Conditional Choice Probability Estimation of Continuous-Time Job Search Models*

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Abstract

This paper applies the insights of dynamic discrete choice models to continuous-time job search models. The proposed framework incorporates preference shocks into the search framework, resulting in a tight connection between value functions and conditional choice probabilities. Incorporating preference shocks results in constructive identification of all the model parameters. Our method also makes it possible to estimate rich nonstationary job search models in a simple and highly tractable way, without having to solve any differential equations. We apply our method to rich longitudinal data from Hungarian administrative records, allowing for nonstationarities in offer arrival rates, wage offers, and unemployment benefits. Longer unemployment durations are associated with worse wage offers, lower offer arrival rates, and negative selection, all of which result in accepted wages falling over time.

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1 Introduction

This paper applies the insights of dynamic discrete choice models to continuous-time job search models. The idea of our approach is to adapt conditional choice probabilities to a continuous-time job search environment. To do so, our framework incorporates preference shocks into the search framework, resulting in a tight connection between value functions and conditional choice probabilities (henceforth CCP). These preference shocks represent the relative attractiveness of a new job compared to the current state of the individual (employed or unemployed), and affect the instantaneous utility associated with that particular job. As a result, and consistent with recent empirical evidence that workers tend to accept particular job offers with probabilities that are significantly different from zero or one (Krueger and Mueller, 2016, and Wiswall and Zafar, 2018), future job offers associated with particular wages will only be accepted probabilistically from the perspective of the worker.

Our approach has two key advantages. The first one is related to identification. We consider a class of nonstationary job search models that incorporates on-the-job search, non-pecuniary job attributes, and involuntary wage transitions. We establish *constructive* identification of all of the model parameters, up to the discount rate. In particular, and in contrast with the non-identification result of Flinn and Heckman (1982), we are able to separately identify the offered wage distribution both from employment and unemployment—the latter allowed to vary over time—without having to assume recoverability of the underlying distribution. Key to our identification strategy is the existence of preference shocks, that allow us to express the employment and unemployment value functions as functions of the conditional choice probabilities. Under this framework, we derive closed-form expressions for most of the model parameters where the expressions depend on the hazard rates associated with the different types of labor market transitions.

The second key advantage is computational. While the empirical labor search literature has been rapidly growing over the last few years, structural estimation of these models often remains challenging. This is particularly true for models in nonstationary environments, which tend to be the norm rather than the exception in the context of job search (van den Berg, 2001, 1990, Cahuc, Carcillo, and Zylberberg, 2014). We provide a novel empirical framework that makes it possible to estimate nonstationary job search models in a simple, tractable, and transparent way.

We apply our method using rich longitudinal administrative data from Hungary. The dataset consists of half of the population, i.e., 4.6 million individuals, who are linked across 900 thousand firms. An important feature of the Hungarian data is that individuals are observed on a monthly basis, making it possible to follow the labor force transitions at a high frequency. In practice we consider a flexible parametric specification that allows for unobserved heterogeneity through worker types, and devise a sequential estimation procedure that builds on the insights of Arcidiacono and Miller (2011) but applies them to a continuous-time search environment.

The data reveal sharp decreases over time in accepted wages out of unemployment. Among those who find a job before benefit expiration, those with the shortest 25% of unemployment durations were a little over half as likely to exit to a minimum wage job than those with the longest 25% of unemployment durations. Estimates of the model show that this in part is the result of the wage offer distribution becoming worse as unemployment duration increases. With the offer arrival rate also declining as unemployment duration increases, workers become increasingly less selective in which jobs they are willing to accept. The decline in accepted wages is then a result both of facing worse wage offer distributions but also changes in the job acceptance rate. These results show that nonstationarities along multiple dimensions play a central role in describing the search environment over the course of unemployment.

This paper fits into several literatures. First, it contributes to the literature on the estimation of dynamic discrete choice models using conditional choice probabilities. Since the seminal work of Hotz and Miller (1993), CCP methods have been increasingly used as a way to estimate complex dynamic discrete choice models at a limited computational cost (see surveys by Aguirregabiria and Mira, 2010 and Arcidiacono and Ellickson, 2011). While CCP methods have been used a variety of settings, they have been mostly used in a discrete time environment. Two recent exceptions are Arcidiacono, Bayer, Blevins, and Ellickson (2016) and Agarwal, Ashlagi, Rees, Somaini, and Waldinger (2021), who apply CCP methods to estimate dynamic equilibrium models of market competition and an equilibrium model of kidney allocations, respectively. CCP methods are also generally used to estimate dynamic discrete choice models in the absence of search frictions, an exception being recent work by Ransom (2021). We contribute to this literature by exploring the use of CCP methods to identify and estimate job search models in continuous time.[1]

This paper also contributes to the empirical job search literature. Since the seminal work of Flinn and Heckman (1982), a large number of papers have structurally estimated various types of job search models (see Eckstein and van den Berg, 2007 for a survey). In this literature, structural parameters are generally estimated via maximum likelihood or indirect inference methods, where the full model needs to be solved within the estimation procedure and often have to incorporate a strict cutoff for whether the offer exceeds the reservation wage. Nonstationarity in job search, which arises in particular when the level of unemployment benefits varies over the unemployment spell, is an important case where the computational demands are especially high. Since the seminal work of van den Berg (1990) who structurally estimated a continuous-time nonstationary search model,[2] examples of structural estimates of nonstationary job search models remain scarce, in part because of the computational burden involved. Important exceptions include Cockx, Dejemeppe, Launov, and Van der Linden (2018), Launov and Walde (2013), Robin (2011), Lollivier and Rioux (2010), Paserman (2008), and Frijters and van der Klaauw (2006).

We contribute to this literature by providing a novel empirical framework, based on a constructive identification strategy, that makes it possible to estimate a rich class of nonstationary job search models in a simple and tractable way (see French and Taber, 2011 for an overview of the literature on the identification of search models). Key to our identification strategy is the existence of preference shocks and, in that sense, our approach is similar in spirit to Sorkin (2018). Our paper also complements recent work by Sullivan and To (2014) and Taber and Vejlin (2020) who consider the identification of search models that allow for non-pecuniary job attributes. In contrast to these papers, we consider a nonstationary environment and show that virtually all of the parameters are obtained as analytical expressions of the underlying hazard rates. On the other hand, an important difference with Taber and Vejlin (2020) is

[1] See also Llull and Miller (2018), who make use of CCP methods to estimate a stationary continuous-time job search model in the context of internal migration in Spain.

[2] See also Wolpin (1987), which is the first study to estimate a (discrete time) nonstationary search model.

that they consider an equilibrium search framework, while our framework is partial equilibrium.

Finally, our application fits into the vast and growing empirical literature that investigates the impact of unemployment benefit levels and duration on labor supply (see, e.g., Johnston and Mas, 2018, Nekoei and Weber, 2017, Le Barbanchon, Rathelot, and Roulet, 2017, Lollivier and Rioux, 2010, Card, Chetty, and Weber, 2007, van den Berg, 1990, and Schmieder and von Wachter, 2016 and Krueger and Meyer, 2002 for overviews of this literature). Consistent with many of these earlier studies, our estimation results provide evidence that nonstationarity plays an important role in describing the search environment over the course of the unemployment spell. A central feature of our empirical strategy is that it leverages the direct links that exist between reduced form hazard rates from unemployment to employment, or from one job to another, and the structural parameters of the model. Beyond the specific application we consider in this paper, a similar approach can be readily used to identify and estimate other types of search models (see Gyetvai, 2021, for an application to occupational mobility).

The rest of the paper is structured as follows. In Section 2, we introduce and discuss the general setup of the nonstationary search model we consider throughout the paper. Section 3 shows identification of the model parameters. In Section 4 we discuss the data used to estimate the model. Section 5 presents our estimation procedure, with Section 6 discussing the estimation results. Section 7 concludes.

2 Model

2.1 The environment

Consider an economy in continuous time with infinitely lived workers, who discount the future at a rate $\rho > 0$. Both employed and unemployed workers are looking for jobs. Job offers are characterized by a wage, w, and a job type, s. Job types capture non-wage characteristics such as firm, occupation, or industry, or any other compensating differentials. The distribution of wages and job types are assumed to be discrete with W and S support points. Denoting $\underline{w}$ and $\overline{w}$ as the minimum and maximum wages, the support for wages and job types is given by $\Omega_w = \{\underline{w}, \ldots, \overline{w}\}$ and $\Omega_s = \{1, \ldots, S\}$, respectively. Conditional on receiving an offer from a particular job type s, the offered wage distributions depend on whether or not one is currently employed and, if not employed, the duration of unemployment, t. The probability mass functions for wage w are given by f_w^s for the employed and $g_w^s(t)$ for the unemployed.

We model job offer arrivals from the different job types as Poisson processes, and allow employed and unemployed workers to sample jobs at different frequencies. While working at a job of type s, the offer arrival rate for jobs of type s' is given by $\lambda^{ss'}$. The offer arrival rate for the unemployed for type-s jobs may vary with the duration of the unemployment spell, t, and is given by $\lambda^s(t)$. Unemployed workers also receive benefits, $b(t)$, that depend on the duration of the spell.[3] The wage offer distribution ($g_w^s(t)$), the unemployed offer arrival rates ($\lambda^s(t)$), and the benefits ($b(t)$) are the three sources of nonstationarity in this model.

While this model shares many of the features of the continuous-time job search models that have been estimated in the literature, a key distinction is that it incorporates preference shocks into the search framework. This feature is instrumental to our approach as it makes it possible to connect the value functions of unemployment and employment to the conditional choice probabilities. Specifically, job offers are associated with a wage and a job type, but also with a preference shock. This preference shock, ε, is drawn independently whenever a new job offer arrives. The preference shock represents the relative attractiveness of a new job compared to the current state of the individual (employed or unemployed), and is supposed to affect the instantaneous utility.

2.2 Value of employment

The flow payoff of working is assumed to be the sum of two parts: the utility of the wage paid, u_w, and the non-pecuniary payoff of working in a job of type s, ϕ^s. Without loss of generality, we normalize $\phi^1 = 0$. Workers employed in a job (w, s)

[3] In practice, following much of the search literature, we treat unemployment and non-participation as a single state.

can experience three different types of transitions. First, they may be laid off and become unemployed, which happens at a rate δ_0^s.[4] Second, within the same firm, they may exogenously transition to a different wage w' and job type s'. These involuntary within-firm changes occur at a rate $\delta_{ww'}^{ss'}$, with the convention that $\delta_{ww}^{ss} = 0$. Third, workers may receive an offer from another firm for a job of type s' at a rate $\lambda^{ss'}$ and then decide whether to accept it or stay with their current job. These voluntary transitions are associated with an instantaneous cost of switching jobs, $c^{ss'}$, and we assume that the switching costs are symmetric ($c^{ss'} = c^{s's}$ for all s, s'). These cross-firm transitions occur both between ($s' \neq s$) and within ($s' = s$) job types.

We now turn to the value of employment, V_w^s. The Bellman equation in this case writes:

$$\begin{aligned}\left(\rho + \delta_0^s + \sum_{w'}\sum_{s'}\delta_{ww'}^{ss'} + \sum_{s'}\lambda^{ss'}\right)V_w^s &= u_w + \phi^s + \delta_0^s V_0(0) + \sum_{w'}\sum_{s'}\delta_{ww'}^{ss'}V_{w'}^{s'} \\ &\quad + \sum_{w'}\sum_{s'}\lambda^{ss'} f_{w'}^{s'}\mathbb{E}_\varepsilon \max\left\{V_{w'}^{s'} - c^{ss'} + \varepsilon, V_w^s\right\}\end{aligned} \tag{2.1}$$

where $V_0(0)$ is the value of unemployment immediately upon entering an unemployment spell ($t = 0$). Following Arcidiacono and Miller (2011) and Arcidiacono et al. (2016), we can re-express Equation 2.1 such that some the value functions on the right-hand side are eliminated. Assuming that the shocks ε are drawn from a logistic distribution, we can rewrite Equation 2.1 as:

$$\begin{aligned}\left(\rho + \delta_0^s + \sum_{w'}\sum_{s'}\delta_{ww'}^{ss'}\right)V_w^s &= u_w + \phi^s + \delta_0^s V_0(0) + \sum_{w'}\sum_{s'}\delta_{ww'}^{ss'}V_{w'}^{s'} \\ &\quad - \sum_{w'}\sum_{s'}\lambda^{ss'} f_{w'}^{s'} \ln\left(1 - p_{ww'}^{ss'}\right)\end{aligned} \tag{2.2}$$

where $p_{ww'}^{ss'}$ denotes the probability of accepting a new job offer of type s' at w' given the current job type s and wage rate w.

Prior to the realization of ε, the probability of a job of type s' paying w' being accepted

[4] Our identification strategy would also readily apply to a more general setup where transitions to unemployment are allowed to be wage-specific. For simplicity, we focus on the case where these transition rates depend on job types only.

given current job type s paying w is then:

$$p_{ww'}^{ss'} = \frac{\exp\left(V_{w'}^{s'} - c^{ss'}\right)}{\exp\left(V_w^s\right) + \exp\left(V_{w'}^{s'} - c^{ss'}\right)} \tag{2.3}$$

2.3 Value of unemployment

We now write the problem of the unemployed individuals. Indexing by t time spent unemployed, we first write the Bellman equation for the unemployment value function $V_0(t)$ in discrete time:[5]

$$\begin{aligned} V_0(t) &= b(t)\Delta t + \frac{\Delta t}{1+\rho\Delta t}\sum_w\sum_s \lambda^s(t) g_w^s(t+\Delta t)\mathbb{E}_\varepsilon \max\left\{V_w^s + \varepsilon, V_0(t+\Delta t)\right\} \\ &\quad + \frac{1-\sum_s \lambda^s(t)\Delta t}{1+\rho\Delta t} V_0(t+\Delta t) \end{aligned}$$

where Δt denotes the discrete time unit and where the equation can be rewritten as:

$$\rho V_0(t) = b(t)(1+\rho\Delta t) + \sum_w\sum_s \lambda^s(t) g_w^s(t+\Delta t)\mathbb{E}_\varepsilon \max\left\{V_w^s - V_0(t+\Delta t) + \varepsilon, 0\right\} + \frac{V_0(t+\Delta t) - V_0(t)}{\Delta t}$$

Next, letting $\Delta t \to 0$, and denoting by $\dot{V}_0(t)$ the derivative of the $V_0(t)$ with respect to unemployment duration and by $p_w^s(t)$ the probability of accepting a job offer of type s and wage w at time t, we obtain the following differential equation in $V_0(\cdot)$:

$$\rho V_0(t) = b(t) - \sum_w\sum_s \lambda^s(t) g_w^s(t) \ln\left(1 - p_w^s(t)\right) + \dot{V}_0(t) \tag{2.4}$$

A couple of remarks are in order. First, Equation 2.4 now involves the derivative of the value of unemployment with respect to duration of unemployment, $\dot{V}_0(t)$. This term represents the change in the option value of job search due to variation over time in the value of unemployment. In the particular case where nonstationarity arises because of over-time changes in the level of unemployment benefits, the option value of searching for a job will decrease as job seekers get closer to the unemployment benefit expiration date.

[5] Note that we rely here on the implicit normalization of a zero switching cost from unemployment to employment. This is an innocuous normalization as the flow utility of unemployment, $b(t)$, can be rewritten wlog. as $b(t) + (\rho+\delta)c_0$, where c_0 denotes the switching cost out of unemployment.

Second, Equation 2.4 is a simple linear first-order differential equation in $V_0(\cdot)$, which admits an exact analytical solution as a function of the structural parameters and the conditional choice probabilities $p_w^s(t)$.[6] The existence of preference shocks ε is key here. Absent these shocks, $V_0(t)$ would satisfy instead the following nonlinear differential equation:

$$\rho V_0(t) = b(t) + \sum_s \sum_w \lambda^s(t) g_w^s(t) \max\left\{V_w^s - V_0(t), 0\right\} + \dot{V}_0(t)$$

This is a highly non-trivial differential equation which would need to be solved numerically, similar to van den Berg (1990) in a simpler context without on-the-job search.

3 Identification

We have shown in the previous section that the unemployment and employment value functions can be expressed as a function of the structural parameters of the model, the wage offer distributions, and the conditional job acceptance probabilities. There are two fundamental differences compared to a Hotz-Miller type approach for dynamic discrete choice models. First, in a search environment, choices (i.e., job offer acceptance or rejection) are generally not observed by the analyst. Second, wage offers are generally unobserved as well. Nonetheless, we provide in the following a simple constructive identification strategy for the parameters of the job search model introduced in Section 2. These results hold in a standard empirical setting where one has access to longitudinal data on (i) across-firm job-to-job transitions, (ii) within-firm transitions, (iii) transitions from unemployment to employment, and (iv) transitions from employment to unemployment.

