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# Mass Consumption, EXCLusion, And unemployment 

Reto Foellmi, University of Zurich*<br>Josef Zweimüller, University of Zurich ${ }^{\dagger \ddagger}$

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#### Abstract

We introduce non-homothetic preferences into a general equilibrium model of monopolistic competition and explore the impact of income inequality on the medium-run macroeconomic equilibrium. We find that (i) a sufficiently high extent of inequality divides the economy into mass consumption sectors (where firms charge low prices and hire many workers) and exclusive sectors (where firms charge high prices and hire few workers). (ii) High inequality may lead to a situation of underemployment and that underemployment could be "Keynesian" in the sense that it cannot be cured by downward-flexible real wages. (iii) A redistribution of income from rich to poor (by means of progressive taxation) leads to higher employment and such a redistribution is Pareto-improving. (iv) An exogenous increase in (minimum) real wages have a cost effect (that lets firms reduce their employment) and a purchasing power effect (that creates an incentive for mass production and raises aggregate employment) with ambiguous net effects. (v) The economy may feature multiple equilibria where full-employment and unemployment equilibria co-exist.


## JEL classification: E25, D30, D42, L16, E24

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[^0]
## 1 Introduction

An old idea in economics holds that a more egalitarian income distribution may be beneficial for aggregate employment via its effect on consumer demand. For instance, early Keynesian models have emphasized the ambiguous role of the real wage for output and employment. As wages are both a major source of production costs and an important source of aggregate demand, an increase in the real wage leads to higher aggregate employment when the demand (or purchasing-power) effect outweighs the cost effect. A related argument, frequently brought forward in the context of economic development, suggests that a high purchasing power of the lower classes is favorable for the emergence of mass consumption sectors creating employment opportunities in the more productive segments of the economy. Hence a more egalitarian society may find it easier to overcome the problem of underemployment.

Central to the above arguments are differences in income elasticities of product demand between rich and poor consumers. A redistribution of income from the rich to the poor raises the aggregate demand for labor because the expansion of demand for the various goods by the poor outweighs the reduction of demand by the rich. Put differently, consumers have nonhomothetic preferences. In this paper we explore the general equilibrium implications of such non-homothetic preferences under the assumption that the various consumer goods are supplied by monopolistically competitive firms. Our analysis helps to rationalize the arguments stated above that, prima facie, appear hard to reconcile with rigorous general equilibrium analysis.

By introducing non-homothetic preferences, our analysis differs crucially from standard general equilibrium models of monopolistic competition. These models typically assume homothetic or constant-elasticity-of-substitution (CES-) preferences, as proposed by Dixit and Stiglitz (1977). Understanding the role of non-homothetic preferences for aggregate outcomes is important for at least three reasons. First, the standard assumption of homothetic preferences is highly unrealistic from an empirical point of view. Previous empirical research on the shape of Engel curves has uniformly rejected the hypothesis of unit income elasticities for all products. ${ }^{1}$ Second, because the representative agent paradigm can no longer be applied, the aggregate implications of non-homothetic preferences are not well understood. Finally, non-homothetic preferences provide us with a richer framework in which firms' mark-ups are no longer determined by consumer preferences alone but depend in a more complex way on the fundamental parameters of the economy.

While Dixit and Stiglitz (1977) themselves were more cautious about the CES-assumption and, in the second part of their seminal article, explored also the implications of variable elasticities of substitution (VES), they abstained from introducing income inequality into their

[^1]model. Our analysis shows that introducing such inequality fundamentally changes the character of the general equilibrium. To keep things simple, we undertake our analysis under the assumption of quadratic preferences. Quadratic preferences feature linear individual demand curves (with potentially binding non-negativity constraints) and hence provides us with a simple and tractable framework of analysis. Nevertheless, the quadratic specification should be viewed as an example which, as discussed at the end of the paper, extends to more general specifications of preferences.

The model yields the following results. First, we find that sufficiently high inequality divides the economy into mass consumption sectors and exclusive sectors. Such an asymmetric equilibrium arises even when all sectors are identical ex ante. Under sufficiently high inequality, firms choose (and in equilibrium are indifferent) between selling only to rich consumers at high prices or selling to all consumers at low prices. Exclusive producers go for the former strategy. They skim the rich's willingness to pay and set prices that "exclude" the poor from the market. Mass producers go for the latter. They set low prices that are affordable not only for the rich but also for the poor.

Second, high inequality may lead to a situation of underemployment. Suppose there is an institutionally fixed minimum wage. When inequality is low, mass consumption is prevalent hence aggregate demand for labor at this wage will be high. In contrast, when inequality is high, many firms will adopt the exclusive strategy depressing aggregate demand for labor. Hence a more egalitarian society is more likely to feature full employment (where the minimum wage does not bind), whereas a more unequal society is more likely to end up in underemployment with a binding minimum wage.

Third, starting from such a situation of unemployment, a redistribution of income from the rich to the poor is Pareto-improving. In an underemployment equilibrium, redistributing income from rich to poor raises aggregate employment, hence aggregate output and incomes are higher in the new equilibrium. Both rich and poor consumers benefit from the higher aggregate income and the rich, while losing in relative terms, enjoy higher consumption in the new equilibrium.

Fourth, our model can rationalize the above mentioned double role of the real wage as a cost and a demand factor. An increase in the real wage has two opposing effects. On the one hand, the higher cost of production induces firms to employ less workers (the "cost effect"). On the other hand, a higher real wage also changes the distribution of income in favor of the poor. The reduced inequality generates incentives for firms to switch from exclusion to mass consumption which increases aggregate employment (the "demand effect"). Our simulations show that the cost effect dominates when wages are initially high, whereas the demand effect dominates at initially low wages.

Fifth, there may be multiple general equilibria. In one equilibrium, wages are low and inequality is high, so that the general equilibrium is characterized by a high fraction of exclusive producers and low aggregate employment. The alternative scenario is one with high wages and low inequality. In this case, the general equilibrium is characterized by a larger fraction of mass producers and by full employment. The reason behind the multiplicity are demand complementarities, reminiscent of "Big Push" arguments emphasized by development economists.

There are several strands of the macroeconomic literature to which the present paper is related. Our analysis is most closely related to a recent paper by Saint-Paul (2006). He shows that monopolistic price setting under non-homothetic preferences may imply that technical progress reduces wages. When the price elasticity of demand decreases along the demand curve, monopolistic price setting implies that an increase in productivity (and hence in consumption) shifts the distribution towards profits. A similar effect shows up in our model. However, while Saint-Paul (2006) sticks to a representative agent framework, our paper analyzes the consequences of an unequal distribution for aggregate outcomes.

