The Wage of Temporary Agency Workers*

[Preliminary , do not circulate]

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Abstract

Using French administrative data we estimate the distribution of the wage gap between in-house and temporary agency workers working in the same firm and the same occupation. The average wage gap is about 3%, but the gap is heterogeneous and can be negative in some commuting zones and occupations. We construct and estimate a search and matching model which shows that the wage gap depends on the cost of job vacancies, on labor market frictions and on the gap in labor management costs between temporary agencies for temp workers and user firms for in-house workers. The model relates the wage gap to the inefficiency of the employment of temp workers. Simulations assess this inefficiency on each labor market and the taxes and subsidies needed to correct it.

Key words: Wage gap, Temporary Work Agency, labor market frictions.

JEL Codes: J24, J31, J64, J65.

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1 Introduction

The expansion of temporary work agency (TWA) contracts in many OECD countries raises important concerns about wage inequalities between in-house and temp workers. Several contributions have indeed shown that temp workers have on average lower wages than in-house workers. Recent access to administrative data that makes possible the comparison of in-house and temp workers in the same workplace shows that temp workers receive much lower workplace-specific pay premia than that earned by in-house workers in user firms (Drenik et al., 2020). However, several key features regarding the different wage policies faced by in-house and temp workers remain poorly understood. Empirically, while studies have focused on estimating the average wage gap, the extent of its dispersion across firms and markets is not well documented. Theoretically, the factors shaping this gap and the welfare interpretation of its sign and magnitude remain understudied.

Our paper aims at making progress, both empirically and theoretically, in our understanding of this wage gap. First, we start by documenting new stylized facts about the wage policies faced by in-house and temp workers. Second, we develop a theory disciplined by these facts which allows us to understand the welfare implications of temp agencies in frictional labor markets.

We use French administrative data, which from 2017 on-wards report both the identifier of the employing firm and the one of the user firm for TWA employees, to estimate the distribution of the wage gap between in-house and TWA workers within the same firms and occupations. In this section we follow closely the approach taken by Drenik et al. (2020), which extends the model of Abowd et al. (1999) by interacting the firm fixed effects with the workers' contract, distinguishing between in-house and TWA arrangements. However, going beyong the average wage gap, we also highlight the substantial heterogeneity that persists across firms and markets, even after controlling for a rich set of observable and unobservable characteristics. Our findings indicate that, although there is, on average, a 3% higher salary for in-house workers compared to TWA workers, the wage gap varies significantly. In fact, the standard deviation is 4.5 times larger than the mean, and in more than one fourth of the cases, the gap even becomes negative. We also show that performing the analysis at the level of commuting zones $\times$ occupations allows us to capture a large portion of the heterogeneity observed in the wage gap across firms, while being much more parsimonious in terms of the number of fixed effects used. This observation motivates the choice of orienting our theoretical analysis at the level of the labor market.

To further explore the reasons behind these differences and evaluate the potential resulting inefficiency, we develop a theoretical model that considers the cost of job vacancies, labor market frictions, and the differences in labor management costs between firms that employ in-house workers versus those that employ temporary workers. Our model suggests that the wage gap is linked to the inefficiency of employing TWA workers.

We considered a directed search and matching model that incorporates TWAs reflecting their crucial role in recruiting workers to provide labor services to firms that are facing difficulties in recruiting workers. The decision to use a directed search model, rather than a random search model, is grounded in both theoretical and empirical considerations. From a theoretical standpoint, markets with directed search operate under a constrained efficient decentralized equilibrium, which has been previously demonstrated in research studies (Moen, 1997; Wright et al., 2021). This approach enables us to identify any potential inefficiencies linked to the activity of TWAs. From an empirical perspective, job seekers can easily search for vacancies posted by TWAs or other firms, especially since the internet has become a critical job search channel, as indicated by recent research (Kircher, 2022).[1]

In our model, firms have two hiring options: they can post job offers to recruit in-house workers and purchase labor services from TWAs to employ temporary workers. Both firms and TWAs compete in the job market by posting job offers, and workers can choose to seek job opportunities from either source. TWAs have two potential comparative advantages compared to other firms: they can find workers faster and manage labor resources at a lower cost. Thus, firms may opt to use TWAs for two primary reasons: to speed up the hiring process and to reduce HR management costs. The model allows us to derive a simple formula for the wage gap, which captures these two key reasons.

More specifically, as labor management costs reduce job surplus, they also reduce wages. As a result, the wage gap between in-house workers and TWA workers decreases in proportion to the difference in labor management costs between firms and TWAs for temporary workers. The gap can become negative if the labor management cost is significantly higher for in-house workers. On the other hand, since the user firm and the TWA contract at the beginning of the search process, the price that the user firm agrees to pay to the TWA decreases with the costs associated with the time

[1] We also compare the properties of the directed search model with those of the random search model and show that the determinants of the wage gap are very close in both models.

expected to fill the vacancy with a temp worker. This implies that the price paid by user firms to TWAs for employing temporary workers decreases with the cost to the user firm of job vacancies advertised in TWAs. This cost depends on the capital mobilized to create and maintain the jobs while awaiting recruitment. Therefore, the sales of the TWAs decrease when this cost is higher, which reduces the surplus of their jobs and the wage of TWA workers. In the limit case, if the cost to the user firms of vacant jobs posted at the TWA and the difference in the labor management cost between TWAs and user firms are all equal to zero, the wage gap becomes null. In the other cases, the wage gap can be either positive or negative. The sign depends on the cost of creation and maintenance of vacant jobs to the user firms, on the filling rate of TWAs' vacant jobs and on the difference between the HR management cost of TWAs and user firms.

The model predicts that the creation and maintenance cost to the user firms of vacant jobs posted at the TWAs is a source of inefficiency. The fundamental reason is that the contracts between the user firms and the TWAs are established at the beginning of the search process to recruit workers, *before the matching between the worker and the firm*, while the employment contracts offered to in-house workers begin *once the matching between the worker and the firm took place*. This implies that the cost of creation and maintenance of vacant jobs to the user firms reduces the price paid by user firms, which directly reduces the surplus of TWA jobs and the wages of temp workers. On the other hand, the competition between firms to recruit in-house workers implies that their wage does not depend directly on the cost of vacant jobs because their contracts start once they are matched with their employer. It depends on it indirectly via the tightness of the labor market. In the limiting case where the maintenance cost of vacant jobs is zero, the recruitment of temp workers is efficient, and the resulting wage gap between in-house and temp workers is consistent with efficient employment of temp workers, which is solely determined by the difference in HR management costs between the TWAs and user firms. However, when the creation and maintenance cost to the user firms of vacant jobs posted at the TWAs is positive, an inefficient use of temp workers arises from two opposing effects. On the one hand, this cost reduces the price paid to the TWAs to employ temp workers and a consequent reduction in wages. In other words, TWA workers bear part of the cost to the user firm of vacant jobs posted at the TWAs through the fall in the price that user firms pay to the TWAs, which reduces their wage and causes firms to hire too many temp workers compared with what is efficient. On the other hand, the wage drop reduces the attractiveness of temp jobs for job seekers, making it more difficult for TWAs to hire workers and create jobs. These two effects in combination lead to an inefficient number of temp workers, which can be either too high or too low, depending on the characteristics of the labor market. These inefficiencies can be corrected by providing wage subsidies to temp workers financed by taxes on the vacancies posted by the TWAs.

We then use our data to calibrate the model. We define more than 20,000 labor markets by considering each pair of commuting zone (*Zone d'emploi* or CZ) and occupation and combine various administrative datasets to measure the average wage of temporary workers, their job finding and filling rates, the average job duration for both in-house and TWA workers. We then use these values to solve the model recursively and estimate market specific parameters. This strategy allows us to decompose the sources of the wage gap and to evaluate the part associated with the inefficient use of temp workers. Then we compare, on each labor market, the wage gap, the unemployment rate, the share of temp workers and total output in the decentralized equilibrium with the constrained efficient solution. We also evaluate the temp workers wage subsidies and taxes on TWAs' which implement the constrained efficient solution.

Related literature: This paper is mainly related two strands of the literature.

A first strand analyzes the consequences of domestic outsourcing (as distinct from international outsourcing or offshoring) on wages. Several contributions analyze situations in which contracting firms provide goods and services (food, security, cleaning or general administrative services) that are not produced by in-house workers. Goldschmidt and Schmieder (2017) find that outsourced workers experience decline in wages which appears to almost entirely explained by a decline in firm-specific wage premia, as captured by AKM fixed-effects. Bilal and Lhuillier (2021) combine an empirical investigation of the productivity and wage effect of exogenous shift in outsourcing with a frictional labor market model in order to gauge the aggregate effects of domestic outsourcing. We depart from these papers by focusing on another type of outsourcing for which the contracting firms, namely the TWAs, provide the same types of labor services as those of in-house workers. Autor (2001) shows that TWAs can be seen as a screening device for user firms. Autor and Houseman (2010) assess the role of TWA as a stepping stone toward more stable employment in user firms. Recently, Drenik et al. (2020) use Argentinian data to show that temp workers get lower wages than in-house workers even conditional on working in the same user firms. We build on this work and expand their results by using newly available data in the French context to confirm their main results (negative wage gap) and contribute by documenting the large dispersion of the in-house temp workers wage gap across firms and markets. We further contribute by developing a simple model of TWA in frictional labor market that give rise to differential wage policies between temp and in-house workers and allows us to understand both the causes and the welfare implications such these discrepancies.

The second strand of the literature is related to the role of intermediaries in markets with frictions. It is well known that the impact of intermediaries on the distribution of gains from trade (Rubinstein and Wolinsky, 1987) and on welfare depends on the characteristics of the market such as the cost of search, the importance of frictions and the structure of competition – see among many others (Biglaiser and Li, 2018; Masters, 2007; Yavaş, 1994). Our contribution consists in proposing a model that takes into account the specific characteristics of a particular type of intermediary on the labor market, namely TWAs, to explain the wage gap between in-house and temp workers. This approach allows us to explain the wage gap distribution and its relation with the inefficiency of the decentralized equilibrium. More generally, taking into account in search and matching models the activity of TWAs, which is omnipresent and growing, is important to understand the sources of inefficiency in the functioning of the labor market.

The rest of the paper is organized as follows. Section 2 provides some descriptive statistics about TWA in France and the characteristics of firms that use this type of contracts, and describes the data used in the main analysis. Section 3 looks at the wage gap and provides various estimates. Section 4 presents our theory and Section 5 our quantitative analysis.

2 Institutions and Data

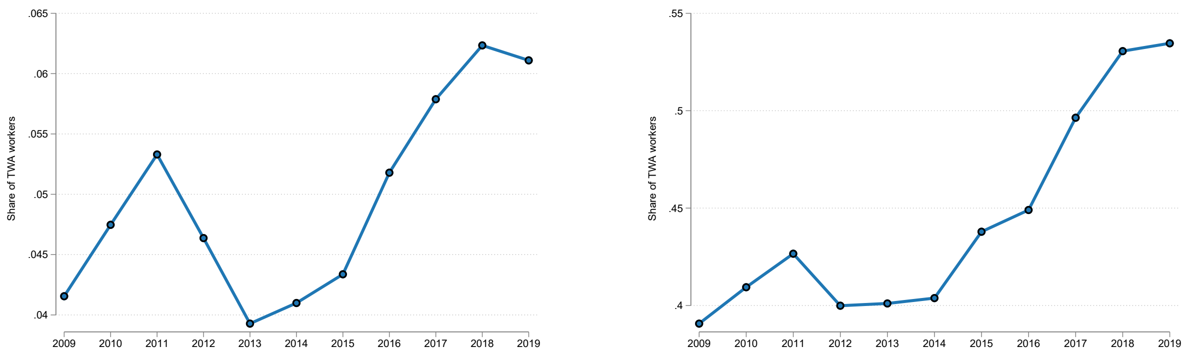
2.1 Temporary Work Agency Workers in France

According to the 2021 Employment Outlook of the OECD (OECD, 2021), TWA employment is becoming increasingly prevalent across Europe. In 2019, France had the 5^{th} largest share of temporary agency employment within its labor force, ranking behind only Spain, the Netherlands, Slovenia, and Slovakia. Using the administrative payroll database (*DADS Postes*), we can look more closely at this trend. Figure VIIa reports the share of TWA worker in the private business sector over total employment. We can see that this share has gone from 4% to 6% of employees between 2009 and 2019 and seems counter-cyclical.

However, the rise in temporary agency employment may be attributed to a greater use of temporary contracts in general. France's labor market is relatively polarized, with 85% of employees working under protected open-ended contracts (*"Contrat à Durée Indéterminée"*, or CDI) while the remaining 15% are in fixed-term contracts, which are primarily taken up by young and low-skilled workers. The dominant form of short-term contract is the CDD (*"Contrat à Durée Déterminée"*), which companies use to hire workers for a specified period of time.[2] Hence, in Figure VIIb, we report the yearly value of TWA worker employment over all temporary contracts (CDD and TWA workers). This confirms that indeed, TWA is becoming an increasingly popular form of fixed-term contract, reaching roughly 54% of all temporary contracts by 2019, from only 40% in 2009. Interestingly, this trend is due to CDD contracts being substituted for TWA contracts, as the share of CDD contracts among all in-house contracts has remained constant over time (if anything it is slightly decreasing since 2016).

FIGURE I. Share of TWA workers since 2009

(a) Over all workers (b) Over temporary workers



Notes: TWA workers are identified using the *"Convention Collective"* 2378 and the *"Contrat de Travail"* 03 from the DADS Postes available since 2009. The shares are calculated using the total number of hours worked during the year. Private business firms only.

While the overall incidence of TWA employment might appear small if compared to the entire labor force, it accounts for a much larger share of employment in the low skill segment, and an even larger portion of job vacancies and of flows in and out of unemployment among low skill workers, rendering TWA a key element for understanding employment dynamics.

TWA workers are formally employed by a temporary work agency but they work for

[2] Precise rules define in which contexts it is possible to hire somebody using a CDD: mainly i) to respond to temporary growth in activity, ii) to replace an employee on leave, iii) and for tasks that are temporary in nature. In addition, it can be renewed maximum 2 times and the total duration cannot be longer than 18 months.

another company that could be operating in any type of activity or industry. The advantage for the worker is to maximize its number of missions over the year while maintaining some degree of flexibility. This is especially relevant among occupations that are relatively low-skilled and on short supply. Table I reports the occupations that make the largest use of TWA contracts. In the list we find occupations in the care sector, such as caregivers, nurses and midwives, and unskilled blue collar professions in construction, industry and maintenance. The table further reports the unconditional relative wage gap between TWA workers and in-house workers, showing substantial differences across categories.

Finally, contrary to other forms of externalisation such as outsourcing, the relation between user firms and TWA takes place predominantly at the local labor market level. In fact, for 75% of TWA contracts the establishment using the worker is located within 20.6 kilometers from the TWA, and only 29% of contracts take place between a user firm and a TWA located in different commuting zone (*"Zone d'emploi"* or CZ).[3] This motivates our choice to consider a labor market as a pair of CZ and occupation. Appendix section A.1 reports additional descriptive evidence of the distribution of TWA contracts across geographic areas.

2.2 Data

We rely on the administrative payroll database (the *DADS*) to study the heterogeneity of the wage gap between in-house and TWA workers across labor markets. In particular, we apply the extension of the Abowd et al. (1999) (AKM) model put forward by Drenik et al. (2020) to recover the wage gap between in-house workers and TWA workers, controlling for differences in using firm characteristics, individual characteristics, and other observable components such as occupations and commuting zones. To apply this methodology, we exploit two important features of the French administrative data. First, we use the information on the user firm of TWA workers available since 2017 to compare them with in-house workers employed by the same firm. This procedure allows to net-out from the wage gap all differences explained by the composition of firms hiring TWA workers, which are typically the most productive ones. Second, we need to follow workers over time and to observe multiple individuals

[3]The commuting zones are purely statistical entities defined by the French statistics office -INSEE- to capture areas encompassing both the place of work and the place of residence of most individuals. There are about 300 commuting zones in France.