Note that we assume throughout that wages are drawn from a discrete distribution with finite support. This distribution can be thought of as a discrete approximation to the underlying continuous wage distribution. We maintain this assumption throughout our analysis for simplicity, but our constructive identification strategy

[6] As we discuss in the section below, $V_0(t)$ and $\dot{V}_0(t)$ can be directly identified (and estimated) from the log-odds ratios out of unemployment, without having to solve any differential equation.

readily applies to the case of continuous wage distributions.[7]

3.1 Assumptions

We first introduce four assumptions that relate to the types of transitions that are observed in the data. Namely, we denote by A1, A2, A3 and A4, respectively, the assumptions that the following hazard rates are identified from the data:

A1 $h_{ww'}^{ss'}$, the hazard rate of moving from a job with wage w and type s to a job with wage w' and type s' (in a different firm);

A2 $h_w^s(t)$, the hazard rate out of unemployment at time t to a job that pays w and is of type s;

A3 $\delta_{ww'}^{ss'}$, the hazard rate of within-firm wage and type changes;

A4 δ_0^s, the hazard rate from a type-s job to unemployment.

As is standard for this class of models, we also maintain the assumption that the discount rate ρ is known.

The involuntary within-job changes ($\delta_{ww'}^{ss'}$) and transitions to unemployment (δ_0^s) are identified directly from the data. We next show that these rates, coupled with the across-firm hazards and transitions out of unemployment, can be used to recover closed-form solutions for the employed and unemployed wage offer distributions (f_w^s and $g_w^s(t)$); the pecuniary and non-pecuniary payoffs of the job (u_w and ϕ^s) each up to a constant; the cost of switching jobs ($c^{ss'}$); the arrival rates for those who are employed and unemployed ($\lambda^{ss'}$ and $\lambda^s(t)$); and unemployment benefits ($b(t)$). All of our identification results are subject to the model specification given in Section 2.

3.2 Identification of the employed-side parameters

We begin by showing identification of the employed wage offer distributions for each job type, f_w^s, proving the following lemma:

[7]Specifically, the hazard rates associated with the transitions to and from any given pair of wages (w, w') are directly identified from the data when wages are continuously distributed. Such hazard rates are also known in the statistical literature as the conditional mark-specific hazard function (see, e.g., Sun et al., 2009, Equation (1) p.395).

Lemma 1 *Assumption A1 is sufficient to identify f_w^s. Further, f_w^s has the following solution:*

$$f_w^s = \frac{h_{ww}^{ss}}{\sum_{w'} h_{w'w'}^{ss}} \tag{3.1}$$

To prove the lemma, first note that the hazard $h_{ww'}^{ss'}$ can be expressed as the product of (i) the arrival rate of offers to job type s' given the current job type is s ($\lambda^{ss'}$), (ii) the pmf of w' for offered wages in job type s' ($f_{w'}^{s'}$), and (iii) the probability of accepting a job of type s' paying wage w' given current job type s and wage w' ($p_{ww'}^{ss'}$):

$$h_{ww'}^{ss'} = \lambda^{ss'} f_{w'}^{s'} p_{ww'}^{ss'} \tag{3.2}$$

Now consider the hazard rate to a job that is of the same type and pays the same amount as the current job (h_{ww}^{ss}). From Equation 2.3, the probability of accepting a job in this case does not depend on w: $p_{ww}^{ss} = p_{w'w'}^{ss}$ for all $\{w, w'\}$. That is, since the wage and job type are the same, so too are the value functions. Hence when the transitions are to same-type and same-pay jobs, the ratio of the hazards for two different initial wages is the ratio of the pmfs for the two wages:

$$\frac{f_w^s}{f_{w'}^s} = \frac{h_{ww}^{ss}}{h_{w'w'}^{ss}}$$

Summing over w' then gives the result:

$$f_w^s = \frac{h_{ww}^{ss}}{\sum_{w'} h_{w'w'}^{ss}}$$

Next consider identification of the employed offer arrival rates ($\lambda^{ss'}$) which then immediately leads to identification of the conditional choice probabilities and switching costs. Closed form solutions exist for each of these as well:

Lemma 2 *(i) A sufficient condition for Assumption A1 to identify λ^{ss} is that there exists a triple $(w, w', \tilde{w}) \in \Omega_w^3$ such that $f_{\tilde{w}}^s h_{ww'}^{ss} h_{w'w}^{ss} \neq f_{w'}^s h_{\tilde{w}w}^s h_{w\tilde{w}}^{ss}$.*

(ii) For $x \in \{w', \tilde{w}\}$, let $A_x = f_x^{s'} f_x^s h_{ww}^{ss'} h_{ww}^{s's} - f_w^{s'} f_w^s h_{xx}^{ss'} h_{xx}^{s's}$, $B_x = f_x^{s'} h_{xx}^{s's} h_{ww}^{ss'} h_{ww}^{s's} - f_w^{s'} h_{ww}^{s's} h_{xx}^{ss'} h_{xx}^{s's}$, and $C_x = f_w^s h_{xx}^{ss'} h_{ww}^{ss'} h_{ww}^{s's} - f_w^s h_{ww}^{ss'} h_{xx}^{ss'} h_{xx}^{s's}$. A sufficient condition for Assumption A1 to identify $\lambda^{ss'}$ for $s \neq s'$ is that there exists a triple $(w, w', \tilde{w}) \in \Omega_w^3$ such that:

(a) $A_{w'} \neq 0$

(b) $B_{w'} A_{\tilde{w}} - B_{\tilde{w}} A_{w'} \neq 0$

(c) $A_{w'} C_{\tilde{w}} - A_{\tilde{w}} C_{w'} \neq 0$

Further, when these conditions are met, there are closed-form expressions for $\lambda^{ss'}$*,* $c^{ss'}$*,* p^s_w*, and* V^s_w*.*

We show identification and the closed-form expression for λ^{ss} in the text with the corresponding proof for $\lambda^{ss'}$ given in Appendix A.1.1. To begin, note that the distributional assumption on the preference shocks ε yields a simple relationship between probabilities of accepting a new job offer, the employment value functions, and the switching cost:

$$\ln\left(\frac{p^{ss}_{ww'}}{1 - p^{ss}_{ww'}}\right) = V^s_{w'} - c^{ss} - V^s_w \tag{3.3}$$

implying:

$$\ln\left(\frac{p^{ss}_{ww'}}{1 - p^{ss}_{ww'}}\right) + \ln\left(\frac{p^{ss}_{w'w}}{1 - p^{ss}_{w'w}}\right) = -2c^{ss} \tag{3.4}$$

Solving Equation 3.2 for $p^{ss}_{ww'}$,

$$p^{ss}_{ww'} = \frac{h^{ss}_{ww'}}{\lambda^{ss} f^s_{w'}} \tag{3.5}$$

it follows that, for any given triple $(w, w', \tilde{w}) \in \Omega^3_w$:

$$\ln\left(\frac{h^{ss}_{ww'}}{\lambda^{ss} f^s_{w'} - h^{ss}_{ww'}}\right) + \ln\left(\frac{h^{ss}_{w'w}}{\lambda^{ss} f^s_{w} - h^{ss}_{w'w}}\right) = \ln\left(\frac{h^{ss}_{w\tilde{w}}}{\lambda^{ss} f^s_{\tilde{w}} - h^{ss}_{w\tilde{w}}}\right) + \ln\left(\frac{h^{ss}_{\tilde{w}w}}{\lambda^{ss} f^s_{w} - h^{ss}_{\tilde{w}w}}\right) \tag{3.6}$$

Solving for λ^{ss} under the assumption that $f^s_{\tilde{w}} h^s_{ww'} h^s_{w'w} \neq f^s_{w'} h^s_{w\tilde{w}} h^s_{\tilde{w}w}$—a condition that can be verified in the data—gives the result:

$$\lambda^{ss} = \frac{\left(f^s_w h^s_{w\tilde{w}} + f^s_{\tilde{w}} h^s_{\tilde{w}w}\right) h^s_{ww'} h^s_{w'w} + \left(f^s_w h^s_{ww'} + f^s_{w'} h^s_{w'w}\right) h^s_{w\tilde{w}} h^s_{\tilde{w}w}}{f^s_w f^s_{\tilde{w}} h^s_{ww'} h^s_{w'w} - f^s_w f^s_{w'} h^s_{w\tilde{w}} h^s_{\tilde{w}w}}$$

Given the solutions for f^s_w and λ^{ss}, closed-form expressions for p^{ss}_{ww} and c^{ss} then immediately result from Equations 3.5 and 3.4, as does the difference in value functions $V^s_{w'} - V^s_w$ from Equation 3.3.

Lemma 3 below states our main identification result for the final set of employed-side parameters, namely the utility of wages, u_w, and the non-pecuniary payoff of working in a job of type s, ϕ^s.

Lemma 3 *Given Assumptions A1, A3, and A4:*

(i) u_w *is identified up to a constant and has a closed-form expression.*

(ii) When workers have CRRA preferences so that $u_w = \frac{\alpha w^{1-\theta}}{1-\theta}$, *both* α *and* θ *are identified.*

(iii) Given the normalization $\phi^1 = 0$, *the non-pecuniary payoffs* ϕ^s *are a known linear function of* $V_0(0)$.

We prove part (i) of Lemma 3 in the text with proofs of the remaining parts in Appendix A.1.2. We begin by eliminating the employed value functions on the right hand side of Equation 2.2. To do this, note that we can use the log-odds ratio to express $V_{w'}^{s'}$ as a linear function of V_w^s, the switching cost $c^{ss'}$, and the conditional choice probabilities:

$$V_{w'}^{s'} = V_w^s - c^{ss'} - \ln\left(p_{ww'}^{ss'}\right) + \ln\left(1 - p_{ww'}^{ss'}\right) \tag{3.7}$$

Equation 2.2 can then be written as:

$$\begin{aligned} V_w^s \;=\; & \Big(u_w + \phi^s + \delta_0^s V_0(0) - \sum_{w'}\sum_{s'} \delta_{ww'}^{ss'}\left[c^{ss'} + \ln\left(p_{ww'}^{ss'}\right) - \ln\left(1 - p_{ww'}^{ss'}\right)\right] \\ & - \sum_{w'}\sum_{s'} \lambda^{ss'} f_{w'}^{s'} \ln\left(1 - p_{ww'}^{ss'}\right)\Big)\Big/(\rho + \delta_0^s) \end{aligned} \tag{3.8}$$

Normalizing the flow payoff of employment in the lowest-paying job, $u_{\underline{w}}$, to zero, we can express $\ln(p_{\underline{w}w}^{ss}/(1 - p_{\underline{w}w}^{ss}))$ as:

$$\begin{aligned} \ln\left(\frac{p_{\underline{w}w}^{ss}}{1 - p_{\underline{w}w}^{ss}}\right) &= V_w^s - V_{\underline{w}}^s - c^{ss} \\ &= \frac{u_w - \sum_{w'}\sum_{s'} \lambda^{ss'} f_{w'}^{s'}\left[\ln\left(1 - p_{\underline{w}w'}^{ss'}\right) - \ln\left(1 - p_{ww'}^{ss'}\right)\right]}{\rho + \delta_0^s} \\ &+ \frac{\sum_{\tilde{w}\in\{\underline{w},w\}}\sum_{w'}\sum_{s'} (-1)^{\tilde{w}\neq\underline{w}} \delta_{\tilde{w}w'}^{ss'}\left[c^{ss'} + \ln\left(p_{\tilde{w}w'}^{ss'}\right) - \ln\left(1 - p_{\tilde{w}w'}^{ss'}\right)\right]}{\rho + \delta_0^s} - c^{ss} \end{aligned} \tag{3.9}$$

As the only unknown in Equation 3.9 is u_w, solving for u_w gives the result.[8]

3.3 Identification of the unemployed-side parameters and main result

We now turn to identification of the parameters governing the transitions out of unemployment. As with the employed-side parameters, we begin with recovering the time-varying offered wage distributions, $g_w^s(t)$.

Lemma 4 *Given Assumptions A1 through A4, the unemployed offer distribution for job type s at time t, $g_w^s(t)$, can be expressed as a linear system of $W-1$ unknowns and $\frac{W(W-1)}{2}-1$ equations. A solution exists when the system is of full rank.*

To prove Lemma 4, we note that, for job type s, the difference in log odds from accepting a job that pays w and accepting a job that pays w' can be written as the difference in the employed value functions:

$$\ln\left(\frac{p_w^s(t)}{1-p_w^s(t)}\right) - \ln\left(\frac{p_{w'}^s(t)}{1-p_{w'}^s(t)}\right) = V_w^s - V_{w'}^s \tag{3.10}$$

The model specified in Section 2 shows that the difference in log odds of accepting any two wage offers is the same *regardless* of how the unemployed offered wage distribution, offer arrival rate, or unemployment benefits vary over time.

The conditional choice probabilities for accepting a job at time t ($p_w^s(t)$) on the left hand side of Equation 3.10 can then be expressed as a function of the hazard out of unemployment ($h_w^s(t)$), the arrival rate ($\lambda^s(t)$), and the probability that the offer pays w ($g_w^s(t)$):

$$p_w^s(t) = \frac{h_w^s(t)}{\lambda^s(t) g_w^s(t)} \tag{3.11}$$

Denote the differenced value function $V_w^s - V_{w'}^s$ as $\kappa_{ww'}^{ss}$. Note that $\kappa_{ww'}^{ss}$ is known from Equation 3.3. Using Equations 3.10 and 3.11, we can then express $\lambda^s(t)$ as:

$$\lambda^s(t) = \frac{h_w^s(t)h_{w'}^s(t)(\exp(\kappa_{ww'}^{ss}-1))}{g_w^s(t)h_{w'}^s(t)\exp(\kappa_{ww'}^{s}) - g_{w'}^s(t)h_w^s(t)} \tag{3.12}$$

[8]Note that the expression substantially simplifies when there are no within-job involuntary changes ($\delta_{ww'}^{ss'} = 0$).

Evaluating the right hand side of Equation 3.12 at $\{\tilde{w}, \tilde{w}'\}$ and differencing yields:

$$\frac{h_w^s(t)h_{w'}^s(t)(\exp(\kappa_{ww'}^{ss}-1))}{g_w^s(t)h_{w'}^s(t)\exp(\kappa_{ww'}^{ss}) - g_{w'}^s(t)h_w^s(t)} - \frac{h_{\tilde{w}}^s(t)h_{\tilde{w}'}^s(t)(\exp(\kappa_{\tilde{w}\tilde{w}'}^{ss}-1))}{g_{\tilde{w}}^s(t)h_{\tilde{w}'}^s(t)\exp(\kappa_{\tilde{w}\tilde{w}'}^{ss}) - g_{\tilde{w}'}^s(t)h_{\tilde{w}}^s(t)} = 0 \quad (3.13)$$

Denote the numerators of the two terms as $A_{ww'}^s(t)$ and $A_{\tilde{w}\tilde{w}'}^s(t)$. These can be calculated from the unemployment hazards and the previously calculated differences in employed-side value functions. Making this substitution and rearranging the terms in Equation 3.13 yields:

$$\begin{aligned} 0 \quad = \quad & A_{\tilde{w}\tilde{w}'}^s(t)h_w^s(t)\exp(\kappa_{ww'}^{ss})g_w^s(t) - A_{\tilde{w}\tilde{w}'}^s(t)h_w^s(t)g_{w'}^s(t) \\ & -A_{ww'}^s(t)h_{\tilde{w}'}^s(t)\exp(\kappa_{\tilde{w}\tilde{w}'}^{ss})g_{\tilde{w}}^s(t) + A_{ww'}^s(t)h_{\tilde{w}}^s(t)g_{\tilde{w}'}^s(t) \qquad (3.14) \end{aligned}$$

This equation is linear in the unknowns, the $g_w^s(t)$'s. Constructing the non-redundant equations by evaluating Equation 3.14 at the following set of wage tuples:

$$\{(w, w', \tilde{w}, \tilde{w}') : w = 1, w' = 2, \tilde{w} < \tilde{w}', (\tilde{w}, \tilde{w}') \neq (1, 2)\}$$

yields a linear system with $W-1$ unknowns and $\frac{W(W-1)}{2} - 1$ equations. When the system of equations is of full rank, a closed-form solution exists for $g_w^s(t)$.
Identification of the remaining unemployed-side parameters is then straightforward:

Lemma 5 *Given Assumptions A1-A4, the offer arrival rates $\lambda^s(t)$, the conditional choice probabilities $p_w^s(t)$, the unemployment benefits $b(t)$, the value functions out of unemployment and their derivatives, $V_0(t)$ and $\dot{V}_0(t)$, are identified.*

An important implication of Lemma 5 is that the parameters ϕ^s, which from Lemma 3 above were only known up to $V_0(0)$, are also identified up to the normalization $\phi^1 = 0$.

We now prove Lemma 5. Identification of $\lambda^s(t)$ follows directly from Equation 3.12 as all the terms on the right hand side are either taken from the data ($h_w^s(t)$) or identified from a previous step ($\kappa_{ww'}^{ss}$ and $g_w^s(t)$). Identification of $p_w^s(t)$ then follows immediately from Equation 3.11.