Another related literature studies how the interaction of non-homothetic preferences and income distribution affect the sectoral distribution of output and employment in the context of economic development. Matsuyama (2002) studies a model where income inequality affects employment in dynamic sectors that generate technical progress via learning-by-doing. He is interested in the dynamic evolution of the economy, whereas our focus is on aggregate employment in a static context. Moreover, firms in the Matsuyama model operate on competitive product markets, whereas in our model firms exert market power and influence the equilibrium outcome via price setting. Murphy, Shleifer, and Vishny (1989) study the effect of income inequality on market size and manufacturing employment under non-homothetic preferences. Their focus is on the entry of firms operating with superior technologies whereas mark-ups and prices are taken as given. In contrast, our analysis focuses on a situation where entry is prohibited and income distribution effects work via endogenous prices and mark-ups. ${ }^{2}$

A further literature addresses the issue of whether there may be unemployment when the labor market is competitive but the product market is not (see Hart 1982, D'Aspremont et al. 1990, Dehez, 1985, and Silvestre, 1990; for a survey of this literature, see Silvestre, 1993). This literature points out that unemployment may occur when firms' revenues are bounded so that labor demand may fall short of labor supply even when the wage rate falls to zero. Such a

[^2]possibility also arises in our model. While these papers have been concerned with the existence of unemployment equilibria in a representative-agent environment, our model focuses on the effect of heterogenous consumers.

The paper is organized as follows. In the next section we present our basic model and derive a households optimal consumption levels and a monopolist's optimal prices and quantities. In section 3 we solve the general equilibrium under the special case that the equilibrium is symmetric. Section 4 studies the existence of an asymmetric equilibrium and gives the equilibrium conditions for such an outcome. Section 5 studies the relationship between inequality and unemployment. Section 6 and 7 study a full employment and multiple equilibria, respectively. In section 8 we study the robustness of our results with respect to the central assumptions and section 9 concludes.

## 2 Monopolistic competition with quadratic preferences

Preferences. Consider a population of consumers of mass 1. Consumers have identical preferences over a continuum of differentiated products $j \in[0, N]$. Consumption of these goods enters total utility in a symmetric and separable way. The utility gain from consuming $c$ units of a particular good $j$ is given by $v(c(j))=-(1 / 2)(s-c(j))^{2}$ with $s$ being the saturation level. The consumer maximizes total utility

$$
\begin{equation*}
u(\{c(j)\})=\int_{0}^{N} v(c(j)) d j=-\int_{0}^{N} \frac{[s-c(j)]^{2}}{2} d j \tag{1}
\end{equation*}
$$

subject to the budget constraint $\int_{0}^{N} p(j) c(j) d j \leq y$. This yields first order conditions

$$
\begin{array}{ll}
c(j)=s-\lambda p(j) & \text { if } p(j) \leq s / \lambda, \text { and }  \tag{2}\\
c(j)=0 & \text { if } p(j)>s / \lambda,
\end{array}
$$

with $\lambda$ as the consumer's marginal utility of income. We note consumers differ in $\lambda$ (because they earn different incomes). While the quadratic utility assumption may seem special, we discuss at the end of the paper that this serves as an example that generalizes to a broader class of preferences.

Technology. All goods are produced with the same technology. Production takes place with labor as the only production factor. We assume a simple linear technology $x(j)=a l(j)$ where $x(j)$ is output of good $j$ and $l(j)$ is the labor input. The productivity parameter $a>0$ is an exogenously given constant.

Endowments. Consumers are heterogenous with respect to their incomes. As the income level is endogenously determined in the model, the distribution we take as given is that of
labor endowments, and that of shares in monopolistic profits. Assume there is a fraction $\beta$ of poor households owning $\Delta_{P}$ units of labor and $1-\beta$ rich owning $\Delta_{R}$ units. Since we have normalized aggregate labor supply to unity we have $\beta \Delta_{P}+\Delta_{R}(1-\beta)=1$. This leaves us with one degree of freedom and we take $\Delta_{P} \equiv \delta$ as exogenous from which $\Delta_{R}=(1-\beta \delta) /(1-\beta)$ is determined. Similarly, we assume that profits distributed to a poor household amount to a fraction $\Gamma_{P}<1$ of profits per capita and profits distributed to a rich household amount to $\Gamma_{R}>1$. Again, we must have $\beta \Gamma_{P}+\Gamma_{R}(1-\beta)=1$ and we take $\Gamma_{P} \equiv \gamma$ as exogenous from which $\Gamma_{R}=(1-\beta \gamma) /(1-\beta)$ is determined. We concentrate on the (realistic) case where the poor rely more heavily on labor income (and suffer more heavily from unemployment) which is the case when $\gamma<\delta$. Occasionally, we will focus on the special case when workers own no firm shares and firm owners do not work which is the case when $\gamma=0$ and $\delta=1 / \beta$; or on the case of equal income composition when $\gamma=\delta .{ }^{3}$

The resource constraint. Aggregate labor supply is equal to unity as each consumer inelastically supplies one unit of labor. Aggregate labor demand is the sum of market demands to produce the various goods. Denote the market demand for good $j$ by $x(j)$, then the market demand for labor in sector $j$ is $x(j) / a$. The economy's resource constraint can then be written as

$$
\begin{equation*}
1 \geq \frac{1}{a} \int_{0}^{N} x(j) d j \tag{3}
\end{equation*}
$$

Equilibrium quantities. We assume the market for each good is monopolistic. There is a mass of $N$ monopolists who are unique suppliers for their respective product and who set prices to maximize profits. Entry is prohibited. Each firm is negligible relative to the aggregate and takes wages and the prices for all other goods as given. The level of market demand faced by firm $j$ is simply the sum of individual demands. Using first order conditions 2 for the respective types of consumers (noting that their $\lambda$ 's are different), the market demand function of this firm, $x(j, p(j))$, can be expressed as

$$
x(j, p(j))=\left\{\begin{array}{lll}
0 & \text { if } p(j) \in\left[s / \lambda_{R}, \infty\right)  \tag{4}\\
(1-\beta)\left[s-\lambda_{R} p(j)\right] & \text { if } p(j) \in\left[s / \lambda_{P}, s / \lambda_{R}\right), \\
s-\left[\beta \lambda_{P}+(1-\beta) \lambda_{R}\right] p(j) & \text { if } \quad p(j) \in\left[0, s / \lambda_{P}\right) .
\end{array}\right.
$$

When the price exceeds the reservation price of the rich, $p(j) \geq s / \lambda_{R}$, market demand is zero; when the price is between the reservation prices of rich and poor, $p(j) \in\left(s / \lambda_{P}, s / \lambda_{R}\right]$,

[^3]only rich consumers purchase; when the price falls short of the reservation price of the poor, $p(j)<s / \lambda_{P}$, both rich and poor consumers purchase (Figure 1).

