibrated	
s	Total separation rate	0.043
$\bar{m}$	Scale parameter of the matching function	0.47529
κ	Vacancy posting cost	1.2184
$\bar{h}$	Upper bound of $\Phi(.)$ distribution	0.65277
γ	CES weight parameters	0.70685
b	Unemployment benefits	0.39177
μ	Job-search efficiency	0.18725
ω	Scale parameter of the leisure utility function	0.0137

estimate of the extensive-margin labor supply (participation) elasticity is around 0.25, which translates into the nonparticipation elasticity at 0.47.[21] In the macro literature, on the other hand, much higher values have been used, and according to these authors, the value 2.3 is the representative "macro" estimate, which is translated into $\nu = 4.3$ in our model. We also consider this value in our model.

Next, we impose that the model match the levels of EU, EN, UE, and NE transition rates in the steady state. In the second column of Table 2, we present the definitions of labor market variables in the model. The restrictions on the steady-state levels of these four rates imply that $f = 0.251$, $s = 0.043$, $\Phi(h^*) = 0.35$, and $\mu = 0.19$. The last value tells us that the nonparticipant's job-finding probability is about one-fifth of an active job seeker's job-finding probability. These pieces of information are sufficient to determine steady-state values of all labor market variables, as presented in Table 2. As is clear from the fifth and sixth rows, the model misses the levels of transition rates that are not directly targeted,

[21]Let $\hat{\nu}$ be the extensive-margin labor supply (participation) elasticity. We convert $\hat{\nu}$ into ν by using $\hat{\nu} = -\frac{N}{1-N}\nu$, where the steady-state nonparticipation rate N is set to 0.348. This value is based on the estimation result of our extended model below and is close to its historical average.

Table 2: Labor Market Variables in Steady State

Data	Model concept	Empirical value	Steady-state value
EU rate	$s\Phi(h^*)$	0.015	0.015
EN rate	$s(1-\Phi(h^*))$	0.028	0.028
UE rate	f	0.251	0.251
NE rate	μf	0.047	0.047
UN rate	$(1-\Phi(h^*))(1-f)$	0.21	0.49
NU rate	$(1-\mu f)\Phi(h^*)$	0.027	0.33
E-pop ratio	E	0.62	0.73
Nonparticipation rate	N	0.33	0.17
Unemployment rate	$\frac{U}{E+U}$	0.06	0.11

namely, UN and NU rates. In particular, the NU rate in the model is grossly overstated. This is expected given the large share of nonparticipants that may not even be available to work in reality (for example, retirees), while our assumption in the model is that all nonparticipants are potentially available for work. Observe that we also miss the average levels of stocks.[22] We nevertheless view the model's steady-state performance as being more than satisfactory for the purpose of illustrating the key economic insights. Again, Section 4 proposes several extensions to overcome the limitations of this model.

We assume that the distribution of $\Phi(h_i)$ is uniformly distributed between 0 and $\bar{h}$ and impose a few more steady-state restrictions to set the remaining parameters. First, we normalize the steady-state value of z_{ht} at 1.[23] Next, the steady-state value of the opportunity cost of employment g is set to 0.71. The literature has used this value as a plausible value of the opportunity cost (e.g., Hall and Milgrom (2008)), lumping together UI benefits b with home production and leisure components. Our paper models these components explicitly, with the latter two being endogenously cyclical (see Equation (10)). We then impose the relative importance of these two components ($\hat{h}z_h$ and z_l) in the steady state. Specifically, we introduce a parameter $\bar{\omega} = \frac{\hat{h}z_h}{z_l}$ and set this to 1 (note also that $z_h = 1$ in the steady state as discussed above). In the absence of the micro evidence, this choice appears to be a plausible benchmark. But we estimate this parameter within our extended model below through the impulse-response matching exercise, which in fact suggests that the value is close to 1. Lastly, the steady-state job-filling rate q is set to 0.9, following, for example, Fujita and

[22]We could alternatively target the steady-state levels of UN and NU transition rates, but we would again miss the levels of the other transition rates and stocks. The model's overall steady-state performance deteriorates in this case.

[23]It is not immediately clear that this is a normalization. However, within our calibration procedure, the model dynamics are indeed invariant to the steady-state level of z_{ht}.

Ramey (2007).[24] These steady-state restrictions, together with the information previously obtained, allow us to set all parameter values. Appendix A.2.2 describes the exact steps of this process in detail.

3.6 Understanding the Cyclical Properties of the Baseline Model

This section analyzes the cyclical properties of the baseline model. This analysis helps us learn about key factors behind the observed cyclicality of labor market transition rates.

3.6.1 Model Concepts and Data

As is clear from Table 2, all transition rates in the model are functions of f (and thus labor market tightness θ), and threshold home-production productivity, h^*. So the question is how the behavior of the six transition rates can be replicated through the movements in these two variables only. Before presenting impulse response functions of the calibrated model, let us first address this question qualitatively.

It is clearly the case that job-finding rates from unemployment and nonparticipation are procyclical in the data. In the model, they both are functions of θ only (which moves procyclically). To the extent that the NE rate moves more or less proportionately along with the UE rate, the model is capable of replicating the cyclicality of these two transition rates. Next, consider EU and EN rates. In the model, the overall separation rate s is constant, but those who separate decide whether to enter the unemployment pool (with probability $\Phi(h_t^*)$) or to exit the labor force (with probability $1 - \Phi(h_t^*)$). Recall that in the data, the EU rate is countercyclical and the EN rate is procyclical. This cyclical pattern puts the restriction that $\Phi(h_t^*)$ and thus h_t^* have to be countercyclical. As mentioned above, the sum of these two transition rates is countercyclical in the data and the current model misses this feature (because the sum s is constant), even though it replicates the broad pattern that these two rates move in opposite directions. The extended model below matches this more nuanced feature of the data as well. Next, the NU rate is countercyclical in the data and is defined as the product of $1 - \mu f_t$ and $\Phi(h_t^*)$ in the model. Taking the countercyclicality of h_t^* and the procyclicality of f_t as given, this product is unambiguously countercyclical. Lastly, the UN rate is empirically procyclical, but its cyclicality in the model, defined as a product of $1 - \Phi(h_t^*)$ and $1 - \mu f_t$, remains ambiguous (because even though the first term is procyclical, the second term is countercyclical). In order for the product to be procyclical, the countercyclical of h_t^* needs to be dominant.

[24] Note, however, that the steady-state level of the job-filling rate is inconsequential for the model dynamics as is well-known for this class of models.

Summing up, *all these relationships between the model and empirical cyclicality point to a strong countercyclicality of* h_t^*. As long as h^* is countercyclical and f_t is procyclical, this bare-bones model can match the overall cyclical pattern of all transition rates (except possibly the UN rate). The cyclicality of h_t^* is determined by the strength of what we call generically the income effect and the substitution effect. If wages are flexible, an expansion will bring increases in both the job-finding rate and the wage rate, thus increasing the returns to market work. If leisure is a normal good, then the income effect will tend to generate a countercyclical h_t^*. At the same time, the increase in the returns to work makes leisure costlier, which makes h^* procyclical. Based on steady-state comparative statics, we show next how the strength of the two effects are affected by wage flexibility and by two parameters of the model, namely, the elasticity of substitution between home and market goods ε and the elasticity of leisure ν.

3.6.2 Steady-State Equilibrium

The steady state of the economy is characterized by the job-creation condition and the participation condition. In the flexible wage version, these conditions are written as:

$$\frac{\kappa}{q(\theta)} = \beta(1-\eta)\frac{y - g(\theta, h^*)}{1 - \beta(1 - s - \eta\hat{f}(\theta, h^*))}, \tag{21}$$

$$z_h(\theta, h^*)h^* + z_l(\theta, h^*) = b + (1-\mu)\frac{\eta}{1-\eta}\kappa\theta. \tag{22}$$

In the rigid wage version, the two conditions are written as:

$$\frac{\kappa}{q(\theta)} = \beta\frac{y - \bar{w}}{1 - \beta(1-s)}, \tag{23}$$

$$z_h(\theta, h^*)h^* + z_l(\theta, h^*) = b + (1-\mu)f(\theta)\beta\frac{\bar{w} - g(\theta, h^*)}{1 - \beta(1 - s - \hat{f}(\theta, h^*))}. \tag{24}$$

Panels (a) and (b) of Figure 3 graphically describe the steady-state equilibrium as an intersection of the two curves in the (θ, h^*) space, when wages are flexible and rigid, respectively. We draw these diagrams using our calibrated model. Panel (a) shows that, under flexible wages, both job-creation and participation conditions are downward sloping. Consider first the job-creation condition. A higher value of h^* lowers the match surplus due to higher values of flow opportunity cost of employment, g, and the effective job-finding probability, $\hat{f}$ (see Equation (21)). Consequently, firms post fewer vacancies, which in turn lowers θ. The participation condition is also downward sloping. Higher θ implies a higher return

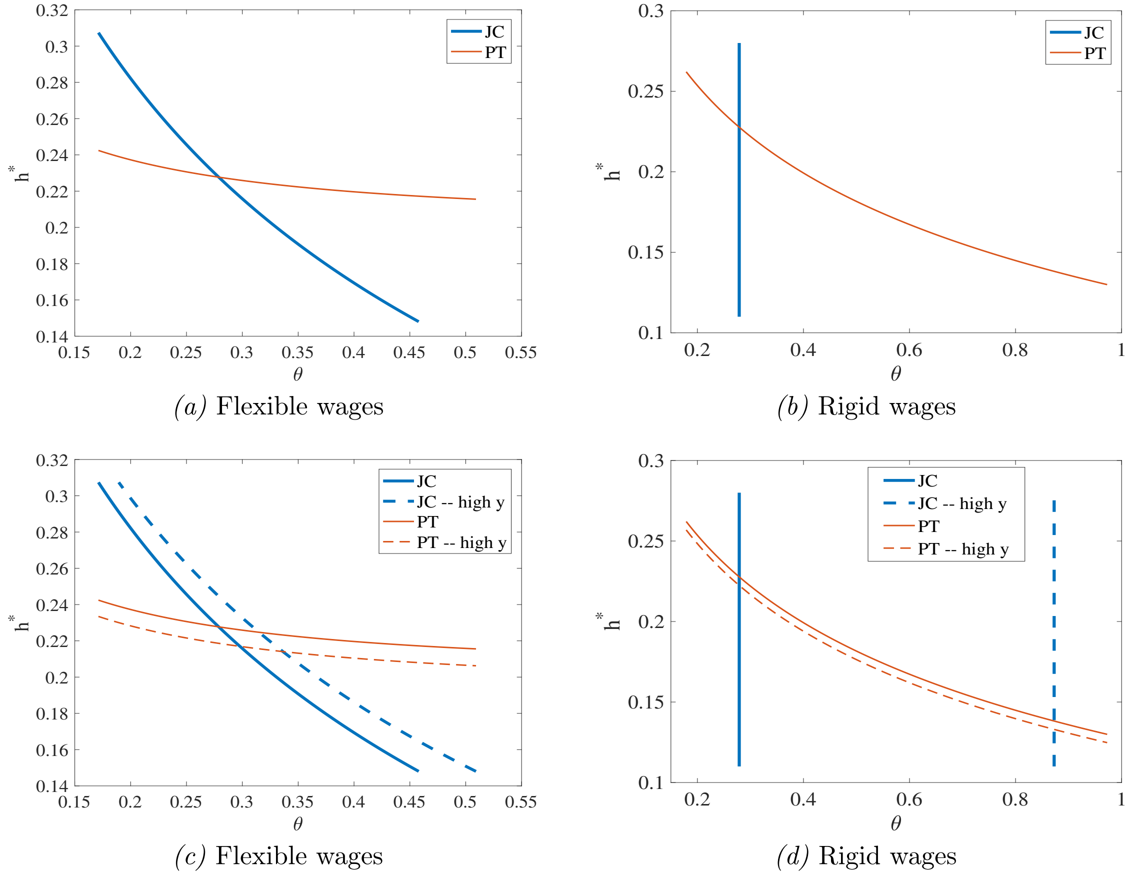


Figure 3: Steady-State Equilibrium and Comparative Statics. *Notes*: JC refers to job-creation condition and PT to participation condition. Dashed lines correspond to the case in which the productivity level is raised permanently.

to market work (because of higher wage and higher probability of receiving a job offer), but higher employment and consumption of market-produced goods raise z_h and z_l, which then offset the incentive to join the labor force, making h^* lower. The increases in the values of leisure and home goods offset the gains from participating along the participation condition curve. As we show below, the participation condition becomes upward sloping when we increase the participation elasticity enough.

Panel (b) presents the case of rigid wage. In this case, the job-creation condition is independent of the participation margin, determining θ by itself. The household then chooses h^*, given the level of θ. Note that the slope of the participation condition remains negative but becomes much steeper than in the flexible wage case. An increase in θ does not have an impact on wages, which eliminates the key part of the substitution effect. The income effect (through higher employment as a result of higher job-finding rates and thus higher market-goods consumption) remains and thus the decline in h^* necessary to restore the equilibrium is larger.

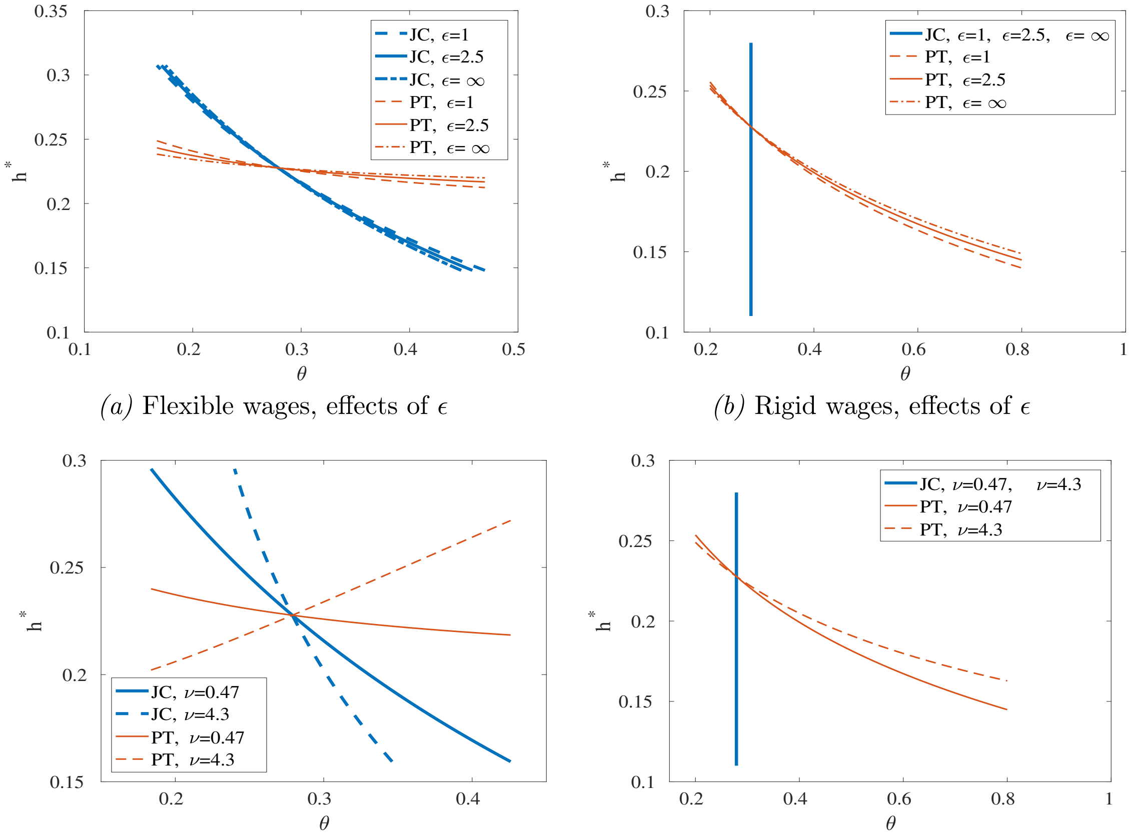
Panels (c) and (d) present graphically the comparative static results of higher productivity when wages are flexible and rigid, respectively.[25] One can see that under our calibration, a higher y unambiguously generates higher θ and lower h^*, but especially so under rigid wages. The constant-wage specification dramatically increases the volatility of θ, as is well-known in the existing literature. Rigid wages also play an important role in lowering h^* upon a higher productivity because the substitution effect is significantly dampened in the participation condition.

As discussed above, the empirical evidence points to a countercyclicality of h^* as well as a procyclicality of θ. Both the slope and the shifting patterns of the two equilibrium conditions in our calibrated model are unambiguously consistent with both of these implications, whether wages are assumed to be flexible or rigid. But the model properties under rigid wages are particularly attractive in that the changes in θ and h^* are of much larger magnitude than under flexible wages.

Next, Figure 4 presents how the equilibrium conditions are affected by different values of ϵ and ν. Panels (a) and (b) show the effects of the different substitutability between home- and market-produced goods (ϵ) when wages are flexible and rigid, respectively. We consider the two alternative cases, $\epsilon = 1$ and $\epsilon = \infty$. As is clear from Equation (8), the response of z_{ht} to changes in $\frac{C_{mt}}{C_{ht}}$ is larger for smaller values of ϵ. Note that $\frac{C_{mt}}{C_{ht}}$ is likely to move procyclically along with market productivity (y_t). Thus, when productivity of market-goods production increases, the value of the home-produced good z_{ht} rises more with a smaller value of ϵ (a larger value of the "elasticity of complementarity"). In the opposite limiting case of $\epsilon = \infty$, z_{ht} is constant over the cycle. In sum, with a smaller value of ϵ, the participation condition becomes steeper and the job-creation condition becomes flatter under the flexible wage case. When wages are rigid, the job-creation condition is independent of h^* and thus not affected by ϵ, and the participation condition rotates clockwise for a higher complementarity.