To recover $V_0(t)$, we express the following log odds ratio by normalizing the future value of working relative to staying at the same job:

$$
\begin{aligned}
\ln\left(\frac{p_w^s(t)}{1-p_w^s(t)}\right) &= V_w^s - V_0(t) \\
&= \Big(u_w + \phi^s + \delta_0^s V_0(0) - \sum_{w'}\sum_{s'} \delta_{ww'}^{ss'}\left[c^{ss'} + \ln\left(p_{ww'}^{ss'}\right) - \ln\left(1-p_{ww'}^{ss'}\right)\right] \\
&\quad - \sum_{w'}\sum_{s'} \lambda^{ss'} f_{w'}^{s'} \ln\left(1-p_{ww'}^{ss'}\right)\Big)\Big/(\rho+\delta_0^s) - V_0(t) \qquad (3.15)
\end{aligned}
$$

where the second line follows directly from Equation 3.8.

Evaluating the previous equation at $t=0$ and solving for $V_0(0)$ yields:

$$
\begin{aligned}
V_0(0) &= \frac{1}{\rho}\Big[u_w + \phi^s - \sum_{w'}\sum_{s'} \delta_{ww'}^{ss'}\left[c^{ss'} + \ln\left(p_{ww'}^{ss'}\right) - \ln\left(1-p_{ww'}^{ss'}\right)\right] \\
&\quad - \sum_{w'}\sum_{s'} \lambda^{ss'} f_{w'}^{s'} \ln\left(1-p_{ww'}^{ss'}\right)\Big] - \frac{\rho+\delta_0^s}{\rho}\ln\left(\frac{p_w^s(t)}{1-p_w^s(t)}\right) \qquad (3.16)
\end{aligned}
$$

Note that at this stage everything on the right hand side is known, so that this equality identifies $V_0(0)$. Plugging $V_0(0)$ into Equation 3.15 then identifies $V_0(t)$ (for all $t \geq 0$), and thus also $\dot{V}_0(t)$. It follows that one can directly identify $b(t)$ using Equation 2.4:

$$
b(t) = \rho V_0(t) + \sum_w \sum_s \lambda^s(t) g_w^s(t) \ln\left(1-p_w^s(t)\right) - \dot{V}_0(t) \qquad (3.17)
$$

A remarkable implication of these results is that, by exploiting the tight connection between value functions and conditional choice probabilities, we are able to recover the structural parameters of our nonstationary job search model without solving any differential equation.

Finally, our main identification result follows from Lemmas 1 through 5:

Theorem 1 *Given Assumptions A1-A4, all of the employed and unemployed-side parameters are identified subject to a normalization of one u_w and one ϕ^s and subject to the rank conditions given in Lemmas 2 and 4.*

3.4 Extensions

Our identification strategy can be extended to more general models than the one described in Section 2. We consider two particular extensions here. The first extension allows for aggregate shocks to the economy. In this case, the economy is in one of K states, $k \in \{1, \ldots, K\}$, with the transition rate from k to k' denoted by $q_{kk'}$. Different states of the economy then affect the job destruction rates, δ_k^s, the within-employer type and wage transitions, $\delta_{ww'k}^{ss'}$, the offer arrival rates, $\lambda_k^{ss'}$, and the offer distributions, f_{wk}^s. Appendix A.2.1 shows that constructive identification holds in this case as well, under the assumption that the econometrician observes the market state, and therefore identifies $q_{kk'}$ and the hazards in A1 through A4, but now by market state. The key insight is that, on the employed side, the introduction of market states has no effect on the identification proof for the offered wage distribution, offer arrival rates, conditional choice probabilities, and switching costs. Given that, identification of the remaining parameters follows trivially.

The second extension allows for the employed offer distribution to depend on the current wage. This substantially increases the number of wage offer parameters for each job type s, from $W-1$ to $W(W-1)$. Nevertheless, we show that, when there is an observed variable $x \in \{1, \ldots X\}$ that affects the job acceptance rates but not the offer rates or the offered wage distribution, one may still recover all the parameters of the model here as well. The exclusion restriction can operate through the costs of switching jobs or through the value of amenities. Identification in this case is discussed in Appendix A.2.2.

4 Application to job search in Hungary: background and data

4.1 Setup

We apply our method to a special case of the job search model described in Section 2, in which there is one job type only ($S = 1$) and no involuntary wage transitions ($\delta_{ww'}^{ss'} = 0$, for all (s, s', w, w')). While this model shares many of the features of nonstationary job search models that have been estimated in the literature (see, in particular, van den Berg, 1990, Lollivier and Rioux, 2010), an important distinction is that it incorporates preference shocks into the search framework.

4.2 Institutional background

Our analysis focuses on the period from January 2003 to October 2005. During this period, Hungary had a two-tier unemployment insurance system. Only those were eligible for second-tier benefits who had a sufficiently long work history, and benefit payments in the second tier were lower than in the first. Those who exhausted benefits in both tiers were eligible for social assistance. Tier 1 benefits expired in 270 days and Tier 2 benefits expired in an additional 90 days. We focus on unemployed workers leaving unemployment in Tier 1, because Tier 2 benefits were very low (almost identical to the amount of social assistance that anyone is eligible for, regardless of prior work history), thus likely did not provide any further incentive to remain in unemployment. In practice, we censor durations at 269 days as our data does not track well those whose unemployment spells last longer than 269 days.

4.3 Data

We estimate the model using matched employer-employee data from Hungarian administrative records, provided by the Center for Economic and Regional Studies at the Hungarian Academy of Sciences (CERS-HAS). The dataset used in this analysis combines data from five administrative sources: (i) the National Health Insurance Fund of Hungary; (ii) the Central Administration of National Pension Insurance; (iii) the National Tax and Customs Administration of Hungary; (iv) the Public Employment Service National Labor Office; and (v) the Educational Authority. This dataset has been used in several recent papers, including DellaVigna, Lindner, Reizer, and Schmieder (2017), Harasztosi and Lindner (2019), and Verner and Gyöngyösi (2020).

The sample consists of half of the population, i.e., 4.6 million individuals, linked across 900 thousand firms. On the individual side, a de facto 50% random sample of the Hungarian population are observed; every Hungarian citizen born on January 1, 1927 and every second day thereafter are included. A key distinctive feature of the Hungarian data is their frequency: job spells are observed on a monthly basis, and unemployment spells are observed at a daily frequency. When working, one individual can be present in at most two work arrangements: labor market measures are observed separately for them. We also have information on demographics, total earnings and days worked (i.e., including tertiary and further work arrangements), as well as benefit payments. On the firm side, all firms are included at which any sampled individuals are observed to have worked. From these data, we can infer the length of their employment spells, as well as job-to-job transitions from changes in firm identifiers.

For estimation we use a sample of employed and unemployed individuals from January 2003 to October 2005.[9] We focus on males of age 18-40: we drop females from our sample to abstract from differential labor market flows resulting from childbearing decisions.[10] Furthermore, we drop older males to abstract from differential search behavior as retirement nears, with a retirement age of 43 for males in certain occupations. Because of some recoding of jobs around the first of the year, for job-to-job transitions we treat spells that go past December 31st of a particular year as right-censored.

Table 1 shows summary statistics for employment spells. In a given year, almost two-thirds of our sample have only one employment spell and about ten percent have two or more employment spells. Seventy-seven percent of employment spells are right-censored. Among those that are not right-censored, a little over one-third are job-to-job transitions with the remaining entailing transitions to unemployment.

Table 1: Summary statistics, employment spells

	1	2	3	4	5	
No. spells (%)	66.13	23.84	7.66	1.91	0.38	
	JJ	EU	RC			
Destination share (%)	7.8	14.9	77.3			
	Mean	Min	p25	p50	p75	Max
Duration (year)	0.754	0.077	0.468	1.000	1.000	1.000
Current wage (HUF)	3,681	1,644	1,874	2,541	4,243	24,643

[9]For the unemployed, we only have data from January 2004 onwards.

[10]The female labor force participation rate in Hungary was 54.0 percent in 2004, 5.8 percentage points lower than the OECD average in the same time period.

For the purposes of estimation we discretize wages into fifty bins. The first bin contains wages around the minimum wage with the remaining bins set to evenly distributed based on the current wage.[11] For the purposes of describing the data, we follow a similar procedure but discretize wages into ten bins.

Table 2 shows the number of job-to-job transitions to particular wage bins given the current wage bin. Excluding transitions to the first bin, the most populous cells are those that involve within-bin transitions, the second most populous cells are ones involving a transition to one bin higher, and the third most populous cells are ones involving a transition to one bin lower. There are also a number of transitions involving substantial wages changes in both directions.

Table 3 takes this analysis one step further by looking at how often a job-to-job transition resulted in wage increases or decreases of particular levels. Over 30% of job-to-job changes involve a wage decrease of more than 5%; this number rises to over 34% if jobs that pay the minimum (which by definition cannot have a wage decrease) are excluded. Over 42% of job-to-job transitions entail a wage increase of more than 5%; 27% of job-to-job transitions results in a wage change between negative five and plus five percent.

The data in Tables 2 and 3 provide support both for and against the model described in Section 2. On the one hand, there is clear evidence of individuals moving to jobs that involve significant wage cuts. This is consistent with a model where individuals value more than just the wage. On the other hand, the large number of transitions along the diagonal in Table 2 suggests that the current wage may affect what wages are offered. Hence in our empirical application we allow for the possibility that the current wage affects the offered wage distribution.

[11] See Appendix B.3 for additional details on the discretization process.

Table 2: Job-to-job transition counts by wage bins

Current wage	Accepted wage: 1	2	3	4	5	6	7	8	9	10
1	18,092	3,066	2,226	2,037	1,615	1,429	1,279	1,100	807	563
2	3,749	2,077	1,179	939	682	602	545	408	287	162
3	2,317	1,261	1,992	1,279	915	713	528	376	268	139
4	1,790	743	1,057	1,853	1,226	904	618	466	285	145
5	1,476	455	600	969	1,570	1,139	798	577	363	158
6	1,097	332	411	507	807	1,475	1,113	754	441	238
7	962	282	305	388	463	740	1,345	1,110	690	269
8	733	215	214	253	299	377	627	1,364	1,229	544
9	569	156	126	187	232	262	345	618	1,788	1,482
10	433	81	104	117	141	233	220	348	757	4,465

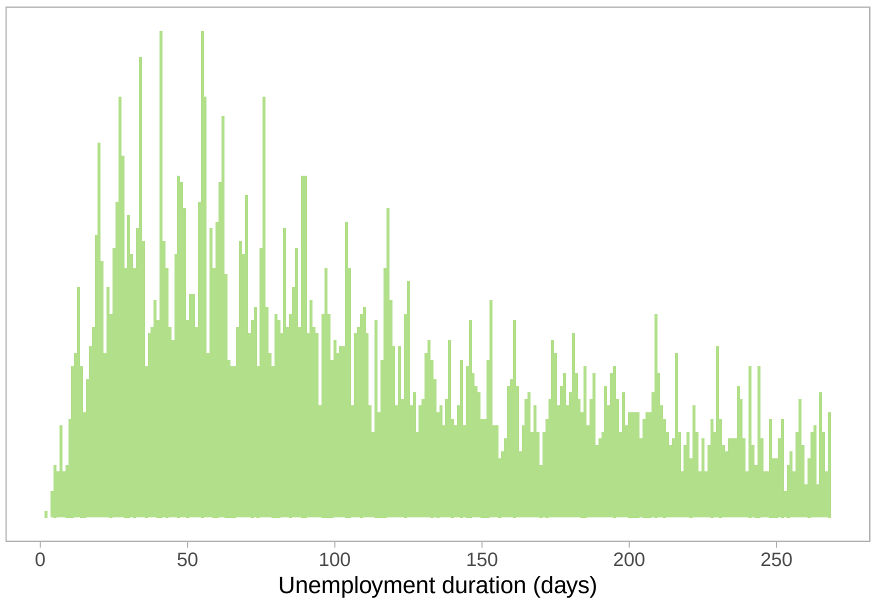
Table 3: Summary statistics, job-to-job transitions

	Overall	**By wage change**		
		Less than −5%	*−5 to 5%*	*More than 5%*
Share (%)				
All E spells	7.8	30.6	27.2	42.2
Cur. wage is min. (21.1)	4.3	–	48.1	51.9
Cur. wage above min. (78.9)	8.7	34.6	23.9	41.4
Mean wage change (%)				
All E spells	19.5	−30.1	−0.3	68.2
Cur. wage is min. (21.1)	40.7	–	0.5	84.0
Cur. wage above min. (78.9)	16.7	−30.1	−0.5	65.8

Notes: Job-to-job transitions, current and accepted wages recoded as $w = \max(w, w_{\min})$.

On the unemployment side, almost 43% of unemployment spells end in employment; most of the remaining spells are right-censored. Figure 1 shows the distribution of unemployment durations for those who exited unemployment. The mean duration is 108.8 days. We divide those who exited unemployment to a job into four quartiles based on their unemployment duration. Summary statistics for accepted wages for those who exited unemployment in each of these quartiles are presented in Table 4. Consistent with unemployed workers willing to take lower wage offers over time, longer durations are associated with lower accepted wages and higher probabilities of accepting a job at the minimum wage. Those whose durations were in the bottom 25% (so they exited the fastest) were a little over half as likely to exit to a job paying the minimum wage as those whose durations were in the top 25%.

Figure 1: Distribution of unemployment durations



Notes: Unemployment spells that end in exiting to employment. Spells censored from the right at 269 days.

Table 4: Summary statistics, unemployment-to-job transitions

	Overall	**By unemployment duration percentiles**					
		0–10	10–25	25–50	50–75	75–90	90–100
Range (days)	[2–268]	[2–27]	[27–50]	[50–93]	[93–161]	[161–216]	[216–268]
Mean U duration (days)	108.8	18.9	38.8	71.1	124.2	189.1	242.0
Mean acc. wage (HUF)	3,044	3,357	3,314	3,072	2,994	2,814	2,719
Share min. wage (%)	19.3	13.0	14.2	18.1	19.2	24.7	29.1

Notes: Unemployment-to-job transitions, accepted wages recoded as $w = \max(w, w_{\min})$.

5 Estimation

We estimate our model using a flexible parametric specification. We do this for two reasons. First, the model is heavily over-identified. The flexible parametric specification allows us to incorporate all the information in a disciplined fashion. Second, the model requires data on job-to-job transitions conditional on the current wage and unemployed-to-job transitions to specific wages at each moment in time. These transitions are inherently noisy.

Consider a workforce populated by N individuals, indexed by n. Workers may differ in their productivities in ways unobserved to the econometrician but possibly observed by the market. We allow for unobserved heterogeneity in the following manner. Each individual belongs to one of R unobserved types with probability q_{nr}; the population probability of type r is given by π_r. In practice, $r = 2$. Each individual experiences S_n employment spells indexed by s and $\tilde{S}_n$ unemployment spells indexed by $\tilde{s}$. The corresponding likelihoods for these spells for individual n of type r are given by $\mathcal{L}^E_{nsr}(\theta^E)$ and $\mathcal{L}^U_{n\tilde{s}r}(\theta^E, \theta^U)$, respectively, where θ^E are the employed-side parameters $\{\delta, \lambda, f_w, u_w, c\}$ and θ^U are the unemployed-side parameters $\{\lambda(t), g_w(t), b(t)\}$. Note that the employed-side parameters enter the likelihood for the unemployment spells but the reverse is not true. We further specify $u_w = \alpha \ln(w)$.

To apply our identification arguments in the case with unobserved heterogeneity requires identifying the type-specific hazard functions, along with the distribution of heterogeneity types. In practice, one can use in an initial step the identification results from Heckman and Singer (1984) for duration models with unobserved heterogeneity but without covariates to recover the distribution of unobserved types along with the type-specific hazards associated with the job-to-job transitions. One can then identify the distribution of the type-specific hazards out of unemployment, taking as given the distribution of heterogeneity types, and assuming that type-specific hazard functions belong to a known parametric family. The previous identification arguments then still apply, resulting in point identification of the structural parameters (arrival rates, switching costs, and wage offer distributions) that are now also a function of unobserved heterogeneity.