TABLE I. Occupation with the highest shares of temporary agency workers

Occup. code	Occup. name	Share of TWA	wage gap
V0Z	Caregivers	0.43	-0.07
J0Z	Unskilled BC workers in maintenance	0.40	0.08
E0Z	Unskilled BC workers in process industries	0.40	0.07
C1Z	Skilled BC workers in electrical and electronic industries	0.36	0.23
V1Z	Nurses, midwives	0.35	0.13
V4Z	Social workers	0.31	0.25
A2Z	Agricultural technicians and managers	0.29	0.30
B0Z	Unskilled BC workers in construction	0.26	-0.01
D0Z	Unskilled BC workers in metal industries	0.25	0.06
B5Z	Machine operators in construction	0.16	0.12

Notes: Shares are calculated by taking the ration of the total hours worked by TWA ("interim") workers over total hours by all workers. The relative wage gap is computed as the average in-house wage minus the average TWA wage, divided by the average in-house wage. It can be thus interpreted as a % difference in favor of in-house workers. Only private sector. Period: average over 2017-2019. Source: DADS.

switching across contracts, occupations and firms in order to identify wage premia net of a rich set of fixed effects. The administrative records published by the French statistics office (INSEE) provide consistent individual identifiers for only $1/12^{th}$ of the total workforce, which results in a connected set that is too sparse to identify our parameters of interest. We therefore apply the procedure described by Babet et al. (2022) to recover the exhaustive worker panel from the *DADS* data over the years 2017 to 2019.[4] In the final dataset obtained from this procedure, we observe over 2 million workers that hold both TWA contracts and in-house contracts over the period, and the vast majority of them also switches across firms and occupations when changing contract type, thus providing substantial variation for the analysis (see Appendix A.2 for more summary statistics).[5]

[4]The individual worker identifiers included in the exhaustive employer-employee data are not consistent across years. However, Babet et al. (2022) showed that the vast majority of individuals can be identified from one year to the next using the available information on their demographics, their firm of employment and their occupation. We therefore apply their codes to construct a worker-level panel of all the French labor force. We finally exclude the public sector because of their different wage setting mechanisms. More details on the data construction are available in Appendix A.2.

[5]One important caveat of the data is that if the TWA worker is employed by different using firms over the year, without changing the TWA agency that employs him, only the user firm where the TWA worker worked the longest number of days will be recorder, assigning to it all of the working hours and salaries perceived while working for other clients as well. This caveat might introduce some noise in the analysis, but we do not expect it to systematically bias our results.

3 the wage of in-house and TWA workers

3.1 Average wage gap

We start by estimating the following AKM model:

$$\log(w)_{ioft} = \beta_1 Inhouse_{ioft} + \beta_2 X_{ioft} + \gamma_t + \gamma_i + \gamma_f + \gamma_o + \epsilon_{ioft} \tag{1}$$

Where $\log(w)_{ioft}$ measures the logarithm of the hourly wage paid to worker i in occupation o, establishment f and time t, $Inhouse_{ioft}$ is a dummy identifying that the worker is under an in-house contract, and X_{ioft} controls for individual and contract-level characteristics such as age and age squared, gender, a dummy for open-ended contracts, and the count of number of days worked in the year. We further include increasingly demanding levels of fixed effects, including year (γ_t), worker (γ_i), user firm (γ_f) and occupation (γ_o) fixed effects.[6] Table II reports the estimation for β_1 when we only control for time FE and the dummy for open-ended contracts (Column 1), when we add all individual and contract level controls in X_{ioft} (Column 2), when we add worker FE (Column 3), user firm FE (Column 4), and occupation FE (Column 5). Column (6) reproduces the specification of Column (5) but restricts the sample of in-house contracts to short-term ones (CDD). The raw comparison reveals that on average in-house contracts pay about 10% more than TWA contracts. When we control for individual fixed effects the difference becomes much smaller, around 0.4%. This suggests that part of the difference is explained by the fact that less productive individuals are more likely to be employed in TWA contracts. However, this low coefficient masks another difference: the fact that large and more productive firms are more likely to use TWA contracts. Once we control for both individual and user firm FE, we obtain a wage-gap of 4.6%, which shrinks to 4.2% once we also control for occupation FE. These estimates are obtained using over 27 million observations spanning roughly 7 million individuals and 1.2 million user firms in the French private sector. Restricting the comparison group to short term contracts only considerably shrinks the number of observations, but give rise to a similar premium although somewhat smaller (3%).

[6]Here by firm we mean every single establishment. Therefore, we do not include commuting zone fixed effects because they are absorbed by the firm fixed effects.

TABLE II. Wage impact of temporary work contracts

	(1) log(w)	(2) log(w)	(3) log(w)	(4) log(w)	(5) log(w)	(6) log(w)
In-house	0.0979***	0.102***	0.00391***	0.0460***	0.0422***	0.0300***
	(0.000239)	(0.000233)	(0.000182)	(0.000237)	(0.000241)	(0.000336)
Controls		yes	yes	yes	yes	yes
Indiv. FE			yes	yes	yes	yes
Using firm FE				yes	yes	yes
Occup. FE					yes	yes
N	27'525'408	27'525'408	27'525'408	27'262'434	27'262'434	10'806'135

Standard errors in parentheses * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.
Column 1 controls for a dummy for open-ended contracts and year fixed effects only. Column 2 adds controls for gender, age and average hours worked per day, Column 3 adds individual fixed effects. Column 4 adds user firm fixed effects. Column 5 adds occupation fixed effects. Finally, Column 6 uses the same model of column 5 but restricts the sample to fixed term contracts.

These results confirm the finding of Drenik et al. (2020): on average, TWA workers are paid less than comparable in-house workers employed in the same firms.

3.2 Heterogeneity in the wage gap

The summary statistics presented in the previous section suggest that the wage gap between in-house and TWA workers presents considerable heterogeneity across occupations and markets. To explore such heterogeneity more rigorously, we extend the AKM model presented in the previous sub-section to recover firm and market-specific wage gaps controlling for possible confounding factors. We compute wage gaps at three different levels of aggregation : i) establishment level, ii) establishment × occupation level, iii) commuting zone × occupation × firm productivity group level.[7] as follows:

$$\log(w)_{ioft} = \beta_2 X_{ioft} + \gamma_f^{C_{ioft}} + \gamma_t + \gamma_i + \epsilon_{ioft} \tag{2}$$

$$\log(w)_{ioft} = \beta_2 X_{ioft} + \gamma_{fo}^{C_{ioft}} + \gamma_t + \gamma_i + \epsilon_{ioft} \tag{3}$$

$$\log(w)_{iozpt} = \beta_2 X_{iozpt} + \gamma_{zop}^{C_{iozpt}} + \gamma_t + \gamma_i + \epsilon_{iozpt} \tag{4}$$

[7]We define productivity based on total factor productivity - TFP - computed under the assumption of a Cobb-Douglas production function and combining all the firms belonging to the same group. We then split all firms into two equally sized groups along median productivity, and we superpose this dimension to the occupation × commuting zone cells.

The superscript $C_{ioft} \in H, T$ indicates whether worker i is employed through a TWA contract T or an in-house contract H. $\gamma_f^{C_{ioft}}$ are contract-specific establishment effects, $\gamma_{fo}^{C_{ioft}}$ are contract-specific establishment-occupation effects, and $\gamma_{zop}^{C_{iozpt}}$ are contract-specific commuting zone-occupation-productivity group effects. The sample is each time restricted to the largest connected set according to the level of analysis.

We then recover the full distribution of cell-specific wage gaps by subtracting the TWA-specific fixed effects from the in-house specific fixed effects : $\gamma_x^H - \gamma_x^T$ for $x \in f, fo, zop$. In practice the wage gaps obtained here are equivalent to extending equation 1 by interacting the TWA contract dummy with the entire battery of firm, firm-occupation or commuting zone-occupation-productivity group fixed effects.

Table III summarizes the wage gap distribution obtained with each one of the three models presented, and Figure III shows the distributions in a graph. Regardless of the level of analysis chosen, we confirm the presence of substantial heterogeneity, with more than 25% of the cells presenting higher wages for TWA workers than for in-house ones.

TABLE III. Summary statistic on firms' and markets' wage gaps

	mean	p25	p50	p75	sd	N
Wage gap by firm	0.040	-0.044	0.036	0.119	0.156	169676
Wage gap by firm - occup	0.045	-0.057	0.035	0.137	0.186	171460
Wage gap by CZ - occup - prod. group	0.024	-0.029	0.020	0.074	0.114	28023

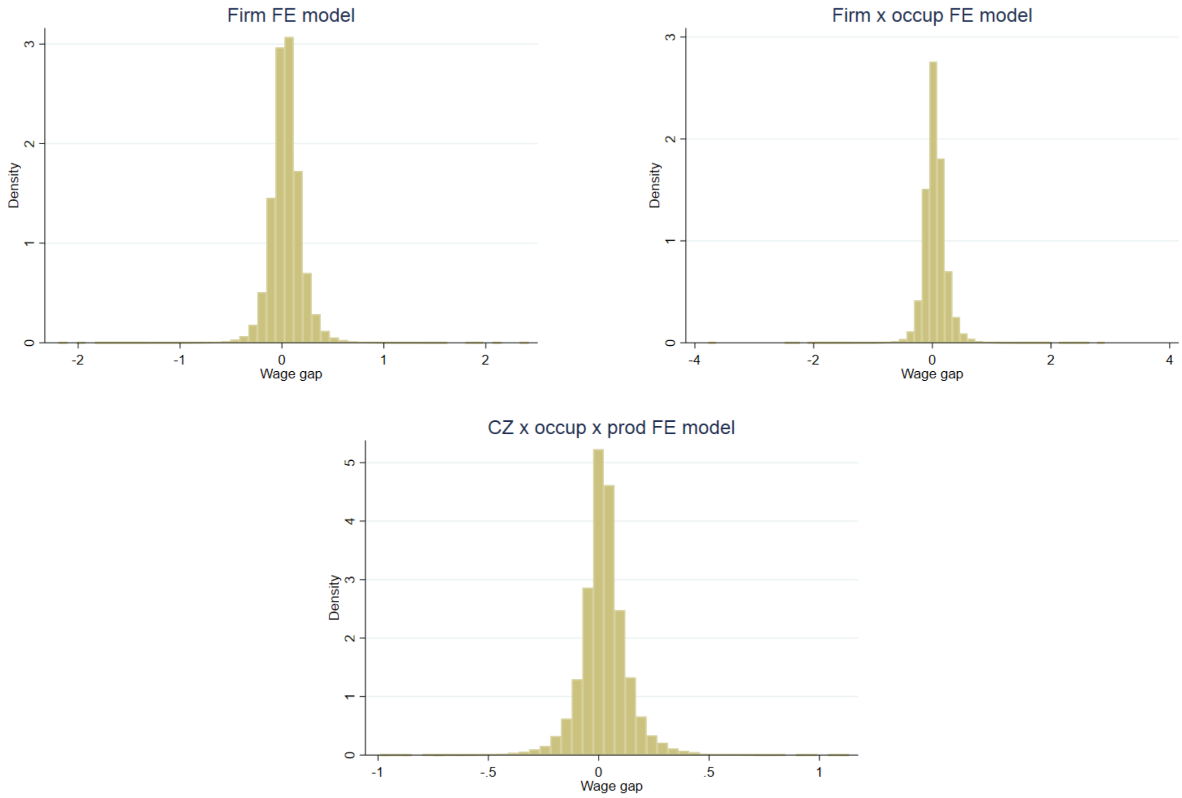
Note: This table reports the summary statistics of the cell-specific wage gaps obtained by subtracting the TWA-specific premium within a given firm, firm-occupation, and CZ-occupation from the in-house specific premium of the same entity, as reported in equations (2), (3) and (4).

3.3 Factors correlated with the wage gap heterogeneity

In appendix A.3 we show that the wage gap is increasing with the in-house premium ($\gamma_x^H - \gamma_x^T$ is positively correlated to γ_x^H), and we present regressions estimating the average level of rent pass-through from in-house to TWA contracts, similarly to Drenik et al. (2020).

However, the common interpretations of the wage gap as evidence of differential rent sharing within using firms or of fairness concerns of companies towards in-house

FIGURE III. Distribution of the wage gaps



Notes: Distribution of the wage gaps between in-house and temporary workers recovered using equations (2), (3) and (4).

workers are unable to explain the considerable proportion of cells with negative wage gaps, since they can only rationalize why TWA workers are paid less than in-house workers. Additionally, both of these explanations only consider the characteristics of the using firms, abstracting away any role played by the temporary work agencies.

To evaluate whether the identity of the TWA matters for explaining the wage of TWA workers, we estimate the following model, restricting the sample to the largest connected set among TWA workers :

$$\log(w)^T_{iofat} = \beta_2 X_{iofat} + \gamma_f + \gamma_a + \gamma_i + \gamma_o + \gamma_t + \epsilon_{iofat} \qquad (5)$$

Where γ_i are the individual fixed effects, γ_f are the using firm fixed effects, and γ_a are the TWA fixed effects. We also control for time and occupation fixed effects, as well as for the same individual and contract characteristics added in the previous models. Table IV reports the portion of the total variance in TWA wages explained by each dimension of fixed effects, and compares it with a "classic" in-house workers wage decomposition into firm and individual components.

TABLE IV. Wage variance decomposition

	variance	share of tot variance
Temp workers		
log wage	0.038	
worker FE	0.013	35%
Firm FE	0.006	17%
TWA FE	0.002	6%
Residual variance	0.015	41%
In-house workers		
log wage	0.232	
worker FE	0.168	72%
Firm FE	0.029	12%
Residual variance	0.035	15%

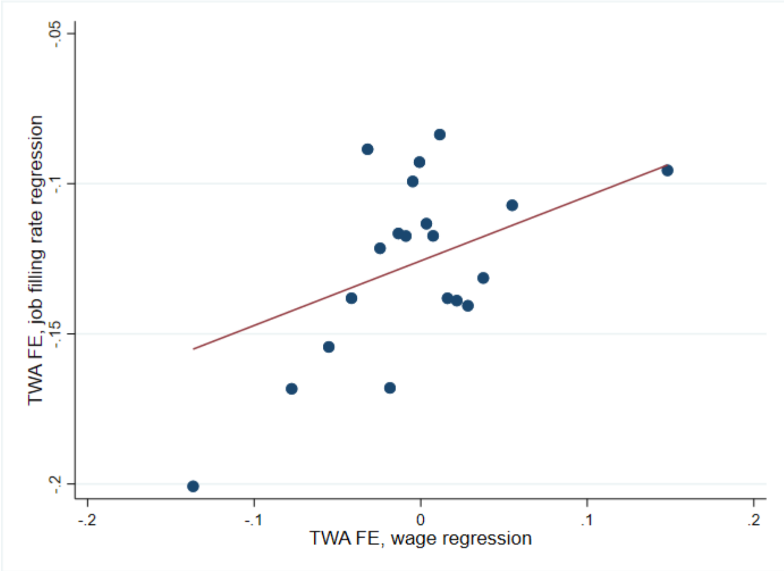
Note: This table reports the decomposition of the wage variance for TWA workers according to the portion explained by individual, firm and TWA effects, following equation 5. It compares the results with the ones obtained from estimating a similar model on in-house workers, dropping the TWA effects.

The variance of TWA workers' wages is much lower than for in-house workers, and a much lower share is explained by individual characteristics (38% vs 72%). A larger share is thus explained by using firm characteristics (17% vs 12%). More importantly, a non-negligible share of TWA worker wages is explained by the TWA fixed effects: 6% of the total, amounting to one third of the role of using firms. This suggests that TWAs are important actors in determining the wage of their employees. Finally, Figure V correlates the estimated TWA rent ($\hat{\gamma}_a$) with an estimation of TWA efficiency in filling vacancies recovered by regressing the job filling rate on TWA fixed effects and client firm fixed effects.[8] The figure shows a clear positive correlation between the ability of the TWA to extract rents and its efficiency in filling job vacancies.

Guided by this empirical evidence, in the next section we put forward a model rationalizing why identical in-house workers and TWA workers can be paid differently within the same firm and occupation, and why the pay gap can vary considerably

[8]This analysis is computed using data from the French employment office *Pôle Emploi* reporting information on all vacancies posted on their platform, including the identifier of the employing firm and, in the case of TWA jobs, the identifier of the client firm. The job filling rate is computed as the log of 1 over the vacancy length. Here there is no scope for worker fixed effects because the match is yet to be realized.

FIGURE V. Correlation between TWA rents and efficiency in filling vacancies



Notes: Correlation between the TWA fixed effects obtained from the job filling rate equation and the TWA effects obtained from the TWA worker wage equation.

across markets, to the point of reversing sign in some instances. The latter will allow for an active role played by TWAs, maximizing output and extracting rents.