Panels (c) and (d) show the effects of a higher value of ν under flexible and rigid wages, respectively. As discussed before, we consider ν equal to 4.3 as an alternative value. The higher value makes the job-creation condition steeper when wages are flexible: When ν gets higher, the value of leisure z_{lt} increases less and, thus, the decline in surplus is smaller in response to the same increase in h^*. A higher value of ν affects the participation condition strongly under flexible wages. It implies that the increase in z_l is smaller and thus the substitution effect is stronger. Consequently, the slope of the participation condition turns

[25] Note that the diagram is drawn by changing productivity by 10% under flexible wages and 5% under rigid wages for the ease of visual inspections.



(a) Flexible wages, effects of ϵ

(b) Rigid wages, effects of ϵ

(c) Flexible wages, effects of ν

(d) Rigid wages, effects of ν

Figure 4: Steady-State Equilibrium, Effects of ν and ϵ.

positive. With this positively-sloped participation condition, it is possible that higher productivity results in higher h^*. Whether h^* indeed increases or not in equilibrium depends on how much each curve shifts, and we will examine this by computing the steady-state elasticities. We also study this point by computing impulse response functions to a temporary but persistent productivity shock.

The effect of ν is more muted when wages are rigid. Again, in this case, the job-creation condition is unaffected by ν, and the participation condition remains downward sloping even when the value of ν is raised significantly to 4.3. The value of ν governs the extent of the substitution effect, but it is largely irrelevant under fixed wages; the participation decision is dominated by the income effect, and thus a higher value of ν has a small impact on the participation condition.

Table 3: Steady State Elasticities w.r.t. 1% Increase in Productivity

	Flexible Wage				Rigid Wage			
ϵ	2.5	1	∞	2.5	2.5	1	∞	2.5
ν	0.47	0.47	0.47	4.3	0.47	0.47	0.47	4.3
θ	2.03	1.76	2.26	2.08	28.76	28.76	28.76	28.76
h^*	−0.65	−0.78	−0.54	−0.43	−11.02	−11.93	−10.26	−10.02
EU	−0.65	−0.78	−0.54	−0.43	−11.02	−11.93	−10.26	−10.02
EN	0.35	0.41	0.29	0.23	5.44	5.85	5.09	4.98
UE	1.01	0.88	1.13	1.04	14.38	14.38	14.38	14.38
UN	0.00	0.12	−0.10	−0.12	0.12	0.53	−0.22	−0.33
NE	1.01	0.88	1.13	1.04	14.38	14.38	14.38	14.38
NU	−0.70	−0.82	−0.59	−0.48	−11.78	−12.70	−11.02	−10.79
u	−1.13	−1.06	−1.19	−1.07	−16.95	−17.30	−16.66	−16.57
E	0.17	0.11	0.21	0.21	2.05	1.91	2.15	2.19
$LFPR$	0.02	−0.02	0.06	0.07	0.08	−0.08	0.23	0.27
w	1.00	1.01	0.99	1.00	–	–	–	–
g	0.85	0.95	0.76	0.81	3.36	3.83	2.96	3.17
z_h	0.58	1.21	0.00	0.66	2.12	4.58	0.00	2.43
z_l	1.31	0.89	1.70	1.19	2.71	1.17	3.99	2.24
C_h	−0.28	−0.09	−0.45	−0.46	−3.14	−2.55	−3.63	−3.79
C_m	1.16	1.12	1.21	1.21	2.16	2.02	2.27	2.30
C	1.02	1.00	1.04	1.04	1.64	1.57	1.69	1.70

3.6.3 Steady-State Elasticities

Table 3 presents steady-state elasticities of model variables with respect to a 1 percent permanent increase in labor productivity y, under various combinations of ϵ, ν, and the wage specification.

Regardless of the calibration, the model generates a correct cyclical pattern for unemployment, employment, and consumption. The participation rate is, in general, procyclical in the model, but the elasticity is small, both of which are consistent with the data. The participation rate changes little even though transition rates are more elastic, due to the offsetting effects of transition rates on the movements in the participation rate, as we discussed earlier in the empirical section. The wage rate and the opportunity cost of employment (g) are procyclical, and the elasticity of g with respect to productivity is of a meaningful size as documented by Chodorow-Reich and Karabarbounis (2016). The procyclicality of g arises because the value of home production z_h increases as consumption shifts towards market-goods consumption C_m and the value of leisure z_l is also procyclical. Again, the procyclicality of z_h and z_l are key to generating countercyclical EU and NU rates.

Qualitatively speaking, our baseline calibration under flexible wages (first column of Table 3) does a good job of generating the correct cyclical patterns for the EU, NE and NU transition rates. As discussed in the previous section, the model is able to generate procyclical labor market tightness (θ) and a countercyclical participation threshold (h^*) that help us explain the cyclical behavior of those transition rates. However, the UN transition rate is essentially acyclical (recall that in the data it is procyclical) because the countercyclicality of h^* is not strong enough to offset procyclical θ and thus f_t.

Rigid wages greatly increase the elasticities because, as expected from the analysis in the previous section, the substitution effect is muted in the participation condition and the job-creation condition becomes more sensitive to changes in aggregate productivity, all of which increase the responses of θ and h^*. In a standard two-state model of the labor market, labor market tightness (labor demand) becomes more cyclically sensitive under rigid wages. But another novel channel in our model is through the countercyclical participation margin, which makes inflow rates to the unemployment rate (EU and NU rates) countercyclical. Remember that the separation rate out of employment is constant in our model. Nevertheless, the share of those who make EU transitions increases in downturns and thus increases the unemployment rate. Similarly, the countercyclicality of the NU rate also contributes to increasing the unemployment pool in downturns. These extra forces do not exist in standard two-state models with exogenous separation. We will quantify the importance of this channel below. Another notable result is that, despite the magnification effect of wage rigidity, the participation rate remains largely insensitive, again because underlying transition rates have offsetting effects on the participation rate.

A well-known resolution to the volatility puzzle within the flexible wage formulation is to increase the level of the opportunity cost of employment, as proposed by Hagedorn and Manovskii (2008). This resolution, however, does not work in our setup because, in our model, the opportunity cost g_t moves procyclically. In order for the model to generate the magnification effect through this channel, the opportunity cost needs to be constant. Chodorow-Reich and Karabarbounis (2016) point out this possibility theoretically and empirically show that it is indeed cyclically sensitive (procyclical) in the data.

The second and third columns present the effects of changing the value of ϵ. With a smaller value of ϵ, C_h and C_m become more complementary, resulting in stronger countercyclicality in h^*. The cyclicality of transition rates changes accordingly. For example, inflow rates to unemployment (EU and NU rates) become more countercyclical, while transitions into nonparticipation (EN and UN) become more procyclical. The opposite pattern holds with $\epsilon = \infty$ (the third column). The fourth column presents the effects of a higher value of ν. Recall from Figure 4 (c) that the high value for ν makes the participation condition upward sloping, implying that a tighter labor market motivates more the workers to join the labor force. However, in equilibrium, h^* remains countercyclical because the job-creation condition shifts out with higher productivity (although the countercyclicality is weaker, resulting in the changes in the elasticity from -0.65 to -0.43).

Under rigid wages, the job-creation condition is affected by neither ϵ nor ν. Thus, UE and NE rates, which are a function of tightness only, are unaffected. The remaining four transition rates are affected through the effects on the participation condition which becomes flatter with higher values ϵ and ν (Figure 4 (b) and (d)). Note that the flatter participation condition in itself implies less countercyclical responses in h^*. Changes in elasticities are consistent with this prediction. A somewhat notable change is in the elasticity of the UN transition rate, which is 0.12 under our baseline calibration, falling to -0.33 under $\nu = 4.3$. Recall that the UN rate is procyclical empirically, and thus the parameter change makes its cyclicality counterfactual. As we discussed before, in order for this variable, defined as $(1 - \Phi(h^*))(1 - f)$, to be procyclical, the countercyclicality of h^* (and thus $\Phi(h^*)$) has to be stronger than the procyclical force of f, and with $\nu = 4.3$, the former effect is weakened, and thus this transition rate becomes countercyclical. Overall, our baseline calibration gives the best performance in terms of replicating the cyclicality of transition rates, but it seems fair to say that, especially under rigid wages, the quantitive impacts of ϵ and ν are relatively small.[26]

3.7 Dynamics

We linearize and solve for the dynamic and stochastic equilibrium of the baseline model under the exogenous stochastic process for productivity (12). Figures 5 and 6 present impulse response functions to a 1 percent positive productivity shock under flexible wages (dashed lines) and rigid wages (solid lines).

From the two figures, one can immediately see the volatility effect of rigid wages. Apart from the volatility, the model behaves similarly under both wage specifications. An issue clearly visible is the cyclicality of the UN rate (Figure 5 (d)). The UN rate is empirically strongly procyclical and quite volatile: Comparing the empirical impulse response functions

[26] The case with $\epsilon = 1$ and $\eta = 0.47$ also performs well. In particular, this case outperforms our baseline calibration in terms of generating stronger procyclical movements in the UN rate. However, the elasticity of the participation rate turns negative.

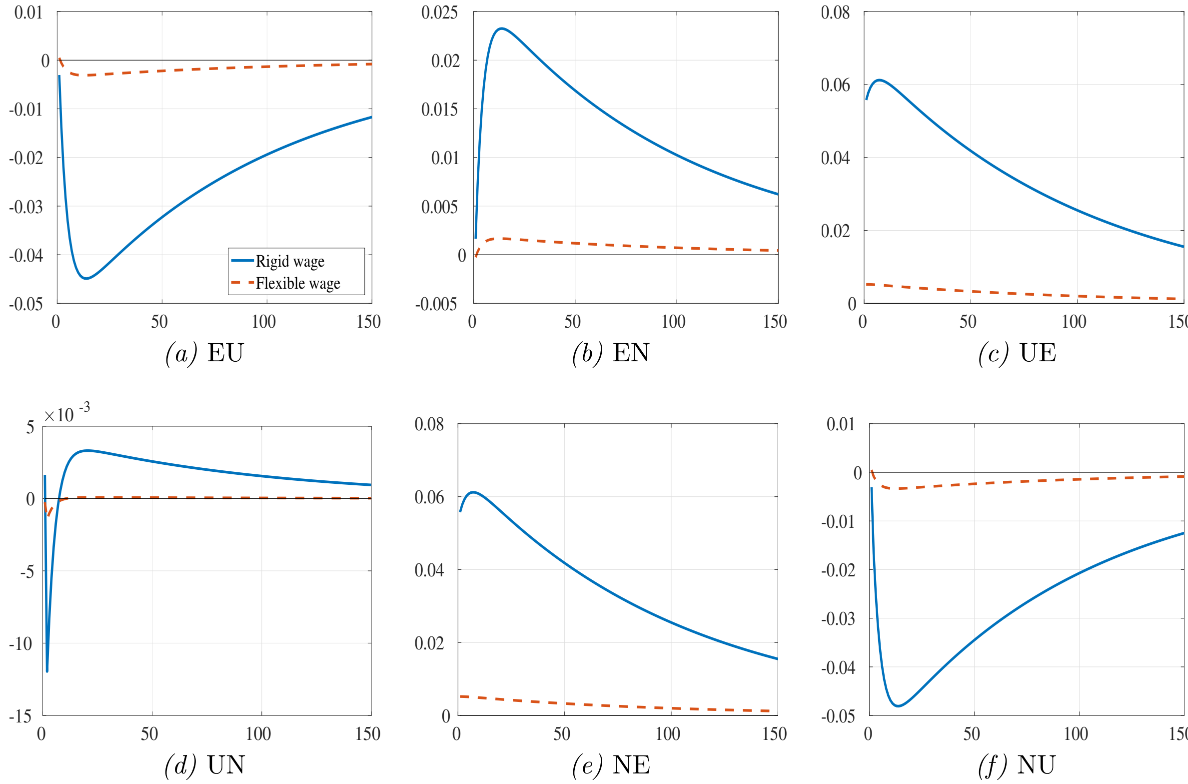
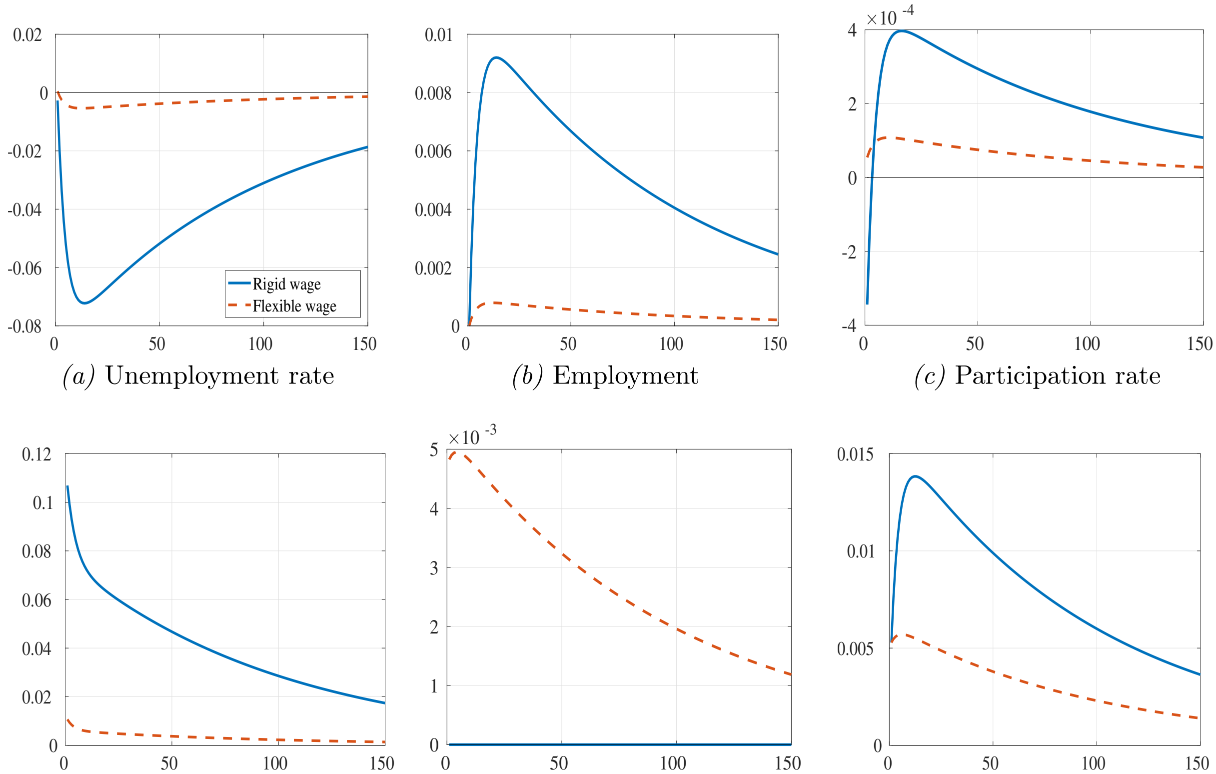


Figure 5: Impulse Response Functions (Baseline Model), Transition Rates. *Notes*: Monthly responses to a one standard deviation positive productivity shock. All responses are expressed as log deviations from steady-state levels.

of UE and UN rates (Figure 1 (c) and (d)), one can see the latter is as procyclical as the former and only slightly less volatile. In the model, the cyclicality of the UN rate is not clearcut and not as volatile as the UE rate, due to the two offsetting effects from h_t^* and f_t, as discussed above.

Turning to the labor market stocks (Figure 6), the unemployment rate (a), the employment-population ratio (b), and vacancies (d) all move in the directions consistent with the data. An important feature of the model that we have not highlighted so far is the procyclicality of vacancies. This result is notable in the sense that the two-state model with endogenous separations that is able to generate countercyclical EU rate fails to replicate this robust empirical regularity. In our model, however, the EU rate is countercyclical, thus contributing to the countercyclical movements in the unemployment rate (as in the data), while maintaining the strong procyclicality of vacancies.[27] Note also that in the data, vacancies exhibit

[27]See Fujita and Ramey (2012) for the quantitative properties of the two-state model with endogenous separations. They show that adding on-the-job search resolves the counterfactual properties of vacancies in



(a) Unemployment rate   *(b)* Employment   *(c)* Participation rate

(d) Vacancies   *(e)* Real wage   *(f)* Output

Figure 6: Impulse Response Functions (Baseline Model), Labor Market Stocks and Real Wage. See Notes to Figure 5.

a hump-shaped response. The current model is not meant to replicate this feature, and we address this issue by introducing curvature in the hiring cost in our extended model.

Figure 6 (c) presents the responses of the participation rate. Under flexible wages, the response remains positive throughout the entire horizon. Under rigid wage, the participation rate falls initially below its steady-state level, before it increases above the steady-state level. Qualitatively speaking, this pattern is line with its empirical counterpart in the sense that the participation rate also tends to fall initially before increasing later in the business cycle. Note also that even though the participation rate moves more under rigid wage than under flexible wage, its variability remains small and is of a similar magnitude to the data. Again, we will provide a more detailed quantitative evaluation of these features in our extended model.