For estimation, the unobserved type must be integrated out of the likelihood function. Note that there is an initial conditions problem here as the initial wage and employment state will be affected by the type. These initial conditions, described in more detail in the next section, will depend on a vector of parameters θ^I. The log-likelihood function for the data is then:

$$\sum_n \ln \left(\sum_r \mathcal{L}^I_{nr}(\theta^I) \prod_{s=1}^{S_n} \mathcal{L}^E_{nsr}(\theta^E) \prod_{\tilde{s}=1}^{\tilde{S}_n} \mathcal{L}^U_{n\tilde{s}r}(\theta^E, \theta^U) \right) \tag{5.1}$$

One way of estimating Equation 5.1 would be to use the Expectation-Maximization (EM) algorithm. As pointed out by Arcidiacono and Jones (2003), this permits the estimation of the parameters in stages. In particular, the EM algorithm treats the unobserved type as known at the maximization stage and weights the log-likelihoods of each observation by the conditional probability of being n being of unobserved type r, q_{nr}. The conditional probability follows directly from Bayes rule:

$$q_{nr} = \frac{\mathcal{L}^I_{nr}(\theta^I) \prod_{s=1}^{S_n} \mathcal{L}^E_{nsr}(\theta^E) \prod_{\tilde{s}=1}^{\tilde{S}_n} \mathcal{L}^U_{n\tilde{s}r}(\theta^E, \theta^U)}{\sum_r \mathcal{L}^I_{nr}(\theta^I) \prod_{s=1}^{S_n} \mathcal{L}^E_{nsr}(\theta^E) \prod_{\tilde{s}=1}^{\tilde{S}_n} \mathcal{L}^U_{n\tilde{s}r}(\theta^E, \theta^U)} \tag{5.2}$$

The maximization step could then proceed in stages, estimating θ^I followed by θ^E. Then, taking the estimates of θ^E as given, estimating θ^U.

Arcidiacono and Miller (2011) extend this approach further by noting that the structure of the model need not be imposed on the employed and unemployed likelihoods. Rather, one can use their empirical counterparts to recover, in this case, θ^I and the q_{nr}'s in a first step. Given the q_{nr}'s, estimation of the θ^E's and θ^U's can be done in (separate) maximization steps. The key difference between this approach and that of Arcidiacono and Jones (2003) is that this last maximization step would only be done once instead of at each EM iteration. In the next subsections, we describe the details of our iterative approach.

5.1 Step 1: recovering the posterior type distribution

We use the initial wage and the job-to-job transitions to estimate the conditional probabilities of being each unobserved type, q_{nr}. We specify the job-to-job transitions as the hazard rate out of a job that pays w times the probability that the accepted wage is w' given that the current was w'. The exact specification is given in C.1.

We specify the likelihood of the initial wage as following a tobit structure. Denote w^I_n as individual n's initial wage level. Let X^I_n denote the characteristics that affect this initial wage. The likelihood contribution of initial wages is

$$\mathcal{L}_{nr}^{I} = \left[\Phi\left(\frac{\ln(\underline{w}) - X_n^I \theta_r^I}{\sigma_r^I}\right)\right]^{\mathbb{1}\{w_n^I = \underline{w}\}} \cdot \left[\frac{1}{\sigma_r^I}\phi\left(\frac{\ln(w_n^I) - X_n^I \theta_r^I}{\sigma_r^I}\right)\right]^{\mathbb{1}\{w_n^I > \underline{w}\}} \tag{5.3}$$

We specify X_n^I as a function of the individual's type and year indicators where the effects of the year indicators are fixed across types:

$$X_n^I \theta_r^I = \theta_{1r}^I + \theta_2^I \mathbb{1}\{y_n = 2004\} + \theta_3^I \mathbb{1}\{y_n = 2005\} \tag{5.4}$$

Given these parameters and the reduced form parameters governing the job-to-job transitions, we can recover the q_{nr}'s.

5.2 Step 2: recovering the employed-side parameters

With the estimated conditional probabilities (the q_{nr}'s) in hand, we now proceed with the estimation of the employed-side parameters. Estimation proceeds as in the case without unobserved heterogeneity but where the q_{nr}'s are used as weights. In each employment spell s we observe its duration t_s and the wage w_s. Let $w_{s+1} = 0$ when individual n transitions to unemployment during their sth employment spell. Estimation of the type-r job separation rate δ_{0r} then directly follows from counting the number of transitions to unemployment and dividing by the time spent in employment:

$$\delta_{0r} = \frac{\sum_{n=1}^{N} q_{nr} \sum_{s=1}^{S_n} \mathbb{1}\{w_{s+1} = 0\}}{\sum_{n=1}^{N} q_{nr} \sum_{s=1}^{S_n} t_s} \tag{5.5}$$

We estimate the other employed-side parameters via maximum likelihood. We construct the type-r structural hazard from moving from a job that pays w to one that pays w' using:

$$h_{ww'r} = \lambda_r f_{ww'r} p_{ww'r} \tag{5.6}$$

The wage offer distribution, $f_{ww'r}$, is parameterized using an ordered logit structure that depends on current wages. First, we specify the wage cutoffs as having the following recursive structure:

$$\phi_w = \begin{cases} \theta_1^{\phi} & \text{if } w = \underline{w} \\ \phi_{w_-} + \exp(\theta_2^{\phi} + \theta_3^{\phi} \ln(w) + \theta_4^{\phi} \ln(w)^2) & \text{if } w > \underline{w} \end{cases} \tag{5.7}$$

where w_- denotes the preceding support point of the pmf. These cutoffs then specify how the large the latent index needs to be to reach a particular wage bin.

We then define the distribution of offered wages using the wage cutoffs as well as current wages w:

$$f_{ww'r} = \begin{cases} \Lambda(\phi_{\underline{w}} + X^f \theta_r^f) & \text{if } w' = \underline{w} \\ \Lambda(\phi_{w'} + X^f \theta_r^f) - \Lambda(\phi_{w'_-} + X^f \theta_r^f) & \text{if } \underline{w} < w' < \overline{w} \\ 1 - \Lambda(\phi_{\overline{w}_-} + X^f \theta_r^f) & \text{if } w' = \overline{w} \end{cases} \tag{5.8}$$

$$X^f \theta_r^f = \theta_1^f \ln(w) + \theta_2^f \mathbb{1}\{w = \underline{w}\} + \theta_{3r}^f \tag{5.9}$$

where $\Lambda(\cdot)$ denotes the logistic function. The log of the current wage then shifts the latent index. Note that unobserved types affect the offered wage distribution only through a level shift given by θ_{3r}^f.

The conditional choice probabilities that enter Equation 5.6 can be written as:

$$p_{ww'r} = \frac{\exp\left(V_{w'r} - V_{wr} - c\right)}{1 + \exp\left(V_{w'r} - V_{wr} - c\right)} \tag{5.10}$$

We iterate the differenced value function in Equation 5.10 to a fixed point using the following contraction mapping: in the $(m+1)$th step,

$$\begin{aligned} (\lambda_r + \delta_{0r} + \rho)\left(V_{w'r}^{(m+1)} - V_{wr}^{(m+1)}\right) &= \alpha(\ln(w') - \ln(w)) + \lambda_r\left(V_{w'r}^{(m)} - V_{wr}^{(m)}\right) \\ &+ \sum_{\tilde{w}} \lambda_r f_{w'\tilde{w}r} \ln\left[1 + \exp\left(V_{\tilde{w}r}^{(m)} - V_{w'r}^{(m)} - c\right)\right] \\ &- \sum_{\tilde{w}} \lambda_r f_{w\tilde{w}r} \ln\left[1 + \exp\left(V_{\tilde{w}r}^{(m)} - V_{wr}^{(m)} - c\right)\right] \end{aligned} \tag{5.11}$$

It follows that the likelihood contribution of a job spell s for a type-r worker n is given by

$$\mathcal{L}_{nsr}^{E} = \prod_{w,w'} \left[(h_{ww'r})^{\mathbb{1}\{w_s=w,w_s'=w'\}} \exp(-h_{ww'r}t_s) \right]^{\mathbb{1}\{w_s=w\}} \tag{5.12}$$

We then estimate the remaining parameters $\left(\lambda_r, \alpha, c, \theta_r^f\right)$ using:

$$\max_{\lambda_r,\alpha,c,\theta_r^f} L^E = \sum_{n=1}^{N} \sum_{r=1}^{R} \sum_{s=1}^{S_n} q_{nr} \ln \left(\mathcal{L}_{nsr}^{E} \right) \tag{5.13}$$

5.3 Step 3: recovering the unemployed-side parameters

In the third and final step, we estimate the distribution of offered wages out of unemployment, $g_{wr}(t)$, and the offer arrival rates, $\lambda_r(t)$, using maximum likelihood. We then use the constructive identification strategy to recover the flow utility of unemployment.

Note that the type-r structural hazard of leaving unemployment at duration t to wage w is given by:

$$h_{wr}(t) = \lambda_r(t) g_{wr}(t) p_{wr}(t) \tag{5.14}$$

In the next subsections, we show how each of these terms are specified.

5.3.1 Specification of $p_{wr}(t)$

We focus first on expressing $p_{wr}(t)$ in a way consistent with the structure of the model. Note that we can write $\kappa_{ww'r} = V_{wr} - V_{w'r}$, thus $\exp(\kappa_{ww'r}) = \exp(V_{wr}) / \exp(V_{w'r})$. Using this identity, we can express the ratio of the conditional choice probabilities as:

$$\begin{aligned} \frac{p_{wr}(t)}{p_{w'r}(t)} &= \frac{\exp(V_{wr})/[\exp(V_{0r}(t)) + \exp(V_{wr})]}{\exp(V_{w'r})/[\exp(V_{0r}(t)) + \exp(V_{w'r})]} \\ &= \exp(\kappa_{ww'r}) \left[1 - p_{wr}(t)\{1 - \exp(-\kappa_{ww'r})\}\right] \end{aligned} \tag{5.15}$$

We can therefore express all conditional choice probabilities relative to one other conditional choice probability, say the one associated with the minimum wage $p_{\underline{w}r}(t)$, and the corresponding $\kappa_{w\underline{w}r}$ terms:

$$p_{wr}(t) = \frac{p_{\underline{w}r}(t)\exp(\kappa_{w\underline{w}r})}{1 - p_{\underline{w}r}(t)\left[1 - \exp(\kappa_{w\underline{w}r})\right]} \tag{5.16}$$

Furthermore, we express the CCPs of accepting an offer from the first wage bin in terms of a parameterized hazard rate out of unemployment to the first bin (the minimum wage):

$$p_{\underline{w}r}(t) = \frac{h_{\underline{w}r}(t)}{\lambda_r(t) g_{\underline{w}r}(t)} \tag{5.17}$$

where[12]

$$h_{\underline{w}r}(t) = \exp(P^h \theta^h_r) \quad \text{with} \tag{5.18}$$

$$P^h = \begin{bmatrix} \underline{1} & \underline{t}^{-1} & \underline{t}^2 & \underline{t}^2 \ln \underline{t} & \underline{t}^3 \end{bmatrix} \tag{5.19}$$

That is, we can express the CCPs as

$$p_{wr}(t) = \begin{cases} \frac{h_{\underline{w}r(t)}}{\lambda_r(t)\, g_{\underline{w}r}(t)} & \text{if } w = \underline{w} \\ \frac{h_{\underline{w}r(t)} \exp(\kappa_{w\underline{w}r})}{\lambda_r(t)\, g_{\underline{w}r}(t) - h_{\underline{w}r(t)}[1-\exp(\kappa_{w\underline{w}r})]} & \text{if } w > \underline{w} \end{cases} \tag{5.20}$$

5.3.2 Specification of $\lambda_r(t)$

We parametrize the offer arrival rates $\lambda_r(t)$ as

$$\lambda_r(t) = \exp(P^\lambda \theta^\lambda \nu^\lambda_r + \psi^\lambda_r) \quad \text{where} \tag{5.21}$$

$$P^\lambda = \begin{bmatrix} \underline{1} & \underline{t} & \underline{t}^2 & \underline{t}^3 \end{bmatrix} \tag{5.22}$$

$$\nu^\lambda_1 = 1 \quad \text{and} \quad \psi^\lambda_1 = 0 \tag{5.23}$$

The type-specific parameters provide a scale and location shift of the common Type 1 profile.

[12] We chose this polynomial because it fits the nonparametric Nelson–Aalen hazard estimates best.

5.3.3 Specification of $g_{wr}(t)$

We parametrize the offered wages $g_{wr}(t)$ using a similar ordered logit structure to that used in the employed offer distribution. We take the wage cutoffs ϕ as given from the employed side in Equation 5.7, and add a type-specific variance-scale parameter β_r. Note that the cutoffs are common across types. We allow the wage offer distribution to vary over time where we specify the offered wage distribution out of unemployment as:

$$g_{wr}(t) = \begin{cases} \Lambda\left(\beta_r \phi_{\underline{w}} + E_r + A_r \ln(t)\right) & \text{if } w = \underline{w} \\ \Lambda\left(\beta_r \phi_w + E_r + A_r \ln(t)\right) - \Lambda\left(\beta_r \phi_{w_-} + E_r + A_r \ln(t)\right) & \text{if } \underline{w} < w < \overline{w} \\ 1 - \Lambda\left(\beta_r \phi_{\overline{w}_-} + E_r + A_r \ln(t)\right) & \text{if } w = \overline{w} \end{cases} \tag{5.24}$$

Note that the only (type-specific) parameters to estimate are β_r, E_r, and A_r.

5.3.4 Estimation of $p_{wr}(t)$, $\lambda_r(t)$, and $g_{wr}(t)$

Putting the three components together, the structural hazards are

$$h_{wr}(t) = \begin{cases} h_{\underline{w}r}(t) & \text{if } w = \underline{w} \\ h_{\underline{w}r}(t)\, \frac{g_{wr}(t)}{g_{\underline{w}r}(t)} \frac{\exp(\kappa_{w\underline{w}r})}{\lambda_r(t)\, g_{\underline{w}r}(t) - h_{\underline{w}r}(t)[1-\exp(\kappa_{w\underline{w}r})]} & \text{if } w > \underline{w} \end{cases} \tag{5.25}$$

We estimate these structural parameters in a maximum likelihood procedure, stratified by types. First, we estimate the parameters $(\theta^\lambda, \theta_1^h, \beta_1, E_1, A_1)$ for Type 1. Then, given these estimates we estimate the parameters $(\nu_r^\lambda, \psi_r^\lambda, \theta_r^h, \beta_r, E_r, A_r)$ for the remaining types. In both cases, we impose that the CCPs are monotonically increasing in t.[13]

The likelihood contribution of a type-r individual n's spell s is

$$\mathcal{L}_{nsr}^U = \prod_w \left\{ [h_{wr}(t_s)]^{\mathbb{1}\{w_s = w\}} \exp\left(-\int_0^{t_s} h_{wr}(u)\, du\right)\right\}. \tag{5.26}$$

[13]Appendices C.2.1 and C.2.2 show these constraints simplify.

We maximize the likelihood for Type 1 as

$$\max_{\theta^\lambda,\theta_1^h,\beta_1,E_1,A_1} L_1^U = \sum_{n=1}^{N} q_{n1} \sum_{s=1}^{S_n} \ln\left(\mathcal{L}_{ns1}^U\right) \tag{5.27}$$

$$\text{s.t. } p_{11}(t) \leq p_{11}(t+1) \quad \text{for } 1 \leq t < T-1 \tag{5.28}$$

$$p_{W1}(T) \leq 1-\varepsilon \tag{5.29}$$

for some small ε

Taking the shape of the offer arrival process as given, we then maximize the likelihood for type $r=2$ as

$$\max_{\nu_2^\lambda,\psi_2^\lambda,\theta_2^h,\beta_2,E_2,A_2} L_2^U = \sum_{n=1}^{N} q_{n2} \sum_{s=1}^{S_n} \ln\left(\mathcal{L}_{ns2}^U\right) \tag{5.30}$$

$$\text{s.t. } p_{12}(t) \leq p_{12}(t+1) \quad \text{for } 1 \leq t < T-1, \tag{5.31}$$

$$p_{W2}(T) \leq 1-\varepsilon \tag{5.32}$$

for some small ε.

5.3.5 Recovering the flow utility of unemployment

For the last remaining parameter, we need to calculate the value function and its first derivative. We calculate $V_{0r}(t)$ pointwise at each duration t as[14]

$$V_{0r}(t) = \begin{cases} \frac{\alpha\ln(w)-\sum_{w'}\lambda_r f_{ww'r}\ln(1-p_{ww'r})}{\rho} - \frac{\delta_{0r}+\rho}{\rho}\ln\left(\frac{p_{wr}(t)}{1-p_{wr}(t)}\right) & \text{if } t=0 \\ \frac{\alpha\ln(w)-\sum_{w'}\lambda_r f_{ww'r}\ln(1-p_{ww'r})+\delta_{0r}V_{0r}(0)}{\delta_{0r}+\rho} - \ln\left(\frac{p_{wr}(t)}{1-p_{wr}(t)}\right) & \text{if } t>0 \end{cases} \tag{5.33}$$

From the time trajectory of the value function, we calculate its first difference as

$$\dot{V}_{0r}(t) = \frac{V_{0r}(t+\Delta\tau) - V_{0r}(t)}{\Delta\tau} \tag{5.34}$$

where $\Delta\tau$ is an arbitrarily small time interval.

[14]This expression would appear as though $V_{0r}(t)$ is overidentified as the expression holds for all w. However, we have already imposed the structure of the model prior to this stage so each value of w leads to the same value of $V_{0r}(t)$.