4 A model

4.1 Framework

The framework is a directed job search model of a one period economy with a numéraire good an labor. The numéraire good is produced with capital and labor. The labor force is composed of $\mathcal{N}$ risk neutral workers who consume the numéraire good. There is an exogenous set of heterogeneous representative firms indexed by $i = 1, ..., I$, which produce the numéraire good. The production function of type-i firms is equal to $x_i F(\mathcal{L}_i)$ where $x_i \in [x_{\text{inf}}, +\infty), x_{\text{inf}} > 0$, and $\mathcal{L}_i \geq 0$ is employment, which can comprise in-house and temp workers. F satisfies the Inada conditions, i.e. $F' > 0, F'' < 0, \lim_{\mathcal{L}\to 0} F'(\mathcal{L}) = +\infty, \lim_{\mathcal{L}\to +\infty} F'(\mathcal{L}) = 0$. The creation of each job requires to invest $k_i \geq 0$ units of the numéraire good, which yields the marginal product $y_i = x_i F'(\mathcal{L}_i)$, once the job is filled. Firms can fill their jobs with in-house workers and temp workers whose labor services are sold by TWAs. The TWAs sell the services of temp workers to firms on a perfectly competitive market. Firms and TWAs need to post vacant jobs to recruit workers.

At the start of the period, all workers are unemployed. Firms and TWAs compete in a frictional job market by posting job offers to recruit workers who can seek jobs offered by the firms and the TWAs. Each vacancy offers a wage that is not renegotiable. There is a submarket for each wage and job type either in-house or temp. Unemployed workers can look for jobs on all submarkets. In each submarket, the number of matches between vacant jobs and unemployed workers is determined by a matching function with constant returns to scale which implies that the $\mathcal{V}_{hi}$ vacant jobs posted by type-i firms to hire in-house workers are filled at endogenous probability $m(\theta_{hi}) \in [0,1)$, $m'(\theta_{hi}) < 0$, $m''(\theta_{hi}) < 0$, $\lim_{\theta_{hi}\to 0} m(\theta_{hi}) = 1, \lim_{\theta_{hi}\to\infty} m(\theta_{hi}) = 0$, where $\theta_{hi} \geq 0$ is the labor market tightness equal to the ratio between the number of job vacancies and the number of unemployed workers looking for jobs in the submarket. The $\mathcal{V}_{ai}$ vacancies posted by the TWAs to fill the jobs of type-i firms are filled at endogenous probability $\alpha m(\theta_{ai}) \in [0,1)$, where θ_{ai} stands for the labor market tightness and $\alpha > 0$ is a positive scalar to account for the difference in search efficiency between firms and TWAs. Remark that α can be smaller than one if the TWAs are less effective than firms. Another possible difference between firms and TWAs is the cost of human resource management. It is represented by a cost per filled job, denoted by $c_a \geq 0$ for the TWAs and by $c_i \geq 0$ for the firms.

To fill a vacant job, a firm can post a vacancy to hire an in-house worker at marginal cost $C'(\mathcal{V}_{hi})$, where $C(\mathcal{V}_{hi})$ is a cost function which satisfies $C(0) = 0$, $C' > 0, C'' > 0$, $\lim_{\mathcal{V}\to 0} C'(\mathcal{V}) = 0$. The convexity hypothesis of the job vacancy cost function is empirically relevant to explain the hiring behavior of firms.[9] Firms can also rely on the TWAs to fill their jobs. In this case, the cost to the firm of posting its vacancy at the TWA is equal to zero and the firm pays the price p_i to the TWA if the job is filled with the temp worker. To ensure that the user firms have no interest looking for in-house workers for the job vacancies posted at the TWAs, contracts between the TWAs and the user firms stipulate that the latter pay compensation to the TWAs if they cancel their demand. This compensation, which is never paid in equilibrium, does not affect the equilibrium price paid to the TWAs.

The increasing marginal cost of posting vacancies for firms seeking to recruit internal workers on their own can stem from the fact that they must mobilize internal human resources that are not necessarily specialized in this type of activity. The specialization of TWAs allows them to increase their activity without being confronted with these

[9]See, among others Coşar et al. (2016); Gavazza et al. (2018); Manning (2006); Merz and Yashiv (2007).

difficulties. Therefore, we assume that the TWAs post vacant jobs at constant marginal cost, equal to κ per vacancy to the TWAs.

Now, we will define the objectives and behaviors of workers, firms and TWAs in a frictional labor market with in-house and temp workers.

4.2 Value functions and offered wages

4.2.1 Workers

Let W_u denote the expected value from unemployment at the start of the period. There are different labor submarkets with different possible values of the wage w and the labor market tightness θ. Workers, all unemployed at the beginning of the period, find jobs with probability $\theta m(\theta)$ on the submarket with labor market tightness θ. Those who do not find a job get the unemployment income b. The arbitrage condition implies that:

$$W_u = b + \theta m(\theta) \left(w - b\right), \quad \forall \left(w, \theta\right) \tag{6}$$

This equation defines a negative relation between the wage and the labor market tightness in each submarket because more unemployed workers are attracted in submarkets in which the wage is higher. Formally, the differentiation of equation (6) implies that:

$$\frac{\partial \theta}{\partial w} = -\frac{\theta}{1-\eta}\frac{1}{(w-b)}; \quad \eta = -\frac{\theta m'(\theta)}{m(\theta)}$$

For the sake of simplicity, it is assumed that m is homogeneous, which implies that η does not depend on θ and therefore that it is identical on all submarkets.

4.2.2 Firms

Firms choose the number of in-house and temp job vacancies. They also choose the wage associated with their in-house job offers. The maximization program of type-i firms is:

$$\max_{(\mathcal{V}_{hi}\geq 0, \mathcal{V}_{ai}\geq 0, w)} x_i F\left(\mathcal{L}_i\right) - \left[\left(w + c_i\right) m(\theta_{hi}) + k_i\right] \mathcal{V}_{hi} - \left[p_i \alpha m(\theta_{ai}) + k_i\right] \mathcal{V}_{ai} - C(\mathcal{V}_{hi}) \text{ s.t } (6)$$

where

$$\mathcal{L}_i = \mathcal{V}_{hi} m(\theta_{hi}) + \mathcal{V}_{ai} \alpha m(\theta_{ai})$$

For the sake of clarity, we describe here the equilibrium in which firms post in-house and temp job vacancies. Firms may hire both temp and in-house workers because the marginal cost of posting vacancies for in-house workers is increasing, while the marginal cost of employing temp workers, equal to the price paid to the TWAs, is constant. The other equilibria are described in appendix. For an interior solution, the first order conditions of the maximization problem of type-i firms implies that they offer the wage

$$w_{hi} = \eta\left(y_i - c_i - b\right) + b, \tag{7}$$

where

$$y_i \equiv x_i F'(\mathcal{L}_i)$$

At the optimum, firms equalize the marginal cost of job creation to its marginal return. To create a job, the type-i firm pays the investment cost k_i plus the vacancy cost $C'(\mathcal{V}_{hi})$ if it looks for a in-house worker. The marginal return of this job is equal to the job filling probability $m(\theta_{hi})$ times its marginal productivity, y_i, minus its labor cost $w_i + c_i$. Therefore, the equalization of the marginal cost of job creation for in-house workers to their marginal return yields, using the definition of the optimal wage:

$$C'(\mathcal{V}_{hi}) + k_i = (1-\eta)m(\theta_{hi})\left(y_i - b - c_i\right) \tag{8}$$

If the firm looks for a temp worker, the marginal cost of job creation is equal to the investment cost k_i and the marginal return to the job filling probability $\alpha m(\theta_{ai})$ times the marginal productivity, y_i, minus the price p_i payed to the TWA if the job is filled. Therefore, the equality between the marginal cost and the marginal return of temp job vacancies yields the demand of type-i firms for temp workers:

$$k_i = \alpha m(\theta_{ai})\left(y_i - p_i\right) \tag{9}$$

4.2.3 TWAs

The temp workers are paid by the TWAs to work in firms which pay the TWAs to buy their labor services. The TWAs post vacancies at unit cost κ and offer the wage w_{ai} to recruit temp workers for type-i firms which pay the price p_i if a temp worker is recruited. The type-i job vacancies are filled with probability $\alpha m(\theta_{ai})$ and the TWAs incur the human resource management cost c_a per temp worker. Thus, the value of type-i vacancies posted by the TWAs satisfies:

$$V_{ai} = \max_{w_{ai}} -\kappa + \alpha m(\theta_{ai}) \left(p_i - c_a - w_{ai}\right) \text{ subject to (6)} \tag{10}$$

The wage offered to temp workers is

$$w_{ai} = \eta \left(p_i - c_a - b\right) + b \tag{11}$$

The free entry condition on the market for temp workers implies that TWAs create vacancies up to the point where the value of their vacant jobs is equal to zero, i.e. $V_{ai} = 0$. This condition defines, together with the expression of the wage w_{ai} the equation of which defines the supply of type-i vacancies for temp workers:

$$\frac{\kappa}{\alpha m(\theta_{ai})} = (1-\eta)\left(p_i - c_a - b\right) \tag{12}$$

4.2.4 Labor market equilibrium

In equilibrium, the demand and supply of vacancies for temp workers are equal and workers looking for a job get the same expected utility in all submarkets.

The equality between the demand of vacancies for temp workers by firms and their supply by TWAs implies that we can substitute the expression of the price p_i paid by type-i firms to employ temp workers defined by the demand of those firms – equation (9) – into the wage equation (11). This provides the wage offered to temp workers in type-i firms:

$$w_{ai} = \eta \left(y_i - b - c_a - \frac{k_i}{\alpha m(\theta_{ai})}\right) + b \tag{13}$$

The comparison of this wage with that of in-house workers yields the following proposition:

Proposition 1. *The wage gap between in-house and temp workers depends on the cost of job creation, on the job filling rate of the TWAs and on the gap in the cost of human resources management between TWAs for temp workers and firms for in-house workers.*

Proof. From equations (7) and (13) the wage gap between in-house and temp workers is equal to:

$$w_{hi} - w_{ai} = \eta \left(\frac{k_i}{\alpha m(\theta_{ai})} + c_a - c_i\right) \tag{14}$$

□

Now, we can define the conditions which determine the equilibrium values of the labor market tightness on all submarkets. Let us denote by γ_{hi} the share of job seekers looking for type-i in-house jobs and by γ_{ai} the share of those looking for type-i temp jobs. The definition of the labor market tightness yields:

$$\theta_{ai} = \frac{\alpha \mathcal{V}_{ai}}{\gamma_{ai}\mathcal{N}}; \theta_{hi} = \frac{\mathcal{V}_{hi}}{\gamma_{hi}\mathcal{N}} \tag{15}$$

The equilibrium values of $\theta_{ai}, \theta_{hi}, \mathcal{V}_{ai}, \mathcal{V}_{hi}, \gamma_{ai}, \gamma_{hi}$ are defined by the identities (15), $\sum_i \gamma_{ai} + \gamma_{hi} = 1$, and the following set of equations:

$$\left\{ \begin{array}{c} \underbrace{\underbrace{\kappa + (1-\eta)k_i}_{\text{Cost of vacant temp job}} = \underbrace{\alpha m(\theta_{ai})(1-\eta)\,(y_i - b - c_a)}_{\text{Profits from vacant temp job}}}_{\text{Supply of type-}i\text{ temp job vacancies}} \\ \underbrace{\underbrace{C'(\mathcal{V}_{hi}) + k_i}_{\text{Cost of vacant in-house job}} = \underbrace{m(\theta_{hi})(1-\eta)\,(y_i - b - c_i)}_{\text{Profits from vacant in-house job}}}_{\text{Supply of type-}i\text{ in-house job vacancies}} \\ \underbrace{\underbrace{\theta_{hi} m(\theta_{hi})\,(y_i - b - c_i)}_{\text{Gains from seeking in-house jobs}} = \underbrace{\theta_{ai'} m(\theta_{ai'}) \left(y_i - b - c_a - \frac{k_i}{\alpha m(\theta_{ai'})} \right)}_{\text{Gains from seeking temp jobs}} ; \forall (i, i')}_{\text{Arbitrage of job seekers}} \end{array} \right. \tag{16}$$

The top equation, which comes from equations (9) and (12), determines the value of the labor tightness for type-i temp workers compatible with the supply of temp job vacancies arising from the interactions between type-i firms and TWAs. The left hand side is the sum of the costs of type-i temp workers vacancies to the TWAs and to the user firms. The cost of job creation to the user firms, k_i, is multiplied by $(1-\eta)$ because the wage of temp workers decreases with the cost of job creation k_i as shown by equation (13). The right hand side is equal to the sum of the expected profits of the user firms and TWAs. It is equal to the probability to fill the vacancy times the joint share $(1-\eta)$ of the job surplus of the user firms and TWAs, times the joint surplus, equal to the production y minus the labor management costs c_a and the income of unemployed workers b.

The middle equation determines the value of the labor tightness for type-i temp workers compatible with the supply to the supply of vacancies derived above – equation (8).

The bottom equation comes from the arbitrage condition (6) which determines, together with the wages, defined by equations (7) and (13), the relation between the labor market tightness of all submarkets for in-house workers and temp workers.

Note that there are as many equations as unknown variables: if there are I firm types, $i = 1, .., I$, there are I times 6 unknown variables $\theta_{ai}, \theta_{hi}, \mathcal{V}_{ai}, \mathcal{V}_{hi}, \gamma_{ai}, \gamma_{hi}$ and I times 2 equations for the first 2 rows, plus I times 2 equations minus one equation for the third row plus two equations for the 2 identities defined by equation (15), plus one equation for the identity $\sum_i \gamma_{ai} + \gamma_{hi} = 1$. Moreover, the convexity of the job vacancy cost function, the properties of the matching function and of the production function imply that the equilibrium is unique, provided it exists.

The number of unemployed workers is equal to

$$\mathcal{U} = \mathcal{N} - \sum_i \alpha m(\theta_{ai})\mathcal{V}_{ai} + m(\theta_{hi})\mathcal{V}_{hi} \tag{17}$$

4.3 Efficiency

4.3.1 Constrained efficient allocation

The constrained efficient allocation can be obtained as the solution of the maximization problem of planner which maximizes the aggregate output minus the costs of vacant jobs and job creations. The constrained efficient values of the number of vacancies for in-house workers and temp workers, of the labor market tightness for their submarket and of the share of job seekers looking for job on each submarket, respectively denoted by $\mathcal{V}^*_{hi}$, θ^*_{hi}, θ^*_{ai} γ^*_{ai}, and γ^*_{hi}, are defined by the identities (15), $\sum_i \gamma^*_{ai} + \gamma^*_{hi} = 1$, and by the following system of equations, when it is optimal to have both in-house and temp workers (see Appendix B.1.2):

$$\begin{cases} \underbrace{\kappa + k_i}_{\text{Cost of vacant temp job}} = \underbrace{\alpha m(\theta^*_{ai})\,(1-\eta)\,(y_i - b - c_a)}_{\text{Gains from vacant temp job}} \\ \underbrace{C'(\mathcal{V}^*_{hi}) + k_i}_{\text{Cost of vacant in-house job}} = \underbrace{m(\theta^*_{hi})(1-\eta)\,(y_i - b - c_i)}_{\text{Gains from vacant in-house job}} \\ \underbrace{\theta^*_{hi} m(\theta^*_{hi})\,(y_i - b - c_i)}_{\text{Gains from seeking in-house jobs}} = \underbrace{\theta^*_{ai'} m(\theta^*_{ai'})\,(y_{i'} - b - c_a)}_{\text{Gains from seeking temp jobs}} ; \forall (i, i') \end{cases} \tag{18}$$

The top equation of each of the two systems (18) and (16) represents the equality between the marginal cost of creation of temp jobs, on the left hand side and the marginal expected gain on the right hand side. For the social planner, the marginal cost includes the cost of job creation k_i plus the cost of vacancies for the TWAs, equal to κ. In the decentralized equilibrium, the marginal cost to the user firms of job creation, equal to $(1-\eta)k$, is lower than for the social planner, because a part of the cost of the job creation is reported on the price of the TWAs and eventually on the wage of temp workers – as shown by equations (9) and (13). The middle equation represents the equality between the marginal creation cost of in-house jobs, on the left hand side, and the expected gains on the right hand side. The bottom equation represents the arbitrage between the job search for an in-house job and a temp job.