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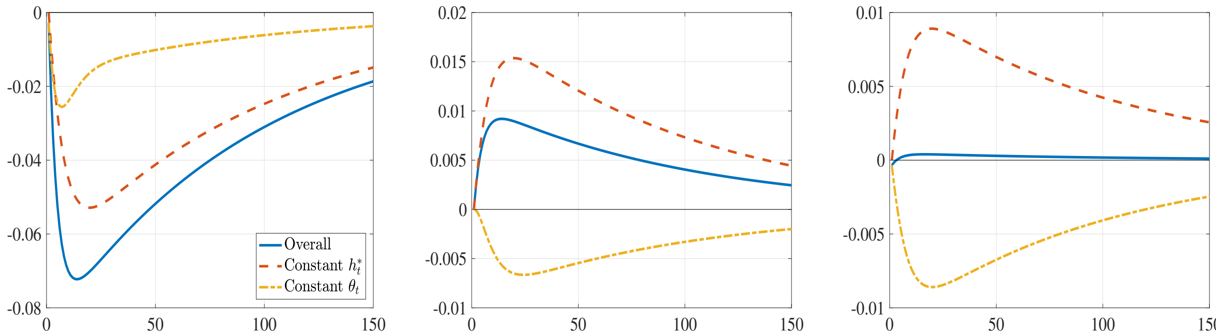
the endogenous separation model.



(a) Unemployment Rate *(b)* E-Pop Ratio *(c)* Participation Rate

Figure 7: Contributions of Job Creation and Participation Margins. *Notes*: Responses to a one standard deviation positive productivity shock. All responses are expressed as log deviations from steady-state levels.

Importance of the two margins. With the endogenous participation decision, the unemployment rate is influenced not only by the response of labor market tightness, but also by the response of the participation margin. In particular, the procyclicality of the UN rate and the countercyclicality of the NU rate imply that the composition of nonemployed population shifts towards (away from) unemployment in a downturn (an expansion). We demonstrate this point by computing the counterfactual paths of three stock variables (the unemployment rate, the employment level and the LFPR) by fixing either the participation margin h_t^* or market tightness at its steady state value.[28] For these experiments, we focus on the results under fixed wage. In Figure 7, the red dashed lines represent the counterfactual paths when h_t^* is fixed, while the yellow dash-dot lines represent the paths when θ_t is fixed.

We can see in Panel (a) that, at the lowest point of the unemployment response, more than 30 percent of the response is due to the change in the participation margin. That is, declines in inflow rates (EU and NU rates) and increases in the UN rate in response to the expansionary shock increase the unemployment response more than 30 percent. Under the counterfactual scenario with the fixed participation margin, the participation rate increases much more strongly (Panel (c)). Note that the only direct channel through which the participation rate moves when fixing h_t^* is direct hiring from nonparticipation (which occurs at rate μf_t) and that this channel only increases the participation rate. The yellow dot-dash line, on the other hand, indicates that variations in h_t^* (while fixing θ_t) result in lower participation, largely offsetting the effects of variations in θ_t. This result demonstrates clearly the underlying reason for the low volatility of the participation rate over the business cycle.

[28] Note that the model and calibration remain the same as before. All we do is to shut down the movements in the two margins within the same model and plot the resulting simulated paths for the stock variables.

4 Extended Model

In the previous section, we studied the properties of the model that is deliberately simplified to convey key economic intuitions, thus necessarily missing some important facts. First of all, our baseline model does not have enough degrees of freedom to match average levels of all transition rates and labor market stocks. Moreover, the model is not quite up for the task of matching closely the full dynamics of the data. In particular, the model was unable to match the strong procyclicality of the UN rate, and the participation rate in the data increase more gradually after an expansionary shock than implied in the model. In this section, we propose several extensions that allow us to overcome these quantitative problems. To be concise, we relegate all details and derivations of the extended model to Appendix A.3.

4.1 Differences From the Baseline Model

We make the following three main modifications to the baseline model. First, we assume that workers out of the labor force can be "available to work" (N_t) or "permanently out of the labor force" ($\bar{N}$). Hence, overall nonparticipants at time t are equal to $N_t + \bar{N}$. This latter group is introduced to accommodate retirees or anybody who has no intention of participating in the labor market and is exogenous to the model. The introduction of these two groups is needed in order to match the average levels of transition rates between unemployment and nonparticipation in the data. We further assume that inactive nonparticipants' productivity at home is equal to the upper bound of $\Phi(h_i)$, $\bar{h}$. To be precise, the "exogeneity" of this group results from our assumption that their home productivity is *permanently* fixed at $\bar{h}$. In our calibration, there is no incentive for these unavailable nonparticipants to take a job even if we allow for a job offer to arrive to them. On the other hand, we also verify that under our calibration, available nonparticipants whose home productivity turns out to be $\bar{h}$ *temporarily* in the current period always accept a job offer in all possible states of the economy.

Second, to capture the fact that unemployed workers tend to be "attached" to the labor market, we assume that unemployment is a persistent state. In particular, we assume that an unemployed worker draws home productivity only with probability $1-\lambda$. This parameter is useful mainly for matching the level of the UN transition rate. Without this parameter, workers switch between unemployment and nonparticipation too often relative to the observed frequency in the data (as was the case in the baseline model). The underlying reason for this persistence can be that unemployed workers are on average more "attached" and thus tend to remain there, once they enter the pool.[29] There are various ways to endogenize this persistence. Krusell et al. (2017) do so by introducing heterogeneity in wealth; Bils et al. (2012) do so by introducing the "comparative advantage" of workers' productivity in market work and home production. By introducing more persistent (or even permanent) heterogeneity in home productivity into our model, our model will be closer to the one by Bils et al. (2012). However, our objective is to develop a representative household framework that can be easily applied to broader analysis, while keeping tractability and generating business cycle moments consistent with the data. We believe on this ground that our reduced-form specification is justified.

Third, the timing of events is modified such that the participation decision is made at the beginning of the period and that those who find jobs can start working in the same period. Specifically, employed workers separate at a constant rate s at the beginning of the period, then draw h_i from $\Phi(h_i)$, and those with $h_i \leq h_t^*(h_i > h_t^*)$ join the unemployment pool (the nonparticipation pool). Job offers arrive with probabilities f_t and μf_t, respectively, and they start working in the same period if they indeed receive a job offer. Among the unemployed, those who renew home-production productivity decide whether to stay in the unemployment pool or exit the labor force. Each choice gives them the opportunity of receiving job offers at the two different rates, and again they will start working in the same period should offers arrive. The same sequence applies to those that are (available) nonparticipants, except that they draw home productivity every period. Table 4 formalizes these transition probabilities. Note that the last three rows in this table are expressed as transition rates for available nonparticipants. To obtain empirically relevant transition rates for overall nonparticipants, we need to weight them by the ratio $\frac{N_t}{N_t+\bar{N}}$. As discussed below in detail, the changes in the definitions of transition rates that result from the timing change have some important effects on the cyclical features of transition rates in the model, making the model's quantitative properties close to the data, even though economic mechanisms in our extended model remain the same.

We make two more minor modifications. To generate persistence and hump-shaped behavior of vacancies, we introduce a convex hiring cost of the form

$$\kappa \frac{(q_t v_t)^{1+\epsilon_v}}{1+\epsilon_v}, \tag{25}$$

where κ is a constant and ϵ_v governs the convexity of this function. Several papers in the

[29] See, for example, Elsby et al. (2015) and Mueller (2017) for differences in the characteristics of unemployed workers and nonparticipants.

Table 4: Transition Probabilities

Initial state (end of $t-1$)	Probability	Terminal state (end of t)
E	$1-s+s\left[\Phi(h_t^*)+(1-\Phi(h_t^*))\mu\right]f_t$	E
E	$s\Phi(h_t^*)(1-f_t)$	U
E	$s(1-\Phi(h_t^*))(1-\mu f_t)$	N
U	$\left[\lambda+(1-\lambda)(\Phi(h_t^*)+(1-\Phi(h_t^*))\mu)\right]f_t$	E
U	$\left[\lambda+(1-\lambda)\Phi(h_t^*)\right](1-f_t)$	U
U	$(1-\lambda)(1-\Phi(h_t^*))(1-\mu f_t)$	N
N	$\left[\Phi(h_t^*)+(1-\Phi(h_t^*))\mu\right]f_t$	E
N	$\Phi(h_t^*)(1-f_t)$	U
N	$(1-\Phi(h_t^*))(1-\mu f_t)$	N

Notes: Last three rows present transition rates for available nonparticipants. E: Employment. U: Unemployment. N: Available nonparticipant.

literature also assume the same convex hiring cost.[30] Lastly, given the crucial role that wages play in the dynamics of the baseline model, we introduce equilibrium wage rigidity proposed by Hall (2005) and assume the following wage evolution:

$$w_t = (1-\delta_w)w_t^* + \delta_w w_{t-1},$$

where δ_w is the degree of wage rigidity and can be interpreted as the fraction of wages that are not renegotiated each period. We consider the period-by-period Nash-bargained wage as the wage norm w_t^*.[31]

Tables A.3 and A.2 in Appendix A.3.2 present all model equations and the steady-state conditions, respectively.

4.2 Calibration and Estimation

As in the baseline model, some of the parameters are exogenously set. They are listed in Panel A of Table 5 and are set to the same values as in the baseline model. We also impose the same

[30]Note that the cost is convex in the number of hires, not in the number of vacancies posted. This specification is used, for example, by Merz and Yashiv (2007), Gertler et al. (2008), and Moscarini and Postel-Vinay (2016). Pissarides (2009) proposes linear cost in the number of hires. The increasing marginal cost is important for our quantitative result in generating hump-shaped responses in vacancies and thus the related labor market variables.

[31]Papers that make uses of similar wage-norm specification in the literature include Krause and Lubik (2007), Blanchard and Galí (2007, 2010), Thomas (2008), Sveen and Weinke (2008), and Shimer (2012).

steady-state restrictions on g, z_h, $\bar{y}$, and q as in the calibration of the baseline model (Panel B). There are 13 more parameters in the extended model. Among these parameters, several parameters are new or carry different interpretations. The parameter κ, previously defined as a cost per vacancy, now represents a scale parameter of the convex hiring cost function (25); the curvature of the hiring cost function is denoted by ϵ_v; and the parameters $\bar{N}$, δ_w, and λ are new. We estimate these 13 parameters by solving a constrained minimization problem of the weighted distance between the median impulse response functions from our VAR and the model impulse response functions. This minimization problem is constrained in the sense that we impose that the steady-state levels of transition rates do not deviate from their historical averages by more than 30 percent.[32] We use six transition rates, vacancies, real wage, and the labor-force participation rate as our observables and weight the impulse response functions of these variables by their unconditional variances in evaluating the fit.[33] Note that, although we estimate 13 parameters, the model's steady-state equilibrium conditions put many restrictions between those parameters. In practice, we set up this minimization problem such that the estimation routine searches the best values of s, $\bar{\omega}$, δ_w, and ϵ_v. The wage rigidity parameter δ_w does not appear in the steady state and thus is determined only from the dynamics of the model. Appendix A.3.2 presents the details of this estimation procedure.

Table 5 Panel C presents the estimated parameter values. Table 6 presents the implied steady-state values of transition rates. First, the relative efficiency of job search among (available) nonparticipants μ is estimated to be 0.31, which is somewhat higher than the calibrated value in the baseline model but not far from it. Next, the size of unavailable nonparticipants ($\bar{N}$) is estimated to be 0.30, which, together with the steady-state values of E and U, implies the steady-state value of available nonparticipants at 0.051. One possible empirical measure that roughly corresponds to this model concept is "persons who want a job" reported in the CPS. In the data, this group is classified as nonparticipants because they did not actively look for a job during the reference week, even though they expressed an interest in having a job. The average size of this pool amounts to 0.025 (as a share of 16+ population) over the period between 1994 and 2016. This is smaller than the model's steady-

[32] The historical means are based on our margin-error adjusted series. We allow for some deviations, because different adjustments, proposed in the literature, lead to different average levels of transition rates, sometimes significantly. See Table A.1 Panel A in Appendix A.1.1. Variations in historical means seem to suggest that allowing for deviations of 30 percent is plausible.

[33] We do not include the employment-population ratio and the unemployment rate in the set of observables. These two variables are largely redundant, given that movements in transition rates imply clear cyclical patterns in those two variables. The behavior of the participation rate, on the other hand, is more subtle, thus including it in the set of observables helps identify some of the parameters more tightly. However, adding the two stock variables or dropping the participation rate does not materially change our results.

Table 5: Parameter Values and Implied Steady-State Values

Parameter	Interpretation	Value
	Panel A: Externally calibrated	
β	Discount factor	0.995
α	Elasticity matching function	0.5
η	Worker's bargaining power	0.5
ϵ	Elasticity of substitution between C_m and C_h	2.5
ν	Elasticity of nonparticipation	0.47
ρ	Autoregressive parameter for log aggregate productivity	0.99
σ	Standard deviation for log aggregate productivity	0.0053
	Panel B: External steady-state restrictions	
g	Flow opportunity cost of employment	0.71
q	Job filling probability	0.9
z_h	MRS between C_h and C_m	1
$\bar{y}$	Steady-state aggregate productivity level	1
	Panel C: Estimated	
s	Total separation rate	0.0498
$\bar{m}$	Scale parameter of the matching function	0.6122
κ	Scale parameter of hiring cost function	8.78e+39
$\bar{h}$	Upper bound of $\Phi(\cdot)$ distribution	0.6178
γ	CES weight parameter	0.6084
b	Unemployment benefits	0.4075
μ	Relative job-search efficiency of nonparticipants	0.3090
λ	Fraction of unemployed workers who do not draw home production values	0.4709
$\bar{N}$	Unavailable nonparticipants	0.2966
ω	Scale parameter of leisure in the utility function	0.0615
δ_w	Wage stickiness	0.9823
ϵ_v	Curvature of hiring cost function	27.5084
$\bar{\omega}$	Ratio between z_l and $\hat{h}z_h$ in the steady state	1.1112

state value of N. However, the empirical measure should be considered as the lower bound, because even outside this group, there are entrants such as new graduates from school that make direct transitions into employment. We view our estimate of available nonparticipants as plausible.

Next, the relative importance of z_l and $\hat{h}z_h$ in the steady state $\bar{\omega}$ is estimated at 1.11. Remember that we set $\bar{\omega} = 1$ in the previous model, and the estimated value in the current quantitative model is close to that value. Similarly, the level of the unemployment insurance benefits is estimated to be 0.41, which is again close to the calibrated value in the baseline model. The upper bound of the $\Phi(h_i)$ distribution is estimated to be 0.62. The implied value of the scale parameter of the leisure function ω is 0.0615.

Note that the steady-state value of the flow opportunity cost of employment in the model g can be written as:

$$\begin{aligned} g &= \Phi(h^*)b + (1-\Phi(h^*))(\hat{h}z_h + z_l) = \Phi(h^*)b + (1-\Phi(h^*))\hat{h}(1+\bar{\omega}) \\ &= 0.407 \times 0.408 + (1-0.407) \times 0.43 \times (1+1.11) = 0.71. \end{aligned} \tag{26}$$

In the second equality, we used $\hat{\omega} = \frac{z_l}{\hat{h}z_h} = 1$ and $z_h = 1$. This calculation implies that about one-quarter of the overall opportunity cost is due to unemployment insurance benefits $(0.407 \times 0.408/0.71)$ and the rest to home production and leisure. Note that, as we discussed with respect to the baseline model, the cyclicality of transition rates suggests h_t^* to be strongly countercyclical. The countercyclicality of h_t^* implies that $\Phi(h_t^*)$ and its conditional mean $\hat{h}_t$ are also countercyclical. The flow opportunity cost g_t in our estimated model is strongly procyclical, as is consistent with the empirical evidence Chodorow-Reich and Karabarbounis (2016), for two reasons. First, as indicated by Equation (26), $b = 0.408 < \hat{h}z_h + z_l = 1.54$ in the steady state, and therefore the procyclicality of the term involving $(1-\Phi(h_t^*))$ is quantitatively more important than the countercyclicality of the term involving $\Phi(h_t^*)$. Second, both the value of home production $(\hat{h}z_h)$ and the value of leisure (z_l) are procyclical.[34]

The total separation rate s is estimated to be around 5 percent per month.[35] The persistence (or "attachment") parameter of the unemployment state is estimated to be 0.47. In the baseline model without this parameter, we were not able to match the steady-state level of the UN rate. However, the introduction of this parameter brings the average level much closer to its empirical counterpart. The scale parameter of the matching function $\bar{m}$ is estimated to be 0.61, and together with the elasticity parameter at 0.5, the implied steady-state value of the contact rate f_t is 0.416.

The wage rigidity parameter δ_w and the curvature parameter of the hiring cost function ϵ_v play important roles for the model dynamics. The estimated value of δ_w (0.98) in the current model implies that the data favor a high degree of wage rigidity. The curvature of the hiring cost is estimated to be very large (27.5), given that job vacancies move gradually and persistently in response to the shock. The large curvature value has also been previously used in the literature and, at the same time, implies a large value of its scale parameter κ.[36]

[34] Note that as mentioned above, $\hat{h}$ is countercyclical. However, $\hat{h}z_h$ remains procyclical.

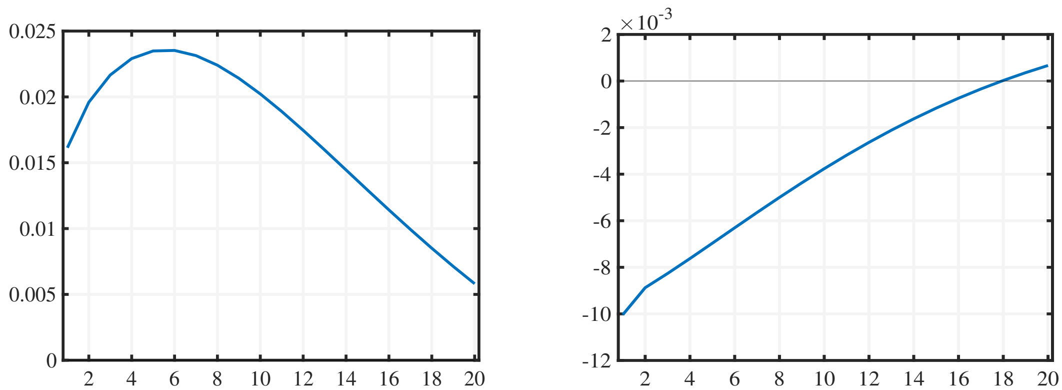
[35] Remember that in our baseline model, this parameter corresponds to the sum of EU and EN rates. But because of the different timing assumptions, s does not equal the sum of the two transition rates in the current model.