Using these pieces, we estimate the flow utility of unemployment as

$$b_r(t) = \rho V_{0r}(t) + \sum_w \lambda_r(t) g_{wr}(t) \ln(1 - p_{wr}(t)) - \dot{V}_{0r}(t) \tag{5.35}$$

6 Estimation results

We now discuss our estimation results. Table 5 shows the point estimates of the employed side structural parameters, along with standard errors from 500 bootstrap repetitions. Roughly 85% of workers are classified as Type 1. These workers begin with lower wages, receive offers at a lower rate, draw from a worse wage offer distribution, and have higher job destruction rates. Type 1 workers receive a job offer once in every 3.7 years (0.271 annually) and have a 22.5% chance of separating from their current job per annum. On the other hand, Type 2 workers receive offers more frequently (one in every 2.7 years or 0.373 per annum) and separate from their jobs less frequently (6.3% probability per annum). It follows that the index of search frictions, which corresponds to the average number of job offers received during any given employment spell (Ridder and Van den Berg, 2003), is substantially higher for this group of individuals (5.9 vs. 1.2 for type 1 individuals) who also tend to have higher initial wages. The mean index of search frictions across types is equal to 1.9, a value which fits in the range of the estimates obtained by Ridder and Van den Berg (2003) using French Labor Force Survey data, but substantially lower than the estimates obtained using US data. The parameter associated with the flow utility of log wages is equal to 0.403, which is very close in magnitude to the cost of switching jobs.

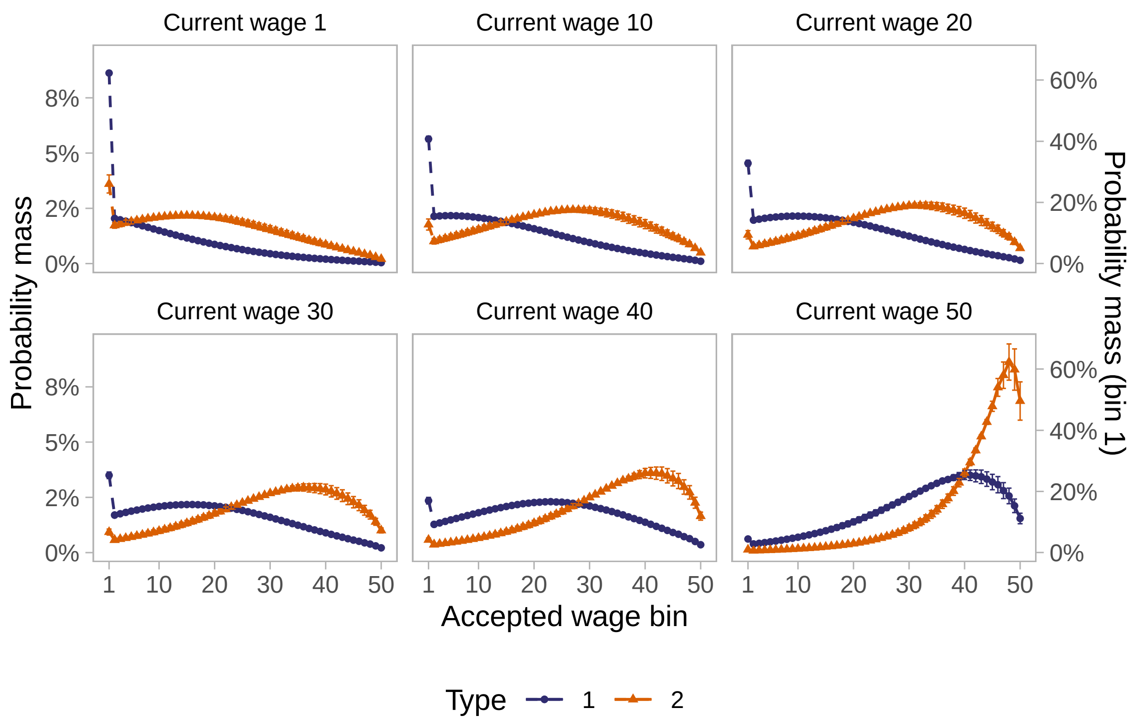
Table 5: Structural parameter estimates, employed side

	Parameter	Estimate	
		Type 1	*Type 2*
λ	Offer arrival rate	0.271	0.373
		(0.004)	(0.015)
δ	Job separation rate	0.225	0.063
		(0.001)	(0.001)
λ/δ	Search friction index	1.201	5.906
		(0.018)	(0.225)
α	Flow utility of log wages	0.403	
		(0.018)	
c	Job switching cost	0.400	
		(0.026)	
π	Type probability	0.851	0.149
		(0.002)	(0.002)

Notes: Bootstrap standard errors in parentheses (500 replications).

As wage offers are allowed to depend on the wage in the current job, Figure 2 shows the offer distributions for workers currently in wage bin 1, 10, 20, 30, 40, and 50. At any current wage, Type 1's face a worse wage offer distribution. However, as the current wage rises, the distribution of offered wages shifts to the right for both types. Hence a Type 1 worker currently working in the 40th wage bin faces a better offer distribution than a Type 2 worker currently making the minimum wage.

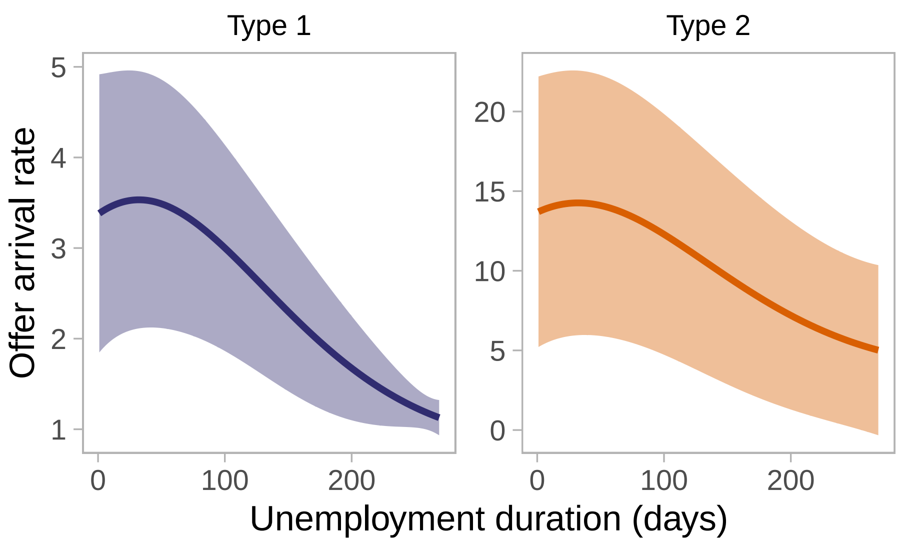
Figure 2: Wage offer distribution, employed side



Notes: Distributions are conditional on the current wage bin. The probability mass of bin 1 offers are represented on the secondary vertical axis (right). Error bars represent 95% bootstrap confidence intervals (500 replications).

Turning to our unemployed side results, our model allows for nonstationarities along multiple dimensions. Figure 3 shows one of these dimensions, revealing how unemployed offer arrival rates evolve over time. For both types, increased unemployment durations are associated with fewer offers. For Type 1's, offers come in at a rate of 3.5 per year at the beginning of the unemployment spell but fall to a rate of 1 per year by the end. Type 2's receive offers at a much higher rate, beginning at a rate of almost 15 per year and falling to around 5.

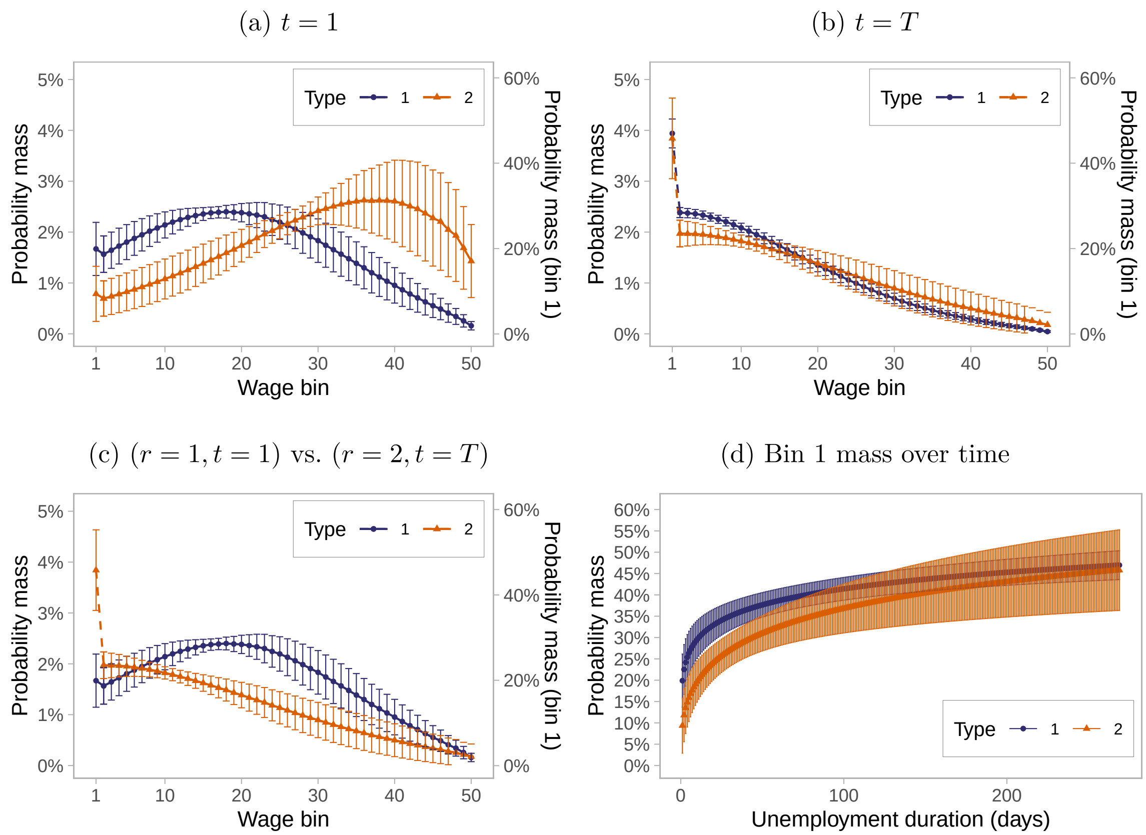
Figure 3: Offer arrival rates out of unemployment



Notes: Shaded regions represent 95% bootstrap confidence intervals (500 replications).

A second source of nonstationarity, shown in Figure 4, is in the offered wage distribution for unemployed workers. Panels (a) and (b) show stark differences in the offer distributions between offers at day 1 of unemployment and day 269. At $t = 1$, Type 2's face a much better offer distribution that Type 1's. But as also shown in Panel (d), this advantage disappears near benefit expiration. As unemployment duration increases, the offer distributions for both types become much worse. Panel (c) shows that this deterioration of the offered wage distribution is much stronger than the selection effect: Type 1's at $t = 1$ face a substantially better wage offer distribution than Type 2's at $t = 269$.

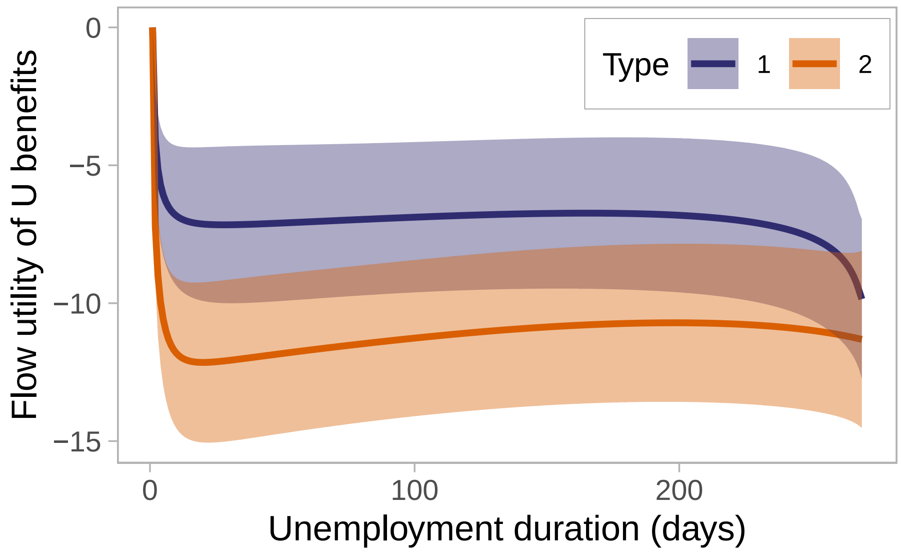
Figure 4: Wage offer distribution, unemployed side



Notes: Distributions are conditional on unemployment duration t. Panel (c) contrasts the Type 1 distribution at duration $t = 1$ to the Type 2 distribution at duration $t = T$. Panel (d) compares the evolution of the probability mass of wage offers from the first bin between types. The probability mass of bin 1 offers are represented on the secondary vertical axis (right). Error bars represent 95% bootstrap confidence intervals (500 replications).

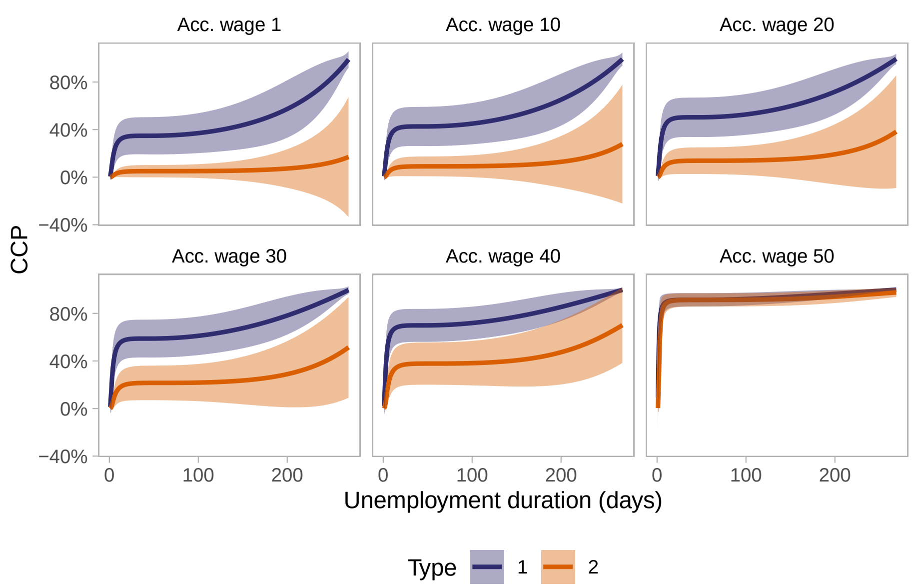
A third source of nonstationarity is unemployment benefits. The evolution of these are displayed in Figure 5. The flow value drops sharply upon entering unemployment and then remains relatively flat. However, for Type 1 individuals, the flow value decreases again close to benefit expiration. In Appendix D we show how these three sources of nonstationarity—offer arrival wages, wage offers, and unemployment benefits—affect the value function and the job acceptance probabilities. As unemployment duration increases, the value function for unemployment falls and, correspondingly, the job acceptance probabilities rise.

Figure 5: Flow utility of unemployment benefits (normalized)



Notes: Flow utility normalized w.r.t. the flow value at $t = 0$ for each type. Shaded regions represent 95% bootstrap confidence intervals (500 replications).

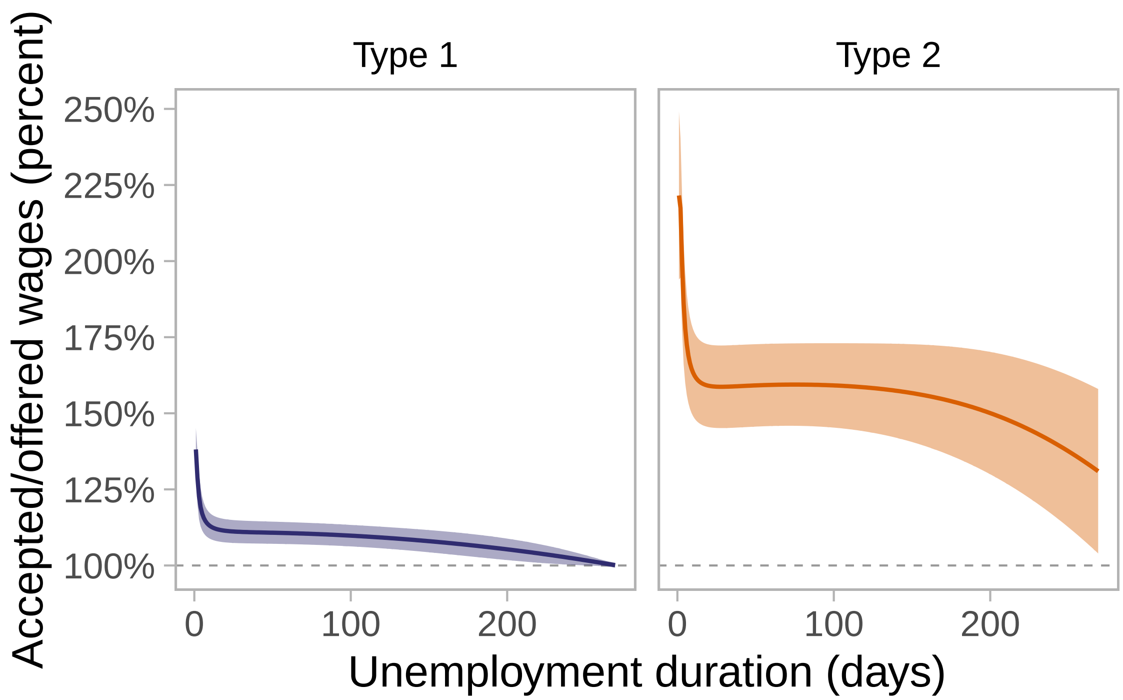
Figure 6: CCPs, unemployment-to-job transitions



Notes: Shaded regions represent 95% bootstrap confidence intervals (500 replications).