4.3.2 Comparison of the decentralized equilibrium with the constrained efficient allocation

The comparison of the systems of equations (16) and (18), shows that, when $k_i = 0$ for all i, the constrained efficient and the decentralized equilibrium value of $(\theta_{hi}, \theta_{ai}, \mathcal{V}_{hi})$ are determined by the same conditions, which means that the decentralized equilibrium is constrained efficient. The only source of inefficiency is the distortion due to the cost of job creation which reduces the wage of temp workers relative to that of in-house workers.

Proposition 2. *The decentralized equilibrium is constrained efficient if and only if the cost of job creation, k_i, is equal to zero for all firms.*

The inefficiency of the decentralized equilibrium can be explained as follows: the firms hires temp workers, whose wage is negatively impact by the cost of job creation k_i, because they share the part η of the cost of job creation – see the left hand side of the top equations of systems (18) and (16) with the TWAs. This arises because k_i reduces the demand for temp workers, which reduces the price paid to the TWAs to hire temp workers – see equation (9) – and then the wages of temp workers – see equation (13).

The expression of the wage gap $w_{hi} - w_{ai}$ – see equation (14). – and the comparison of systems (18) and (16) that define the decentralized equilibrium values of $(\theta_{hi}, \theta_{ai}, \mathcal{V}_{hi})$ and their constrained efficient values, imply the following result.

Proposition 3. *The wage gap between in-house and temp workers arises from the inefficiency induced by the cost of job creation and from the gap in the cost of human resources management between the TWAs for temp workers and firms for in-house workers which does not induce inefficiency of the decentralized equilibrium.*

4.3.3 Implementation of the constrained efficient allocation

Let us show that the constrained efficient allocation can be implemented with wage subsidies for temp workers and taxes on the vacancies posted by the TWAs. Let us denote by σ_i the wage subsidy, such that temp workers get the income $w_{ai} + \sigma_i$ when the TWAs pay the wage w_{ai} and by τ_i the tax on each vacancy posted by the TWAs. We find that the wage of temp workers and the value of vacancies posted by the TWAs are defined by

$$\begin{aligned} w_{ai} + \sigma_i &= \eta \left(p_i - c_a + \sigma_i - b\right) + b \\ V_{ai} &= \kappa - \tau_i + \alpha m(\theta_{ai})(1-\eta)\left(p_i - c_a + \sigma_i - b\right) \end{aligned}$$

Solving the model as before, we get the set of 3 equations which define the equilibrium values of $\theta_a, \theta, \mathcal{V}$:

$$\begin{cases} \kappa + (1-\eta)k_i + \tau_i = \alpha m(\theta_{ai})(1-\eta)\left(y_i + \sigma_i - b - c_a\right) \\ \theta_{hi} m(\theta_{hi})\left(y_i - b - c_i\right) = \theta_{ai'} m(\theta_{ai'})\left(y_i + \sigma_i - b - c_a - \frac{k_i}{\alpha m(\theta_{ai'})}\right); \forall (i, i') \\ C'(\mathcal{V}_{hi}) + k_i = m(\theta_{hi})(1-\eta)\left(y_i - b - c_i\right) \end{cases}$$

The comparison of this system of equations with the system (18) which defines the constrained efficient values $\left(\theta^*_{hi}, \theta^*_{ai}, \mathcal{V}^*_{hi}\right)$ implies that the constrained efficient allocation is implemented with

$$\begin{aligned} \sigma_i &= \frac{k_i}{\alpha m(\theta^*_{ai})} \\ \tau_i &= k_i \end{aligned}$$

Since all vacancies posted by TWAs pay the tax $\tau_i = k_i$ and the share $\alpha m(\theta_{ai})$ of them are filled, the total amount of taxes on temp job vacancies – paid by TWAs or user firms – is equal to the total amount of paid subsidies to temp workers. This establishes the following proposition:

Proposition 4. *The constrained efficient allocation is implemented with wage subsidies for temp workers financed by a tax on the vacancies posted by TWAs.*

When the constrained efficient allocation is implemented, the labor income gap between in-house and temp workers is equal to

$$w_{hi} - (w_{ai} + \sigma_i) = \eta(c_i - c_a)$$

Thus, we can claim:

Proposition 5. *When the constrained efficient allocation is implemented with subsidies and taxes in the decentralized equilibrium, the wage gap between in-house workers and external workers only depends on the gap between the costs of human resources management of TWAs for temp workers and firms for in-house workers.*

5 Quantitative analysis

5.1 Wage gap

Figure A.1.3 reports the distribution of the average daily wage by markets, respectively for TWA and in-house workers. This corresponds to a wage gap which in the infinite horizon version of the theoretical model is equal to (see (**??**)):

$$w - w_a = \underbrace{\eta(c_a - c)}_{\text{Efficient wage gap}} + \underbrace{\eta(r+q)\frac{rk}{\alpha m(\theta_a)}}_{\text{Inefficiency}}. \tag{19}$$

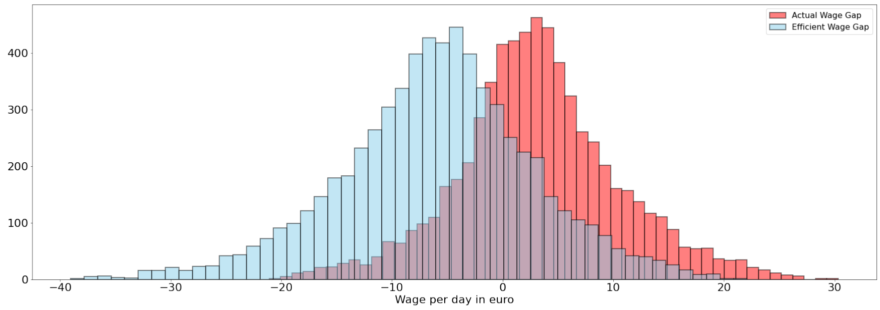
Before moving to a full calibration of the model market by market, we can simply use direct observations on the separation rate q, the job filling rate of TWA workers $\alpha m(\theta_a)$ and the stock of capital k and set the values of η and r to evaluate the size of the inefficiency (see next Section for details on the data). Figure VI reports the distribution of both the actual wage gap $w - w_a$ and the efficient wage gap defined as the difference between the actual wage gap and the inefficiency presented in equation (19).

The distribution of efficient wage gaps is noticeably translated to the left compared to the distribution of actual wage gaps which suggests that the wage of TWA workers is too small compared to the social optimum. In fact, in the majority of markets, the efficient wage gap is negative due to the fact that c_a is smaller than c, i.e. that TWA are more efficient at managing their human resources than the average firm.

5.2 Calibration of the model

We now turn to a full calibration the infinite horizon model. To do so, we gather information on each labor market. As explained in Section 2, a labor market is a combination of an occupation code (we use the 3-digit FAP classification which considers

FIGURE VI. Efficient and actual wage gaps



Notes: Wage gaps are taken as the difference between the average wage of in-house temporary workers (CDD) and TWA workers for each market and are given in euros per day. Efficient wage gap is calculated as the difference between the actual wage gap and the inefficiency defined in equation (19).

86 different categories) and a commuting zone (306 different areas). In what follows, labor markets are indexed by i. To avoid confusion with other indices, we adopt the following convention: we denote X_i^h and X_i^a the value of variable X in a market i respectively for in-house and temporary workers.

Data On top of the administrative data on payroll and firm balance sheet that we presented in Sections 2 and 3, we use data from the French public employment agency ("*Pole Emploi*") and from the fichier ForCE ("*Formation, Chomage, Emploi*") for the year 2019. These datasets allow us to measure the job finding rates $\varphi_i^{(h)}$ and $\varphi_i^{(a)}$ respectively for in-house and TWA workers. They also allow to measure the corresponding job filling rates $\chi_i^{(h)}$ and $\chi_i^{(a)}$ and the total unemployment $\mathcal{U}_i$.

Cleaning and variable construction We use historic public employment services records available in the ForCE dataset to construct the unemployment histories for the universe of French job seekers. Within each job seeker's unemployment history we concatenate spells whose ending and starting date respectively are separated by less than 30 days. After a basic cleaning step where we ensure that all observations have non-missing information on crucial variables (most notably, age, gender, location and search occupation), we drop all job seekers who declare not to be immediately available for a job in public employment services' files. All retained job seekers are thus either looking for a full-time permanent position, a permanent part-time position or a regular definite duration/temporary work agency job. To construct a measure of local unemployment and job finding rates, we first measure the monthly stock of job seekers observed within each commuting zone ("zone d'emploi") × occupation ("Famille

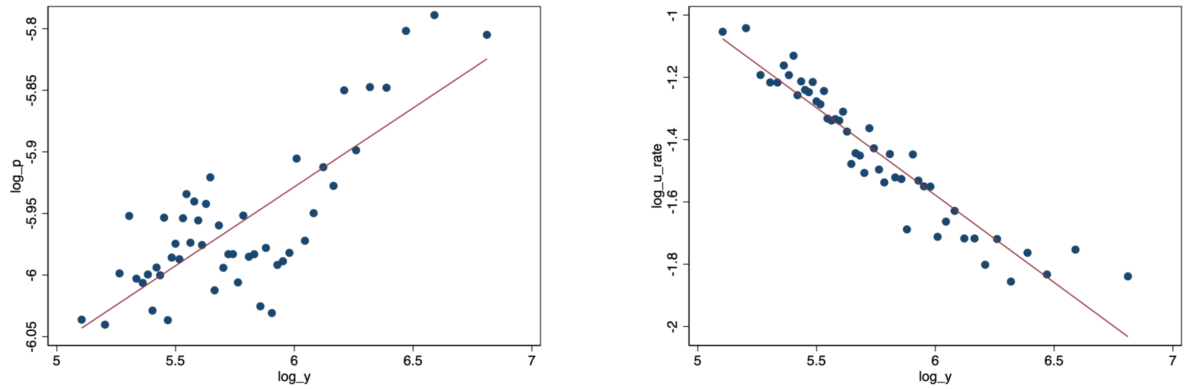
Professionnelle") cell. We average this monthly measure of the stock of registered job seekers over 2019. In a second step we use related exhaustive contract level DSN data ("Déclarations Sociales Nominatives") from the same ForCE datatset to construct the average monthly flow of new contracts in these same commuting zones $\times$ occupations cells. While doing so we distinguish between in-house permanent (CDI), in-house definite duration (CDD) and temporary work agency type contracts (Interim). Combining the average monthly stocks of registered job seekers and the average monthly flows of new contracts pertaining to the same underlying population of job seekers we are able to construct monthly local labor market level job finding rates by contract type. We add to this information exhaustive information on vacancies posted by firms on public employment services' website. Each vacancy contains information on the location, user firm identifier, contract type, posting firm identifier in the case of temporary work arrangement, occupation, as well as a creation and destruction rate. We use vacancy duration at the occupation, commuting zone and contract type level to construct measures of local job filling rates. Finally we use exhaustive data on establishment/individual level employment (DADS Postes) to measure average monthly employment by commuting zone, contract and occupation as well as commuting zone, contract and occupation specific wage premia. For the latter, we use the premia obtained from estimating equation (3) as presented in Section **??**. Given that in this specification we control for firm fixed effects without interacting them with contract type, our market-specific wage gaps still include variation due to the effect of firm characteristics on the relative wage-gap between in-house and TWA workers. Such variation thus incorporates the elements from our model : i) the cost of creation and maintenance of vacant jobs to the user firms, ii) the difference in job-filling rates between regular firms and TWAs, and iii) the difference in HR management cost between regular firms and TWAs. Finally, we recover occupation/commuting zone specific productivity by averaging firm level value added per worker according to each firm's occupation/commuting zone employment share. Figure VIIIa and VIIIb show that our local measure of productivity correlates well with in-house job finding rates (positively) and local unemployment rate (negatively). Overall, we recover and use complete labor market data for 8,508 occupation/commuting zone cells which account for 13 millions individuals (a little over a third of French labor force).

Common parameters. We set two parameters which are common across labor markets. First the elasticity η is set to 0.5 which is a standard value used in the literature (Petrongolo and Pissarides, 2001). Second, the real-interest rate r is set to 0.01337% to

FIGURE VII. Local labor market productivity, job finding and unemployment rate

(a) Correlation between job finding rate and productivity

(b) Correlation between unemployment rate and productivity



Notes: These binned scatter plots represent the raw local labor market level correlation between productivity computed as average value added per worker on the one hand, and the in-house (a) job finding rate and (b) unemployment rate on the other hand.

match a daily value corresponding to a yearly interest rate of 5%.

Market specific inputs In addition to $\varphi_i^{(h)}$, $\varphi_i^{(a)}$, $\chi_i^{(h)}$, $\chi_i^{(a)}$ and u_i, we use market specific values for y_i, k_i, b_i, $w_i^{(h)}$, $w_i^{(a)}$, q_i, $l_i^{(h)}$ and $l_i^{(a)}$. Details are given in Table V.

TABLE V. Market specific inputs

Variable	Description	Source	Average
y	Value added of a worker (in euro per day)	FARE and DADS	362
k	Stock of capital (in euro per day)	Assuming $rk_i = 0.3y_i$	$102/r$
$w^{(h)}$	Average gross daily wage of a in-house worker (in euro)	DADS (see Section 3)	104
$w^{(a)}$	Average gross daily wage of a TWA worker (in euro)	DADS (see Section 3)	101
b	Unemployment compensation (in euro)	$b_i = 0.5 \min\left(w_i^{(h)}, w_i^{(a)}\right)$	49
$l^{(h)}$	Total employment (full time equivalent) of in house workers	DADS	1090
$l^{(a)}$	Total employment (full time equivalent) of in TWA workers	DADS	77
q	Inverse of the average duration of a contract	DADS	0.004 (252 days)

Most of these inputs are directly measured in the data, but some requires additional assumptions. Both wages are estimated using a wage equation which is detailed in Section 3.

y cannot be directly measured at the occupation and commuting zone level. We estimate its value by calculating the average value added per worker for each firm in a sector s located in commuting zone l.[10] We then use the share of occupation k in sector s to input a value of y_i.

[10]We do know the location of the workforce but do not observe the value added at the establishment

Finally, q is estimated by taking the inverse of the average contract duration for all temporary workers (TWA and CDD).

Calibration of other parameters We make two functional form assumptions. First, we assume a matching function can be written as

$$m(\theta) = m_0\theta^{-\eta}$$

where m_0 is a parameter that can vary by market. Second, we assume that the cost of vacancy for in house worker can be written as:

$$C(\mathcal{V}) = \nu_0\mathcal{V}^{\nu},$$

and we thus have $C(\mathcal{V})' = \nu_0\nu\mathcal{V}^{\nu-1}$. Contrary to the matching function, we assume that the cost parameters ν and ν_0 are similar across markets.

The goal of the calibration is therefore to start from a vector of observable

$$\Theta_i = \left(y_i, k_i, w_i^{(h)}, w_i^{(a)}, b_i, l_i^{(h)}, l_i^{(a)}, q_i^{(h)}, q_i^{(a)}, \varphi_i^{(h)}, \varphi_i^{(a)}, \chi_i^{(h)}, \chi_i^{(a)}, \mathcal{U}_i\right)$$

and parameters r and η to estimate a vector of model specific parameters:

$$\Gamma_i = \left(\mathcal{V}_i^{(h)}, \mathcal{V}_i^{(a)}, \mathcal{U}_i^{(a)}, \mathcal{U}_i^{(h)}, m_{0i}, \kappa_i, c_i^{(h)}, c_i^{(a)}, \alpha_i, \theta_i^{(h)}, \theta_i^{(a)}, p_i\right)$$

as well as ν_0 and ν.

The model can be solved recursively in order to estimate Γ_i, ν and ν_0. The procedure is detailed in Appendix B.3

Results The average values of each component of Γ is given in Table VI. The values of ν and ν_0 are respectively 1.73 and 8.46. The average value of α is about 1.5 which means that TWA are 50% more efficient in finding workers, this value is lower than 1 in 24% of the markets.

The ratio of the price charge by the TWA to the firm p and the wage received by the worker w_a is on average equal to 3.4.

level which matters for multi plant firms. When such case arises, we split the value added assuming that each establishment is equally productive.