[36] For example, Moscarini and Postel-Vinay (2016) use the same hiring cost function and set the curvature

Table 6: Steady-State Performance

Empirical Concept	Model Concept	Target Values	Steady-State Values
E-pop ratio	E	0.62	0.619
Unemployment rate	$\frac{U}{E+U}$	0.064	0.050
Participation rate	$E+U$	0.63	0.652
EU transition rate	$s\Phi(h^*)(1-f)$	0.015	0.012
EN transition rate	$s(1-\Phi(h^*))(1-\mu f)$	0.028	0.026
UE transition rate	$[\lambda+(1-\lambda)(\Phi(h^*)+(1-\Phi(h^*))\mu)]\, f$	0.251	0.326
UN transition rate	$(1-\lambda)(1-\Phi(h^*))(1-\mu f)$	0.214	0.273
NE transition rate	$[\Phi(h^*)+(1-\Phi(h^*))\mu]\, f\frac{N}{N+\bar{N}}$	0.047	0.036
NU transition rate	$\Phi(h^*)(1-f)\frac{N}{N+\bar{N}}$	0.027	0.035

Note: Steady-state values for $\Phi(h^*) = 0.407$, $f = 0.416$, $h^* = 0.251$, and $N = 0.051$.



(a) Labor Market Tightness *(b)* Participation Margin

Figure 8: Impulse Response Functions (Extended Model), Labor Market Tightness and Participation Margin. *Notes*: Responses to a one standard deviation positive productivity shock. Quarterly averages of monthly responses, expressed as log deviations from steady-state levels.

One can see in Table 6 that, although the model is unable to match perfectly the steady-state values of these labor market stocks and flows, all values are near the empirical counterparts. The model performs much better in the steady state than our previous model thanks to the extensions we introduced.

value at 50. Note that the value κ has no impact on our model dynamics, since it only shows up as a constant term in the log-linearized model. The same is true for the values of the scale parameters of the matching function $\bar{m}$ and the leisure utility function ω.

4.3 Model Dynamics

We now show that this extended model is capable of replicating the cyclical features of all labor market variables including both stocks and transition rates. As emphasized above, we use values of the extensive-margin elasticity of labor supply and the elasticity of substitution between home and market goods that are consistent with the micro evidence.

First, we show in Figure 8 the responses of the two key endogenous variables in the model: labor market tightness and the participation margin. As emphasized earlier, in order for the model to be consistent with the cyclicality of labor market variables, the participation margin must be countercyclical and tightness must be procyclical. Our baseline model was able to achieve this property, which is maintained in our extended model. Note, however, that having these two features is not sufficient to match the behavior of all transition rates. In particular, the relationships between these two variables and transition rates are more complex in the current model.

In Figures 9 and 10, we compare the model impulse response functions to the empirical counterparts. These figures demonstrate that our model performs excellently. In particular, the model successfully replicates the two key cyclical features of the LFPR. First, it tends to fall initially before it increases above its steady-state level. Second, its variations over the business cycle are small even though the underlying transition rates are much more volatile. Let us delve deeper into the quantitative results of our model.

First, consider separation rates into unemployment and nonparticipation (Figure 9 (a) and (d)). As emphasized before, the data show the former being countercyclical and the latter being procyclical. In the baseline model, these two rates are defined as $s\Phi(h*_t)$ and $s(1-\Phi(h_t^*))$, and the countercyclicality of h_t^* makes it possible to match the opposite movements in these two transition rates. In that simpler model, however, the net effect is zero by construction. In the current model, incorporating the richer timing assumptions allows us to match the fact that the overall separation rate (sum of the two rates) is countercyclical, even though the underlying separation probability s remains constant. For the EU rate defined by $s\Phi(h_t^*)(1-f_t)$, the last term appears in the current model, since we allow for finding a job within the same period after entering the unemployment pool at the beginning of the period: Those who enter the unemployment pool (with probability $s\Phi(h_t^*)$) remain in the pool only if they fail to find a job in that period (with probability $1-f_t$). In expansions, not only does the probability of entering into the unemployment pool ($s\Phi(h_t^*)$) fall, but also the probability of remaining there $1-f_t$ falls.[37] Regarding the EN rate, the time aggregation

[37]Shimer (2012) emphasizes this time aggregation effect in the countercyclicality of the observed EU transition rate. Although this alone does not explain the countercyclicality (see Fujita and Ramey (2009)),

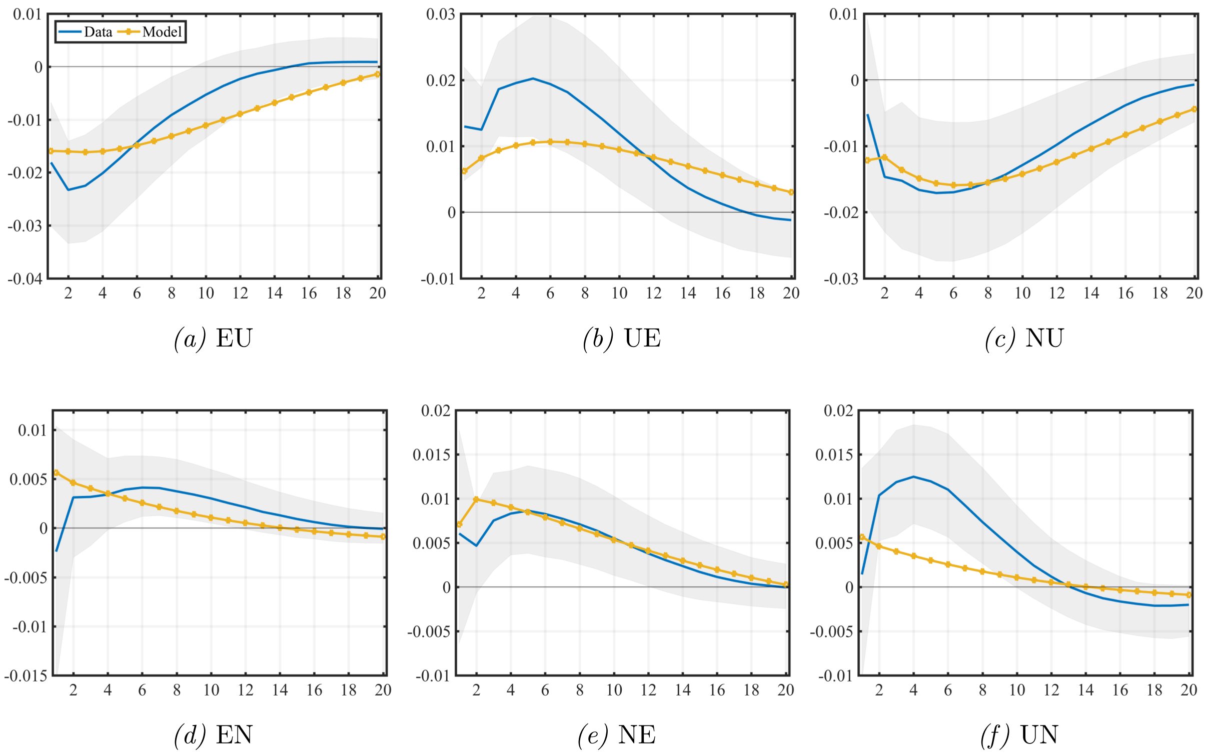


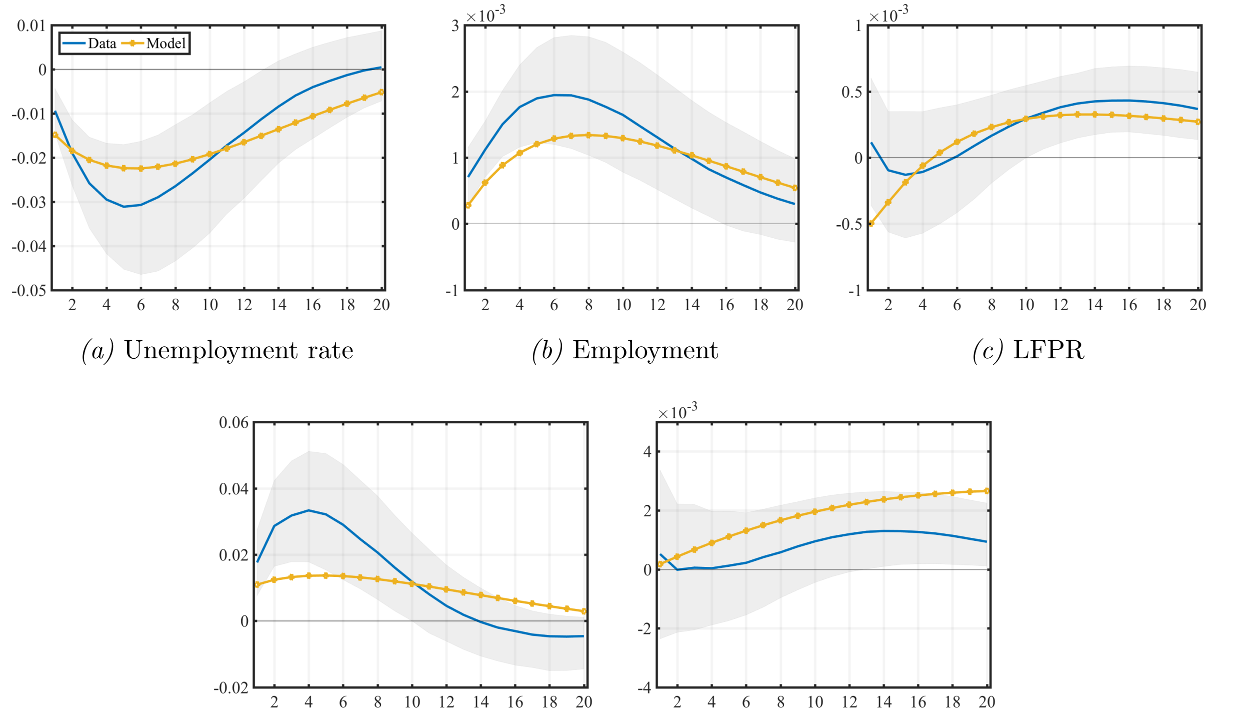
Figure 9: Impulse Response Functions (Extended Model), Transition Rates. *Notes*: See notes to Figure 8. Yellow solid dotted lines represent responses in the model. Blue solid lines are the median empirical responses, and shaded areas are error bands that represent the 16th and 84th percentiles of the posterior distribution.

effect weakens the procyclicality due to the movements in $1 - \Phi(h_t^*)$. Thus, the overall separation rate remains countercyclical (i.e., the countercyclicality of the EU rate dominates the procyclicality of the EN rate).

Next, consider job-finding rates from unemployment and nonparticipation (Figure 9 (b) and (e)). Again, the definitions of these two variables are simple in the baseline model, namely, f and μf, respectively, and thus their log-deviations from their respective steady state levels are identical. But that is no longer the case in the current model, due to the extra terms. Nevertheless, the cyclical movements in these two variables remain dominated by the changes in f_t, although the procyclicality of these transition rates are somewhat mitigated by the presence of $\Phi(h_t^*) + (1 - \Phi(h_t^*)\mu$ because this term is countercyclical given the countercyclicality of $\Phi(h_t^*)$ together with $\mu < 1$.

Let us now discuss transition rates between unemployment and nonparticipation (Fig-

it plays some role, and our model does incorporate the effect.



(a) Unemployment rate *(b)* Employment *(c)* LFPR

(d) Vacancies *(e)* Real wage

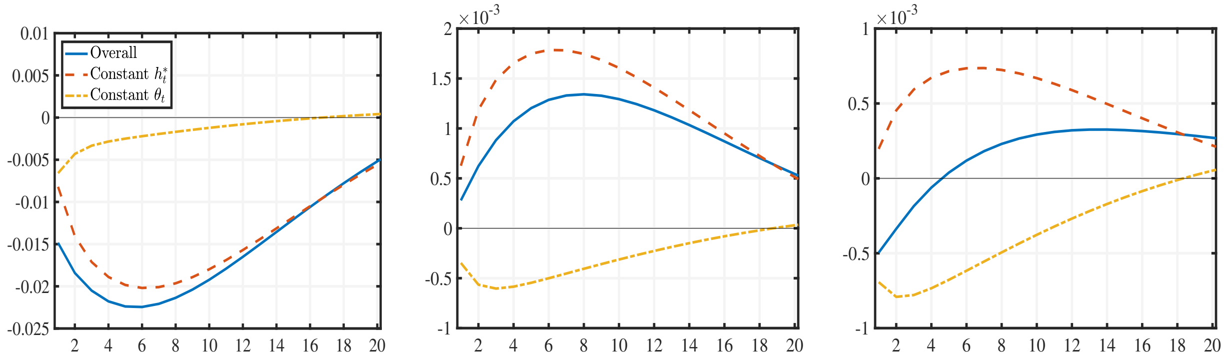
Figure 10: Impulse Response Functions (Extended Model), Stocks and Real Wages. *Note*: See notes to Figure 9.

ure 9 (c) and (f)). Regarding the NU rate, it is straightforward to replicate the observed countercyclicality as far as f_t is procyclical and thus $(1 - f_t)$ is countercyclical and h_t^* is countercyclical (see Table 6 for its definition). The same mechanism was discussed for the baseline model. The definition in the current model is different from the one in the baseline model only due to the presence of the term $\frac{N_t}{N_t+\bar{N}}$. The fluctuation of this term is relatively small and thus is not quantitatively important for the cyclical movements in this rate. On the other hand, replicating the behavior of the UN rate is not as straightforward. In the baseline model where it is defined as a product of $1-\Phi(h_t*)$ and $(1-f_t)$, we emphasized that in order for the model to replicate the observed procyclicality, the volatility of $1 - \Phi(h_t*)$ has to be much larger than that of $(1 - f_t)$. We found that it was difficult to achieve this property (Figure 5 (f)). In the extended model, the UN rate is clearly procyclical, although the model misses the hump-shaped pattern in the data. In our baseline model, our timing assumption is that only after an unemployed worker fails to find a job (which occurs with probability $1 - f_t$), a transition to nonparticipation occurs with probability $1 - \Phi(h_t^*)$. That timing assumption implies that an unemployed worker is more likely to make a UE transition in expansions and that the lower $(1-f_t)$ is, the lower the chance of making a transition to nonparticipation. Both implications made it hard to generate the procyclicality of the exit rate. Conditional on not receiving a job offer, the probability of the worker dropping out of the labor force, $1-\Phi(h_t^*)$ is procyclical, but this effect is strongly mitigated by the countercyclicality of $(1-f_t)$. In our extended model, however, the worker draws the home production value first and drops out of the labor force if it is below h_t^*. It is possible that those who drop out of the labor force then can find a job within the period with probability μf_t, in which case the worker is observed to make a UE transition. The UN rate is proportional to the product of $1-\Phi(h_t^*)$ and $(1-\mu f_t)$ (instead of $(1-f_t)$ in the baseline model) because of the timing assumption. Thus, although a qualitatively similar offsetting effect through $(1-\mu f_t)$ exists in the extended model, this effect is significantly weakened, thanks to $\mu < 1$. Moreover, the smaller the value of μ is, the more dominant the effect of $1-\Phi(h_t^*)$ is. The estimation picks the value of μ at 0.31, which generates the strong procyclicality of the UN rate.

Figure 10 presents the responses of the stock variables. A high degree of wage rigidity is manifested in the impulse response in Panel (e). This feature is of first-order importance for results on both volatilities of labor demand (i.e., f_t) and thus the unemployment rate, but also the cyclical patterns in transition rates into and out of nonparticipation. The model matches the countercyclical unemployment rate and the procyclical employment-to-population ratio, and their volatilities are roughly of the same magnitude as in the data.[38] Vacancies increase in a hump-shaped manner: The sluggishness of the vacancy response is a direct result of our convex hiring cost and its large curvature.[39] The responses of the unemployment rate and vacancies in opposite directions form the Beveridge curve. As we discussed earlier with respect to our simpler model, in the two-state labor matching model, there is a tension between the model's capability of replicating the negative correlation between unemployment and vacancies and how the separation rate is modeled (i.e., whether separation is assumed to be exogenous or endogenous). In our setup, the underlying separation rate s is an exogenous parameter, and thus it is perhaps not surprising that our model is able to replicate the strong negative correlation between the two variables. It is nevertheless remarkable that our model replicates both the strongly countercyclical EU rate and the Beveridge curve at the same time. In addition, our model matches the fact that the overall separation rate (sum of EU

[38]Matching the cyclical patterns of the unemployment rate and the employment-to-population ratio is perhaps not surprising. However, Veracierto (2008) finds that adding the participation margin to his search model makes the unemployment rate procyclical. Our model does not suffer this problem.

[39]We could adopt a more structural modeling of this margin as in Fujita and Ramey (2007), but the focus of this paper is different, and we thus make use of a reduced-form specification with a steeply-increasing marginal hiring cost.