With job acceptance probabilities rising, the ratio of average accepted wages to average offered wages falls over time. This is displayed in Figure 7. Like with unemployment benefits, we see a sharp drop in the accepted/offered wage ratio immediately after entering unemployment. But now as duration increases, workers become less and less selective over which jobs they accept. By the time benefits are about to expire, Type 1's find almost all jobs acceptable. This is not the case for Type 2's even though the offered wage distribution is similar between the two types at benefit expiration. As benefits near expiration, Type 2's still accept jobs that pay on average 25% more than the average offer. The reason for this is that, as shown in Figure 3, Type 2's receive offers at a much higher rate than Type 1's even at benefit expiration.

Figure 7: Offered vs. accepted wages out of unemployment



Notes: Dashed lines: mean level of offered wages. Solid lines: mean level of accepted wages.

Table 6 summarizes the unemployment-side findings. The first row shows that, because Type 2's receive offers at a much higher rate than Type 1's, Type 2's who exit unemployment do so faster than their Type 1 counterparts, making up 17% of those who leave in the shortest durations but only 10.6% of those who leave in the longest durations. As shown in the first column, this translates to Type 2's who exit to a job having unemployment durations that are on average 10 days shorter than their Type 1 counterparts.

Type 2's also exit unemployment to higher wage jobs. Overall, Type 2's who exit unemployment do so to jobs that pay 70% more than their Type 1 counterparts. For both types, however, we see sharp drops in accepted wage over time.

Table 6: Summary statistics by type, unemployment-to-job transitions

	Overall	**By unemployment duration percentiles**					
		0–10	10–25	25–50	50–75	75–90	90–100
Pop. prob. of Type 2 (%)	12.1	17.0	14.3	11.5	11.0	10.2	10.6
Type 1							
Range (days)	[4–268]	[4–28]	[28–52]	[52–95]	[95–163]	[163–217]	[217–268]
Mean U duration (days)	110.0	19.6	40.4	72.7	125.8	189.8	241.9
Mean acc. wage (HUF)	2,805	2,968	3,007	2,830	2,790	2,646	2,551
Share min. wage (%)	21.0	15.2	15.9	19.5	20.9	26.3	30.5
Type 2							
Range (days)	[5–268]	[5–24]	[24–41]	[41–82]	[82–147]	[147–211]	[211–268]
Mean U duration (days)	99.8	16.5	32.1	60.6	111.6	178.7	238.8
Mean acc. wage (HUF)	4,781	5,322	5,176	4,917	4,798	4,211	4,096
Share min. wage (%)	7.5	4.0	5.3	7.6	4.8	10.0	17.2

Notes: Unemployment-to-job transitions, accepted wages recoded as $w = \max(w, w_{\min})$. Summary statistics are weighted by type probabilities.

7 Conclusion

In this paper, we extend the canonical continuous-time job search model with on-the-job search to allow for preference shocks. Incorporating preference shocks and using the insights from conditional choice probability methods results in constructive identification of the model parameters even in nonstationary settings. Nonstationary search models typically require solving a nonlinear differential equation within the maximization routine. But in our setting no differential equation needs to be solved to estimate the parameters of the model. As a result, the computational costs are small for the class of nonstationary search models we consider.

We apply our methods to administrative data from Hungary. Nonstationarities when unemployed operate through three sources: the offered wage distribution, the offer arrival rates, and unemployment benefits. The estimates of the model show that the wage offer distribution becomes worse and offer arrivals slow substantially as the duration of unemployment increases. Workers then become less selective in the jobs they are willing to accept as unemployment duration increases, implying that the gap between accepted and offered wages shrinks with unemployment duration.

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A Mathematical appendix

A.1 Proof of Theorem 1

A.1.1 Proof of Lemma 2 (ii)

Akin to Equation 3.6, for any triple $(w, w', \tilde{w}) \in \Omega_w^3$:

$$\begin{pmatrix} \ln\left(\frac{h_{ww}^{ss'}}{\lambda^{ss'} f_w^{s'} - h_{ww}^{ss'}}\right) + \ln\left(\frac{h_{ww}^{s's}}{\lambda^{s's} f_w^{s} - h_{ww}^{s's}}\right) \\ \ln\left(\frac{h_{ww}^{ss'}}{\lambda^{ss'} f_w^{s'} - h_{ww}^{ss'}}\right) + \ln\left(\frac{h_{ww}^{s's}}{\lambda^{s's} f_w^{s} - h_{ww}^{s's}}\right) \end{pmatrix} = \begin{pmatrix} \ln\left(\frac{h_{w'w'}^{ss'}}{\lambda^{ss'} f_{w'}^{s'} - h_{w'w'}^{ss'}}\right) + \ln\left(\frac{h_{w'w'}^{s's}}{\lambda^{s's} f_{w'}^{s} - h_{w'w'}^{s's}}\right) \\ \ln\left(\frac{h_{\tilde{w}\tilde{w}}^{ss'}}{\lambda^{ss'} f_{\tilde{w}}^{s'} - h_{\tilde{w}\tilde{w}}^{ss'}}\right) + \ln\left(\frac{h_{\tilde{w}\tilde{w}}^{s's}}{\lambda^{s's} f_{\tilde{w}}^{s} - h_{\tilde{w}\tilde{w}}^{s's}}\right) \end{pmatrix} \tag{A.1}$$

Note that now we exploit transitions across job types s and s', thus we are able to use the same wage in the old and new jobs. This nonlinear system of two equations and two unknowns—$\lambda^{ss'}$ and $\lambda^{s's}$—can be rewritten as follows:

$$\begin{pmatrix} B_{w'}\lambda^{ss'} + C_{w'}\lambda^{s's} - A_{w'}\lambda^{ss'}\lambda^{s's} \\ B_{\tilde{w}}\lambda^{ss'} + C_{\tilde{w}}\lambda^{s's} - A_{\tilde{w}}\lambda^{ss'}\lambda^{s's} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{A.2}$$

where the A, B, C coefficients are defined in Lemma 2 (ii). Assuming $A_{w'} \neq 0$ (Condition (a) from Lemma 2 (ii)) and replacing $\lambda^{ss'}\lambda^{s's}$ in the second equation by its expression from the first equation identifies the ratio of the arrival rates, with:

$$\lambda^{s's} = \left(\frac{B_{w'}A_{\tilde{w}} - B_{\tilde{w}}A_{w'}}{A_{w'}C_{\tilde{w}} - A_{\tilde{w}}C_{w'}}\right)\lambda^{ss'}$$

where $A_{w'}C_{\tilde{w}} - A_{\tilde{w}}C_{w'} \neq 0$ from Condition (c). Finally, substituting for $\lambda^{s's}$ in the first equation identifies, under Condition (b), $\lambda^{ss'}$ and then $\lambda^{s's}$, which admit the following closed-form expressions:

$$\lambda^{ss'} = \frac{B_{w'}C_{\tilde{w}} - B_{\tilde{w}}C_{w'}}{B_{w'}A_{\tilde{w}} - B_{\tilde{w}}A_{w'}} \qquad \text{and} \quad \lambda^{s's} = \frac{B_{w'}C_{\tilde{w}} - B_{\tilde{w}}C_{w'}}{A_{w'}C_{\tilde{w}} - A_{\tilde{w}}C_{w'}} \tag{A.3}$$

Having identified the arrival rates $\lambda^{ss'}$ and the wage offer distribution f_w^s, identification of the CCPs $p_{ww'}^{ss'}$ follows. Then, we can identify $c^{ss'} + c^{s's}$, and together with the assumption that switching costs are symmetric (i.e., $c^{ss'} = c^{s's}$), $c^{ss'}$ is identified.

A.1.2 Proof of Lemma 3 (ii)–(iii)

(ii) Identification of CRRA preferences. We assume that workers are endowed with CRRA preferences, such that:

$$u(w) = \alpha \frac{w^{1-\theta}}{1-\theta}$$

From the prior identification result in Lemma 3 such that u_w is identified up to a constant, it follows that for $\tilde{w} > w' > w$, the following ratio is identified:

$$\frac{u_{w'} - u_w}{u_{\tilde{w}} - u_w} = \frac{{w'}^{1-\theta} - w^{1-\theta}}{\tilde{w}^{1-\theta} - w^{1-\theta}} \tag{A.4}$$

In order to establish identification of the risk aversion parameter θ, we show that the function $\theta \mapsto \frac{y^{1-\theta}-x^{1-\theta}}{z^{1-\theta}-y^{1-\theta}}$, where $z > y > x > 0$, is monotonically increasing on $(0, \infty)$.

$$\begin{aligned}
f(\theta) &= \frac{y^{1-\theta} - x^{1-\theta}}{z^{1-\theta} - y^{1-\theta}} && \text{(A.5)}\\
f'(\theta) &= \left(z^{1-\theta} - y^{1-\theta}\right)^{-2} \cdot \Big[\left(x^{1-\theta}\ln x - y^{1-\theta}\ln y\right)\left(z^{\theta} - y^{\theta}\right) \\
&\quad - \left(y^{1-\theta} - x^{1-\theta}\right)\left(y^{1-\theta}\ln y - z^{1-\theta}\ln z\right)\Big] && \text{(A.6)}\\
f'(\theta) &> 0 && \text{(A.7)}\\
&\Leftrightarrow \left(x^{1-\theta}\ln x - x^{1-\theta}\ln y\right)\left(z^{1-\theta} - y^{1-\theta}\right) + \left(x^{1-\theta}\ln y - y^{1-\theta}\ln y\right)\left(z^{1-\theta} - y^{1-\theta}\right) \\
&> \left(z^{1-\theta}\ln y - z^{1-\theta}\ln z\right)\left(y^{1-\theta} - x^{1-\theta}\right) + \left(y^{1-\theta}\ln y - z^{1-\theta}\ln y\right)\left(y^{1-\theta} - x^{1-\theta}\right) \\
& && \text{(A.8)}\\
&\Leftrightarrow \left[x^{1-\theta}\ln(x/y)\right]\left(z^{1-\theta} - y^{1-\theta}\right) > \left[z^{1-\theta}\ln(y/z)\right]\left(y^{1-\theta} - x^{1-\theta}\right) && \text{(A.9)}\\
&\Leftrightarrow \ln(y/x)\left[1 - (y/z)^{1-\theta}\right] < \ln(z/y)\left[(y/x)^{1-\theta} - 1\right] && \text{(A.10)}\\
&\Leftrightarrow \quad (y/x)^{1-\theta}\ln(z/y) + \ln(y/x)(y/z)^{1-\theta} > \ln(y/x) + \ln(z/y) && \text{(A.11)}
\end{aligned}$$

The above condition holds if and only if $g(\theta) > g(1)$, where, for all $\theta > 0$, $g(\theta) \equiv (y/x)^{1-\theta}\ln(z/y) + (y/z)^{1-\theta}\ln(y/x)$. The derivative of $g(\cdot)$ is given by:

$$g'(\theta) = \ln(y/x)\ln(z/y)[(y/z)^{1-\theta} - (y/x)^{1-\theta}]$$

It follows that $g'(\theta) < 0$ on $(0, 1)$ and $g'(\theta) > 0$ on $(1, \infty)$. Identification of θ follows.

Having identified θ, it follows that the utility coefficient α is identified and given by the following closed-form expression:

$$\alpha = \frac{u_{\tilde{w}} - u_w}{\tilde{w}^{1-\theta} - w^{1-\theta}} \tag{A.12}$$

which yields full identification of the flow utility of wages.

(iii) Identification of ϕ^s up to $V_0(0)$. We can express the log odds ratio in terms of the structural parameters using Equation 3.8:

$$\begin{aligned}
\ln\left(\frac{p^{s\tilde{s}}_{w\tilde{w}}}{1 - p^{s\tilde{s}}_{w\tilde{w}}}\right) &= V^{\tilde{s}}_{\tilde{w}} - c^{s\tilde{s}} - V^s_w \\
&= \Big(u_{\tilde{w}} + \phi^{\tilde{s}} + \delta^{\tilde{s}}_0 V_0(0) - \sum_{w'}\sum_{s'} \delta^{\tilde{s}s'}_{\tilde{w}w'} \left[c^{\tilde{s}s'} + \ln\left(p^{\tilde{s}s'}_{\tilde{w}w'}\right) - \ln\left(1 - p^{\tilde{s}s'}_{\tilde{w}w'}\right)\right] \\
&\quad - \sum_{w'}\sum_{s'} \lambda^{\tilde{s}s'} f^{s'}_{w'} \ln\left(1 - p^{\tilde{s}s'}_{\tilde{w}w'}\right)\Big)\Big/(\rho + \delta^{\tilde{s}}_0) \\
&\quad - \Big(u_w + \phi^s + \delta^s_0 V_0(0) - \sum_{w'}\sum_{s'} \delta^{ss'}_{ww'} \left[c^{ss'} + \ln\left(p^{ss'}_{ww'}\right) - \ln\left(1 - p^{ss'}_{ww'}\right)\right] \\
&\quad + \sum_{w'}\sum_{s'} \lambda^{ss'} f^{s'}_{w'} \ln\left(1 - p^{ss'}_{ww'}\right)\Big)\Big/(\rho + \delta^s_0) - c^{s\tilde{s}}
\end{aligned} \tag{A.13}$$

Collecting all known terms on the left hand side, the equation can be rearranged as:

$$\kappa^{s\tilde{s}}_{w\tilde{w}} = \frac{1}{\rho + \delta^{\tilde{s}}_0}\,\phi^{\tilde{s}} - \frac{1}{\rho + \delta^s_0}\,\phi^s + \left(\frac{\delta^{\tilde{s}}_0}{\rho + \delta^{\tilde{s}}_0} - \frac{\delta^s_0}{\rho + \delta^s_0}\right) V_0(0) \tag{A.14}$$

where

$$
\begin{aligned}
\kappa_{w\tilde{w}}^{s\tilde{s}} &= \ln\left(\frac{p_{w\tilde{w}}^{s\tilde{s}}}{1-p_{w\tilde{w}}^{s\tilde{s}}}\right) + c^{s\tilde{s}} \\
&- \frac{u_{\tilde{w}} - \sum_{w'}\sum_{s'}\delta_{\tilde{w}w'}^{\tilde{s}s'}\left[c^{\tilde{s}s'} + \ln\left(p_{\tilde{w}w'}^{\tilde{s}s'}\right) - \ln\left(1-p_{\tilde{w}w'}^{\tilde{s}s'}\right)\right] - \sum_{w'}\sum_{s'}\lambda^{\tilde{s}s'} f_{w'}^{s'} \ln\left(1-p_{\tilde{w}w'}^{\tilde{s}s'}\right)}{\rho+\delta_0^{\tilde{s}}} \\
&+ \frac{u_{w} - \sum_{w'}\sum_{s'}\delta_{ww'}^{ss'}\left[c^{ss'} + \ln\left(p_{ww'}^{ss'}\right) - \ln\left(1-p_{ww'}^{ss'}\right)\right] - \sum_{w'}\sum_{s'}\lambda^{ss'} f_{w'}^{s'} \ln\left(1-p_{ww'}^{ss'}\right)}{\rho+\delta_0^{s}}
\end{aligned}
\tag{A.15}
$$

Now, since $\phi^1 = 0$, writing Equation A.14 for $s = 1$ yields:

$$
\tilde{\kappa}_{w\tilde{w}}^{1\tilde{s}} = \frac{1}{\rho+\delta_0^{\tilde{s}}}\,\phi^{\tilde{s}} + \left(\frac{\delta_0^{\tilde{s}}}{\rho+\delta_0^{\tilde{s}}} - \frac{\delta_0^{1}}{\rho+\delta_0^{1}}\right) V_0(0) \tag{A.16}
$$

Thus, we can write ϕ^s as a known linear function of $V_0(0)$. Furthermore, note that when the job destruction rates are not specific to job types, i.e., $\delta_0^s = \delta_0$ for all s, the non-pecuniary payoffs ϕ^s are directly identified from Equation A.16.