TABLE VI. Results in decentralized equilibrium

	$\mathcal{V}^{(h)}$	$\mathcal{V}^{(a)}$	m_0	κ	$c^{(h)}$	$c^{(a)}$	α	$\theta^{(h)}$	$\theta^{(a)}$	p
Mean	241	11	0.07	191	7.87	7.91	1.48	0.18	0.12	362
p25	29	1	0.03	135	3.88	3.91	1.05	0.09	0.08	241
p50	75	3	0.05	168	5.70	5.72	1.30	0.13	0.11	308
p75	218	9	0.08	215	10.16	10.16	1.66	0.21	0.16	434

Notes: Average value and first, second and third quartiles of the component of Γ. Averages are weighted by the size of each market (total employment and unemployment). 6,678 markets.

5.3 Constrained efficient equilibrium

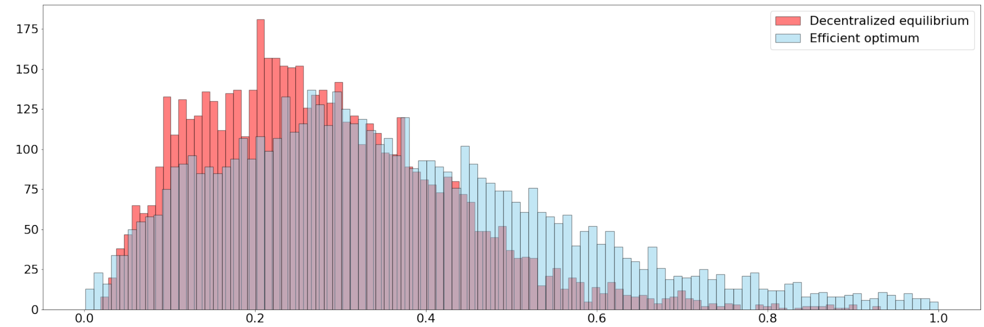
We now use the calibrated parameters from the previous section to calculate the optimal values of the number of vacancies $\mathcal{V}$ and $\mathcal{V}_a$, the labor market tightnesses θ and θ_a, the share of TWA worker γ and unemployment $\mathcal{U}$ for each market as defined by equations (B.37). We solve recursively starting with θ_a, then θ, $\mathcal{V}$, $\mathcal{U}$, γ and finally $\mathcal{V}_a$. Compared to the decentralized equilibrium, the (constrained) social optimum will generally chose higher tightnesses both for in-house and TWA workers and will increase the number of vacancies for in-house workers.

The unemployment rate in each market, both in the social optimal and in the decentralized equilibrium is reported in Figure IX. In most markets, the social planner wants to increase unemployment rate.

Finally, the constrained efficient allocation is characterized by a larger prevalence of job seekers looking for temp jobs. In 90% of the markets, the share of job seekers looking for temp jobs is larger than in the decentralized equilibrium. The average value of $\gamma^\star$ at the constrained efficient solution is 0.47, which is much higher than the unweighted average of 0.08 in the data (see Figure X).

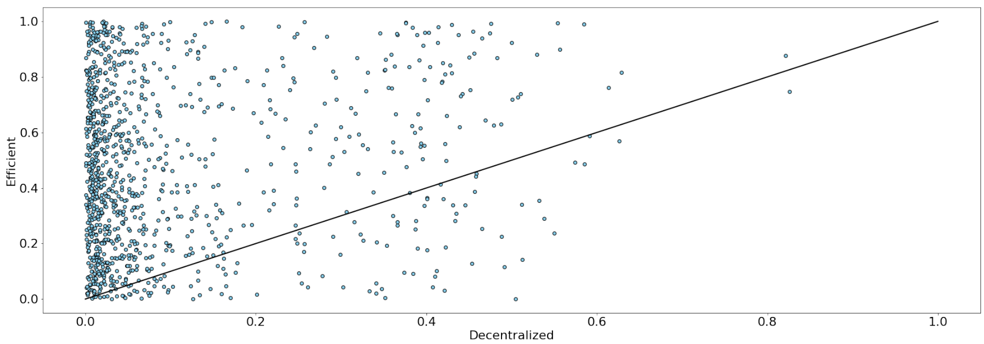
Implementing this equilibrium can be done as described in Section 4.3.3. Using the calibrated values, this implies an average subsidy (σ) of 18.9% of the TWA workers wage.

FIGURE IX. Distribution of unemployment rates in the decentralized equilibrium and the constrained efficient allocation



Notes: unemployment rate is calculated over the total size of the market (unemployed, TWA workers and in-house). Only markets with an optimal unemployment rate between 0 and 1 are kept.

FIGURE X. Share of job seekers looking for temp jobs in the decentralized equilibrium and at the constrained efficient allocation



Notes: only markets with a value of γ between 0 and 1 are kept.

6 Conclusion

TBD

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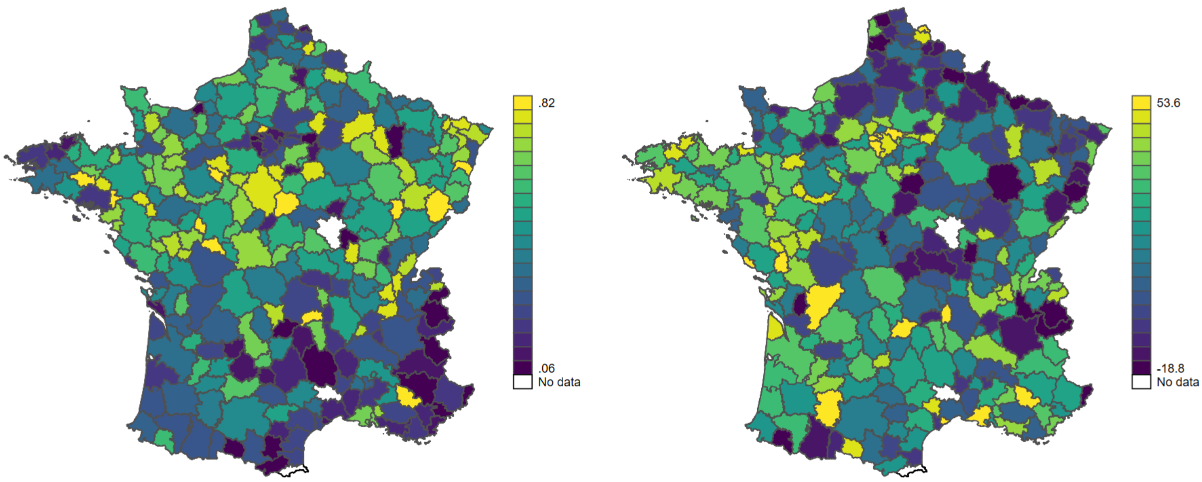
A Empirical Appendix

A.1 TWA workers across labor markets

Figure A.1.2a reports the share of TWA workers over total temporary work contracts (both in house and TWA). We see that there exists some degree of heterogeneity across CZ, however this variance is much lower than across occupation. Considering the 21,307 pairs with at least one temporary contracts[11] we find that an occupation fixed effect explains 42.5% of the variance while a CZ fixed effect only explains 6.4%.

FIGURE A.1.1. Geographical distribution of TWA contracts and wage gap

(a) Share of TWA workers (b) Relative wage gap (%)

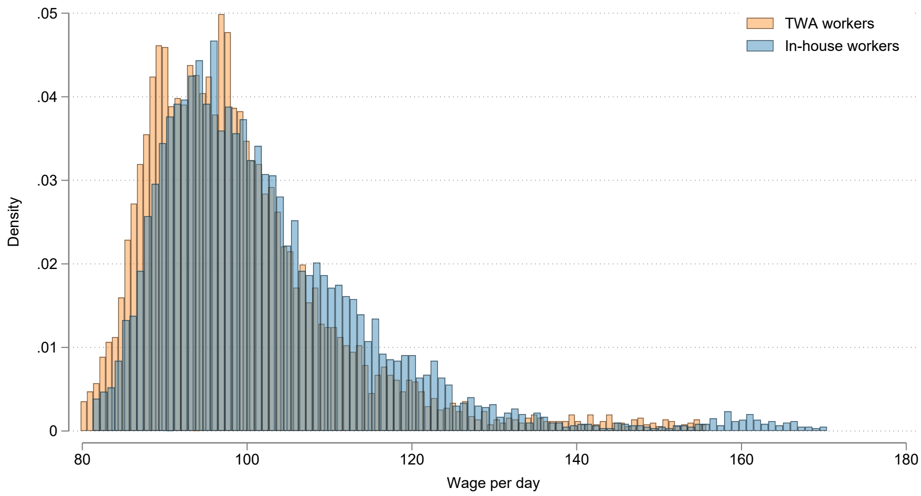


Notes: TWA workers are located using the establishment in which they work (as opposed to the TWA that employ them). The share is calculated using total number of hours worked over all temporary workers (CDD and TWA). The wage gap is calculated as the relative difference between the average hourly wage of TWA workers and the average hourly wage of CDD workers in each CZ. Average 2017-2019.

Figure A.1.2b shows that the relative wage gap - the difference between the average hourly wage of a TWA worker and the average hourly wage of an in-house temporary worker - is also heterogeneous across CZ and is negative for 40% of them. If we consider individual markets, we find a negative wage gap in a little less than half of the cases. Figure A.1.3 plots the entire distributions respectively for in-house and TWA temporary workers. These distributions reports the unconditional average wages which do not take into account the fact that TWA and in-house workers can be different and work in different firms. In Section 3, we investigate further this wage heterogeneity.

[11]There are 84 occupations and 297 CZ which in theory amount to 24,948 possible pairs but some occupations are non-existent in some CZ.

FIGURE A.1.3. Distribution of the wages



Notes: Average wage per day for in-house temporary workers (CDD) and TWA workers respectively by CZ-occupation. Average 2017-2019.

A.2 Data Construction

The most commonly used French administrative employer-employee data is derived from the payroll tax registry called *DADS postes*. In this dataset, firms and their establishments have consistent identifiers across all yearly waves, and can thus be followed over time. On the other hand, workers have anonymized identifiers that connect them to the different jobs that they perform in a given yearly wave, but are changed from one wave to the next precluding the possibility of following workers over a longer period of time. The original work by Abowd et al. (1999), as well as most papers using individual panel data from France, rely on the narrower *DADS panel*. The latter is provided by the French statistical office INSEE and consists in a sub-sample of 1/12th of the French workforce, selected based on their month of birth, that is complemented with individual identifiers that are consistent across years.[12] This dataset is not optimal for performing AKM-type regressions, because it amplifies the issue of the limited connected set.

Nevertheless, each yearly file of the full *DADS postes* contains some information related to the previous year (t-1). In particular, for each job post present in a given year t, we know whether the same individual was already working for the firm the year before, and in that case we know the occupation, the wage, the number of hours worked, the municipality of work and residence, etc. Even for individuals that were working in a different firm at time t-1, we still find the information relative to t-1 in the yearly wave of t, only with missing values for the variables relative to the present (t). This overlap over consecutive years allows for matching between yearly files, based on the common information inserted in the variables at t for the previous year and the variables at t-1 for the consecutive year. The paper by Babet et al. (2022) shows that such procedure gives a single match to 98% of the individuals. The matching cannot be established in the rare cases were several individuals have the exact same information, or were the information for a given individual were corrected from one wave

[12] Before 2002 the sample only included 1/24th of the entire workforce, and was increased to 1/12th from 2002 onward.

to the next. Finally, individuals that go through career breaks that last more than one year cannot be connected, so are identified as different individuals if they reappear later on in the data. We follow the codes made available on the authors' website and construct the full panel for the years 2017-2019, corresponding to the years where we can find information on both the employing firm and the using firms for TWA work arrangements.

We perform some additional cleaning on the data. First of all, we exclude the agricultural sector and the public sector, because of the difference in wage setting procedures, and we restrict the sample to firms belonging to the legal category of "commercial companies". We only keep the 3 most important types of contract : open-ended contracts (CDI), fixed-term contracts (CDD), and TWA contracts (interim), to avoid other minor regimes such as apprenticeships and contracts subsidized by the state to reinsert the long-term unemployed. We further restrict to workers aged 18 to 67 active in Metropolitan France, earning an hourly wage between 10 and 500 Euros and working between 1 and 14 hours per day.

Table A.2.1 reports some summary statistics related to our final data. In total, the DADS postes from 2017 to 2018 report roughly 19 million employees in the private sector. 75% of them are observed in more than one job, and on average we observe 2.7 jobs per worker. 36% of them are observed in more than one firm, and on average we observe 1.35 firms per worker. Finally, 86% of them are observed in more than one year.

TABLE A.2.1. Number of jobs, firms and years observed per worker

	mean	sd	share >1	N. workers
N. jobs	2.74	1.69	75%	19’134’589
N. firms	1.67	1.35	36%	19’134’589
N. years	2.18	0.86	70%	19’134’589

Note: This table reports the average number of distinct jobs that workers have in our sample (defined by the French statistical office as the interaction between a contract, a firm and an occupation), the average number of firms for which they work, and the average number of years in which we observe them, over the period from 2017-2019.

Table A.2.2 further explores the average characteristics of the individuals in the full sample, the sample of workers observed in more than one job, and the sample of workers observed in more than one firm (for TWA workers, we consider the user firm). Workers observed in more than one job are very similar to the average characteristics of the full sample. Workers observed in more than one firm are on average younger, are almost twice more likely to hold TWA contracts over the period, and have slightly lower wages. Finally, workers holding both types of contracts over the three years – TWA and in-house – are younger, more likely to be male, and earn lower wages, in line with the descriptive statistics presented in the main text showing that such contracts are more prevalent among low-skill occupations.

TABLE A.2.2. Characteristics of workers across sample-selections

	all workers	workers > 1 job	workers > 1 firm	workers TWA & in-house contr.
	mean/(sd)	mean/(sd)	mean/(sd)	mean/(sd)
age	38	39	35	32
	(12.52)	(12.04)	(11.67)	(11.00)
share women	0.38	0.36	0.36	0.32
	(0.48)	(0.48)	(0.48)	(0.47)
share TWA contracts	0.15	0.14	0.24	0.47
	(0.33)	(0.29)	(0.35)	(0.19)
wage	19.4	20.2	18.1	13.9
	(15.3)	(15.2)	(12.3)	(3.38)
N. workers	19'134'589	14'341'490	6'925'713	2'101'577

Note: This table reports the average characteristics of workers observed in different sub-samples: those observed in more than one job, those observed in more than one firm, and those observed in both types of contracts: TWA and in-house. Source: DADS postes 2017-2019.

Given that our identification of contract-specific wage premia relies on individuals that move across contracts, firms and markets over the period, Table A.2.2 summarizes how many individuals are concerned by such transitions. About 2 million workers are observed holding both TWA and in-house contracts, which corresponds to 11% of workers in the sample, and 30% of workers that work in more than one firm. The large majority of them transitions also across firms and occupations when changing contract type. This feature ensures that we have enough variation to identify contract-specific wage premia across markets and firms. In our AKM-type analysis we restrict the sample to individuals working in at least 2 user firms, which helps us reducing the data size without influencing the identification of our parameters of interest.

TABLE A.2.3. Number of workers observed under both TWA and in-house contracts

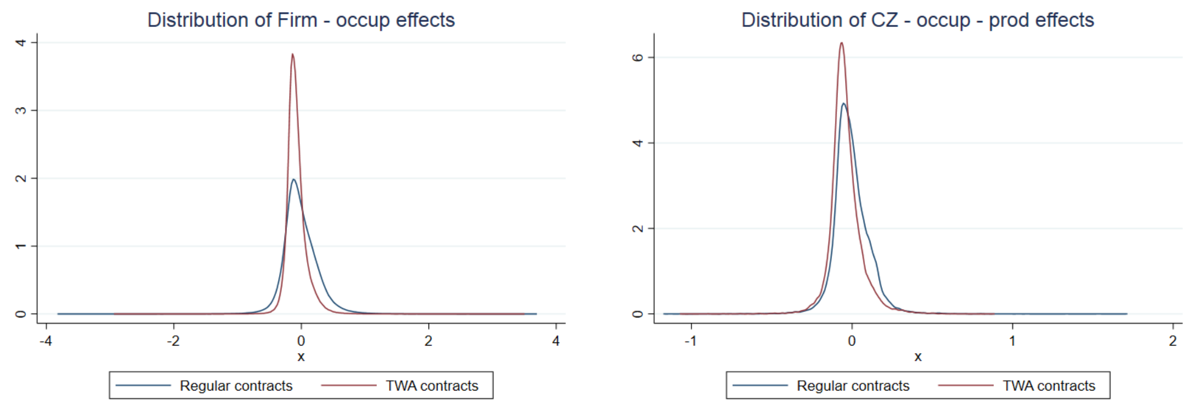
	N. workers	sh. of workers	sh. of workers >1 firm
TWA & in-house	2'101'577	11%	30%
TWA & in-house in diff. firms	1'927'573	10%	28%
TWA & in-house in diff. firms x occup.	2'034'009	11%	29%
TWA & in-house in diff. CZ x occup.	1'926'603	10%	28%

Note: This table reports the number of workers observed across both TWA and in-house contracts, distinguishing between whether the transition happens across firms, occupations and markets. Source: DADS postes 2017-2019.