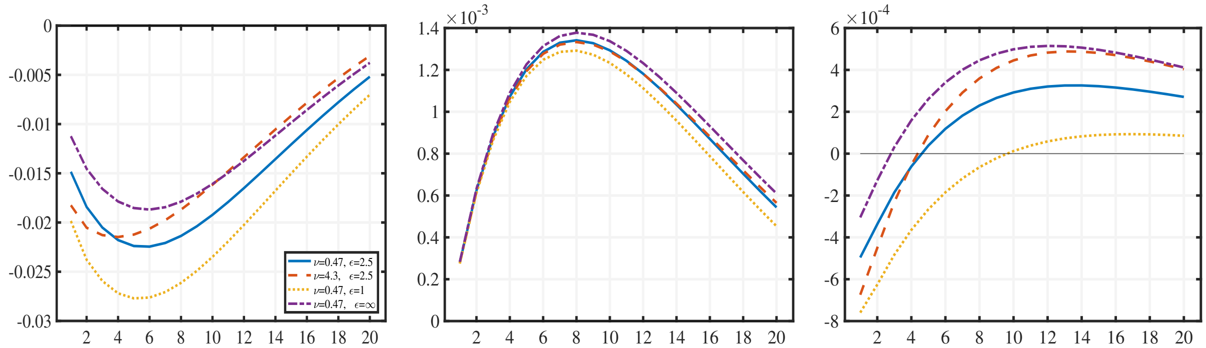
(a) Unemployment Rate *(b)* E-Pop Ratio *(c)* Participation Rate

Figure 11: Contributions of Job Creation and Participation Margins. *Notes*: Responses to a one standard deviation positive productivity shock. All responses are expressed as log deviations from steady-state levels.

and EN rates) is much less cyclical than either of the two but remains countercyclical. We believe that our model's quantitative performance shows significant improvements over the existing literature.

Contributions of the two margins. As we did for the baseline model, Figure 11 simulates counterfactual paths of the three labor market stock variables, while fixing either θ_t or h_t^*. Through this exercise, we gauge the contribution of each variable to the fluctuations of the three labor market stock variables. The same intuitions we discussed for our earlier experiments apply here. For the unemployment rate, both θ_t and h_t^* contribute to lowering it in response to the positive shock, while for the other two variables, the two margins move them in opposite directions. Especially for the participation rate, contributions of one variable are largely offset by those of the other, and thus the net movements are small. With respect to timing, the initial declines are due to the fact that the fall in the participation margin kicks in immediately, before increases in labor market tightness become quantitatively more important. Once the latter becomes more dominant, the participation rate tends to rise above its steady-state level.

The role of elasticity parameters. In Figure 12, we study the effects of changing the values of the two elasticity parameters, while holding the remaining parameters at the same values as before. We can see that these parameters do have some impacts on model dynamics (and the directions of the changes are intuitive) but do not change the overall qualitative patterns. This is expected from our earlier exercises in the baseline model, given that wages are highly rigid in the current model as well. In Figure 13, we consider the same



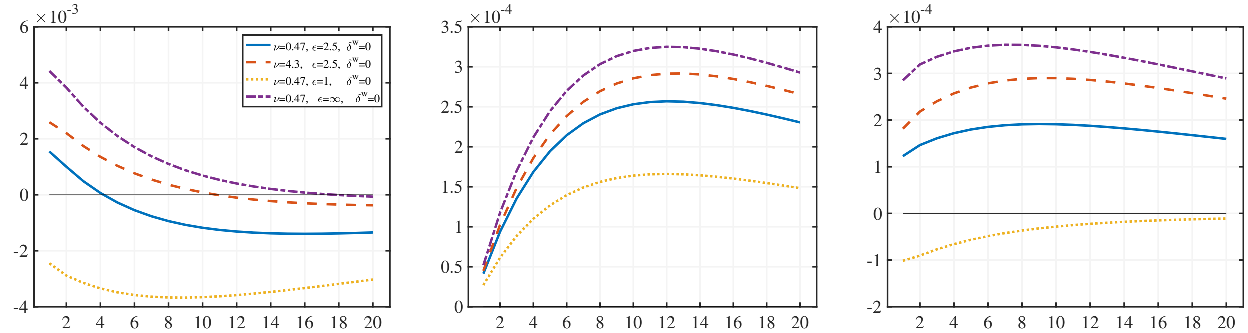
(a) Unemployment Rate *(b)* E-Pop Ratio *(c)* Participation Rate

Figure 12: Effects of Changing Elasticity Parameters (Estimated Extended Model).

experiments but in the model without wage rigidity ($\delta_w = 0$). In that environment, these parameters have much more pronounced impacts on the model dynamics. First of all, setting $\delta_w = 0$ obviously changes the model dynamics significantly even without changing elasticity values (see blue solid lines): The volatility of the economy is considerably reduced; the unemployment rate increases initially before it falls slightly below the steady state level, due to the strong substitution effect that brings more workers into the unemployment pool; consistent with this, the participation rate increases immediately and stays above the steady-state level throughout the horizon. These results are amplified with higher labor supply elasticity $\nu = 4.3$ and the higher substitutability between home and market consumption goods ($\epsilon = \infty$). In the case with $\nu = 4.3$ and $\epsilon = 2.5$ or similarly in the case with $\nu = 0.47$ and $\epsilon = \infty$, the unemployment rate becomes largely procyclical and the participation rate increases further. Observe also that when we make the two types of consumption goods more complimentary ($\epsilon = 1$), while keeping ν at 0.43, the model can match the cyclical patterns of these three stocks. All underlying transition rates also behave in a consistent manner in that case. However, the remaining issue in this case is the lack of overall volatility.

5 Conclusion

This paper studies qualitative and quantitative properties of a labor search and matching model with endogenous labor force participation. The model is capable of generating realistic cyclical movements in *all* labor market transition rates and labor market stocks. In particular, our model generates substantial cyclical variations in transitions into and out of the labor force, along with small and relatively weak procyclical variations in the LFPR, consistent with the data. We achieve the successful quantitative performance, while using



(a) Unemployment Rate *(b)* E-Pop Ratio *(c)* Participation Rate

Figure 13: Effects of Changing Elasticity Parameters (Extended Model with Flexible Wages).

extensive-margin labor supply elasticities consistent with micro-level evidence. Our results underscore the importance of the procyclical opportunity cost of employment, together with wage rigidity, in understanding the cyclicality of labor market flows and stocks. In the paper, we spell out these economic mechanisms at work through various exercises.

Search frictions open the door to decouple shifts in labor demand from shifts in labor force participation: Changes in the participation margin influence the composition of nonemployed individuals (between nonparticipants and the unemployed) and firms pull workers from the pool of individuals available (or waiting) to work. In such an environment, small values of labor supply elasticities are consistent with the observed cyclical behavior of transition rates between unemployment and nonparticipation. The unemployment pool expands in downturns not only because the pace of job loss increases and the pace of hiring slows down, but also because the entry rate into the pool from nonparticipation increases and the exit rate to nonparticipation slows down. Therefore, transitions from and into nonparticipation make important contributions to the countercyclicality of the unemployment pool. These movements appear to lend more important roles to income effects than previously perceived in the literature.

This paper considers the simplest possible models in which only a neutral technology shock hits labor-only linear technology. Thus, our model lacks features necessary to address policy relevant questions discussed, for example, in Bernanke (2012) and Yellen (2014). An advantage of our representative-household framework is that it can easily be extended to a full-fledged DSGE model that enables us to address those questions. This paper lays out an essential foundation for this next step, which we leave for future research.

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Appendices

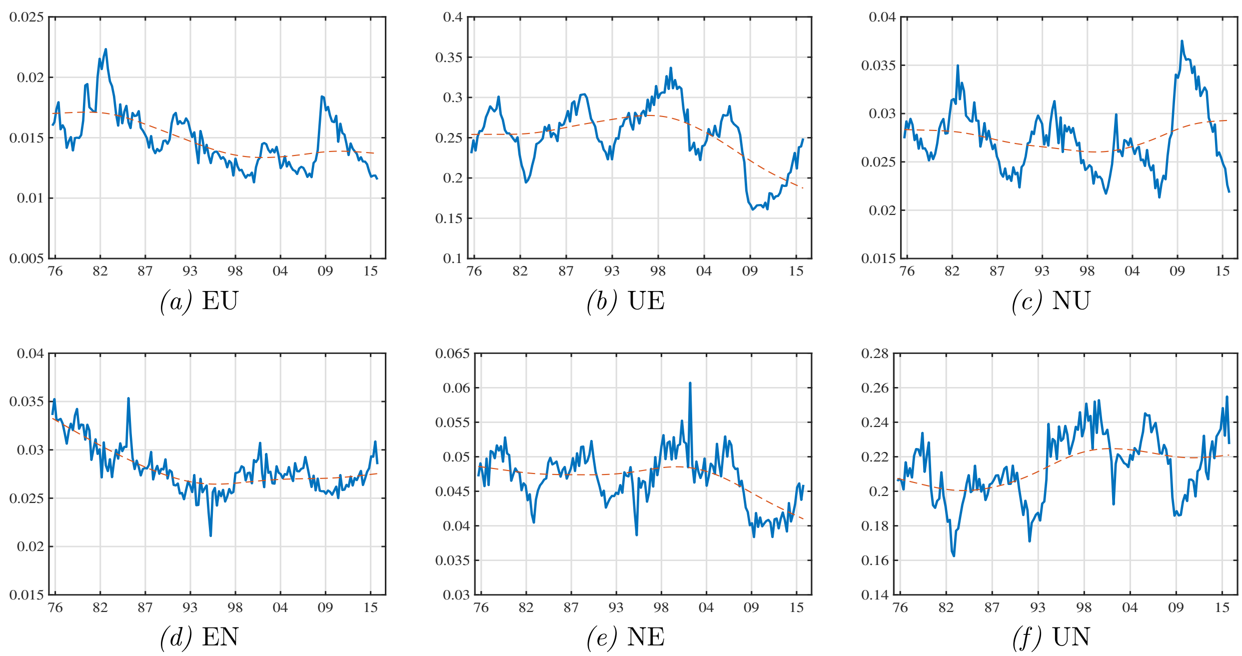
A.1 Additional Empirical Evidence

We show that our empirical results are robust with respect to various alternative data series and specifications of the VAR.

A.1.1 Data

Figure A.1 presents transition rate series used in our analysis, adjusted by margin error, between 1976 and 2016.

Corrections to CPS transition rates. The literature has proposed various adjustments to the CPS gross flow data. We show here that our main results are robust with respect to these alternative datasets. We consider a total of nine different datasets following Krusell et al. (2017), and Table A.1 presents unconditional first and second moment statistics of these datasets. Although Krusell et al. (2017) look at similar statistics, our filtering method and the sample period are different from theirs, and thus we present the results for completeness. As pointed out by Krusell et al. (2017), the adjusted flows using the Abowd and Zellner (1985) misclassification correction are systematically below their unadjusted counterparts.



(a) EU *(b)* UE *(c)* NU

(d) EN *(e)* NE *(f)* UN

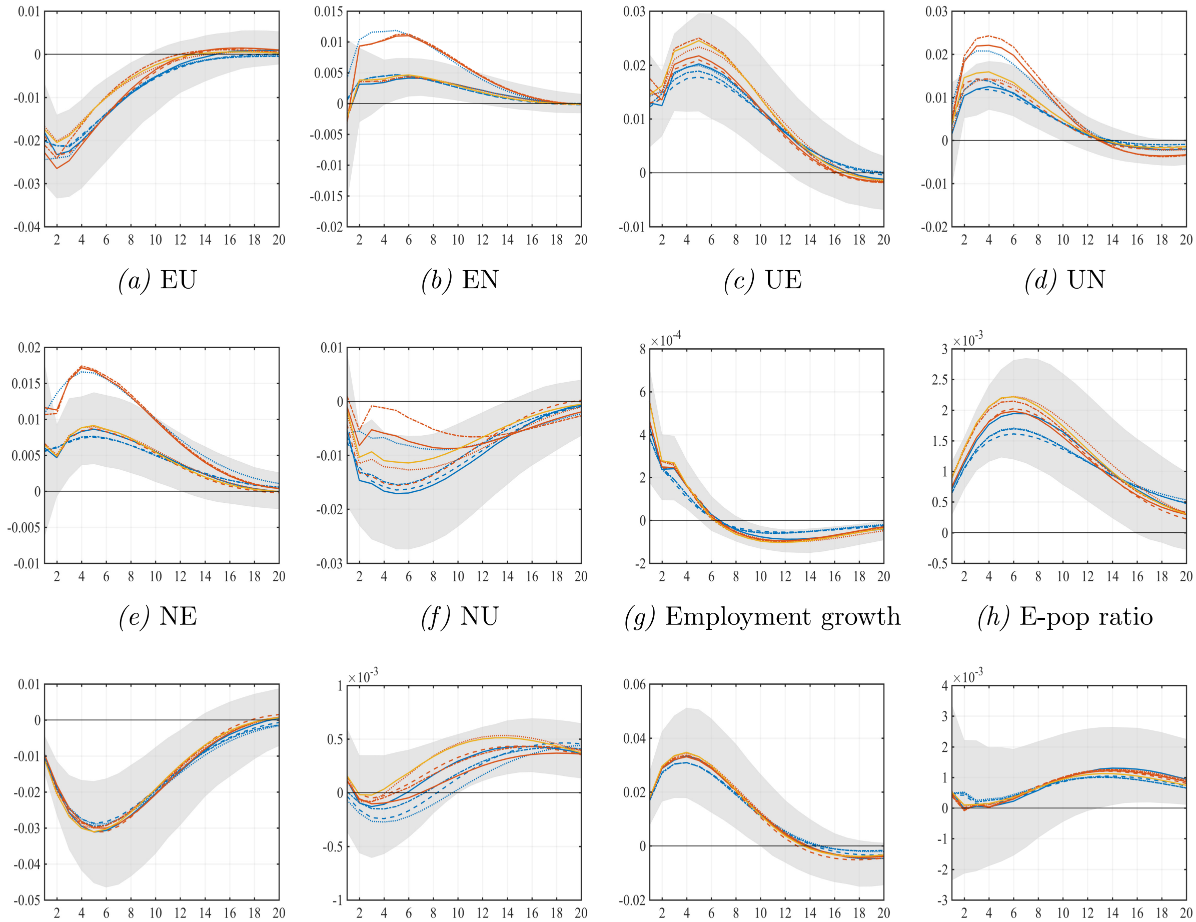
Figure A.1: Transition Rates. *Notes*: Solid blue lines are quarterly averages of monthly data. Red dashed lines are the HP filtered trend.

Table A.1: U.S. Data, 1976-2016

Class. error adj.	Other adj.	EU	EN	UE	UN	NE	NU
				Panel A: Mean			
Unadjusted		0.015	0.029	0.253	0.224	0.045	0.026
Abowd-Zellner		0.014	0.015	0.228	0.150	0.021	0.019
DeNUNified		0.015	0.029	0.258	0.158	0.045	0.018
Unadjusted	ME	0.015	0.028	0.251	0.214	0.047	0.027
Abowd-Zellner	ME	0.014	0.014	0.226	0.138	0.022	0.020
DeNUNified	ME	0.015	0.028	0.254	0.149	0.047	0.019
Unadjusted	ME and TA	0.019	0.030	0.300	0.253	0.049	0.036
Abowd-Zellner	ME and TA	0.017	0.014	0.263	0.153	0.023	0.025
DeNUNified	ME and TA	0.019	0.029	0.304	0.168	0.049	0.024
				Panel B: Standard deviation			
Unadjusted		0.112	0.047	0.127	0.078	0.061	0.120
Abowd-Zellner		0.121	0.106	0.140	0.141	0.138	0.102
DeNUNified		0.112	0.047	0.131	0.096	0.061	0.118
Unadjusted	ME	0.107	0.051	0.127	0.074	0.066	0.117
Abowd-Zellner	ME	0.116	0.114	0.141	0.139	0.142	0.103
DeNUNified	ME	0.107	0.051	0.131	0.088	0.066	0.111
Unadjusted	ME and TA	0.087	0.052	0.146	0.084	0.068	0.098
Abowd-Zellner	ME and TA	0.098	0.116	0.159	0.150	0.144	0.096
DeNUNified	ME and TA	0.088	0.052	0.151	0.096	0.068	0.093
				Panel C: Correlation with GDP			
Unadjusted		-0.760	0.596	0.774	0.653	0.792	-0.755
Abowd-Zellner		-0.734	0.675	0.766	0.664	0.764	-0.537
DeNUNified		-0.760	0.596	0.772	0.595	0.791	-0.728
Unadjusted	ME	-0.758	0.545	0.769	0.650	0.762	-0.732
Abowd-Zellner	ME	-0.717	0.626	0.755	0.661	0.738	-0.487
DeNUNified	ME	-0.749	0.544	0.765	0.633	0.755	-0.718
Unadjusted	ME and TA	-0.706	0.542	0.768	0.650	0.758	-0.676
Abowd-Zellner	ME and TA	-0.655	0.628	0.756	0.662	0.735	-0.317
DeNUNified	ME and TA	-0.701	0.548	0.764	0.635	0.752	-0.651

Notes: The volatility and correlation is computed using all series in logs and HP filtered with a smoothing parameter equal to 10^5. ME stands for margin error correction and TA for time-aggregation correction.

Also, the deNUNified correction proposed by Elsby et al. (2015) has a similar effect for the average transition rates between unemployment and nonparticipation. Importantly, the cyclicality of flows are similar for the unadjusted and adjusted flows, except perhaps for the NU rate in that its cyclicality falls in the adjusted data.