Figure 1
Now consider a firm's profit maximizing price. The monopolist being very small relative to the aggregate economy takes the wage rate and the prices of all other goods as given (i.e. it takes the consumers' $\lambda$ 's as given.) The firm chooses the price $p(j)$ that maximizes the profit function $[p(j)-w / a] x(j, p(j))$. As the market demand function (4) is piecewise linear, there are two candidates for that price. The profit-maximizing price along the steep segment (where only the rich buy) or the profit-maximizing price along the flat segment (where all consumers buy). Given $\bar{p}$, the monopoly price along the steep segment and $\underline{p}$, the equilibrium quantities along the respective segments can be calculated

Lemma 1 Denote by $x^{E}$ and $x^{M}$ the equilibrium quantities supplied along the steep and the flat segment of the market demand curve, respectively. These quantities are given by $x^{E}=$ $(1-\beta) s(\bar{p}-w / a) /(2 \bar{p}-w / a)$ and $x^{M}=s(\underline{p}-w / a) /(2 \underline{p}-w / a)$.

Proof. See Appendix A.

## 3 The symmetric equilibrium

Recall that all firms face the same demand and cost functions. This implies that, in equilibrium, all firms must earn the same profit. There are two possible outcomes: (i) a symmetric equilibrium where all firms charge the low price and all consumers purchase all goods; (ii) an asymmetric equilibrium where firms are indifferent between the high and the low price. In the latter case some firms charge high prices and sell only to the rich; and some firms charge a low price and serve the whole customer base. In other words, pricing decisions lead to a particular industry structure that divides the economy into "mass consumption sectors" and "exclusive goods sectors".

Let us briefly discuss the symmetric case. In the symmetric equilibrium, all firms operate along the flat segment of the market demand curve and supply $x^{M}$ in equilibrium. We are free to choose a numeraire, so let us set $\underline{p}=1$ and interpret $w$ as the real wage. Aggregate demand $L^{D}$ for labor is given by

$$
\begin{equation*}
L^{D}(w)=\frac{s N}{a} \frac{1-w / a}{2-w / a} . \tag{5}
\end{equation*}
$$

When there is a perfectly competitive labor market, the general equilibrium is characterized by $L^{D}\left(w^{*}\right)=1$ where $w^{*}$ is the market clearing wage, given by

$$
\begin{equation*}
w^{*}=a \frac{s N-2 a}{s N-a} . \tag{6}
\end{equation*}
$$

As pointed out by Saint-Paul (2006), this equilibrium has two interesting features. First, for the existence of an equilibrium where the labor force is fully employed we must have $s N / 2>a$. Notice, however, that $N, s$, and $a$ are exogenous and nothing prevents parameter values to be such that the full employment is not feasible. When the full employment condition is violated, the real wage falls to zero and even with a zero real wage, labor demand falls short of labor supply. The reason is intuitive. $s / 2$ is the level of output that maximizes a firm's revenue and, when $w=0$, the output level $s / 2$ also maximizes profits. Hence $s N / 2$ is the highest aggregate output level that is consistent with profit-maximization. $a$ is the full employment output. When $s N / 2<a$ firms are not willing to supply the full employment output even when the production of output is costless. In other words, there is no wage $w^{*} \geq 0$ that clears the labor market.

Second, technical progress (an increase in $a$ ) may not be as favorable and even harmful for workers. At low levels of $a$, technical progress leads to increases in $w$ but less than proportional. At high levels of $a$ technical progress may reduce real wages. (And if $a$ gets very large, the full employment condition gets violated.) The reason is the following: When demand functions feature decreasing price elasticities (such as in the linear case), a higher feasible output level is associated with higher mark-ups and a lower real wage. In such a situation, an increase in $a$ increases the wedge between a worker's productivity and the wage rate. In the present context, technical progress may even drive down the wage to very low levels, a situation reminiscent of Marx's vision of technical progress as a cause of exploitation and the pauperization of the proletariat.

## 4 The asymmetric equilibrium

An asymmetric outcome arises from the combination of two features: non-homothetic preferences and a sufficiently high extent of economic inequality. In this section we solve the model in the asymmetric case. We derive a sufficient condition for an asymmetric outcome, characterize the equilibrium conditions, and describe how the model can be solved. In later sections we then explore the effects changes in inequality under alternative equilibrium scenarios - an unemployment equilibrium, a full employment equilibrium, and multiple equilibria.

### 4.1 Existence of an asymmetric equilibrium

In the asymmetric equilibrium some firms sell to rich consumers at high prices and other firms sell to all household. To check when this is actually the case, we consider a single firm's incentive to deviate (= sell to the rich at the high price), given a situation where all other firms sell to all households. It turns out that the exclusion strategy is worthwhile if the rich are much
wealthier than the poor. This is very intuitive. Were rich and poor almost identical, the steep segment of the market demand curve would become irrelevant. The following proposition gives a sufficient condition that a symmetric equilibrium does not exist. In that case, the equilibrium is asymmetric.

Proposition 1 If $\gamma \leq \delta$, an asymmetric equilibrium exists if $\beta>\frac{4 \delta(1-\zeta)^{2}}{4 \delta^{2} \zeta(1-\zeta)-4 \delta \zeta+(1+\delta)^{2}}$ with $\zeta \equiv a /(s N)$.

## Proof. See Appendix B.

It is easy to check that the right hand side of this inequality goes to zero if $\delta \rightarrow 0$, goes to unity if $\delta \rightarrow 1$, and is monotonically increasing over this range as can be easily checked. Hence, and confirming our intuition, the above condition in Proposition 1 is more likely to hold with higher inequality, that is, when $\beta$ is large and/or $\delta$ is small. This confirms our claim that, when inequality is sufficiently high, an asymmetric outcome will prevail.