A.3 Robustness and additional results AKM Analysis

Figure A.3.1 displays the distribution of the occupation $\times$ firm effects and the occupation $\times$ commuting zone $\times$ productivity group effects for each contract type obtained from the estimation of equations 3 and 4 respectively. Beyond the lower average observed within TWA contracts, we also observe a much narrower distribution, revealing that there is also much less variation in TWA workers' pay across firms and markets relative to what we observe among in-house contracts.

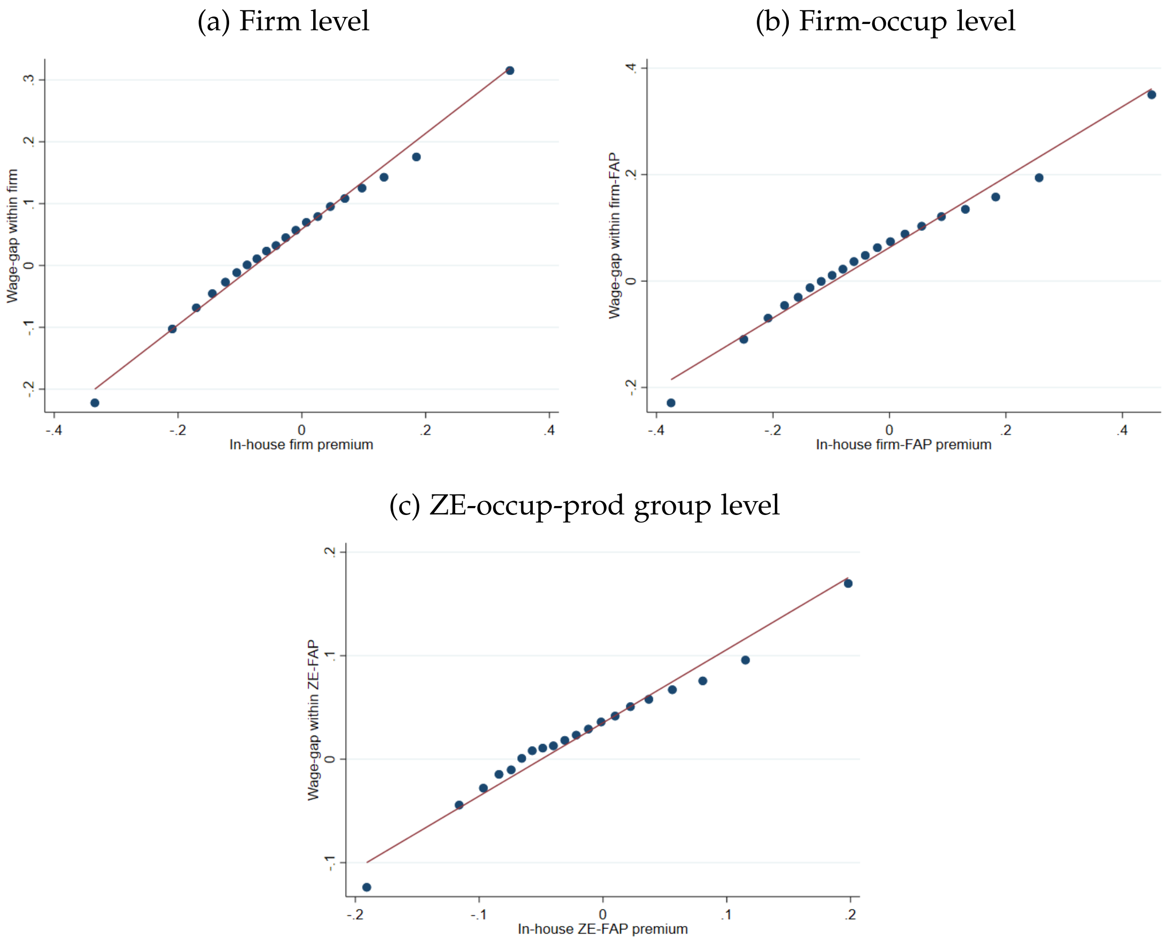
FIGURE A.3.1. Firms' and markets' wage premia by contract type



Note: This graph represents the density of firms' and market wage premia by contract type. Source: DADS POSTES 2017-2019.

figure A.3.3 computes the correlation between the in-house specific premium within a firm and market and the wage-gap between in-house workers and TWA workers. We see that regardless of the level of aggregation taken into account, there is a strong positive correlation between the two.

FIGURE A.3.3. Correlation in-house premium and in-house to TWA wage-gap



Note: This figure correlates the wage gap obtained with $\gamma_x^H - \gamma_x^T$ with the in-house specific premium γ_x^H. Panel a) computes it at the firm level, panel b) at the firm-occupation level, panel c) at the CZ-occupation-productivity group level. Source: DADS POSTES 2017-2019.

Finally, table A.3.1 computes the amount of pass-through obtained from regressing the premium for TWA contracts on the premium for in-house contracts within firms (Column 1), firm-occupations (Column 2), and employment zones-occupations-productivity groups (Column 3). This exercise is similar to the main result shown in Drenik et al. (2020). At the firm-level the pass-through is of 45%, in line with the one found by Drenik et al. (2020) in Argentina (approaching 50%). When we compare within the same occupation and firm we find a slightly higher pass-through than when the occupation dimension is omitted, highlighting the importance of taking this level into consideration. The pass-through obtained at the market level is on the contrary lower, around 24%.

TABLE A.3.1. Wage pass-through by contract type

	(1)	(2)	(3)
VARIABLES	γ_x^H	γ_x^H	γ_x^H
γ_x^T	0.446***	0.534***	0.238***
	(0.00323)	(0.00275)	(0.00520)
$x = f$	yes		
$x = fo$		yes	
$x = zop$			yes
Observations	169,676	171,460	28,023
R-squared	0.101	0.180	0.070

Note: This table reports the result of three separate regressions. Column (1) regresses TWA wage premia within firms on in-house premia. Column (2) does the same but computes the premia within firms and occupations. Column (3) regresses TWA wage premia within markets, as defined by the occupation and commuting zone. The regression sample is restricted to the set of establishments and markets for which we could identify the two types of wage premia. Clustered standard errors (occupation*CZ) are reported in parentheses. Source: DADS POSTES 2017-2019.

B Model Appendix

B.1 Solution of the static directed search model

B.1.1 Properties of the optimal contract between TWAs and user firms

To maximize the value of their job offers, TWAs must offer contracts to user firms that ensure that user firms are not seeking in-house workers for these offers. To ensure that user firms do not look for in-house workers for the vacancies posted at the TWAs, the contract between TWAs and user firms stipulates the payment of a compensation, denoted by ψ, if the user firms cancel their demand to the TWAs. In this context, the expected value of a vacant job of firms which look for in-house workers for their jobs posted at the TWAs is

$$\tilde{V} = -C'(\mathcal{V}) + \max_{w} m(\theta)\left(y - w - c - \psi\right) + V_t$$

The wage offered to in-house workers by firms that also post their vacancies at the TWAs satisfies

$$\tilde{w} = \eta\left(y - b - c - \psi\right) + b \tag{B.20}$$

This equation, compared with equation (7), shows that the wage offered to in-house workers by firms which look for temp workers to fill the same job is lower than that offered by firms which do not simultaneously use both recruitment channels.

To ensure that user firms do not look for in-house workers for the vacancies posted at the TWAs, the compensation ψ has to fulfill the following incentive compatible constraint:

$$V_t \geq \tilde{V} \tag{B.21}$$

In equilibrium, the free entry condition implies that $V_t = 0$ and

$$V = \max_{w} -C'(\mathcal{V}) - k + m(\theta)\left(y - w - c\right) = 0 \tag{B.22}$$

Therefore, we can write $\tilde{V}$ as follows

$$\tilde{V} = -C'(\mathcal{V}) + m(\tilde{\theta})\left(y - \tilde{w} - c - \psi\right)$$

or, using the free entry condition (B.22)

$$\tilde{V} = -m(\theta)\left(y - w - c\right) + m(\tilde{\theta})\left(y - \tilde{w} - c - \psi\right)$$

The incentive compatible constraint (B.21) can be written, using the expressions (7) and (B.20) of offered wages

$$m(\theta)\left(y - b - c\right) \geq m(\tilde{\theta})\left(y - b - c - \psi\right) \tag{B.23}$$

Since the difference $w - \tilde{w}$ is equal to zero when $\psi = 0$ and increases with ψ, the arbitrage equation (6) implies that $\theta = \tilde{\theta}$ if $\psi = 0$ and that $\theta - \tilde{\theta}$ increases with ψ. Since $\theta = \tilde{\theta}$ if $\psi = 0$ and the job filling rate decreases with the labor market tightness, the ratio $m(\theta)/m(\tilde{\theta})$ is equal to one when $\psi = 0$ and increases with ψ. This implies that the incentive compatible constraint is satisfied for any $\psi > 0$.

B.1.2 Constrained efficient solution of the static model

This appendix presents the computation of the solution to the maximization problem which defines the constrained efficient allocation. The social planner solves

$$\max_{(\gamma\in[0,1],\mathcal{V}_{hi}\geq 0,\mathcal{V}_{ai}\geq 0)} \mathcal{U}b + \sum_i x_i F(\mathcal{L}_i) - [\kappa + \alpha m(\theta_{ai})c_a]\,\mathcal{V}_{ai} - C(\mathcal{V}_{hi}) - m(\theta_{hi})c_i\mathcal{V}_{hi} - k_i\,(\mathcal{V}_{hi}+\mathcal{V}_{ai})$$

$$\mathcal{L}_i = \mathcal{V}_{hi}m(\theta_{hi}) + \mathcal{V}_{ai}\alpha m(\theta_{ai})$$

subject to

$$\mathcal{U} = \mathcal{N} - \sum_i \mathcal{L}_i \text{ where } \theta_{ai} = \frac{\alpha\mathcal{V}_{ai}}{\mathcal{N}\gamma_{ai}}; \theta_{hi} = \frac{\mathcal{V}_{hi}}{\mathcal{N}\gamma_{hi}}; \sum_i \gamma_{ai} + \gamma_{hi} = 1$$

Let λ be the multiplier associated with the law of motion of unemployment, χ_i the multiplier associated with the constraint $\mathcal{V}_{ai} \geq 0$ and γ the multiplier associated with the contraint $\sum_i \gamma_{ai} + \gamma_{hi} = 1$. Assuming $C(0) = 0$, $\lim_{\mathcal{V}\to 0^+} C'(\mathcal{V}) = 0$ implies that the optimal value of $\mathcal{V}$ is always positive if $\kappa > 0$ and $y_i - b - c_i > 0$, and therefore that $\gamma_{hi} > 0$. The Lagrangian of the planner's problem is written

$$\begin{aligned} H = {} & \sum_i \mathcal{L}_i y_i + \mathcal{U}b - [\kappa + \alpha m(\theta_{ai})c_a]\,\mathcal{V}_{ai} - C(\mathcal{V}_{hi}) - m(\theta_{hi})c_i\mathcal{V}_{hi} - k_i\,(\mathcal{V}_{hi}+\mathcal{V}_{ai}) + \\ & \lambda\left(\mathcal{N} - \sum_i \mathcal{L}_i - \mathcal{U}\right) + \gamma\left(1 - \sum_i(\gamma_{ai} + \gamma_{hi})\right) + \sum_i \chi_i\mathcal{V}_{ai} \end{aligned}$$

The first order conditions are

$$\begin{aligned} \frac{\partial H}{\partial \mathcal{V}_{hi}} &= (1-\eta)m(\theta_{hi})\,(y_i - c_i) - \left[C'(\mathcal{V}_{hi}) + k_i\right] + \lambda\,(1-\eta)\,m(\theta_{hi}) = 0 \\ \frac{\partial H}{\partial \mathcal{V}_{ai}} &= (1-\eta)\alpha m(\theta_{ai})\,(y_i - c_a) - (\kappa + k_i) + \lambda\alpha m(\theta_{ai})\,(1-\eta) + \chi_i = 0 \\ \frac{\partial H}{\partial \gamma_{hi}} &= \mathcal{N}\eta\theta_{hi}m(\theta_{hi})\,(y_i - \lambda - c_i) - \gamma = 0 \\ \frac{\partial H}{\partial \gamma_{ai}} &= \mathcal{N}\eta\theta_{ai}\alpha m(\theta_{ai})\,(y_i - \lambda - c_a) - \gamma = 0 \\ \frac{\partial H}{\partial \mathcal{U}} &= b - \lambda = 0 \end{aligned}$$

The exclusion relation is

$$\chi_i\mathcal{V}_{ai} = 0$$

Let us first look for an interior solutions $\mathcal{V}_{ai} > 0$. Substituting the value of $\lambda = b$ in the other first-order conditions, yields the values of $(\theta_{hi}, \theta_{ai}, \mathcal{V}_{hi})$ at the constrained efficient allocation

$$\kappa + k_i = \alpha m(\theta^*_{ai})\,(1-\eta)\,(y_i - b - c_a) \tag{B.24}$$

$$\theta^*_{hi} m(\theta^*_{hi})\,(y_i - b - c_i) = \theta^*_{ai'} m(\theta^*_{ai'})\,(y_{i'} - b - c_a)\,; \forall (i, i') \tag{B.25}$$

$$C'(\mathcal{V}^*_{hi}) + k_i = m(\theta^*_{hi})(1-\eta)\,(y_i - b - c_i) \tag{B.26}$$

Equation (B.24) defines a positive value for $m(\theta^*_{ai})$ if and only if $y_i - b - c_a > 0$. This implies that the solution is interior, with $\mathcal{V}_{ai} > 0$, if and only if the surplus of temp jobs to the social planner is positive, i.e. $y_i - b - c_a > 0$. Otherwise, there are in-house workers only.

B.2 Infinite horizon model

This appendix presents the infinite horizon version of the static model presented in the main text.

B.2.1 Framework

We consider the same economy as in the static model presented in the main text except that the horizon is infinite and individuals have an infinite lifespan. Time is continuous. There is a numéraire good an labor. The numéraire good is produced with capital and labor. The labor force is composed of $\mathcal{N}$ risk neutral workers who consume the numéraire good. Firms can create jobs, hire in-house workers and buy the labor services of temp workers hired by the TWAs. The TWAs sell the services of temp workers to firms on a perfectly competitive market.

Firms and TWAs compete in the job market by posting job offers to recruit workers who can seek jobs offered by the firms and the TWAs. There is a set of representative firms indexed by $i = 1, ..., I$. The production function of type-i firms is equal $x_i F(\mathcal{L})$ where $x_i \in [x_{\inf}, +\infty), x_{\inf} > 0$, and $\mathcal{L} \geq 0$ is employment and F satisfies the Inada conditions, i.e. $F' > 0, F'' < 0, \lim_{L \to 0} F'(\mathcal{L}) = +\infty, \lim_{\mathcal{L} \to +\infty} F'(\mathcal{L}) = 0$.

The creation of each job requires to invest $k_i \geq 0$ units of the numéraire good, which yields the marginal product $x_i F'(\mathcal{L}_i)$, once the job is filled. The net marginal production of a job per unit of time, is equal to $y_i = x_i F'(\mathcal{L}_i) - rk_i$.To fill a vacant job, a firm can post a vacancy to hire an in-house worker at marginal cost $C'(\mathcal{V}_{hi})$, where $\mathcal{V}_{hi}$ stands for the number of vacancies posted to hire in-house workers in the firm and $C(\mathcal{V}_{hi})$ is a cost function which satisfies $C(0) = 0$, $C' > 0, C'' > 0, \lim_{\mathcal{V} \to 0} C'(\mathcal{V}) = 0$. The capital k_i is required whether the job is filled of vacant. Filled jobs are destroyed at exogenous Poisson rate q_i.