A.2 Extensions

A.2.1 Aggregate shocks

One can extend our identification strategy to accommodate aggregate shocks. Specifically, consider the case where the market economy can be in one of K different states, where the job offer arrival rates, the job destruction rates, the rates of involuntary wage mobility, the offered wage distributions, and the flow payoff of unemployment are allowed to depend on the state of the economy. We further assume that the econometrician perfectly observes the state of the economy. We denote the rate at which the economy transitions from state k to k' by $q_{kk'}$, which is identified from the observed transition rates across market states.

On the employment side, identification of the state-specific offer arrival rates, destruction and involuntary wage mobility rates, offered wage distribution and conditional choice probabilities, along with the switching cost all follow directly from the baseline case, leaving the flow payoffs of employment as the only unknown parameters. The value function of employment V^s_{wk} is given by:

$$\begin{aligned}\Big(\rho + \sum_{k'} q_{kk'} + \delta^s_{0k} + \sum_{s'} \lambda^{ss'}_k\Big) V^s_{wk} &= u_w + \phi^s + \delta^s_{0k} V^{(k)}_0(0) + \sum_{k'} q_{kk'} V^s_{wk'} \\ &+ \sum_{w'} \sum_{s'} \delta^{ss'}_{ww'k} \left[V^{s'}_{w'k} - V^s_{wk}\right] + \sum_{s'} \lambda^{ss'}_k \sum_{w'} f^{s'}_{w'k} \ln\left(1 - p^{ss'}_{ww'k}\right) \end{aligned} \tag{A.17}$$

where $V^{s'}_{w'k} - V^s_{wk} = \ln\left(p^{ss'}_{ww'k}\right) - \ln\left(1 - p^{ss'}_{ww'k}\right) + c^{ss'}$.

Subtracting off the corresponding expression for $V^s_{\tilde{w}k}$ (with $\tilde{w} \neq w$) yields:

$$\begin{aligned}\Big(\rho + \sum_{k'} q_{kk'} + \delta^s_{0k} + \sum_{s'} \lambda^{ss'}_k\Big) \left[V^s_{wk} - V^s_{\tilde{w}k}\right] &= u_w - u_{\tilde{w}} + \sum_{k'} q_{kk'} \left[V^s_{wk'} - V^s_{\tilde{w}k'}\right] \\ &+ \sum_{w'} \sum_{s'} \left(\delta^{ss'}_{ww'k} \left[V^{s'}_{w'k} - V^s_{wk}\right] - \delta^{ss'}_{\tilde{w}w'k} \left[V^{s'}_{w'k} - V^s_{\tilde{w}k}\right]\right) \\ &+ \sum_{s'} \lambda^{ss'}_k \sum_{w'} f^{s'}_{w'k} \left(\ln\left(1 - p^{ss'}_{ww'k}\right) - \ln\left(1 - p^{ss'}_{\tilde{w}w'k}\right)\right) \end{aligned} \tag{A.18}$$

where the difference in value functions on the left and right-hand sides are given by the sum of the log odds ratio and the switching cost. This identifies the wage component of the flow utility payoffs up to a constant. Identification of the non-pecuniary components ϕ^s then proceeds in a similar fashion, using instead the job-to-job transitions across job types.

Identification of the unemployment-side parameters then follows from similar arguments as in Section 3.3. The same strategy applies to a context with aggregate shocks, after conditioning the hazard rates out of unemployment on the (observed) states of the economy.

A.2.2 Current wage-specific wage offer distribution

One can extend our identification framework by allowing the on-the-job wage offer distribution to depend on current wages. The important role played by Equation 3.4 extends beyond our baseline model. Namely, using these restrictions, along with the existence of an observed taste (or switching cost) shifter, x, one may still jointly identify in this case the offer arrival rates $\lambda^{ss'}$ and of the offer wage distributions. Note that we now observe hazard rates conditional on x, $h^{ss'}_{ww'x}$. We use the within job-type transitions to recover the within job-type offer arrival rates, λ^{ss}, as well as the job-type specific offered wage distributions that now depend on the current wage w, $f^s_{ww'}$. Assuming that x only affects job-to-job transition rates through the acceptance probabilities, the following equality holds for any quadruple $\{w, w', \tilde{w}, \tilde{w}'\} \in \Omega^4_w$ and any $x \in \{1, \dots, X\}$:

$$\ln\left(\frac{h^{ss}_{ww'x}}{\lambda^{ss} f^s_{ww'} - h^{ss}_{ww'x}}\right) + \ln\left(\frac{h^{ss}_{w'wx}}{\lambda^{ss} f^s_{w'w} - h^{ss}_{w'wx}}\right) \tag{A.19}$$

$$= \ln\left(\frac{h^{ss}_{\tilde{w}\tilde{w}'x}}{\lambda^{ss} f^s_{\tilde{w}\tilde{w}'} - h^{ss}_{\tilde{w}\tilde{w}'x}}\right) + \ln\left(\frac{h^{ss}_{\tilde{w}'\tilde{w}x}}{\lambda^{ss} f^s_{\tilde{w}'\tilde{w}} - h^{ss}_{\tilde{w}'\tilde{w}x}}\right) \tag{A.20}$$

The set of non-redundant wage tuples for which we can evaluate Equation A.19 is given by:

$$\{(w, w', \tilde{w}, \tilde{w}') : w = 1, w' = 2, \tilde{w} < \tilde{w}', (\tilde{w}, \tilde{w}') \neq (1, 2)\}$$

Collecting all admissible tuples yields a linear system with $1 + W(W-1)$ unknowns and $X\left(\frac{W(W-1)}{2} - 1\right)$ equations. Therefore the system (over)identifies the offer arrival rate λ^{ss} and the wage offer probabilities $(f^s_{ww'}, f^s_{w'w}, f^s_{\tilde{w}\tilde{w}'}, f^s_{\tilde{w}'\tilde{w}})$ if the following condition holds:

$$\frac{2X + 2}{X - 1} \leq W(W - 1)$$

For example, with 3 support points for x, 4 wage support points are sufficient for identification.

B Data appendix

B.1 Sample creation

We define our analysis sample as follows:

1. Flip primary and secondary work arrangements (PWAs, SWAs)
    - In the raw data, PWA is defined as the arrangement with the highest earnings in the month. This setup may result in PWAs and SWAs flipping in the raw data, e.g. when a worker works only a few days in their PWA.

- *Solution:* Looping through all worker-months, we flip variables related to PWAs and SWAs as follows:

month	firmid1	var1	firmid2	var2		month	firmid1	var1	firmid2	var2
$t-1$	A	x_{t-1}	B	y_{t-1}	$\rightsquigarrow$	$t-1$	A	x_{t-1}	B	y_{t-1}
t	B	x_t	A	y_t		t	A	y_t	B	x_t

2. Calculate durations

   (a) Employed: we calculate or infer spell-year durations in PWA. See Appendix B.2 for details.

   (b) Unemployed: we observe daily unemployment durations in the raw data.

3. Define JJ, EU, UE, EN, NE transitions

4. Calculate wages

   (a) Calculate counterfactual minimum wage earnings: how much the worker would have earned in a day working full time in a minimum-wage job

   (b) Calculate daily wages as total earnings in a spell-year, divided by spell-year durations

   (c) Discretize wages: see Appendix B.3 for details

   (d) Calculate accepted wages

5. Define covariates for population probabilities

6. Save analysis sample

B.2 Correcting employment spell durations

The raw data on employment spells are recorded at a monthly frequency. In each month, the total number of days worked (`days`) and total earnings are known. Furthermore, days worked and earnings at PWAs and SWAs (`days_1`, `days_2`) are known if the arrangement was ongoing on the 15th of the month. We focus on PWAs only. Table 7 summarizes the possible ways in which JJ transitions show up in the raw data when observations on PWAs are not missing. When `days` equals `days_1`, we know with certainty that the transition happened on the boundary of the month: we label this as a clean JJ transition (see Panel a). When `days` does not equal `days_1`, we need to make some assumptions about the uncovered days: Panels b-d illustrate these cases that we label fuzzy. The bottom tables summarize our assumptions on the number of days worked in each PWA.

Table 7: JJ scenarios in raw data, no missing PWAs

(a) Clean JJ

days	days_1	firmid1
31	31	A
30	30	A
31	31	B

⇓

no assumption needed

(b) Fuzzy JJ 1

days	days_1	firmid1
31	31	A
30	16	A
31	31	B

⇓

days_1	firmid1
31	A
16	A
14	B
31	B

(c) Fuzzy JJ 2

days	days_1	firmid1
31	31	A
30	16	B
31	31	B

⇓

days_1	firmid1
31	A
14	A
16	B
31	B

(d) Fuzzy JJ 3

days	days_1	firmid1
31	31	A
30	16	B
31	31	C

⇓

days_1	firmid1
31	A
$a < 14$	A
16	B
$30 - 16 - a$	C
31	C

Table 8 summarizes our assumptions when PWA data are missing.

Table 8: JJ scenarios in raw data, missing PWAs

(a)

days	days_1	firmid1
31	31	A
25	.	.
31	31	A

⇓

days_1	firmid1
31	A
25	A
31	A

(b)

days	days_1	firmid1
31	31	A
25	.	.
31	31	B

⇓

days_1	firmid1
31	A
$d < 15$	A
$25 - d$	B
31	B

(c)

days	days_1	firmid1
31	31	A
10	.	.
7	.	.
30	30	B

⇓

days_1	firmid1
31	A
10	A
7	B
31	B

(d)

days	days_1	firmid1
31	31	A
20	.	.
25	.	.
31	31	B

⇓

days_1	firmid1
31	A
$a < 15$	A
$20 - a + 25 - b$	X
$b < 15$	B
31	B

B.3 Discretizing wages

We discretize the continuously observed wages in the data into W bins, with $W = 50$ for our main results. Figure 8 demonstrates our discretization method. The first bin contains wages between 75 and 107 percent of the effective minimum wage.[15] We drop wage observations below 75 percent of the effective minimum wage because we cannot

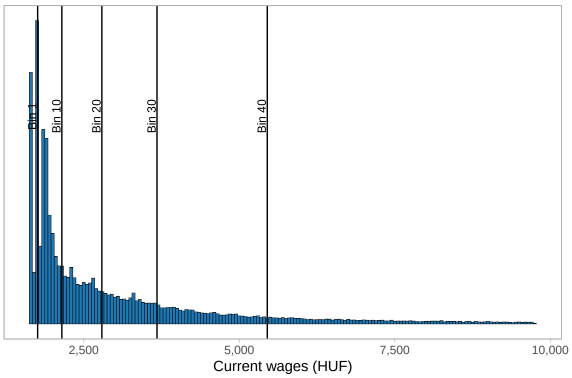
[15]During our sampling period, Hungary had a simple minimum wage policy: 53,000 HUF in 2004 and 57,000 HUF in 2005. (1 USD was worth around 200 HUF in 2005.)

distinguish between full-time and part-time earners in the data. Furthermore, we add a 7 percent padding to the right cutoff of the first bin to ensure that we include all minimum wage earners in the first bin. We then split the other wage observations, censored at the 99th percentile, evenly across the remaining $W - 1$ bins.

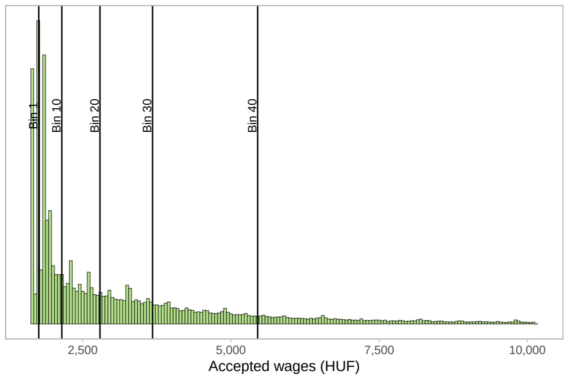
Figure 9 plots the resulting discrete distribution of current wages. Current wages for employment spells that lead to a job-to-job transition, on the left panel, have a mean of 3,459 HUF (percentiles: 25th 1,857 50th 2,356 75th 3,741). Current wages for all employment spells, on the right panel, have a mean of 3,667 HUF (percentiles: 25th 1,859 50th 2,541 75th 4,243). Similarly, Figure 10 plots the discrete distribution of accepted wages for job-to-job and unemployment-to-employment transitions. Accepted wages for job-to-job transitions have a mean of 3,696 HUF (percentiles: 25th 1,863 50th 2,549 75th 4,169). The accepted wages out of unemployment are right-tailed, with a mean of 2,689 HUF (percentiles: 25th 1,856 50th 2,099 75th 3,036), in line with the notion that the unemployed tend to move to lower-paying jobs.

Figure 8: Discretizing observed wages

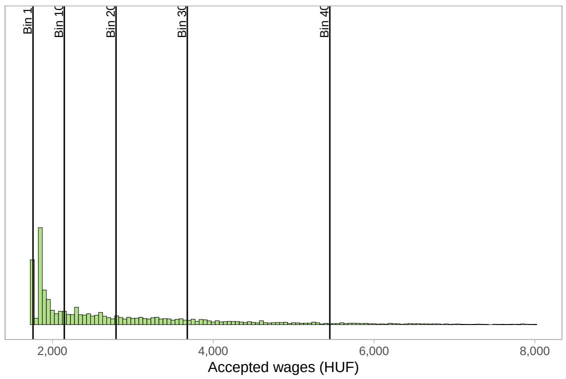
(a) Current wages



(b) Accepted wages (job-to-job trans.)



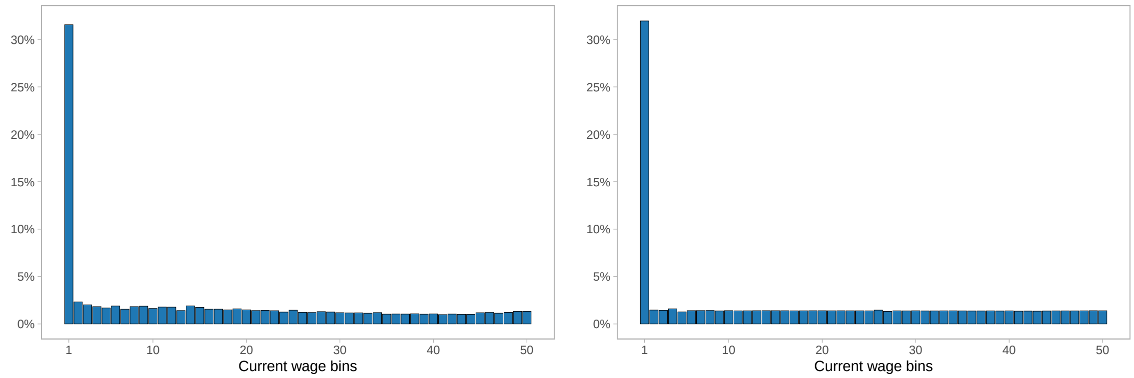
(c) Accepted wages (unemployment-to-job trans.)



Notes: Histograms of daily wage rates with 50 HUF bin width, truncated at the 95th percentile. Vertical lines denote selected wage bin cutoffs. Panel a: current daily wages for employment spells that lead to a job-to-job transition. Panel b: accepted daily wages for employment spells after a job-to-job transition. Panel c: accepted daily wages for unemployment spells after an unemployment-to-job transition.

Figure 9: Discrete distribution of current wages

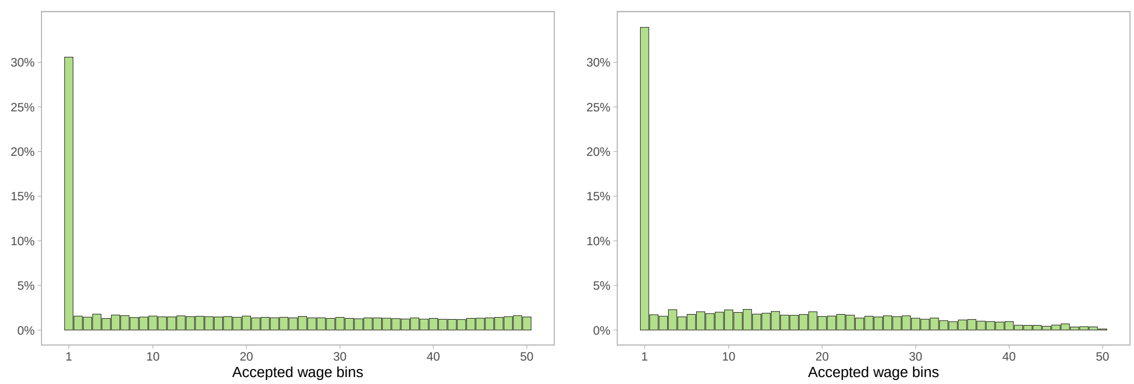
(a) Spells with job-to-job transitions (b) All spells



Notes: Panel a: discrete distribution of current wages for employment spells that lead to a job-to-job transition. Panel b: discrete distribution of current wages for all employment spells.