It is shown in the proof of Proposition 1 that the right-hand-side of the condition decreases in $\zeta$. A rise in $\zeta$ implies there are more $(\beta, \delta)$-combinations for which the condition in the proposition is violated, so that an asymmetric equilibrium becomes more likely. The reason is the following. A higher level of $a /(s N)$ means - in equilibrium - higher production per firm and allows an increase in consumption for both groups. This increases mark-ups as both types of consumers purchase at a less elastic point on their individual demand curves. However, since rich consumers are closer to their saturation point than the average consumer this causes a disproportionate decrease in their demand elasticity. In other words, when $a /(s N)$ increases mark-ups increase more strongly when firms sell exclusively to the rich and increase less strongly when they sell on mass markets. As a result, the exclusion strategy becomes more attractive.

### 4.2 Equilibrium conditions

Let us now characterize the asymmetric equilibrium. There is a fraction of $n<1$ firms that sells to the whole customer base at the low price (mass producers) and a fraction of $1-n$ firms who sell only to the rich at the high price (exclusive producers). As we are free to order goods, we arrange the index $j$ in such a way that goods $j \in[0, n N]$ are mass consumption goods and goods $j \in(n N, N]$ are exclusive goods.

The asymmetric equilibrium can be represented in terms of three conditions. The first condition states that firms are indifferent between the mass consumption and the exclusive strategy, $\Pi^{M}=\Pi^{E}$. Profits are $\Pi^{M}=(1-w / a) x^{M}$ for a mass producer and $\Pi^{E}=(\bar{p}-w / a) x^{E}$ for an exclusive producer. Using Lemma 1 this equilibrium condition can be expressed in terms
of the endogenous variables $w$ and $\bar{p}$

$$
\begin{equation*}
s(1-\beta) \frac{(\bar{p}-w / a)^{2}}{2 \bar{p}-w / a}=s \frac{(1-w / a)^{2}}{2-w / a} \tag{7}
\end{equation*}
$$

It is straightforward to verify that equation (7) can be solved for the price of the exclusive good $\bar{p}$ and expressed as a function of the real wage $w .{ }^{4}$ For further use express the equilibrium condition (7) as $\bar{p}=g(w)$ with $g^{\prime}(w)<0$. The negative relationship between $\bar{p}$ and $w$ is very intuitive. A reduction in the wage rate $w$ increases profits per unit of output by the same (absolute) amount both for exclusive producers and for mass producers. With prices unchanged, their larger market size lets profits of mass producers increase more strongly than the profits of exclusive producers. To restore equilibrium, a higher price $\bar{p}$ is required to prevent exclusive producers from switching to the mass consumption strategy.

The second equilibrium condition follows from the fact that consumers' budget constraints have to be exhausted. As a results the relative incomes of rich to poor must be equal to their relative consumption expenditures. Formally, we must have $y_{R} / y_{P}=\left[n N c_{R}^{M}+(1-n) N \bar{p} c_{R}^{E}\right] / n N c_{P}^{M}$. It is easy to show that relative incomes of rich to poor $y_{R} / y_{P}$ can be written as a function of the wage $w$ and the unemployment rate $u$. The income level of a poor consumers as $y_{P}=\delta w(1-$ $u)+\gamma N \Pi(w)$ and of a rich consumer as $y_{R}=(1-\beta \delta) /(1-\beta) \cdot w(1-u)+(1-\beta \gamma) /(1-\beta) \cdot N \Pi(w)$ where we observe that equilibrium profits are $\Pi(w)=(1-w / a)^{2} /(2-w / a)$ from (7). Notice that earnings from labor depend not only on the wage rate but also on the extent to which a consumer's labor force is utilized. ${ }^{5}$ Hence we can write $y_{R} / y_{P}=\phi(w, u)$, with $\partial \phi / \partial w<0$ and $\partial \phi / \partial u>0$. The partial derivatives of $\phi$ indicate that the poor suffer more from a wage cut and/or from an increase in unemployment than the rich because they have to rely more heavily on labor income that the rich, $\delta>\gamma$. Relative consumption expenditures can be expressed in

$$
\begin{aligned}
& { }^{4} \text { Equation (7) is a quadratic equation in } \bar{p} \text { with the relevant root } \\
& \qquad \bar{p}=w / a+\left[(1-w / a)^{2}+(1-w / a) \sqrt{1-\beta(2-w / a) w / a}\right] /[(1-\beta)(2-w / a)] .
\end{aligned}
$$

## Clearly, $\bar{p}$ is decreasing in $w$.

${ }^{5}$ When there is unemployment, we assume all households are equally affected. This is, unemployment takes the form of a reduction in hours worked, the same for all households. We make this assumption for analytical convenience. It ensures that also in an equilibrium with unemployment there are only two types of consumers. Alternatively, we could assume that only a fraction $u$ of all households are affected by unemployment whereas the remaining households are fully employed. This would complicate our analysis by creating four types of consumers: the unemployed poor, the unemployed rich, the employed poor and the employed rich. In such an equilibrium, firms would not only the choice between mass consumption and exclusion but between selling only the richest, to the richest and second richest, to the richest, second richest and third richest, or to all households. Obviously, this would complicate the exposition considerably without adding much substance to the analysis. The equilibrium presented in the text is identical to the one obtained from a four group economy, where selling only to the unemployed poor (instead of all poor) and selling only to the unemployed rich (instead of all rich) is never a profitable option.
terms of the endogenous variables $w$ and $n$ using the expressions for $c_{R}^{M}, c_{R}^{E}$, and $c_{P}^{M}$ in the proof of Lemma 1 and condition (7) to rewrite the price of exclusive goods as $\bar{p}=g(w)$

$$
\begin{equation*}
\phi(w, u)=\frac{n(2 g(w)-1-w / a) /(2 g(w)-w / a)+(1-n) g(w)(g(w)-w / a) /(2 g(w)-w / a)}{n[1-(1 / \beta)(1 /(2-w / a)-(1-\beta) /(2 g(w)-w / a))]} \tag{8}
\end{equation*}
$$

We note that equation (8) is linear in $n$ and can be expressed as a function of $w$ and $u$. For further use we express this relationship as $n=h(w, g(w), \phi(w, u)) \equiv n(w, u)$.