To fill a vacant job, a firm can post a vacancy to hire an in-house worker at marginal cost $C'(\mathcal{V})$, where $\mathcal{V}$ stands for the number of vacancies posted to hire in-house workers in the firm and $C(\mathcal{V})$ is a cost function which satisfies $C(0) = 0$, $C'(\mathcal{V}) > 0, C''(\mathcal{V}) > 0$. Firms can also rely on the TWAs to fill their jobs. In this case, the cost to the firm of posting its vacancy at the TWA is equal to zero and the firm pays the price p to the TWA if the job is filled with the temp worker.

Each vacancy offers a wage that is not renegotiable. There is a submarket for each wage. Unemployed workers can look for jobs on all submarkets. In each submarket, the number of matches between vacant jobs and unemployed workers is determined by a matching function with constant returns to scale which implies that vacant jobs posted by firms to hire in-house workers are filled at endogenous Poisson rate $m(\theta) > 0$, $m'(\theta) < 0$, $m''(\theta) < 0$, where $\theta \geq 0$ is the labor market tightness equal to the ratio between the number of job vacancies and the number of unemployed workers looking jobs in the submarket. Vacant jobs posted by TWAs cost κ per vacancy to the TWAs and are filled at endogenous rate $\alpha m(\theta) > 0$, where $\alpha > 0$ is a positive scalar to account for the difference in search efficiency between firms and TWAs. Remark that α can be smaller than one if the TWAs are less effective than firms. Another possible difference between firms and TWAs is the cost of human resource management. It is represented by a fixed cost per filled job, denoted by $c_a \geq 0$ for the TWAs and by $c_i \geq 0$ for type-i firms.

B.2.2 Value functions and offered wages

Workers Denoting by W_u the expected value from unemployment and by b the income when unemployed, the arbitrage condition implies that

$$rW_u = b + \theta_i m(\theta_i) (W_i - W_u), \text{ for all } (w_i, \theta_i) \tag{B.27}$$

This equation defines a relation between the utility W and the labor market tigthness in each submarket:

$$\frac{\partial \theta_i}{\partial w_i} = -\frac{\theta_i}{1-\eta} \frac{1}{(W_i - W_u)}, \eta = -\frac{\theta_i m'(\theta_i)}{m(\theta_i)}$$

The labor market tightness decreases with the utility w_{hi} because more unemployed workers are attracted in submarkets in which the promised utility is higher. We have

$$rW_i = w_i + q_i(W_u - W_i)$$

Firms Firms choose the number of in-house and temp job vacancies. They also choose the wage associated with their in-house job offers. Let us denote by $\mathrm{d}t \to 0$ a small interval of time. The value function of type-i firms satisfies

$$\begin{aligned}(1 + r\mathrm{d}t) \Pi_i (\mathcal{L}_i) \quad = \quad & \max_{(\mathcal{V}_{hi} \geq 0, \mathcal{V}_{ai} \geq 0, w)} [x_i F(\mathcal{L}_i) - (w_{hi} + c_i) \mathcal{L}_{hi} - C(\mathcal{V}_{hi}) - rk_i (\mathcal{L}_i + \mathcal{V}_{hi} + \mathcal{V}_{ai}) - p_i \mathcal{L}_{ai}] \mathrm{d}t \\ & + \Pi_i (\mathcal{L}_i^+)\end{aligned}$$

subject to (B.27) and the law of motion of in-house and temp jobs:

$$\begin{aligned}\mathcal{L}_{hi}^+ &= (1 - q_i \mathrm{d}t) \mathcal{L}_{hi} + \mathcal{V}_{hi} m(\theta_{hi}) \mathrm{d}t \\ \mathcal{L}_{ai}^+ &= (1 - q_i \mathrm{d}t) \mathcal{L}_{ai} + \alpha \mathcal{V}_{ai} m(\theta_{ai}) \mathrm{d}t\end{aligned}$$

with

$$\mathcal{L}_i = \mathcal{L}_{hi} + \mathcal{L}_{ai}$$

The offered wage satisfies

$$w_{hi} = \eta (y_i - c_i - rW_u) + rW_u \tag{B.28}$$

where $y_i = x_i F(\mathcal{L}_i) - rk_i$.

The first order conditions yield

$$\begin{aligned} -C'(\mathcal{V}_{hi}) - rk_i + \Pi_i'\left(\mathcal{L}_i^+\right) m(\theta_{hi}) &= 0 \\ -rk_i + \Pi_i'\left(\mathcal{L}_i^+\right) \alpha m(\theta_{ai}) &= 0 \end{aligned}$$

The envelope conditions yield

$$\begin{aligned} (1 + r\mathrm{d}t)\, \Pi_i'(\mathcal{L}_i) &= (y_i - w_{hi} - c_i)\, \mathrm{d}t + (1 - q_i \mathrm{d}t)\, \Pi_i'(\mathcal{L}_i) \\ (1 + r\mathrm{d}t)\, \Pi_i'(\mathcal{L}_i) &= (y_i - p_i)\, \mathrm{d}t + (1 - q_i \mathrm{d}t)\, \Pi_i'(\mathcal{L}_i) \end{aligned}$$

or

$$\Pi_i'(\mathcal{L}_i) = \frac{y_i - w_{hi} - c_i}{r + q_i} = \frac{y_i - p_i}{r + q_i}$$

Thus, the equality between the marginal cost and the marginal return of in-house job vacancies yields

$$C'(\mathcal{V}_{hi}) + rk_i = (1-\eta) m(\theta_{hi}) \frac{y_i - rW_u - c_i}{r + q_i} \tag{B.29}$$

The equality between the marginal cost and the marginal return of temp job vacancies yields

$$rk_i = \alpha m(\theta_{ai}) \frac{y_i - p_i}{r + q_i} \tag{B.30}$$

TWAs The TWAs get the price p per temp worker. The value of vacancies posted by the TWAs satisfies:

$$rV_{ai} = \max_{w_{ai}} -\kappa + \alpha m(\theta_{ai}) \left(\frac{p_i - w_{ai} + c_a}{r + q_i} - V_{ai} \right) \text{ subject to (B.27)}$$

The offered wage satisfies

$$w_{ai} = \eta\left(p_i - c_a - rW_u\right) + rW_u \tag{B.31}$$

B.2.3 Equilibrium with temp and in-house workers

In equilibrium, the free entry condition implies that $V_{ai} = 0$. From the definitions of V_{ai} and w_{ai} we get

$$\frac{\kappa}{\alpha m(\theta_{ai})} = (1-\eta) \left(\frac{p_i - c_a - rW_u}{r + q_i} \right) \tag{B.32}$$

This is the equation of supply of TWAs' vacancies arising from the free entry condition on the market for temp workers.

From the artibrage equation (B.27), the wage equations and the labor demand equations (B.29) and (B.32) we get the relations

$$rW_u = b + \frac{\eta\theta_{ai}}{\alpha(1-\eta)}\kappa \tag{B.33}$$

$$rW_u = b + \frac{\eta\theta_{hi}}{(1-\eta)}\left[C'(\mathcal{V}_i) + rk_i\right] \tag{B.34}$$

From conditions (B.32), (B.30) and (B.33), the equality between the supply and demand of TWAs' vacancies:

$$\frac{\kappa}{\alpha m(\theta_{ai})} = \frac{1-\eta}{r+q_i+\eta\theta_{ai}m(\theta_{ai})}\left(y_i - c_a - b - \frac{r+q_i}{\alpha m(\theta_{ai})}rk_i\right) \tag{B.35}$$

The left hand side is the expected cost of vacancies for the TWA and the right hand side the expected gains from filled vacancies. The arbitrage condition (B.27) defines the relation between the labor market tightness of the submarkets and the number of vacancies for in-house workers.

From the definition (B.34) of the reservation wage rW_u of unemployed workers, the equality between the supply and demand for in-house workers (B.30) can be rewritten as follows:

$$\frac{C'(\mathcal{V}_{hi}) + rk_i}{m(\theta_{hi})} = \frac{1-\eta}{r+q_i+\eta\theta_{hi}m(\theta)}\left(y_i - c_i - b\right) \tag{B.36}$$

The equalization of the reservation wage of unemployed workers defined by equations (B.33) and (B.34) yields, together with equations (B.35) and (B.36) the relation between labor market tighnesses arising from the arbitrage of unemployed workers between submarkets:

$$\frac{\theta_{hi'}m(\theta_{hi'})\left(y_{i'} - c - b\right)}{r+q_i+\eta\theta_{hi'}m(\theta_{hi'})} = \frac{\theta_{ai'}m(\theta_{ai'})}{r+q_i+\eta\theta_{ai'}m(\theta_{ai'})}\left(y_{i'} - c_a - b - \frac{r+q_i}{\alpha m(\theta_{ai'})}rk_{i'}\right), \forall(i,i')$$

Eventually, the equilibrium values of are defined by

$$\begin{aligned}
\frac{\kappa}{\alpha m(\theta_{ai})} &= \frac{1-\eta}{r+q_i+\eta\theta_{ai}m(\theta_{ai})}\left(y_i - c_a - b - \frac{r+q_i}{\alpha m(\theta_{ai})}rk_i\right) \\
\frac{\theta_{hi'}m(\theta_{hi})\left(y_i - c_i - b\right)}{r+q_i+\eta\theta_{hi}m(\theta_{hi})} &= \frac{\theta_{ai'}m(\theta_{ai'})}{r+q_i+\eta\theta_{ai'}m(\theta_{ai'})}\left(y_{i'} - c_a - b\right), \forall(i,i') \\
\frac{C'(\mathcal{V}_{hi}) + rk_i}{m(\theta_{hi})} &= \frac{1-\eta}{r+q_i+\eta\theta_{hi}m(\theta_{hi})}\left(y_i - c_i - b\right)
\end{aligned}$$

and by the equality between the number of entries and exits in each job type:

$$\begin{aligned}
q_i\mathcal{L}_{hi} &= \mathcal{V}_{hi}m(\theta_{hi}) \\
q_i\mathcal{L}_{ai} &= \alpha\mathcal{V}_{ai}m(\theta_{ai})
\end{aligned}$$

The number $\mathcal{U}_{hi}$ of workers looking for type-i in-house jobs and the number $\mathcal{U}_{ai}$ of those looking for type-i temp jobs are defined by

$$\theta_{ai} = \frac{\alpha \mathcal{V}_{ai}}{\mathcal{U}_{ai}}; \theta_{hi} = \frac{\mathcal{V}_{hi}}{\mathcal{U}_{hi}}; \sum_i \mathcal{U}_{ai} + \mathcal{U}_{hi} = \mathcal{U}$$

Unemployment is equal to

$$\mathcal{U} = \mathcal{N} - \sum_i (\mathcal{L}_{hi} + \mathcal{L}_{ai})$$

The wage gap is

$$w_{hi} - w_{ai} = \eta (r + q_i) \left(\frac{rk_i}{\alpha m(\theta_{ai})} + \frac{c_a - c_i}{r + q_i} \right)$$

B.2.4 Constrained efficient solution

The social planner maximizes the discounted net output subject to the law of motion of in-house and temp employment respectively denoted by $\mathcal{L}$ and $\mathcal{L}_a$. The maximization problem which defines the constrained efficient allocation is:

$$\begin{aligned}
\max_{(\mathcal{U}_{hi}, \mathcal{U}_{ai}, \mathcal{V}_{hi}, \mathcal{V}_{ai})} \quad & \int_0^\infty \left[\left(\mathcal{N} - \sum_i \mathcal{L}_i \right) b \right] e^{-rt} \mathrm{d}t + \\
& + \int_0^\infty \left[\sum_i \left[x_i F(\mathcal{L}_i) - \mathcal{L}_{ai} c_a - \mathcal{L}_{hi} c_i - \kappa \mathcal{V}_{ai} - C(\mathcal{V}_{hi}) - rk_i \left(\mathcal{L}_{ai} + \mathcal{L}_{hi} + \mathcal{V}_{hi} + \mathcal{V}_{ai} \right) \right] \right] e^{-rt} \mathrm{d}t
\end{aligned}$$

subject to

$$\begin{aligned}
\dot{\mathcal{L}}_{hi} &= \mathcal{V}_{hi} m(\theta_{hi}) - q_i \mathcal{L}_{hi} \\
\dot{\mathcal{L}}_{ai} &= \alpha \mathcal{V}_{ai} m(\theta_{ai}) - q_i \mathcal{L}_{ai} \\
\mathcal{N} &= \sum_i \mathcal{L}_{ai} + \mathcal{L}_{hi} + \mathcal{U}_{ai} + \mathcal{U}_{hi} \\
\text{where } \theta_{ai} &= \frac{\alpha \mathcal{V}_{ai}}{\mathcal{U}_{ai}}; \theta_{hi} = \frac{\mathcal{V}_{hi}}{\mathcal{U}_{hi}}; \mathcal{L}_i = \mathcal{L}_{ai} + \mathcal{L}_{hi}
\end{aligned}$$

Let λ_{hi} and λ_{ai} be the multiplier associated with the law of motion of in-house and temp employment, χ_{ai} the multiplier associated with the constraint $\mathcal{V}_{ai} \geq 0$ and γ the multiplier associated with the contraint $\mathcal{N} = \sum_i \mathcal{L}_{ai} + \mathcal{L}_{hi} + \mathcal{U}_{ai} + \mathcal{U}_{hi}$.(Assuming $C(0) = 0$, $\lim_{\mathcal{V} \to 0^+} C'(\mathcal{V}) = 0$ implies that the optimal value of $\mathcal{V}$ is always positive if $\kappa > 0$ and $y_i - b - c_i > 0$, and therefore that $\gamma_{hi} > 0$.) The current Hamiltonian of the planner's problem is written

$$
\begin{aligned}
H &= \left(\mathcal{N}-\sum_i \mathcal{L}_i\right) b+\sum_i \left[x_i F(\mathcal{L}_i)-\mathcal{L}_{ai}c_a-\mathcal{L}_{hi}c_i-\kappa\mathcal{V}_{ai}-C(\mathcal{V}_{hi})-rk_i\left(\mathcal{L}_{ai}+\mathcal{L}_{hi}+\mathcal{V}_{hi}+\mathcal{V}_{ai}\right)\right] \\
&\quad +\sum_i \lambda_{hi}\left[\mathcal{V}_{hi}m(\theta_{hi})-q_i\mathcal{L}_{hi}\right]+\sum_i \lambda_{ai}\left[\alpha\mathcal{V}_{ai}m(\theta_{ai})-q_i\mathcal{L}_{ai}\right]+\gamma\left(\mathcal{N}-\sum_i\left(\mathcal{L}_{ai}+\mathcal{L}_{hi}+\mathcal{U}_{ai}+\mathcal{U}_{hi}\right)\right) \\
&\quad +\sum_i \chi_{ai}\mathcal{V}_{ai}
\end{aligned}
$$

The first order conditions are

$$
\begin{aligned}
\frac{\partial H}{\partial \mathcal{V}_{hi}} &= -\left[C'(\mathcal{V}_{hi})+rk_i\right]+\lambda_{hi}\left(1-\eta\right)m(\theta_{hi})=0 \\
\frac{\partial H}{\partial \mathcal{V}_{ai}} &= -\left(\kappa+rk_i\right)+\lambda_{ai}\alpha m(\theta_{ai})\left(1-\eta\right)+\chi_{ai}=0 \\
\frac{\partial H}{\partial \mathcal{U}_{hi}} &= \lambda_{hi}\eta\theta_{hi}m(\theta_{hi})-\gamma=0 \\
\frac{\partial H}{\partial \mathcal{U}_{ai}} &= \lambda_{ai}\eta\alpha\theta_{ai}m(\theta_{ai})-\gamma=0 \\
\frac{\partial H}{\partial \mathcal{L}_{hi}} &= y_i-c_i-b-\lambda_{hi}q_i=r\lambda_{hi} \\
\frac{\partial H}{\partial \mathcal{L}_{ai}} &= y_i-c_a-b-\lambda_{ai}a_i=r\lambda_{ai}
\end{aligned}
$$

and the exclusion relation is:

$$\chi_{ai}\mathcal{V}_{ai}=0$$

Let us first look for an interior solution $\mathcal{V}_{ai}>0$ for all i. We get:

$$
\begin{aligned}
\frac{\partial H}{\partial \mathcal{V}_{hi}} &= 0\Leftrightarrow C'(\mathcal{V}_{hi})+rk_i=\left(1-\eta\right)m(\theta_{hi})\lambda_{hi} \\
\frac{\partial H}{\partial \mathcal{V}_{ai}} &= 0\Leftrightarrow \kappa+rk_i=\lambda_{ai}\alpha m(\theta_{ai})\left(1-\eta\right) \\
\frac{\partial H}{\partial \mathcal{U}_{hi}} &= 0\Leftrightarrow \lambda_{hi}\eta\theta_{hi}m(\theta_{hi})=\gamma \\
\frac{\partial H}{\partial \mathcal{U}_{ai}} &= 0\Leftrightarrow \lambda_{ai}\eta\alpha\theta_{ai}m(\theta_{ai})=\gamma \\
\frac{\partial H}{\partial \mathcal{L}_{hi}} &= r\lambda_{hi}\Leftrightarrow y_i-c_i-b=\left(r+q_i\right)\lambda_{hi} \\
\frac{\partial H}{\partial \mathcal{L}_{ai}} &= r\lambda_{ai}\Leftrightarrow y_i-c_a-b=\left(r+q_i\right)\lambda_{ai}
\end{aligned}
$$