(a) EU *(b)* EN *(c)* UE *(d)* UN

(e) NE *(f)* NU *(g)* Employment growth *(h)* E-pop ratio

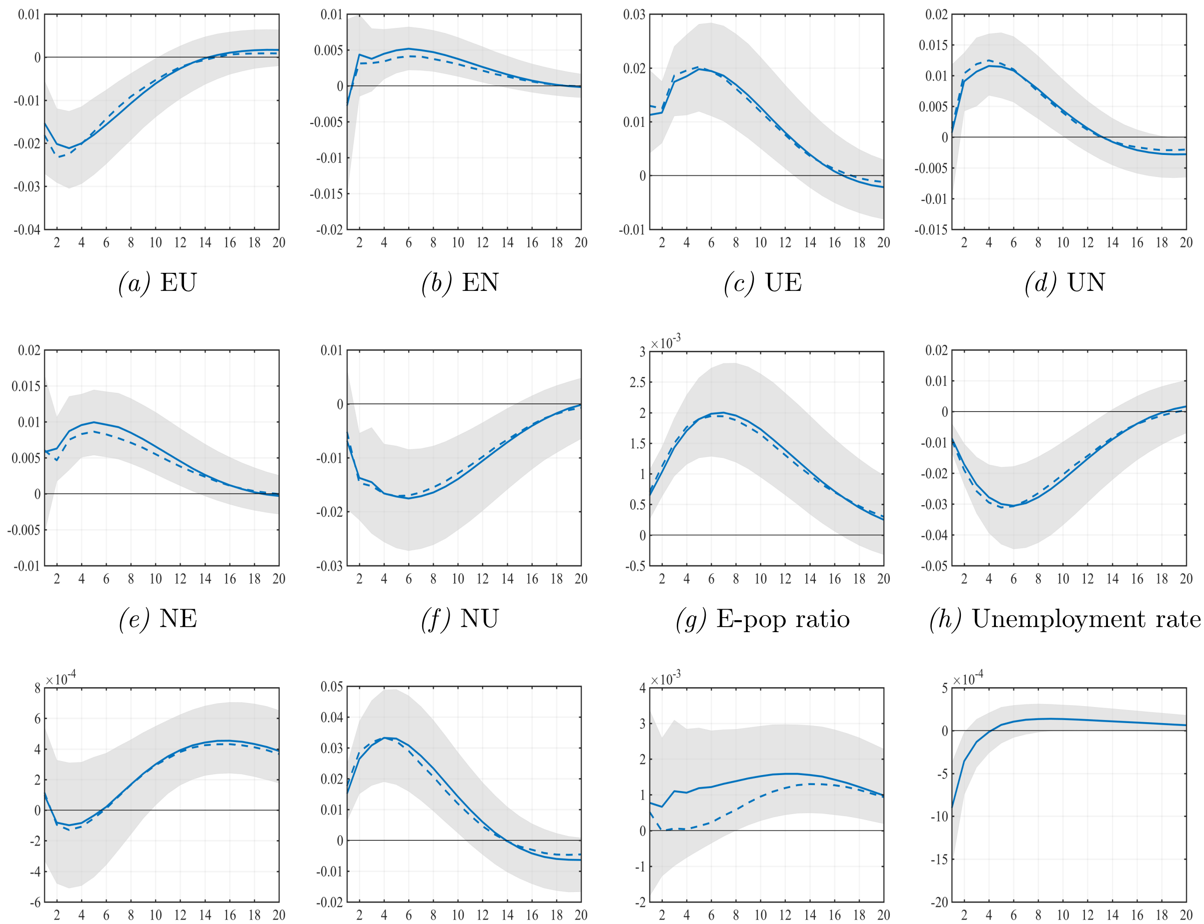
(i) Unemployment rate *(j)* Participation rate *(k)* Vacancies *(l)* Real wage

Figure A.2: Results with Alternative Datasets. *Notes*: Shaded areas are error bands for our baseline results (16th and 84th percentiles of the posterior distribution). Solid blue: baseline results; Blue dashed: unadjusted; Blue dotted: adjusted for MC (AZ flows); Blue dash-dotted: adjusted for MC (DeNUNified flows); Red solid: adjusted for MC+ME (AZ flows); Red dashed: adjusted for MC+ME (DeNUNified flows); Red dotted: adjusted for ME+TA; Red dash-dotted: adjusted for MC+ME+TA (AZ flows); Yellow solid: adjusted for MC+ME+TA (DeNUNified flows). MC stands for misclassification error correction, ME for margin error correction and TA for time-aggregation correction.

A.1.2 Robustness of the VAR

We show in this section that our results are robust with respect to (i) alternative datasets, (ii) structural shocks, and (iii) alternative detrending methods.

VARs with alternative datasets. Figure A.2 presents the results of re-estimating the VAR using the various datasets. Overall, our results are fairly robust with respect to the



(a) EU *(b)* EN *(c)* UE *(d)* UN

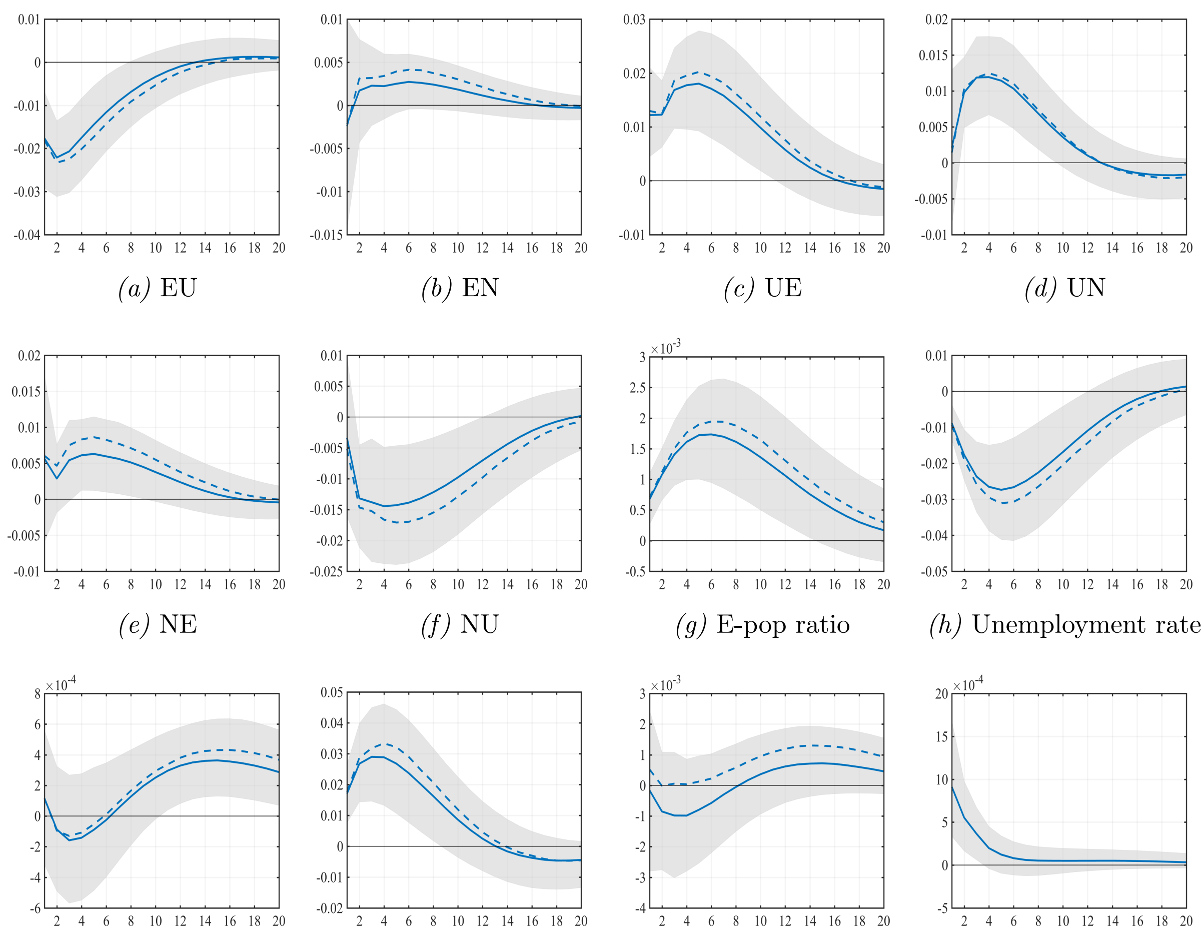
(e) NE *(f)* NU *(g)* E-pop ratio *(h)* Unemployment rate

(i) Participation rate *(j)* Vacancies *(k)* Real wage *(l)* Inflation

Figure A.3: Responses to a Technology Shock. *Notes*: Shaded areas are error bands for the baseline model. Except for inflation, impulse responses are expressed as log deviations. Inflation responses are expressed as level differences in the quarterly rates.

different adjustment procedures.

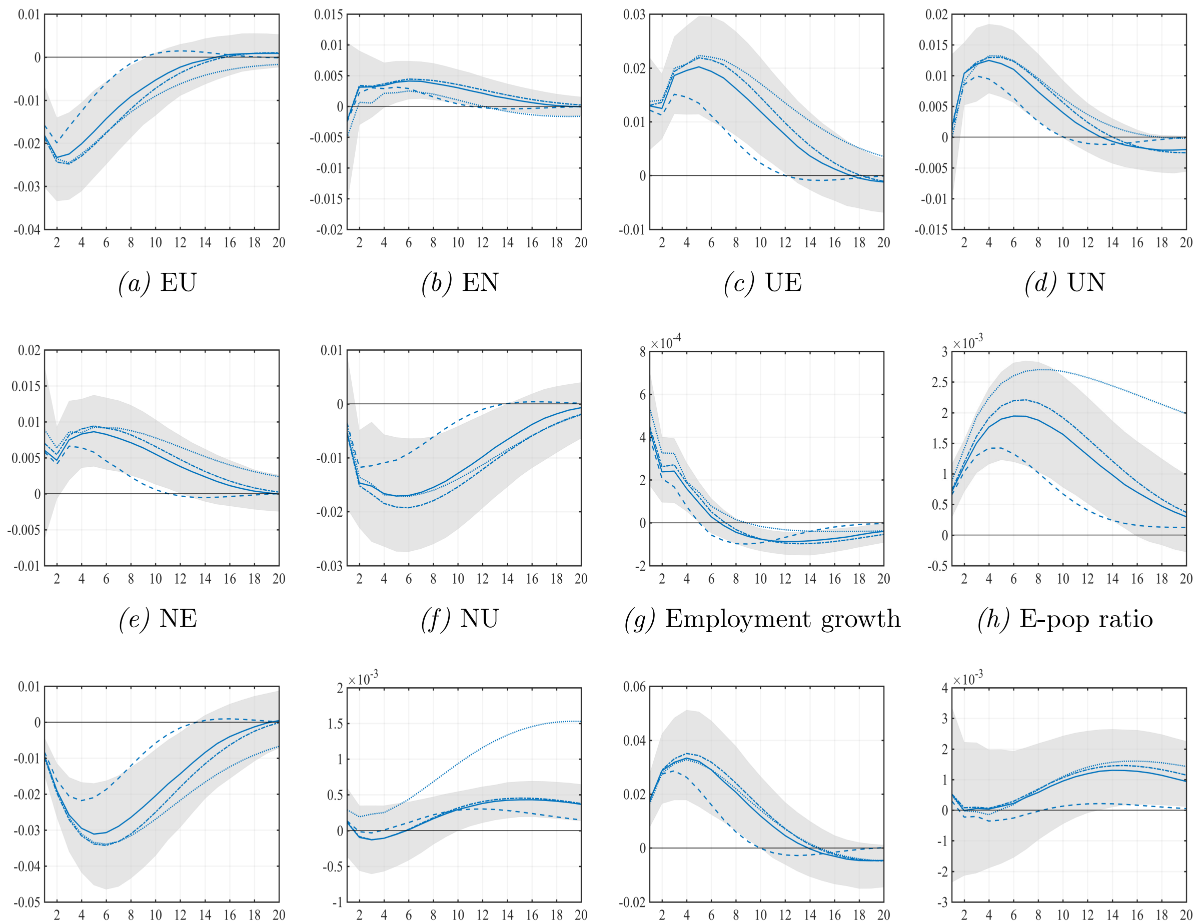
Labor market responses to different structural shocks. In the main body of the paper, our VAR considers a generic shock, which we call "aggregate shock", by claiming that labor market responses with respect to more fundamental shocks are similar. We demonstrate that similarity in this section. Our VAR includes the same eight variables as before (six transition rates, vacancies and wages) plus the headline PCE inflation rate. The PCE inflation rate is computed by taking log differences of price levels between the two adjacent quarters and thus should be understood as a quarterly rate. All series are seasonally adjusted, logged, and HP-filtered with a smoothing parameter of 10^5, and we set the lag length of the VAR at two quarters.



(a) EU, *(b)* EN, *(c)* UE, *(d)* UN, *(e)* NE, *(f)* NU, *(g)* E-pop ratio, *(h)* Unemployment rate, *(i)* Participation rate, *(j)* Vacancies, *(k)* Real wage, *(l)* Inflation

Figure A.4: Responses to a Demand Shock. *Notes*: Shaded areas are error bands for the baseline model. Except for inflation, impulse responses are expressed as log deviations. Inflation responses are expressed as level differences in the quarterly rates.

We identify two shocks that one can call "demand shock"' and "technology or supply shock." We impose exactly the same sign restrictions on the responses of EU and UE transition rates, vacancies, and employment growth for both the demand- and supply-side shocks. We distinguish between demand and supply shocks based on the pattern in the inflation response. In response to a positive demand shock, the price level increases for the first two quarters. Note that confining the responses of the price level rather than the inflation rate is less restrictive in the sense that the price level restriction allows for the inflation rate to fall at any point after the first period, as long as the price level restriction is satisfied in the following periods. This restriction together with the previous restrictions on labor market variables imply a short-run "Phillips-curve-like" relationship, i.e., negative relationship between inflation and the unemployment rate. The unemployment rate tends



(a) EU *(b)* EN *(c)* UE *(d)* UN

(e) NE *(f)* NU *(g)* Employment growth *(h)* E-pop ratio

(i) Unemployment rate *(j)* Participation rate *(k)* Vacancies *(l)* Real wage

Figure A.5: Results with Alternative Detrending Methods. *Notes*: Solid lines and shaded areas represent the median responses and error bands of our baseline VAR. Dashed lines: HP filtered with smoothing parameter of 1,600; Dotted lines: linear trend; Dash-dotted lines: cubic trend

to fall in response to a positive demand shock in our identification because of the higher UE transition rate and lower EU transition rate. Next, we assume that a positive supply shock leads to a fall in the price level for the two quarters after the shock. Note that this supply innovation can include both technology and cost-push shocks. We also studied the properties of the VAR that further adds labor productivity growth to the system and obtained very similar results. Results are available upon request. The identification in this case follows the same procedure as in Fujita (2011).

Figures A.3 and A.4 present the responses to positive supply and demand shocks, respectively. Solid lines represent the median responses and shaded areas represent 16 and 84 percentiles of the posterior distributions. Dashed lines are the median responses in our baseline specification presented in Figure 2. The overall pattern of responses of labor market variables is remarkably similar to our main results except for the fact that the demand shock implies the Phillips-curve relationship.

Alternative detrending methods. In our baseline specification, all variables were detrended using the HP filter with a smoothing parameter equal to 10^5. We show here the results based on different detrending methods. Figure A.5 plots the median responses that would result after detrending the data using (i) the HP filter with a smoothing parameter equal to 1,600 (dashed lines), (ii) a linear trend (dotted lines), and (iii) a cubic trend (dash-dotted lines). Solid lines give the median responses, and shaded areas are error bands from our baseline specification.

A.2 Baseline Model

In this section, we present details of the derivations in our baseline model and its calibration procedure.