We get our third equilibrium condition from the economy's resource constraint. Aggregate labor demand is given by $L^{D}=\left[n N x^{M}+(1-n) N x^{E}\right] / a$. We use the expressions in Lemma 1 to replace $x^{M}$ and $x^{E}$, and equilibrium conditions (7) and (8) we replace $n$ by $h(w, \bar{p}, \phi(w, u) \equiv$ $n(w, u)$. Aggregate labor demand in general equilibrium can be expressed as

$$
\begin{equation*}
L^{D}(w, u)=\frac{s N}{a}\left(n(w, u) \frac{1-w / a}{2-w / a}+(1-n(w, u))(1-\beta) \frac{g(w)-w / a}{2 g(w)-w / a}\right) \tag{9}
\end{equation*}
$$

The above equation has a very intuitive interpretation. Labor demand is proportional to $s N / a$, the number of workers needed to produce at the economy's saturation point. The term in parenthesis reflects the "distance" to saturation in this monopolistic economy. This depends on the percentage mass consumption firms $n(w, u)$ and the demand for labor in these firms (proportional to $(1-w / a) /(2-w / a))$ as well as on the percentage exclusive firms $1-n(w, u)$ and the demand for labor in these firms (proportional to $(g(w)-w / a) /(2 g(w)-w / a))$. Notice that the term in parenthesis has its maximum at $1 / 2$ when $n=1$ and $w=0$. In this case all firms are mass producers and supply the revenue maximizing quantity.

To understand the shape of this general-equilibrium-labor-demand curve it is instructive to compare the aggregate labor-demand curve (9) with some well-known standard cases. In the case of perfect competition labor demand is horizontal (since marginal costs are constant) at $w=a$. In the case of monopolistic competition with CES preferences, labor demand is horizontal but at a lower level reflecting the fact that part of a worker's output is appropriated by the monopolistic firm. In the present context, where preferences exhibit a decreasing elasticity of substitution and where there is economic inequality, there is a much more complex relationship between employment and the real wage. This relationship is the result of two different channels. First, a higher demand for labor reflects higher production in each market. With a decreasing price elasticity of demand this implies higher equilibrium mark-ups and a lower real wage. Second, to the extent that a lower real wage is associated with a more uneven distribution of income there will be less mass consumption sectors and more exclusive sectors which, ceteris paribus, reduces employment. As these two channels imply contradicting relationships, the aggregate relationship between employment and the real wage is not a priori clear. Hence, at a given real wage, the aggregate labor-demand curve may be upward or downward sloping.

In equilibrium, total labor supply is not necessarily fully employed and only a fraction $1-u$ of all workers may have a job, where $u$ is the unemployment rate. So, our third general equilibrium condition becomes

$$
\begin{equation*}
1-u=L^{D}(w, u) \tag{10}
\end{equation*}
$$

In a full employment equilibrium, we have $u=0$ and equation (10) solves the model for the real wage $w$. In an unemployment equilibrium we either have a situation where there is a positive minimum wage $w=\bar{w}>0$ or, in the absence of a minimum wage, we have a situation where the labor market does not clear even when the wage falls to zero (see Figure 2). Once we have solved for either $u$ or $w$, the remaining endogenous variables $\bar{p}$ and $n$ can be determined using (7) and (8).

Figure 2

## 5 Economic inequality and unemployment

In this section we explore the asymmetric equilibrium when there is unemployment. In particular, we examine the impact of economic inequality (as captured by the parameters $\beta, \gamma$, and $\delta)$ on the general equilibrium. We start with the special case when the labor market is perfect and parameters are such that unemployment arises even when the real wage falls to zero. While one might argue this is not an empirically relevant case, it is nevertheless instructive because it yields a very simple and intuitive solution which carries over to the more relevant case when unemployment is associated with a positive minimum wage.

### 5.1 A special case: unemployment with a zero wage

In the symmetric equilibrium, we saw that unemployment with a zero wage arises when $N s / 2<$ $a$. In the asymmetric equilibrium, unemployment arises under weaker conditions. While mass producers reach their profit (and revenue) maximizing output still at $s / 2$, the profit maximizing output of exclusive firms is already reached at output level $(1-\beta) s / 2$. When $n N$ firms are mass producers and $(1-n) N$ firms are exclusive producers, the highest level of output that firms are willing to supply in the asymmetric equilibrium is $n N s / 2+(1-n) N(1-\beta) s / 2<s N / 2$.

We can easily solve for the asymmetric unemployment equilibrium by setting $w=0$ in equilibrium conditions (7), (8), and (10). Condition (7) simplifies to

$$
\bar{p}=1 /(1-\beta)
$$

The left-hand side of condition (8) becomes $y_{R} / y_{P}=\phi(0, u)=(1-\beta \gamma) /(1-\beta)$. When wages are zero, the relative income of rich to poor consumers is solely determined by relative
profit shares. Using $w=0$ and $\bar{p}=1 /(1-\beta)$ on the right-hand-side of (8) and solving for $n$ yields

$$
n=\frac{\gamma}{\beta} .
$$

Finally, plug $w=0, n=\gamma / \beta$, and $\bar{p}=1 /(1-\beta)$ into equilibrium condition (10) to get the equilibrium level of unemployment

$$
u=1-\frac{s N}{2 a}(1+\gamma-\beta) .
$$

These results are very intuitive. If inequality increases because relative income of the poor $\gamma$ goes down, or because the group size of the poor $\beta$ increases, more firms find it optimal to charge a price that the poor cannot afford and sell exclusively to the rich. This reduces the share of mass producers, decreases the demand for labor even further and increases unemployment.

This result has striking welfare implications. Consider a redistribution of endowments from the rich to the poor such that $\gamma$ rises and $\beta$ remains constant. This implies that $n=\gamma / \beta$ increases and more products are sold to all consumers. Using the expressions in the proof of Lemma 1 and setting $w=0$ yields $c_{R}^{M}=(1+\beta) s / 2$ and $c_{P}^{M}=\beta s / 2$ for mass consumption goods $j \in[0, n N]$ and $c_{R}^{E}=s / 2$ and $c_{P}^{E}=0$ for exclusive goods $j \in(n N, N]$.

A redistribution of income from the rich to the poor which increases $\gamma$ increases the share of mass consumption goods but has not effect on the equilibrium quantities on mass market and on exclusive markets. Such a redistribution clearly benefits the poor. They can purchase more mass consumption goods, and purchase any given mass consumption good in the same quantity as before. More surprisingly, such a redistribution also benefits the rich. When the share of mass consumption sectors $n$ increases, there are more sectors with a low price (where the rich purchase in high quantity), and less sector with a high price (where the rich purchase in low quantity). This allows to rich to increase their overall consumption and welfare. Stated differently, a redistribution of (firm share) endowments from the rich to the poor creates additional demand. This increases the degree of resource utilization and creates additional income. Both groups of consumers benefit from the higher income and can increase their consumption.

We summarize these results in the following
Proposition 2 a) In an asymmetric unemployment equilibrium with a zero wage, a redistribution of income from rich to poor increases aggregate output and employment. b) Such a redistribution is Pareto-improving.