These first order conditions, together with the definition of the labor market tightnesses and the labor market flows equilibrium, yields the system of equations which defines the constrained efficient values of $\theta, \theta_a, \mathcal{V}, \mathcal{V}_a, \gamma, \mathcal{U}$:

$$\begin{aligned}
\frac{\kappa + rk_i}{\alpha m(\theta^*_{ai})} &= \frac{1-\eta}{r+q_i+\eta\theta^*_{ai}m(\theta^*_{ai})}\left(y_i - c_a - b\right) \\
\frac{\theta^*_{hi}m(\theta^*_{hi})\left(y_i - c - b\right)}{r+q_i+\eta\theta^*_{hi}m(\theta^*_{hi})} &= \frac{\theta^*_{ai'}m(\theta^*_{ai'})}{r+q_i+\eta\theta^*_{ai'}m(\theta^*_{ai'})}\left(y_{i'} - c_a - b\right), \forall (i,i') \\
\frac{C'(\mathcal{V}^*_{hi}) + rk_i}{m(\theta^*_{hi})} &= \frac{1-\eta}{r+q_i+\eta\theta^*_{hi}m(\theta^*_{hi})}\left(y_i - c_i - b\right) \\
\theta^*_{ai} &= \frac{\alpha\mathcal{V}^*_{ai}}{\mathcal{U}^*_{ai}}; \theta^*_{hi} = \frac{\mathcal{V}^*_{hi}}{\mathcal{U}^*_{hi}} \\
\mathcal{N} &= \sum_i (\mathcal{L}^*_{ai} + \mathcal{L}^*_{hi} + \mathcal{U}^*_{ai} + \mathcal{U}^*_{hi})
\end{aligned} \tag{B.37}$$

B.2.5 Implementation of the constrained efficient allocation

Let us show that the constrained efficient allocation can be implemented with wage subsidies for temp workers and taxes on the vacancies posted by the TWAs. Let us denote by σ_i the wage subsidy, such that temp workers get the income $w_{ai} + \sigma_i$ when the TWAs pay the wage w_{ai} and by τ_i the tax on each vacancy posted by the TWAs.

Solving the model as before, we get the set of 3 equations which define the equilibrium values of $\theta_a, \theta, \mathcal{V}$:

$$\begin{aligned}
\frac{\kappa + \tau_i}{\alpha m(\theta_{ai})} &= \frac{1-\eta}{r+q_i+\eta\theta_{ai}m(\theta_{ai})}\left(y_i - c_a + \sigma_i - b - \frac{r+q_i}{\alpha m(\theta_{ai})}rk_i\right) \\
\frac{\theta_{hi'}m(\theta_{hi})\left(y_i - c - b\right)}{r+q_i+\eta\theta_{hi}m(\theta_{hi})} &= \frac{\theta_{ai'}m(\theta_{ai'})}{r+q_i+\eta\theta_{ai'}m(\theta_{ai'})}\left(y_{i'} - c_a - b\right), \forall (i,i') \\
\frac{C'(\mathcal{V}_{hi}) + rk_i}{m(\theta_{hi})} &= \frac{1-\eta}{r+q_i+\eta\theta_{hi}m(\theta_{hi})}\left(y_i - c_i - b\right)
\end{aligned}$$

The comparison of this set of equations with those defining the value of those variables at the constrained efficient optimum shows that

$$\begin{aligned}
\sigma_i &= \frac{r+q_i}{\alpha m(\theta^*_{ai})}rk_i \\
\tau_i &= rk_i
\end{aligned}$$

B.3 Details on the quantitative calibration

This section details the calibration of the vector of parameter Γ defined in section 5.2

- We first estimate $\theta_i^{(h)}$ and $\theta_i^{(a)}$ using the fact that

$$\theta_i^{(h)} = \frac{\varphi_i^{(h)}}{\chi_i^{(h)}} \quad \text{and} \quad \frac{\theta_i^{(h)}}{\theta_i^{(a)}} = \left(\frac{\varphi_i^{(h)}}{\varphi_i^{(a)}}\right)^{\frac{1}{1-\eta}}$$

- Using the value of total unemployment $\mathcal{U}_i$, we can estimate $\mathcal{U}_i^{(a)}$ as

$$\mathcal{U}_i^{(a)} = \frac{\mathcal{U}_i^{(h)}}{\frac{l_i^{(h)}}{l_i^{(a)}}\left(\frac{\theta_i^{(h)}}{\theta_i^{(a)}}\right)^{\eta-1} + 1}$$

- From this we can measure $\mathcal{V}_i^{(h)}$, $\mathcal{V}_i^{(a)}$ and α_i

$$\mathcal{V}_i^{(h)} = \theta_i^{(h)}\left(\mathcal{U}_i - \mathcal{U}_i^{(a)}\right) \qquad \alpha_i = \frac{\chi_i^{(h)}}{\chi_i^{(a)}}\left(\frac{\theta_i^{(h)}}{\theta_i^{(a)}}\right)^{-\eta} \qquad \mathcal{V}_i^{(a)} = \frac{\theta_i^{(a)}\mathcal{U}_i^{(a)}}{\alpha_i}$$

- m_{0i} can be estimated using the Beveridge curve

$$m_{0i} = \frac{(l_i^{(h)} + l_i^{(a)})q_i}{\mathcal{U}_i^{(a)}{\theta_i^{(a)}}^{1-\eta} + (\mathcal{U}_i - \mathcal{U}_i^{(a)}){\theta_i^{(h)}}^{1-\eta}}$$

- We can estimate the value of unemployment Wu as:

$$rWu_i = \frac{\varphi_i^{(a)}w_i^{(a)} - \varphi_i^{(h)}w_i^{(h)}}{\varphi_i^{(a)} - \varphi_i^{(h)}}$$

  The values of rWu define "impossible" mixed markets in the sense that $w^{(a)} < rWu$ (there should not be any TWA worker) or $w^{(h)} < rWu$ (there should not be any in-house worker).

- Focusing on the set of markets with at least one in-house worker, according to the above definition, we can calculate

$$(r+q_i)c_i^{(h)} = \frac{y_i - rWu_i}{r+q_i} - \frac{rWu_i - b_i}{\eta m_{0i}{\theta_i^{(h)}}^{1-\eta}}$$

- And focusing on the set of market with TWA workers, we estimate

$$\kappa_i = \alpha_i(rWu_i - b_i)\frac{1-\eta}{\eta\theta_i^{(a)}}$$

and

$$(r+q_i)c_i^{(a)} = \frac{y_i - rWu_i}{r+q_i} - \frac{\kappa_i}{\alpha_i m_{0i} \theta_i^{(a)^{-\eta}}(1-\eta)} - rk_i \frac{r + \alpha_i m_{0i} \theta_i^{(a)^{-\eta}}}{\alpha_i m_{0i} \theta_i^{(a)^{-\eta}}}$$

- We can now calculate the price p_i:

$$p_i = \frac{y_i}{r+q_i} - rk_i \frac{r + \alpha_i m_{0i} \theta_i^{(a)^{-\eta}}}{\alpha_i m_{0i} \theta_i^{(a)^{-\eta}}}$$

Finally, in order to calibrate a value for ν and ν_0 that is common across markets, we use the fact that:

$$C'(\mathcal{V}_i^{(h)}) = rk_i + (1-\eta) \frac{rWu_i}{\eta \theta_i^{(h)}}$$

for all markets i. Taking this relationship in log yields:

$$\log \left(rk_i + (1-\eta) \frac{rWu_i}{\eta \theta_i^{(h)}} \right) = (\nu - 1) \log(\mathcal{V}_i^{(h)}) + \log(\nu_0 \nu),$$

where the right-hand-side can be calculated for all i. We estimate ν and ν_0 using the above equation and an IV estimator, where the value of $\mathcal{V}^{(h)}$ is instrumented by an analog taken from ForCE to mitigate measurement bias (the average number of vacancies per firm for in-house workers).

B.4 Random search model

This appendix presents the random search version of the directed search model presented in the main text. All assumptions are the same as in the directed search version except that search is random and wages are bargained instead of set by firms and TWAs. In this framework, there is only one labor market tightness, instead of different labor market tightenesses for temp and in-house vacancies submarkets. The labor market tightness is defined as

$$\theta = \frac{\alpha \mathcal{V}_a + \mathcal{V}}{\mathcal{N}} \tag{B.38}$$

where $\mathcal{V}_a$ is the number of vacancies posted by the TWAs and $\mathcal{V}$ the number of vacancies posted by firms. $\mathcal{N}$ is the total number of unemployed workers at the beginning of the period.

B.4.1 Value functions and bargaining

Workers. Denoting by W_u the expected value from unemployment and by b the income when unemployed, W_u satisfies the following equation

$$W_u = b + \theta m(\theta) \left[\mu w_a + (1-\mu) w - b\right] \tag{B.39}$$

where

$$\mu = \frac{\alpha \mathcal{V}_a}{\alpha \mathcal{V}_a + \mathcal{V}} \tag{B.40}$$

is the endogenous share of temp workers vacancies in total vacancies.

Firms. The value V of marginal vacancies for in-house workers satisfies

$$V = -C'(\mathcal{V}) - k + m(\theta)\left(y - w - c\right)$$

Wages bargaining implies that workers get the share β of the surplus:

$$w = \beta\left(y - b - c\right) + b \tag{B.41}$$

Therefore we get

$$V = -C'(\mathcal{V}) - k + m(\theta)\left(1-\beta\right)\left(y - b - c\right) \tag{B.42}$$

If the firm relies on the TWAs, it agrees to pay the price p if the job is filled. The value to the firm of vacancies posted at the TWA, denoted by V_t, is

$$V_t = -k + \alpha m(\theta)\left(y - p\right) \tag{B.43}$$

TWAs. The TWAs hire temp workers and gets the price p per temp worker who works in user firms. Temp workers are paid the wage w_a by the TWAs. The value of vacancies posted by the TWAs is

$$V_a = -\kappa + \alpha m(\theta)\left(y - w_a - c_a\right)$$

Wage bargaining implies that

$$w_a = \beta\left(y - b - c_a\right) + b \tag{B.44}$$

Therefore

$$V_a = -\kappa + \alpha m(\theta)(1-\beta)\left(p - c_a - b\right) \tag{B.45}$$

B.4.2 Labor market equilibrium

Equilibrium with temp and in-house workers. In equilibrium, the free entry condition implies that $V_t = V_a = V = 0$.

From $V_a = 0$ we get

$$p = b + c_a + \frac{\kappa}{(1-\beta)\alpha m(\theta)}$$

From $V_t = 0$ we get

$$p = y - \frac{k}{\alpha m(\theta)} \tag{B.46}$$

From this expression of the price p, the equations (B.38) and (B.38) which define the wages of in-house and temp workers, we get the wage gap:

$$w - w_a = \beta\left(\frac{k}{\alpha m(\theta)} + c_a - c\right) \tag{B.47}$$

From $V_t = 0$ and $V_a = 0$ we get

$$\frac{\kappa + (1-\beta)k}{\alpha m(\theta)} = (1-\beta)\left(y - c_a - b\right) \tag{B.48}$$

From $V = 0$ we get

$$\frac{C'(\mathcal{V}) + k}{m(\theta)} = (1-\beta)\left(y - b - c\right) \tag{B.49}$$

The equilibrium values of $\theta, \mathcal{V}, \mathcal{V}_a$,and $\preceq$ are defined by 4 equations: (B.38), (B.40), (B.48) and (B.49), and unemployment is equal to

$$\mathcal{U} = \mathcal{N} - \left(\alpha\mathcal{V}_a + \mathcal{V}\right) m(\theta)$$

Equilibrium with in-house workers only The equilibrium with in-house workers only occurs if

$$p = y - \frac{k}{\alpha m(\theta)} \leq 0$$

The equilibrium values of $\theta, \mathcal{V}$ are defined by two equations:

$$\begin{aligned} \frac{C'(\mathcal{V}) + k}{m(\theta)} &= (1-\beta)(y - c - b) \\ \theta &= \frac{\mathcal{V}}{\mathcal{N}} \end{aligned}$$

Equilibrium with temp workers only. This equillibrium can be neglected assuming that $C(0) = 0$, $\lim_{\mathcal{V}\to 0^+} C'(\mathcal{V}) = 0$ and $y - b - c > 0$.

B.4.3 Constrained efficient solution

The social planner solves

$$\max_{\mathcal{V},\mathcal{V}_a \geq 0} (\mathcal{N} - \mathcal{U})\, y + \mathcal{U}b - [k + \kappa + \alpha m(\theta) c_a]\, \mathcal{V}_a - C(\mathcal{V}) - [k + m(\theta)c]\, \mathcal{V}$$

subject to

$$\mathcal{U} = \mathcal{N} - (\alpha \mathcal{V}_a + \mathcal{V})\, m(\theta) \text{ where } \theta = \frac{\alpha \mathcal{V}_a + \mathcal{V}}{\mathcal{N}}$$

Let λ be the multiplier associated with the constraint and χ the multiplier associated with the constraint $\mathcal{V}_a \geq 0$ (assuming $C(0) = 0$ and $\lim_{\mathcal{V}\to 0^+} C'(\mathcal{V}) = 0$ implies that the optimal value of $\mathcal{V}$ is always positive if $h + \kappa > 0$). The Lagangrian of the planner's problem is written

$$H = (\mathcal{N} - \mathcal{U})\, y + \mathcal{U}b - [k + \kappa + \alpha m(\theta) c_a]\, \mathcal{V}_a - C(\mathcal{V}) - [k + m(\theta)c]\, \mathcal{V} + \lambda\, [\mathcal{N} - (\alpha \mathcal{V}_a + \mathcal{V})\, m(\theta)] + \chi \mathcal{V}_a$$

The first order conditions are

$$\begin{aligned} \frac{\partial H}{\partial \mathcal{V}} &= -C'(\mathcal{V}) - k + \eta \mu m(\theta) c_a - c m(\theta)\, [1 - (1-\mu)\, \eta] - \lambda\, (1-\eta)\, m(\theta) = 0 \\ \frac{\partial H}{\partial \mathcal{V}_a} &= -k - \kappa - c_a \alpha m(\theta)\, (1 - \mu\eta) + c \alpha m(\theta)\, (1-\mu)\, \eta - \lambda\, (1-\eta)\, \alpha m(\theta) + \chi = 0 \end{aligned}$$

For an interior solution $\mathcal{V}_a > 0$ we get the equation which defines the constrained efficient value of the labor market tightness, denoted by θ^*:

$$\frac{C'(\mathcal{V}) + k}{m(\theta^*)} + c = \frac{k + \kappa}{\alpha m(\theta^*)} + c_a \tag{B.50}$$

For the sake of comparison with the decentralized equilibrium, it is useful to write, from equations (B.48) and (B.49), the equation that defines the labor market tightness in decentralized equilibrium as follows:

$$\frac{C'(\mathcal{V})+k}{m(\theta)}+(1-\beta)\,c=\frac{\kappa+(1-\beta)k}{\alpha m(\theta)}+(1-\beta)c_a \tag{B.51}$$