A.2.1 Derivations

The household problem in this model is written as follows:

$$V(\mathbf{\Omega}_t) = \max_{\{C_{mt}, A_{t+1}, E_{t+1}, h_t^*\}} \ln C_t + \omega \frac{L_t^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}} + \beta \mathbb{E}_t V(\mathbf{\Omega}_{t+1}),$$

where $C_t = \left(\gamma C_{mt}^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma) C_{ht}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$ with $C_{ht} = \int_{h_t^*}^{\bar{h}} h_i \Phi(h_i)(1-E_t)$, and $\mathbf{\Omega}_t = \{E_t, A_t; y_t\}$ is a set of state variables in t. Note also that aggregate leisure L_t is equal to the number of nonparticipants N_t. This problem is subject to the following two constraints:

$$\begin{aligned} E_{t+1} &= E_t - sE_t + f_t \Phi(h_t^*)(1-E_t) + \mu f_t (1-\Phi(h_t^*))(1-E_t), \\ A_{t+1} + C_{mt} &= w_t E_t + b\Phi(h_t^*)(1-E_t) + (1+r_t)A_t + \Pi_t - T_t. \end{aligned}$$

As mentioned above, E_t is the predetermined variable and h_t^* determines the split between unemployment and nonparticipants with no dependence on their past values, namely,

$$\begin{aligned} U_t &= \Phi(h_t^*)(1-E_t), \\ N_t &= (1-\Phi(h_t^*))(1-E_t). \end{aligned}$$

For the threshold home-production productivity level, $\frac{\partial V(\mathbf{\Omega}_t)}{\partial h_t^*} = 0$ results in:

$$\frac{1-\gamma}{C_t} \left(\frac{C_t}{C_{ht}}\right)^{\frac{1}{\epsilon}} h_t^* + \omega N_t^{-\frac{1}{\nu}} = \Lambda_t^{C_m} b + (1-\mu) f_t \Lambda_t^E, \tag{A.1}$$

where Λ_t^E and $\Lambda_t^{C_m}$ are the Lagrange multipliers associated with the two constraints above. The first-order condition with respect to employment is:

$$\frac{\partial V(\mathbf{\Omega}_t)}{\partial E_{t+1}} = \beta \mathbb{E}_t \frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial E_{t+1}} - \Lambda_t^E = 0, \tag{A.2}$$

where the marginal value of employment evolves according to:

$$\begin{aligned} \frac{\partial V_t}{\partial E_t} =& \Lambda_t^{C_m}(w_t - \Phi(h_t^*)b) - \frac{1-\gamma}{C_t}\left(\frac{C_t}{C_{ht}}\right)^{\frac{1}{\epsilon}} \hat{h}_t(1-\Phi(h_t^*)) - \omega N_t^{-\frac{1}{\nu}}(1-\Phi(h_t^*)) \\ &+ \Lambda_t^E[1 - s - f_t\Phi(h_t^*) - \mu f_t(1-\Phi(h_t^*))]. \end{aligned} \tag{A.3}$$

For market-goods consumption, the following standard Euler equation characterizes its path.

$$\Lambda_t^{C_m} = \beta \mathbb{E}_t \Lambda_{t+1}^{C_m}(1 + r_{t+1}).$$

By dividing Equation (A.1) by $\Lambda_t^{C_m}$, we can express it in terms of C_t^m:

$$z_{ht} h_t^* + z_{lt} = b + (1-\mu) f_t \mathbb{E}_t \hat{\beta}_{t,t+1} \mathcal{V}_{t+1}^E. \tag{A.4}$$

where $z_{ht} = \frac{1-\gamma}{\gamma}\left(\frac{C_{mt}}{C_{ht}}\right)^{1/\epsilon}$, $z_{lt} = \frac{\omega N_t^{-1/\nu}}{\frac{\gamma}{C_t}\left(\frac{C_t}{C_{mt}}\right)^{1/\epsilon}}$, $\hat{\beta}_{t,t+1} = \beta \frac{\Lambda_{t+1}^{C_m}}{\Lambda_t^{C_m}}$, and $\mathcal{V}_{t+1}^E = \frac{\frac{\partial V_{t+1}}{\partial E_{t+1}}}{\Lambda_{t+1}^{C_m}}$.

Using Equation (A.2), we can rewrite Equation (A.3), again, in terms of C_t^m:

$$\mathcal{V}_t^E = w_t - \Phi(h_t^*)b - (1-\Phi(h_t^*))\hat{h}_t z_{ht} + (1-\Phi(h_t^*)) z_{lt} + [1 - s - f_t\Phi(h_t^*) - \mu f_t(1-\Phi(h_t^*))]\mathbb{E}_t \hat{\beta}_{t,t+1} \mathcal{V}_{t+1}^E. \tag{A.5}$$

As described above, Equation (A.5) is written compactly as:

$$\mathcal{V}_t^E = w_t - g_t + (1 - s - \hat{f}_t)\mathbb{E}_t \hat{\beta}_{t,t+1} \mathcal{V}_{t+1}^E, \tag{A.6}$$

where g_t and $\hat{f}_t$ are defined by Equations (10) and (11).

The labor demand side is straightforward. The value of the job is:

$$\mathcal{V}_t^J = y_t - w_t + (1-s)\mathbb{E}_t \hat{\beta}_{t,t+1} \mathcal{V}_{t+1}^J, \tag{A.7}$$

and the free entry condition is:

$$\frac{\kappa}{q_t} = \mathbb{E}_t \hat{\beta}_{t,t+1} \mathcal{V}_{t+1}^J. \tag{A.8}$$

Flexible Wage. In this case, we assume that surplus is shared by fixed proportions between the firm and the household. The surplus is defined by:

$$\mathcal{S}_t = \mathcal{V}_t^J + \mathcal{V}_t^E, \tag{A.9}$$

and assume $\mathcal{V}_t^J = (1-\eta)\mathcal{S}_t$, and $\mathcal{V}_t^E = \eta\mathcal{S}_t$. Using Equations (A.6) and (A.7) in $\eta\mathcal{V}_t^J = (1-\eta)\mathcal{V}_t^E$, we obtain:

$$w_t = \eta y_t + (1-\eta)g_t + (1-\eta)\hat{f}_t\mathbb{E}_t\hat{\beta}_{t,t+1}\mathcal{V}_{t+1}^E.$$

Using the free entry condition (A.8) and the definition of $\hat{f}_t$ (see Equation (11)), the wage equation is written as:

$$w_t = \eta y_t + (1-\eta)g_t + \eta\kappa\theta_t\psi(h_t^*).$$

Note that, under the situation where $\Phi(h_t^*) = 1$ and thus $\psi(h_t^*) = 1$ and $g = b$, the above equation reduces to the wage equation in the simple two-state search model with continuously-renegotiated Nash bargaining, namely, $w_t = \eta y_t + (1-\eta)b + \eta\kappa\theta_t$. Wage is not "allocative" and not relevant for model equilibrium. Using Equations (A.6) and (A.7) in Equation (A.9) gives the evolution of surplus:

$$\mathcal{S}_t = y_t - g_t + (1-s-\eta\hat{f}_t)\mathbb{E}_t\hat{\beta}_{t,t+1}\mathcal{S}_{t+1}. \tag{A.10}$$

Rewriting the free entry condition (A.8) as $\mathbb{E}_t\hat{\beta}_{t,t+1}\mathcal{S}_{t+1} = \frac{\kappa}{(1-\eta)q_t}$ and using it in Equation (A.10), one obtains the job-creation condition:

$$\frac{\kappa}{q_t} = \mathbb{E}_t\hat{\beta}_{t,t+1}\left[(1-\eta)(y_{t+1}-g_{t+1}) + \kappa\left(\frac{1-s}{q_{t+1}} - \eta\theta_{t+1}\psi(h_t^*)\right)\right]. \tag{A.11}$$

On the labor supply side, using $\mathcal{V}_t^E = \eta\mathcal{S}_t$ together with the free entry condition, one can rewrite the participation condition (A.4) as:

$$z_{ht}h_t^* + z_{lt} = b + (1-\mu)\frac{\eta}{1-\eta}\kappa\theta_t. \tag{A.12}$$

Aside from the variations in z_{ht} and z_{lt}, the equilibrium under the flexible Nash bargaining is determined by Equations (A.11) and (A.12).

Fixed Wage. Assume that wage is fixed at $\bar{w}$. In this case, the model is characterized by the following conditions:

$$\mathcal{V}_t^E = \bar{w} - g_t + (1-s-\hat{f}_t)\mathbb{E}_t\hat{\beta}_{t,t+1}\mathcal{V}_{t+1}^E, \tag{A.13}$$

$$z_{ht}h_t^* + z_{lt} = b + (1-\mu)f_t\mathbb{E}_t\hat{\beta}_{t,t+1}\mathcal{V}_{t+1}^E, \tag{A.14}$$

$$\mathcal{V}_t^J = y_t - \bar{w} + (1-s)\mathbb{E}_t\hat{\beta}_{t,t+1}\mathcal{V}_{t+1}^J, \tag{A.15}$$

$$\frac{\kappa}{q_t} = \mathbb{E}_t\hat{\beta}_{t,t+1}\mathcal{V}_{t+1}^J. \tag{A.16}$$

Combining Equations (A.13) and (A.14), we can obtain the following participation condition:

$$\frac{z_{ht}h_t^* + z_{lt} - b}{f_t} = \mathbb{E}_t\hat{\beta}_{t,t+1}\left[(1-\mu)[\bar{w} - g_{t+1}] + \frac{(1-s-\hat{f}_{t+1})(z_{ht+1}h_{t+1}^* + z_{lt+1} - b)}{f_{t+1}}\right]. \tag{A.17}$$

On the job-creation side, combining Equations (A.15) and (A.16) together with $w_t = \bar{w}$ results in

$$\frac{\kappa}{q_t} = \mathbb{E}_t\hat{\beta}_{t,t+1}\left[y_{t+1} - \bar{w} + \frac{(1-s)\kappa}{q_{t+1}}\right]. \tag{A.18}$$

Again, aside from the variations in z_{ht} and z_{lt}, Equations (A.17) and (A.18) determine the behavior of the participation margin and market tightness. In particular, if the wage is fixed, the behavior of θ_t is independent of the participation margin.

A.2.2 Solving for the Steady-State Equilibrium and the Calibration

We first calibrate the labor market block of the model by targeting the level of transition rates in the steady state. However, it is clear that this model lacks enough degrees of freedom to match the levels of all labor market flows. Thus, our calibration matches the levels of the subset of these transition rates, namely, $EU = s\Phi(h^*) = 0.015$, $EN = s(1-\Phi(h^*)) = 0.028$, $UE = f = 0.251$, and $NE = \mu f = 0.047$. These conditions imply $s = 0.043$, $\frac{EU}{EU+EN} = \Phi(h^*) = 0.349$ and $\mu = 0.187$. The levels of nontargeted transition rates, UN and NU rates are then computed as $UN = (1-f)(1-\Phi(h^*)) = 0.49$ and $NU = (1-\mu f)\Phi(h^*) = 0.33$. Both rates are far too large relative to the empirical counterparts for the reason explained the main text. The information so far also allows us to solve the steady-state law of motion for E for the employment stock through $E = \frac{[\Phi(h^*)+\mu(1-\Phi(h^*))]f}{s+[\Phi(h^*)+\mu(1-\Phi(h^*))]f} = 0.73$. The remaining labor market variables can be obtained by $U = \Phi(h^*)(1-E) = 0.09, u = \frac{U}{(E+U)} = 0.11, N = (1-\Phi(h^*))(1-E) = 0.17, S = U + \mu N = 0.126$. We also target the steady-state job-filling rate q at 0.9, implying that the steady state labor market tightness $\theta = \frac{f}{q}$ from which we can determine $V = \theta S$. The scale parameter of the matching function $\bar{m}$ is then backed out from $\overline{m} = \frac{f}{\theta^{1-\alpha}}$. This completes the calibration of the labor market flows and stocks.

Next, the steady-state free entry condition, $\kappa = q\beta(1-\eta)\mathcal{S}$, can be combined with the steady-state surplus Equation (A.10), $\mathcal{S} = \frac{\bar{y}-g}{1-\beta(1-s-\eta\hat{f})}$, to obtain:

$$\kappa = q\beta(1-\eta)\frac{\bar{y} - g}{1-\beta(1-s-\eta\hat{f})},$$

where $\bar{y}$ is the steady-state value of labor productivity and normalized at 1. Recall also $\hat{f} = \Phi(h^*)f + (1-\Phi(h^*))\mu f$. Thus, the unknowns in the above equation are the vacancy posting cost κ and g. The latter represents the flow outside option value and is set to 0.71 in the steady state, allowing us to back out the vacancy positing cost κ from the above equation.

Values of the remaining parameters are determined from the participation condition and related equations. Using the steady-state surplus expression in the participation condition, we obtain:

$$z_h h^* + z_l = b + (1-\mu) f \beta \eta \frac{\bar{y} - g}{1 - \beta(1 - s - \eta \hat{f})}. \tag{A.19}$$

We assume that the distribution of $\Phi(.)$ is uniformly distributed between 0 and $\bar{h}$. Further, it is also convenient to define the relative importance of z_l and z_h in the steady state as:

$$\bar{\omega} = \frac{z_l}{z_h \hat{h}},$$

where $\hat{h}$ is the conditional mean of h_i, $\mathbb{E}(h|h_i > h^*) = \frac{\bar{h}+h^*}{2}$. The steady-state value of z_h is irrelevant for the model dynamics under our calibration procedure and thus is normalized to one. Then Equation (A.19) can be rewritten as:

$$\left(\frac{2\Phi(h^*)}{1+\Phi(h^*)} + \bar{\omega} \right) \hat{h} = b + (1-\mu) f \beta \eta \frac{\bar{y} - g}{1 - \beta(1 - s - \eta \hat{f})}. \tag{A.20}$$

Note also that g can also be rewritten as:

$$g = \Phi(h^*) b + (1 - \Phi(h^*)) \hat{h} (1 + \bar{\omega}). \tag{A.21}$$

One can think of Equations (A.20) and (A.21) having three unknowns, $\bar{\omega}$, $\hat{h}$, and b (we have already determined all other parameter and steady-state values. For the value of $\bar{\omega}$, we use the estimated value in the extended model. In the current context, it is therefore set to some value exogenously. We then can solve Equations (A.20) and (A.21) for the level of UI benefits b and the conditional mean of home production $\hat{h}$. Because $\Phi(h_i)$ is assumed to be uniform, knowing the values of $\hat{h}$ and $\Phi(h^*) = \frac{h^*}{\bar{h}}$ allows us to set the upper bound of the distribution $\bar{h}$.

Note that normalizing z_h at 1 implies:

$$1 = \frac{1-\gamma}{\gamma} \left(\frac{C_m}{C_h} \right)^{\frac{1}{\epsilon}}. \tag{A.22}$$

From the household budget constraint (together with the equilibrium conditions) and the definition of home production, we can solve for C_m and C_h as:

$$C_m = \bar{y} E - \kappa v \text{ and } C_h = \hat{h} N. \tag{A.23}$$

Using these values in (A.22), we can determine the value of the CES weight parameter γ given the value of the elasticity of substitution ϵ. Lastly, recall that z_l is defined as:

$$z_l = \frac{\omega N^{-\frac{1}{\nu}}}{\frac{\gamma}{C} (\frac{C}{C_m})^{\frac{1}{\epsilon}}}.$$

The value of z_l is known from $\bar{\omega} = \frac{z_l}{\hat{h}}$, and thus we can solve this equation for ω.

A.3 Extended Model

As discussed in the main text, we extend the baseline model along the following five dimensions: (i) timing of hiring, (ii) introduction of the "attachment" parameter, (iii) distinction between available nonparticipants and unavailable nonparticipants, (iv) introduction of the convex hiring cost, and (v) refinement of wage rigidity.

A.3.1 Description and Derivation of the Model

Labor market transitions. In the extended model, we split nonparticipants into two groups, available nonparticipants (N) and unavailable nonparticipants ($\bar{N}$). The latter group consists of workers who are permanently out of the labor force and thus are not available for work. The total population is normalized to one, i.e., $E_t + U_t + N_t + \bar{N} = 1$. The mass of permanent nonparticipants is assumed to be constant. The decisions of the permanent nonparticipants are not explicitly modeled, as described in the main text. The main effect of having $\bar{N}$ in the model is through the scaling of NU and NE rates.

Job separation occurs at the beginning of each period, and we allow the separated workers to be rehired within the same period. As in the baseline model, the separated workers draw h_i from the distribution $\Phi(h_i)$ and decide whether to actively engage in job search (U) or to exit the labor force while being available to work (N). Job finding probabilities for these two groups are f_t and μf_t, respectively, as in the baseline model. If a family member is nonparticipant at the start of t, she draws h_i and makes the participation decision. Depending on the labor market states, the worker finds a job with f_t or with μf_t.

The decision problem is slightly modified for those who are unemployed at the start of the period, due to the introduction of the attachment parameter λ. That is, they draw h_i only with probability $1 - \lambda$. If the unemployed worker gets to renew h_i, then the worker makes the same participation decision as just described. With probability λ, the worker continues to be an active job seeker in the current period and receives the job offer with probability f_t. These modifications also imply that the effective number of job seekers in t is given by:

$$S_t = [\Phi(h_t^*) + (1 - \Phi(h_t^*))\mu]\{sE_{t-1} + (1 - \lambda)U_{t-1} + N_{t-1}\} + \lambda U_{t-1}.$$

Under the modified timing assumption together with the introduction of permanent nonparticipants and the attachment parameter, the labor market stocks evolve according to the following laws of motion:

$$\begin{bmatrix} E_t \\ U_t \\ N_t \end{bmatrix} = \Gamma_t' \begin{bmatrix} E_{t-1} \\ U_{t-1} \\ N_{t-1} \end{bmatrix}, \tag{A.24}$$

where

$$\Gamma_t = \begin{bmatrix} 1-s+s\left[\Phi(h_t^*)+(1-\Phi(h_t^*))\mu\right]f_t & s\Phi(h_t^*)(1-f_t) & s(1-\Phi(h_t^*))(1-\mu f_t) \\ \left[\lambda+(1-\lambda)(\Phi(h_t^*)+(1-\Phi(h_t^*))\mu)\right]f_t & \left[\lambda+(1-\lambda)\Phi(h_t^*)\right](1-f_t) & (1-\lambda)(1-\Phi(h_t^*))(1-\mu f_t) \\ \left[\Phi(h_t^*)+(1-\Phi(h_t^*))\mu\right]f_t & \Phi(h_t^*)(1-f_t) & (1-\Phi(h_t^*))(1-\mu f_t) \end{bmatrix}. \tag{A.25}$$

Household's problem. The preferences of the household are the same as in the baseline model, but aggregate leisure hours are now given by $L_t = N_t + \bar{N}$. As in the baseline model, total consumption is a CES aggregator of the market- and home-produced goods, but home production is now given by:

$$C_{ht} = \hat{h}_t N_t + \bar{h}\bar{N}, \tag{A.26}$$

where it is assumed that unemployed workers' productivity at home is fixed at zero and inactive nonparticipants' productivity is equal to $\bar{h}$. The term $\hat{h}_t$ is the conditional mean of h_i, $\mathbb{E}(h_i|h_i > h_t^*) = \hat{h}_t \equiv \frac{\int_{h_t^*} h_i d\Phi(h_i)}{(1-\Phi(h_t^*))}$. The household problem is formally written as:

$$V(\mathbf{\Omega}_t) = \max_{\{C_{mt}, A_{t+1}, E_t, U_t, N_t, h_t^*\}} \ln C_t + \omega \frac{L_t^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}} + \beta \mathbb{E}_t V(\mathbf{\Omega}_{t+1}),$$

where the set of state variables is now given by $\mathbf{\Omega}_t = \{E_{t-1},\ U_{t-1},\ N_{t-1},\ A_t;\ y_t\}$. This problem is subject to the same budget constraint as in the baseline model (6) and the laws of motion (A.24). Notice that in the current model, the labor market stocks dated with subscript t are defined as of the end of that period, whereas in the baseline model they were defined as of the beginning of the period. This timing change is made to accommodate the contemporaneous hiring.