### 5.2 Unemployment with positive minimum wage

The case where wages become literally zero may sound implausible to many readers. The real wage in modern economies is far from zero. In contrast, most workers participate in
achieved societal standards of living. Labor market institutions such as union bargaining and, in particular, minimum wage legislation prevent the real wage from falling to zero.

It is therefore interesting to study how inequality affects aggregate output and employment in the more relevant case when there is a positive minimum wage $\bar{w}>0 .{ }^{6}$ Equilibrium unemployment can be determined from (10)

$$
1-u=L^{D}(\bar{w}, u) .
$$

Recall that we used the function $n=n(u ; \bar{w})$ to express the resource balance condition (10) in terms of $u$ as the unique endogenous variable. From equation (8), it is easy to show that $n$ and $u$ are negatively related. When there is higher unemployment, the extent of exclusion is larger. The reason is that higher unemployment increases relative incomes of rich to poor (because $\delta>\gamma$ ). The higher relative income of rich to poor makes it more attractive for firms to sell exclusively to the rich at the high price rather than serving the entire customer base which decreases the share of mass consumption sectors.

The situation is drawn in Figure 2 above. The Figure is drawn in such a way that there is no intersection between labor demand and labor supply curve. In other words, there exists no positive wage that clears the market. The labor supply curve is a vertical line. Under the parameter values chosen for Figure 2, the labor demand curve first falls and then bends backwards. With a minimum wage $\bar{w}>0$, the intersection of this minimum wage floor with the labor demand curve determines aggregate employment (and output).

Is Proposition 2 still valid? The answer is yes. To see these employment effects consider a redistribution of income from rich to poor such that $\gamma$ and/or $\delta$ increase. We see from (7) that such a change leaves relative prices of exclusive to mass consumption goods $g(\bar{w})$ unaffected. We also see that an increase in $\gamma$ and/or $\delta$ raises the relative income of rich to poor $\phi(\bar{w}, u)$. We know from (8) that the share of mass consumption sectors $n=h(\bar{w}, g(\bar{w}), \phi(\bar{w}, u))$ unambiguously increases in $\phi$. We know further that, holding the unemployment rate constant, aggregate labor demand increases if the share of mass consumption sectors $n$ increases. This implies that, in the new equilibrium, aggregate labor demand is higher and unemployment is lower. Notice that there is a reinforcing effect on labor demand that comes from the effect of the unemployment rate on relative incomes $\phi(\bar{w}, u)$. Since the poor depends more heavily on labor income, $\delta>\gamma$, the reduction in unemployment benefits the poor are more strongly than the rich and increases relatives incomes even further. Hence the unemployment-reducing effect of the redistribution reinforces the positive employment effect.

[^4]Figure 3
Now consider the effect of this redistribution on welfare. We see from equation (7) that profits in the new equilibrium do not depend on $\delta$ and/or $\gamma$ and remain unchanged. From (7) we also see that both the prices $g(\bar{w})$ and output levels $(g(\bar{w})-\bar{w} / a) /(2 g(\bar{w})-\bar{w} / a)$ of exclusive goods remain unchanged as well. This implies that the rich consume exclusive goods in the same quantity as before. From equation (2) it must be that the rich consume also mass consumption goods in the same quantity as before. Since in the new equilibrium there are more mass consumption sectors, there are more sector where the rich consume in high quantity and less sectors where the rich consume in low quantity. Just like before, the welfare level of the rich increases.

Less surprisingly, also the welfare level of the poor increases. They consume mass consumption goods in the same quantity as before and because there are more mass consumption sector in the new equilibrium, they consume more in total which increases their welfare. We summarize the above discussion in the following

Proposition 3 a) In an asymmetric unemployment equilibrium with a positive minimum wage $\bar{w} \geq 0$, a redistribution of endowments from rich to poor increases aggregate output and employment. b) Such a redistribution is Pareto-improving.

The above proposition states that a more equal distribution of income may be favorable for aggregate output and employment. One obvious way how such a redistribution can be achieved is progressive taxation.

A different way to influence the distribution of income which is adopted in many countries is minimum wage legislation. It is therefore suggestive to ask how an increase in the minimum wage affects macroeconomic outcomes. A minimum wage increase has two opposing effects: a cost effect and a demand (purchasing power) effect. The cost effect lets firms move up their individual labor demand curves. This clearly decreases employment. The purchasing power effect arises because increasing the minimum wage leads to a more equitable distribution of income. This induces former exclusive producers to become mass producers. As switching from exclusion to mass consumption is associated with an increase in output and employment for the individual firm, an increase in the number of mass producers is associated with higher output and employment in the aggregate.

The two effects can be readily seen from inspection of equation (10). The cost effect shows up in the labor demands of mass and exclusive producers which are, respectively, given by $x^{M}(\bar{w})=(1-\bar{w} / a) /(2-\bar{w} / a)$ and $x^{E}(\bar{w})=(g(\bar{w})-\bar{w} / a) /(2 g(\bar{w})-\bar{w} / a) .^{7}$ Since $g(\bar{w})$ is

[^5]falling in $\bar{w}$, it is immediately clear that increasing $\bar{w}$ reduces the labor demands of exclusive and mass producers. The purchasing power effect shows up in the function $n(u ; \bar{w})$. As long as the increase in $\bar{w}$ increases in $n$ (which holds for most parameter values), the purchasing power effect stimulates employment and output. The reason is the same as above. When a minimum wage increase changes the distribution in favor of the poor, the higher purchasing power of the poor gives firms an additional incentive to supply mass consumption rather than exclusive goods and switching from exclusion from mass consumption raises employment.

We can easily make these arguments using Figure 3. An increase in the minimum wage $\bar{w}$ shifts the minimum wage floor in Figure 3 upwards, but does not affect the labor demand curve. The relative importance of cost- and purchasing power effect shows up in the slope of the labor demand curve. In Figure 3, the labor demand curve is downward sloping at the point of intersection. In that situation increasing the minimum wage reduces employment. In other words, the cost effect dominates. At lower wage levels, the labor demand curve has a positive slope. In that case, the purchasing power effect dominates and minimum wages increase aggregate employment.

Now consider the effects on consumer welfare. Recall that we have assumed that all household are equally affected by unemployment. When the demand effect dominates and unemployment decreases, all consumers benefit as they can better utilize their labor force and increase their income. When the cost effect dominates and unemployment increases, all households lose as their income is reduced.