Figure 10: Discrete distribution of accepted wages

(a) Job-to-job transitions (b) Unemployment-to-employment transitions



Notes: Panel a: discrete distribution of accepted wages for employment spells that lead to a job-to-job transition. Panel b: discrete distribution of accepted wages for unemployment spells that lead to an employment spell.

C Estimation appendix

This appendix details our estimation procedure, outlined in Section 5.

C.1 Posterior type distribution

Rather than imposing the structure of the model when classifying types, we instead choose a flexible functional form for the likelihood of job-to-job transitions. In particular, we obtain estimates of θ^I by maximizing an alternative objective function:

$$\sum_n \ln\left(\sum_r \mathcal{L}^I_{nr}(\theta^I)\prod_{s=1}^{S_n}\tilde{\mathcal{L}}^E_{nsr}(\tilde{\theta}^E)\right) \tag{C.1}$$

where $\mathcal{L}^I_{nr}(\theta^I)$ was defined in Equation 5.3 and where specify the reduced-form likelihood associated with employment spell s below.

We break the hazard of going from w to w' into two parts: (i) the hazard of leaving w-paying job for any other job, and (ii) the probability that the accepted job pays w'. We specify the reduced-form hazard of n leaving a w-paying job given the individual is of type-r as:

$$\tilde{h}_{wr} = \exp(\tilde{\theta}^h_{1r} + \tilde{\theta}^h_{2r}\ln(w_s) + \tilde{\theta}^h_{3r}\mathbb{1}\{w_s = \underline{w}\} + \tilde{\theta}^h_{4r}\mathbb{1}\{y_s = 2004\} + \tilde{\theta}^h_{5r}\mathbb{1}\{y_s = 2005\}) \tag{C.2}$$

where y_s refers to the calendar year of spell s.

Conditional on moving to a new job, for the reduced form we model the accepted wage as a tobit like in Equation 5.3 but where one of the conditioning variables is the log of the current wage. $\tilde{\mathcal{L}}^E_{nr}$ is then given by:

$$\begin{aligned}\tilde{\mathcal{L}}^E_{nr}(\tilde{\theta}^h,\tilde{\theta}^E) &= \prod_{s=1}^{S_n}\left[\prod_w \tilde{h}_{wr}\exp(-\tilde{h}_{wr}t_s)\right]^{\mathbb{1}\{w_s=w\}} \\ &\times\left[\Phi\left(\frac{\ln(\underline{w}) - \tilde{X}^E_s\tilde{\theta}^E_r}{\tilde{\sigma}^E_r}\right)\right]^{\mathbb{1}\{w_{s+1}=\underline{w}\}}\cdot\left[\frac{1}{\tilde{\sigma}^E_r}\phi\left(\frac{\ln(w_{s+1}) - \tilde{X}^E_s\tilde{\theta}^E_r}{\tilde{\sigma}^E_r}\right)\right]^{\mathbb{1}\{w_{s+1}>\underline{w}\}}\end{aligned} \tag{C.3}$$

where $\tilde{X}_s\tilde{\theta}^E_r$ is given by:

$$\tilde{X}^E_s\tilde{\theta}^E_r = \tilde{\theta}^E_{1r} + \tilde{\theta}^E_2\ln(w_s) + \tilde{\theta}^E_3\mathbb{1}\{y_s = 2004\} + \tilde{\theta}^E_4\mathbb{1}\{y_s = 2005\} \tag{C.4}$$

We then estimate the parameters $\left(\theta^I, \tilde{\theta}^h, \tilde{\theta}^E\right)$ using:

$$\max_{\theta^I, \tilde{\theta}^h, \tilde{\theta}^E} \sum_n \ln \left(\sum_r \mathcal{L}_{nr}^I(\theta^I) \tilde{\mathcal{L}}_{nr}^E(\tilde{\theta}^h, \tilde{\theta}^E) \right) \tag{C.5}$$

and recover the conditional type probabilities using:

$$q_{nr} = \frac{\mathcal{L}_{nr}^I(\theta^I) \tilde{\mathcal{L}}_{nr}^E(\tilde{\theta}^h, \tilde{\theta}^E)}{\sum_r \mathcal{L}_{nr}^I(\theta^I) \tilde{\mathcal{L}}_{nr}^E(\tilde{\theta}^h, \tilde{\theta}^E)} \tag{C.6}$$

C.2 Unemployed-side structural parameters

C.2.1 Optimization constraints for Type 1

The first set of constraints in Equation 5.28 simplify to the following nonlinear constraints:

$$p_{11}(t) \leq p_{11}(t+1) \tag{C.7}$$

$$\frac{h_{11}(t)}{\lambda_1(t)\, g_{11}(t)} \leq \frac{h_{11}(t+1)}{\lambda_1(t+1)\, g_{11}(t+1)} \tag{C.8}$$

$$\frac{\exp(P_{t+1}^\lambda \theta^\lambda)}{\exp(P_t^\lambda \theta^\lambda)} \frac{\Lambda\left(\beta_1 \phi_1 + E_1 + A_1 \ln(t+1)\right)}{\Lambda\left(\beta_1 \phi_1 + E_1 + A_1 \ln(t)\right)} \leq \frac{\exp(P_{t+1}^h \theta_1^h)}{\exp(P_t^h \theta_1^h)} \tag{C.9}$$

$$\begin{aligned} &\left(P_{t+1}^\lambda - P_t^\lambda\right)\theta^\lambda - \left(P_{t+1}^h - P_t^h\right)\theta_1^h \\ &+ \ln\left[1 + \exp\left(-\beta_1\phi_1 - E_1 - A_1 \ln(t)\right)\right] \\ &- \ln\left[1 + \exp\left(-\beta_1\phi_1 - E_1 - A_1 \ln(t+1)\right)\right] \leq 0 \end{aligned} \tag{C.10}$$

The second constraint simplifies as follows:

$$p_{W1}(T) \leq 1 - \varepsilon \tag{C.11}$$

$$\frac{h_{11}(T) \exp(-\kappa_{1W1})}{\lambda_1(T)\, g_{11}(T) - h_{11}(T)[1 - \exp(-\kappa_{1W1})]} \leq 1 - \varepsilon \tag{C.12}$$

$$\exp(P_T^h \theta_1^h) \left[1 + \frac{\varepsilon}{1-\varepsilon} \exp(-\kappa_{1W1})\right] \leq \exp(P_T^\lambda \theta^\lambda)\, \Lambda\left(\beta_1 \phi_1 + E_1 + A_1 \ln(T)\right) \tag{C.13}$$

$$P_T^h \theta_1^h + \ln\left[1 + \frac{\varepsilon}{1-\varepsilon} \exp(-\kappa_{1W1})\right] \leq P_T^\lambda \theta^\lambda - \ln[1 + \exp(-\beta_1\phi_1 - E_1 - A_1 \ln(T))] \tag{C.14}$$

C.2.2 Optimization constraints for Type $r = 2$

The first set of constraints in Equation 5.31 simplify to the following nonlinear constraints:

$$p_{12}(t) \leq p_{12}(t+1) \tag{C.15}$$

$$\frac{h_{12}(t)}{\lambda_2(t)\, g_{12}(t)} \leq \frac{h_{12}(t+1)}{\lambda_2(t+1)\, g_{12}(t+1)} \tag{C.16}$$

$$\frac{\exp(P^\lambda_{t+1}\theta^\lambda\nu^\lambda_2 + \psi^\lambda_2)}{\exp(P^\lambda_t\theta^\lambda\nu^\lambda_2 + \psi^\lambda_2)} \frac{\Lambda\left(\beta_2\phi_1 + E_2 + A_2\ln(t+1)\right)}{\Lambda\left(\beta_2\phi_1 + E_2 + A_2\ln(t)\right)} \leq \frac{\exp(P^h_{t+1}\theta^h_2)}{\exp(P^h_t\theta^h_2)} \tag{C.17}$$

$$\begin{aligned} &\left(P^\lambda_{t+1} - P^\lambda_t\right)\theta^\lambda\nu^\lambda_2 - \left(P^h_{t+1} - P^h_t\right)\theta^h_2 \\ &+ \ln\left[1 + \exp\left(-\beta_2\phi_1 - E_2 - A_2\ln(t)\right)\right] \\ &- \ln\left[1 + \exp\left(-\beta_2\phi_1 - E_2 - A_2\ln(t+1)\right)\right] \leq 0 \end{aligned} \tag{C.18}$$

The second constraint which ensures that the CCPs are less than one simplifies to the following nonlinear constraint:

$$p_{W2}(T) \leq 1 - \varepsilon \tag{C.19}$$

$$\frac{h_{12}(T)\exp(-\kappa_{1W2})}{\lambda_2(T)\, g_{12}(T) - h_{12}(T)[1 - \exp(-\kappa_{1W2})]} \leq 1 - \varepsilon \tag{C.20}$$

$$\exp(P^h_T\theta^h_2)\left[1 + \frac{\varepsilon}{1-\varepsilon}\exp(-\kappa_{1W2})\right] \leq \exp(P^\lambda_T\theta^\lambda\nu^\lambda_2 + \psi^\lambda_2)\,\Lambda\left(\beta_2\phi_1 + E_2 + A_2\ln(T)\right) \tag{C.21}$$

$$P^h_T\theta^h_2 + \ln\left[1 + \frac{\varepsilon}{1-\varepsilon}\exp(-\kappa_{1W2})\right] \leq P^\lambda_T\theta^\lambda\nu^\lambda_2 + \psi^\lambda_2 - \ln[1 + \exp(-\beta_2\phi_1 - E_2 - A_2\ln(T))] \tag{C.22}$$

Table 9: Computation time

Step	**Elapsed time**
Estimate posterior probabilities	44.03 min
Estimate job-to-job structural parameters	7.38 min
Estimate unemployment-to-job hazards	7.02 sec
Estimate unemployment-to-job structural parameters	28.61 sec

Notes: Benchmarked on a 32-core Intel® Xeon® Gold 6134 3.20GHz CPU with 96GB RAM, running MathWorks® MATLAB® R2018b (9.5.0.1033004).

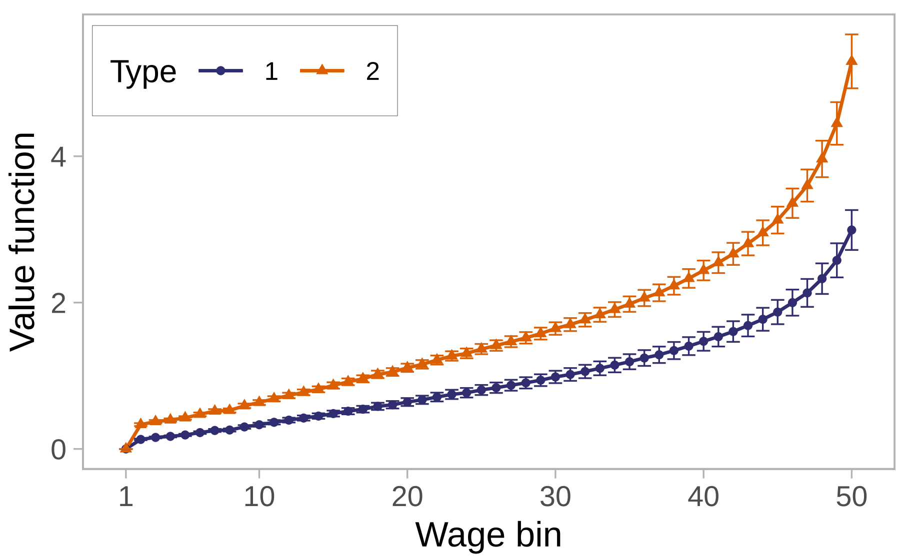
D Additional results

Table 10: Type probabilities

Initial wage bin	Type probability	
	Type 1	*Type 2*
1	99.7%	0.3%
	(1.4e-06%)	(1.4e-06%)
10	98.7%	1.3%
	(2.0e-05%)	(2.0e-05%)
20	95.1%	4.9%
	(2.0e-04%)	(2.0e-04%)
30	83.2%	16.8%
	(1.5e-03%)	(1.5e-03%)
40	55.7%	44.3%
	(4.7e-03%)	(4.7e-03%)
50	1.6%	98.4%
	(1.9e-04%)	(1.9e-04%)

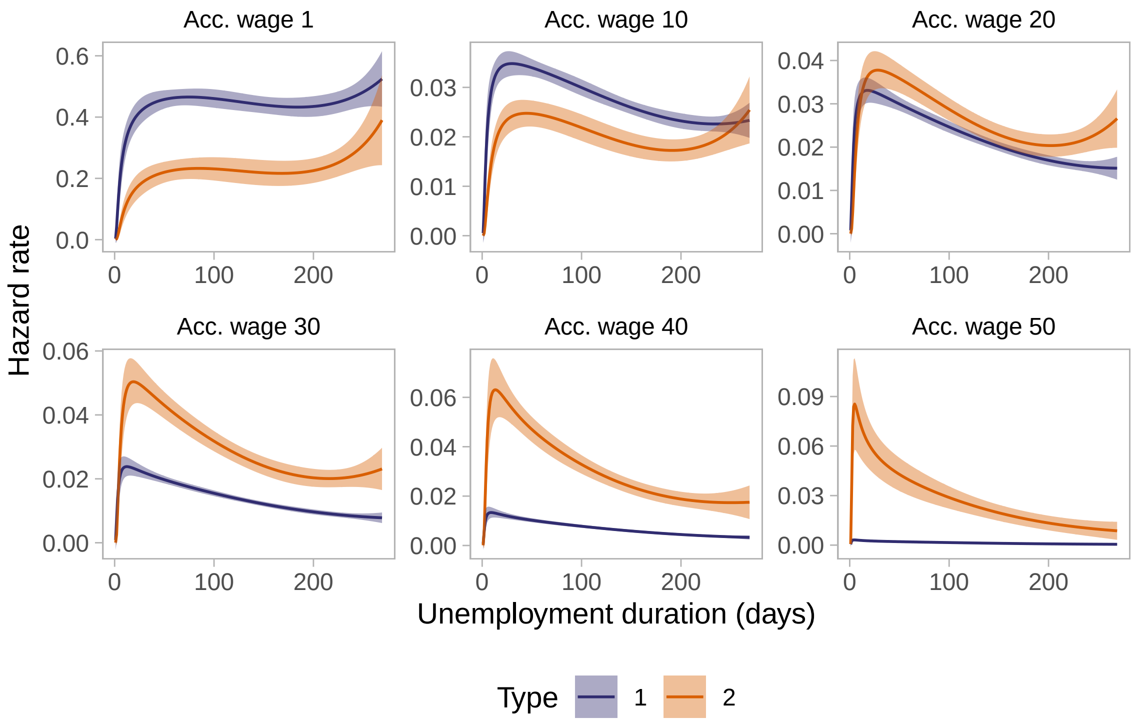
Notes: Bootstrap standard errors in parentheses (500 replications).

Figure 11: Value function of employment (normalized)



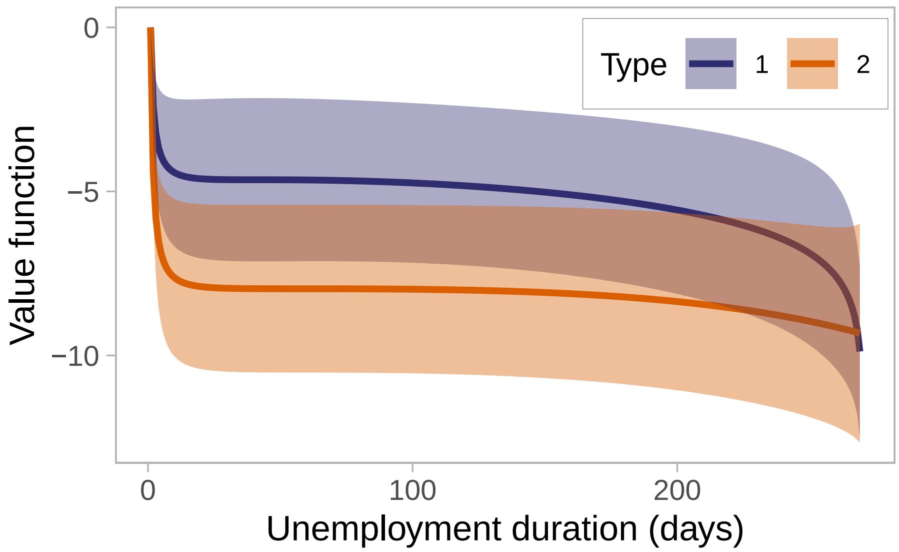
Notes: Value function normalized w.r.t. the value of a job in the first wage bin for each type. Error bars represent 95% bootstrap confidence intervals (500 replications).

Figure 12: Structural unemployment-to-job hazards



Notes: Shaded regions represent 95% bootstrap confidence intervals (500 replications).

Figure 13: Value function of unemployment (normalized)



Notes: Value function normalized w.r.t. the value of unemployment at $t = 0$ for each type. Shaded regions represent 95% confidence intervals.