The FOCs for this problem are given by:

$$-\Lambda_t^{C_m} + \frac{1}{C_t}\frac{\partial C_t}{\partial C_{mt}} = 0, \tag{A.27}$$

$$-\Lambda_t^{C_m} + \beta \mathbb{E}_t \frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial A_{t+1}} = 0, \tag{A.28}$$

$$\Lambda_t^{C_m} w_t + \beta \mathbb{E}_t \frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial E_t} - \Lambda_t^E = 0, \tag{A.29}$$

$$\Lambda_t^{C_m} b + \beta \mathbb{E}_t \frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial U_t} - \Lambda_t^U = 0, \tag{A.30}$$

$$\frac{1}{C_t}\frac{\partial C_t}{\partial C_{ht}}\hat{h}_t + \omega L^{-\frac{1}{\nu}} + \beta \mathbb{E}_t \frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial N_t} - \Lambda_t^N = 0, \tag{A.31}$$

$$\frac{1}{C_t}\frac{\partial C_t}{\partial C_{ht}}\frac{\partial C_{ht}}{\partial h_t^*} + \Lambda_t^E \frac{\partial E_t}{\partial h_t^*} + \Lambda_t^U \frac{\partial U_t}{\partial h_t^*} + \Lambda_t^N \frac{\partial N_t}{\partial h_t^*} = 0, \tag{A.32}$$

where $\Lambda_t^{C_m}$ is the Lagrange multiplier associated with the budget constraint; and Λ_t^E, Λ_t^U

and Λ_t^N are the Lagrange multipliers associated with the constraints on E, U, and N, i.e., Equation (A.24), respectively. Equations (A.27) and (A.28) together with the associated envelope condition give the usual consumption Euler equation. Note also that we can write the marginal value functions with respect to the predetermined labor market stocks as:

$$\begin{bmatrix} \frac{\partial V(\mathbf{\Omega}_t)}{\partial E_{t-1}} \\ \frac{\partial V(\mathbf{\Omega}_t)}{\partial U_{t-1}} \\ \frac{\partial V(\mathbf{\Omega}_t)}{\partial N_{t-1}} \end{bmatrix} = \Gamma_t' \begin{bmatrix} \Lambda_t^E \\ \Lambda_t^U \\ \Lambda_t^N \end{bmatrix}.$$

Using these three equations to substitute out $\frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial E_t}$, $\frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial U_t}$, and $\frac{\partial V(\mathbf{\Omega}_{t+1})}{\partial N_t}$ in Equations (A.29), (A.30), and (A.31), respectively, and dividing them through marginal utility of market-goods consumption $\Lambda_t^{C_m}$, we obtain:

$$\begin{bmatrix} \mathcal{V}_t^E \\ \mathcal{V}_t^U \\ \mathcal{V}_t^N \end{bmatrix} = \begin{bmatrix} w_t \\ b \\ z_{lt}+z_{ht}\hat{h}_t \end{bmatrix} + \mathbb{E}_t \tilde{\beta}_{t+1} \Gamma_{t+1}' \begin{bmatrix} \mathcal{V}_{t+1}^E \\ \mathcal{V}_{t+1}^U \\ \mathcal{V}_{t+1}^N \end{bmatrix}, \tag{A.33}$$

where $\mathcal{V}_t^i = \frac{\Lambda_t^i}{\Lambda_t^{C_m}}$ with $i \in \{E, U, N\}$. Note that the variable $\mathcal{V}_t^E$ is previously defined as the value *net* of the value of nonemployment in the baseline model, whereas in the current model we write $\mathcal{V}_t^E$ as the gross value.

Next, the optimal participation condition (A.32) can be rewritten as:

$$z_{ht}\frac{\partial C_{ht}}{\partial h_t^*} + \mathcal{V}_t^E \frac{\partial E_t}{\partial h_t^*} + \mathcal{V}_t^U \frac{\partial U_t}{\partial h_t^*} + \mathcal{V}_t^N \frac{\partial N_t}{\partial h_t^*} = 0. \tag{A.34}$$

We can use the following expressions in (A.34):

$$\begin{aligned}
\frac{\partial C_{ht}}{\partial h_t^*} &= \frac{-h_t^*\Phi'(h_t^*)(1-\Phi(h_t^*)), +\Phi'(h_t^*)\int_{h_t^*} h_i d\Phi(h_i)}{(1-\Phi(h_t^*))^2} N_t, \\
\frac{\partial N_t}{\partial h_t^*} &= -\frac{\Phi'(h_t^*)}{(1-\Phi(h_t^*))} N_t, \\
\frac{\partial E_t}{\partial h_t^*} &= \frac{\Phi'(h_t^*)}{(1-\Phi(h_t^*))}\frac{(1-\mu)f_t}{(1-\mu f_t)} N_t, \\
\frac{\partial U_t}{\partial h_t^*} &= \frac{\Phi'(h_t^*)}{(1-\Phi(h_t^*))}\frac{(1-f_t)}{(1-\mu f_t)} N_t.
\end{aligned}$$

The participation condition can then be written as:

$$(1-\mu f_t) z_{ht}(\hat{h}_t - h_t^*) = \left[(1-\mu f_t)\mathcal{V}_t^N + \mu f_t \mathcal{V}_t^E\right] - \left[(1-f_t)\mathcal{V}_t^U + f_t \mathcal{V}_t^E\right], \tag{A.35}$$

which characterizes the worker's indifference margin between unemployment and nonparticipation. Our timing assumption in the current model makes the interpretation of this condition less straightforward. But we can further rewrite this condition as:

$$(1-\mu f_t)(z_{ht}h_t^*+z_{lt}+\tilde{\mathcal{V}}_t^N)+\mu f_t(w_t+\tilde{\mathcal{V}}_t^E) = (1-f_t)(b+\tilde{\mathcal{V}}_t^U)+f_t(w_t+\tilde{\mathcal{V}}_t^E), \tag{A.36}$$

where

$$\mathcal{V}_t^E \equiv w_t+\tilde{\mathcal{V}}_t^E, \mathcal{V}_t^U \equiv b+\tilde{\mathcal{V}}_t^U, \text{ and } \mathcal{V}_t^N \equiv z_{ht}\hat{h}_t+z_{lt}+\tilde{\mathcal{V}}_t^N.$$

Equation (A.36) is intuitive: The left-hand side is the value of passive job search evaluated at the threshold productivity h_t^*, while the right-hand side is the value of active job search. Both sides are written as of the beginning of the current period, thus allowing for the possibility that the worker receives a job offer and starts working within period t.

Firm's problem. The description of the representative firm is similar to our baseline model, with the exception of the hiring cost function. We assume a strictly convex hiring cost function as in Equation (25). Given the new timing and the convex hiring cost function, the job-creation condition is now given by:

$$\kappa\,(q_tV_t)^{\epsilon_v} = \mathcal{V}_t^J. \tag{A.37}$$

The value of a filled job ($\mathcal{V}_t^J$) is identical to our baseline model (see Equation (14)).

Wages. As discussed in the main text, we assume the following form of real wage rigidity:

$$w_t = (1-\delta_w)w_t^*+\delta_w w_{t-1}.$$

The wage norm w_t^* is set by the period-by-period Nash-bargained wage and can be derived as follows. For the firm, the surplus is given by $\mathcal{V}_t^J$. For the worker, the surplus is given by the difference between the value of employment $\mathcal{V}_t^E$ and the value of the outside option, which equals:

$$\mathcal{V}_t^A = \Phi(h_t^*)\mathcal{V}_t^U+(1-\Phi(h_t^*))\mathcal{V}_t^N. \tag{A.38}$$

The period-by-period Nash-bargained wage is implicitly defined by:

$$\eta\mathcal{V}_t^J = (1-\eta)(\mathcal{V}_t^E-\mathcal{V}_t^A). \tag{A.39}$$

As mentioned above, in the baseline model $\mathcal{V}_t^E$ was written as the value net of the outside value, but in the current model we write it as the gross value.

A.3.2 Estimation Procedure

As explained in the main text, the parameters listed in Panel C of Table 5 are all estimated. We describe here the procedure through which the estimation is implemented in practice.

First, Table A.2 presents the steady-state equilibrium equations. The "endogenous" variable associated with each equation is listed in parentheses. Some of these variables are model parameters, but they are endogenously solved. There are 25 equations and 25 associated unknowns listed in the table.

These 25 endogenous variables are a nonlinear function of 13 "parameters." Some of these parameters are steady-state values of some model variables such as f_t and z_{ht}. The first five of these 13 parameters are set exogenously as explained in the main text and are listed in Panel A of Table 5. In addition, we impose the following steady-state restrictions: $g = 0.71$, $q = 0.9$, and $z_h = 1$ (listed in Panel B of Table 5). The remaining five parameters $\{\epsilon^v, f, \bar{N}, \Phi(h^*), s\}$ plus the degree of wage rigidity (δ_w), which matters only for the dynamics of the model, are estimated by solving a constrained minimization problem. Specifically, those parameters are estimated such that they minimize the weighed square difference between the model and data generated impulse response functions, where the weight of each one of these responses is given by the inverse of the unconditional variance in the data. We implement the Nelder-Mead algorithm for this minimization process, but this procedure is constrained such that the steady-state values of labor market transition rates and stocks stay within a 30% range of our target values (see Table 6). The remaining model parameters listed in Panel C of Table 5 are endogenously determined by the equations listed in Table A.2.

Table A.2: Steady-State Equilibrium

$$\begin{bmatrix} E \\ U \end{bmatrix} = \begin{bmatrix} s+(1-s)\left[\Phi(h^*)+(1-\Phi(h^*)\mu)\right] f & -\lambda(1-\mu)(1-\Phi(h^*))f \\ (1-s)\Phi(h^*)(1-f) & 1-(1-f)(1-\Phi(h^*))\lambda \end{bmatrix}^{-1} \times \begin{bmatrix} \left[\Phi(h^*)+(1-\Phi(h^*))\mu\right] f \\ \Phi(h^*)(1-f) \end{bmatrix} (1-\bar{N}) \tag{E,U}$$

$$N = 1-\bar{N}-E-U \tag{N}$$

$$\begin{bmatrix} \mathcal{V}^E \\ \mathcal{V}^U \\ \mathcal{V}^N \end{bmatrix} = (I-\beta\Gamma')^{-1} \begin{bmatrix} w \\ b \\ \hat{h}(1+\bar{\omega}) \end{bmatrix} \tag{$\mathcal{V}^E, \mathcal{V}^U, \mathcal{V}^N$}$$

$$\mathcal{V}^J = \frac{1-w}{(1-s)\beta} \tag{$\mathcal{V}^J$}$$

$$\kappa(qV)^{\epsilon_v} = \mathcal{V}^J \tag{κ}$$

$$(1-\mu f)(\hat{h}-h^*) = \left[(1-\mu f)\mathcal{V}^N+\mu f\mathcal{V}^E\right]-\left[(1-f)\mathcal{V}^U+f\mathcal{V}^E\right] \tag{h^*}$$

$$\eta\mathcal{V}^J = (1-\eta)\left[\mathcal{V}^E-(\Phi(h^*)\mathcal{V}^U+(1-\Phi(h^*))\mathcal{V}^N)\right] \tag{w}$$

$$C_m = E-\tfrac{\kappa(qV)^{1+\epsilon_v}}{1+\epsilon_v} \tag{C_m}$$

$$C_h = \hat{h}N+\bar{h}\bar{N} \tag{C_h}$$

$$C = \left(\gamma C_m^{\frac{\varepsilon-1}{\varepsilon}}+(1-\gamma)C_h^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{C}$$

$$z_h = \tfrac{1-\gamma}{\gamma}\left(\tfrac{C_m}{C_h}\right)^{1/\epsilon} \tag{γ}$$

$$\hat{h}z_h\bar{\omega} = \frac{\omega(N+\bar{N})^{-1/\nu}}{\frac{\gamma}{C}\left(\frac{C}{C_m}\right)^{1/\epsilon}} \tag{ω}$$

$$\Phi(h^*) = \frac{h^*}{\bar{h}} \tag{$\bar{h}$}$$

$$\hat{h} = \frac{\bar{h}+h^*}{2} \tag{$\hat{h}$}$$

$$\bar{\omega} = \frac{z_l}{\hat{h}z_h} \tag{z_l}$$

$$g = \Phi(h^*)b+(1-\Phi(h^*))\hat{h}(1+\bar{\omega}) \tag{b}$$

$$\frac{f}{q} = \theta \tag{θ}$$

$$S = (\Phi(h^*)+(1-\Phi(h^*))\mu)(sE+(1-\lambda)U+N)+\lambda U \tag{S}$$

$$\theta = \frac{V}{S} \tag{V}$$

$$f = \bar{m}\theta^{1-\alpha} \tag{$\bar{m}$}$$

Note: Γ represents the steady-state transition probability matrix (A.25).

Table A.3: Model Equations for the Extended Model

$\begin{bmatrix} \mathcal{V}_t^E \\ \mathcal{V}_t^U \\ \mathcal{V}_t^N \end{bmatrix} = \begin{bmatrix} w_t \\ b \\ z_{lt}+z_{ht}\hat{h}_t \end{bmatrix} + \mathbb{E}_t \hat{\beta}_{t+1} \Gamma'_{t+1} \begin{bmatrix} \mathcal{V}_{t+1}^E \\ \mathcal{V}_{t+1}^U \\ \mathcal{V}_{t+1}^N \end{bmatrix}$	(Household values)
$\mathcal{V}_t^J = y_t - w_t + (1-s)\mathbb{E}_t \hat{\beta}_{t,t+1} \mathcal{V}_{t+1}^J$	(Job value)
$\kappa\,(q_t v_t)^{\epsilon_v} = \mathcal{V}_t^J$	(Job creation)
$(1-\mu f_t) z_{ht}(\hat{h}_t - h_t^*) = \left[(1-\mu f_t)\mathcal{V}_t^N + \mu f_t \mathcal{V}_t^E\right] - \left[(1-f_t)\mathcal{V}_t^U + f_t \mathcal{V}_t^E\right]$	(Participation)
$w_t = (1-\delta_w) w_t^* + \delta_w w_{t-1}$	(Wage evolution)
$\eta \mathcal{V}_t^J = (1-\eta)\left[\mathcal{V}_t^E - (\Phi(h_t^*)\mathcal{V}_t^U + (1-\Phi(h_t^*))\mathcal{V}_t^N)\right]$	(Surplus sharing for w_t^*)
$C_{mt} = y_t E_t - \kappa \frac{(q_t v_t)^{1+\epsilon_v}}{1+\epsilon_v}$	(Resource constraint)
$C_{ht} = \hat{h}_t N_t + \bar{h}\bar{N}$	(Home production/consumption)
$g_t = \Phi(h_t^*) b + (1-\Phi(h_t^*))(\hat{h}_t z_{ht} + z_{lt})$	(Opportunity cost of employment)
$z_{ht} = \frac{\Lambda_t^{C_h}}{\Lambda_t^{C_m}}$	(MRS between C_{ht} and C_{mt})
$z_{lt} = \frac{\Lambda_t^{L}}{\Lambda_t^{C_m}}$	(MRS between leisure and C_{mt})
$\Lambda_t^{C_h} = \frac{1-\gamma}{C_t}\left(\frac{C_t}{C_{ht}}\right)^{\frac{1}{\varepsilon}}$	(MU of C_{ht})
$\Lambda_t^{C_m} = \frac{\gamma}{C_t}\left(\frac{C_t}{C_{mt}}\right)^{\frac{1}{\varepsilon}}$	(MU of C_{mt})
$\Lambda_t^L = \omega L_t^{-\frac{1}{\nu}}$	(MU of leisure)
$\hat{h}_t = \frac{\int_{h_t^*}^{\bar{h}} h_i d\Phi(h_i)}{1-\Phi(h_t^*)}$	(Conditional mean of h_i)
$L_t = N_t + \bar{N}$	(Aggregate leisure)
$f_t = \bar{m}\theta_t^{1-\alpha}$	(Job finding rate)
$q_t = \bar{m}\theta_t^{-\alpha}$	(Job filling rate)
$S_t = [\Phi(h_t^*) + (1-\Phi(h_t^*))\mu]\,[sE_{t-1} + (1-\lambda)U_{t-1} + N_{t-1}] + \lambda U_{t-1}$	(Aggregate job search)
$\theta_t = {}^{V_t}/_{S_t}$	(Market tightness)
$\begin{bmatrix} E_t \\ U_t \\ N_t \end{bmatrix} = \Gamma'_t \begin{bmatrix} E_{t-1} \\ U_{t-1} \\ N_{t-1} \end{bmatrix}$	(Labor market stock evolutions)
$E_t + U_t + N_t + \bar{N} = 1$	(Population normalization)
$\ln y_t = (1-\rho)\ln \bar{y} + \rho \ln y_{t-1} + \varepsilon_t$	(Aggregate productivity)

Note: Γ_t represents the transition probability matrix (A.25).