Proposition 4 a) An increase in the minimum wage has a cost effect, decreasing aggregate output and employment; and a demand effect increasing output and employment. b) An increase in the minimum wage is Pareto-improving if the demand effect dominates and is Paretoinferior when the cost effect dominates.
unit of labor $\bar{w}$. In reality, governments want to ensure a minimum wage income for the low qualified. Hence legistlation sets a wage floor on (hourly) earnings. Suppose minimum wage legistlation requires firms to pay at least $\bar{w}$ to a full time worker and that the poor supply less efficiency units than the rich $\delta<1$. In that case the legislation ensures a minimum wage income of $\bar{w}$ for the poor and - since the poor supply $\delta$ efficiency units of labor - implicitely establishes a compensation of $\bar{w} / \delta$ per efficiency unit of labor. If the minimum wage is binding, market forces will lead to a situation where (rich and poor) workers earn the same wage per efficiency unit of labor. Since the rich supply $(1-\beta \delta) /(1-\beta)$ efficiency units their compensation is $\bar{w}(1-\beta \delta) /(\delta(1-\beta))$. Hence, in the present context, setting a minimum labor labor income for low-income workers is identical to fixing the minimum wage per efficiency unit of labor which is discussed in the text.

## 6 Full employment

Now consider a full employment equilibrium. In such an equilibrium minimum wages do not bind and we have $u=0$ and $w=w^{*}$ where $w^{*}$ is the market clearing wage. From (10) we get an equation in the endogenous variable $w^{*}$

$$
\begin{equation*}
1=L^{D}\left(w^{*}, 0\right) . \tag{11}
\end{equation*}
$$

Figure 4 shows the equilibrium graphically. Labor supply, the left-hand-side of equation (11) is horizontal and the right-hand-side of equation (11) is downward sloping at high wage but may bend backward at lower wages. Figure 4 is drawn in such a way that there is unique equilibrium at point A. At this point the labor demand curve slopes downward. The latter situation also guarantees stability (in the sense that wages increases due to excess demand for labor pushes the economy closer towards the market clearing wage).

Figure 4
It is interesting to see how, under full employment, the fraction of mass consumption sectors and the distribution of income between wages and profits are affected by the extent of economic inequality. Let us again consider the effect of a redistribution from rich to poor by increasing $\gamma$ and/or $\delta$. Such a redistribution increases $\phi(w)$ and unambiguously increases $n$, see equilibrium condition (8). This implies that, for a given wage $w$, a larger share of mass products increases the aggregate demand for labor. In terms of Figure 4, the labor demand curve shifts to the right. Since, the labor demand schedule cuts the labor supply curve from above, the new equilibrium is associated with a higher market clearing real wage $w^{*}$. A higher real wage is directly associated with a lower mark-up for mass producers $1-w^{*} / a$. Because the price of exclusive goods $g(w)$ depends negatively on $w$ (see condition (7)), it follows that the exclusive producers' mark-up $g\left(w^{*}\right)-w^{*} / a$ falls.

Proposition 5 a) In an asymmetric equilibrium with full employment, an increase in inequality increases the extent of exclusion (decreases n). b) Increasing inequality increases mark-ups and profits, and decreases the real wage.

Proof. See Appendix.
The economic intuition behind this result follows immediately from our previous analysis. A redistribution of income in favor of the poor creates an incentive for firms to adopt the mass consumption strategy. The rich loose purchasing power which, for a given $n$, decreases the profit of exclusive producers. Hence some firms will switch to the mass consumption strategy and increase their demand more labor. This drives up the real wage and decreases profit margins.

## 7 Multiple equilibria

When the income ratio $y_{R} / y_{P}=\phi(w, u)$ decreases in $w$, there is an additional factor affecting aggregate labor demand (9). For lower levels of the real wage $w$, inequality is higher and thus the number of mass consumption goods $n$ is lower. This additional effect decreases labor demand. As simulations show, this effect may be so strong such that the labor demand schedule bends backwards giving rise to multiple equilibria. Intuitively, in an equilibrium with low wage $w$ inequality is high. This implies that many firms choose to sell to the rich only. However, this causes labor demand to be low which supporting an equilibrium where the wage rate is low. Instead, if the equilibrium wage is high, inequality and exclusion is on a low level such that labor demand is high supporting a high-wage equilibrium with full employment. As shown in Figure 5, equilibria with full employment and unemployment coexist: two equilibria feature full employment and one equilibrium is characterized by unemployment.

## Figure 5

## 8 How general are our results?

In this paper we have presented an model where consumers have non-homothetic preferences and where the distribution of income plays a central role for aggregate employment. Our model has started out from simplifying assumptions. Let us briefly discuss the robustness of our results with respect to these assumptions.

Preferences In our model we have assumed a quadratic subutility function. We used the quadratic specification because it keeps the analysis simple and yields closed form solutions. The quadratic subutility function has two crucial properties. First, the marginal utility from consuming the first unit is finite, $v^{\prime}(0)=s<\infty$. This is a necessary condition for an equilibrium where poor consumers are not able to afford all goods (i.e. the non-negativity constraint may become binding). Second, the quadratic specification implies that a linear demand curve of a particular consumer and a price elasticity of demand which decreases in consumed quantity. Denoting by $\eta(c)$ the price elasticity of demand we have $\eta(c)=(s-c) / c$ which is decreasing in $c .^{8}$

Our analysis extends in a straightforward way to the subclass of hyperbolic absolute risk aversion (HARA) preferences that feature $v^{\prime}(0)<\infty$. HARA-preferences with that property

[^6]also feature decreasing price elasticities along individual demand curves. Provided that this elasticity falls below unity at a finite $c$, both unemployment and full employment equilibria are possible under appropriate parameter values. We elaborate this in more detail in appendix D. Going beyond HARA, things become more complicated because the distribution of income does not only affect consumption along the extensive margin (how many consumers can purchase a certain good) but also the intensive margin. As Engel-curves are no longer affin-linear, market demand curves depend on the distribution of income even in symmetric equilibria. ${ }^{9}$

Entry An important assumption of our analysis was a fixed number of firms. What happens if we allow for entry? As the demand for labor is increasing in the number of firms, allowing entry is an obvious ways of eliminating unemployment. To see this, consider the original Dixit and Stiglitz (1977) framework in which homogeneous labor is used both in setting up new firms and in producing final output. In that case, low wages would eliminate unemployment by making entry of new firms very cheap. However, in reality may be difficult even if wages for homogenous labor are very low. Product market regulations and/or scarce resources (such as entrepreneurial talent, ideas, specific skills/technologies) could be reasons which entry is prohibitively expensive.

To illustrate that the flavor of our results survives also in more elaborate contexts, appendix C sketches a simple model with skilled and unskilled workers. Skilled workers are needed to create a new firm and both types of workers can be used to produce final output. We show that, if skilled workers are not a necessary input in final goods production, unemployment can arise even if the unskilled wage goes to zero. In that case, we are back in the model studied in section 6 and all results discussed there go through.

Factor substitution Our model has assumed that labor is the only production factor and that there is no possibility of factor substitution. Introducing capital into the picture does lead to a substantial change of our results. To see this, suppose output is produced with homogenous labor and physical capital using the production function $a \tilde{F}(k, l)$. With a given number of firms (and associated maximum level of employment), eliminating unemployment would require to reduce labor productivity. However, as long as labor has a positive marginal product, which increases in the capital stock, capital and labor will "compete" for employment. A higher (fully employed) capital stock will make things even worse: By increasing the productivity of workers the demand for labor will become even smaller. Therefore, not considering capital as

[^7]a production factor and the associated possibility of factor substitution, while being essential for the details of the equilibrium, does not affect our general conclusion.

More general distributions A simplifying assumption of our analysis was that there are only two types of consumers, rich and poor. How would the analysis change by allowing for arbitrarily many groups? To get the intuition how the analysis extends to many groups, consider three groups. A candidate for a general equilibrium would be a situation where some firms sell only to the rich, other firms sell to the rich and the middle class and a final group of firms sells to all consumers. Whether or not such an equilibrium arises depends on how different the various groups are. When rich, middle class and poor differ only slightly, a symmetric equilibrium will arise. When the rich and the middle class are very similar, there will be a situation where the poor but not the middle class are excluded from some markets. When the poor and the middle class are very similar, the poor and the middle class are excluded from the same markets, and so on. It is obvious that this line of reasoning can be extended to the general case with $x$ different groups of households. The equilibrium will be characterized by $z \leq x$ different types of firms, where $z$ is weakly smaller than $x$ reflecting the fact that the market equilibrium merges very similar groups. Furthermore, a redistribution of income from richer to poorer households has analogous effects as the redistribution discussed in the two-group economy, provided the redistribution occurs between groups which are sufficiently different.

## 9 Conclusions

Recent macroeconomic analysis has focused on the role of consumer heterogeneity on aggregate outcomes. Our analysis extends this literature along two dimensions. First, our analysis explores the combination of non-homothetic preferences and monopolistic market power as the important channel by which economic inequality affects the general equilibrium. This channel has not been much studied in the literature which has emphasized capital market imperfections (e.g. Galor and Zeira, 1993) or political-economy considerations (e.g. Bertola, 1993) or some combination of the two (e.g. Bénabou, 1996). Second, we study how economic inequality affects the medium run, in particular, the level of aggregate employment and the allocation of labor across industries. This is different from the recent literature which has predominantly studied the effect of inequality in the context of long-run economic growth.

We introduced non-homothetic preferences in a very stylized way. Instead of CES-preferences we have assumed that preferences are quadratic. This seemingly minor change in assumptions changes the character of the general equilibrium. Sufficiently high inequality divides an oth-
erwise symmetric economy into mass consumption industries and exclusive industries; and it may lead to underemployment of the work force. Moreover, the model predicts that underemployment can be cured by redistributive policies; and that the effect of incomes policies (which increase wages at the expense of profits) is ambiguous due to the dual role of real wages as a cost- and a demand-factor. We have also shown that our results generalize to more general assumptions on preferences.

Our model could be extended in various directions. First, our model is static and it may be worthwhile to extend the analysis to a dynamic context. Allowing for innovation decisions brings interesting new elements into the picture. When new products are introduced, unemployment will eventually disappear. When more efficient production processes are implemented, however, unemployment will even increase for two reasons. On the one hand, higher productivity makes workers increasingly redundant. On the other hand, as unemployment hurts the poor disproportionately, the resulting increase in inequality depresses aggregate employment even further. A second potentially interesting extension concerns international trade. Our model is closed and opening it up for international trade would allow to explore the interaction between increasing returns and economic inequality as a determinant of trade flows. Inter alia this may provide a rationale for why terms of trade may be affected by demand considerations (such as the relative size of home markets) and income distribution.

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## Appendix

A. Proof of Lemma (1) Maximizing the profit function $[p(j)-w / a] x(p(j))$ using () it is straightforward to calculate the respective monopoly prices along these two segments as

$$
p(j)= \begin{cases}\bar{p}=\frac{1}{2}\left[w / a+s / \lambda_{R}\right] & \text { if only the rich buy, } \\ \underline{p}=\frac{1}{2}\left[w / a+s /\left(\beta \lambda_{P}+(1-\beta) \lambda_{R}\right)\right] & \text { if all consumers buy. }\end{cases}
$$

We express the $\lambda$ 's in terms of $\bar{p}, \underline{p}$, and $w$ and substitute the resulting expressions into the individual demands (2). This yields

$$
\begin{array}{ll}
c_{P}^{E}=0, & c_{P}^{M}=s-\frac{s}{\beta}\left[\frac{1}{2 \underline{p}-w / a}-\frac{1-\beta}{2 \bar{p}-w / a}\right] \underline{p}, \\
c_{R}^{E}=s-s\left[\frac{1}{2 \bar{p}-w / a}\right] \bar{p}, & c_{R}^{M}=s-s\left[\frac{1}{2 \bar{p}-w / a}\right] \underline{p},
\end{array}
$$

where $c_{i}^{E}$ denotes the quantity purchased by a consumer of type $i \in\{R, P\}$ when the firm chooses the exclusive strategy (= charges the high price) and $c_{i}^{M}$ denotes the respective quantities when the firm chooses the mass consumption strategy ( $=$ charges the low price). The equilibrium output of exclusive producers is $x^{E}=(1-\beta) c_{R}^{E}$ and of mass producers, $x^{M}=\beta c_{P}^{M}+(1-\beta) c_{R}^{M}$. Using the above expressions for $c_{P}^{M} c_{R}^{E}$, and $c_{R}^{M}$ yields the values for $x^{E}$ and $x^{M}$.
B. Proof of Proposition 1 Selling exclusive goods yields market demand $(1-\beta)\left(s-\lambda_{R} p\right)$. The profit maximizing price is $\left(w / a+s / \lambda_{R}\right) / 2$, which yields profits

$$
\Pi^{E}=(1-\beta)\left(s-\lambda_{R} w / a\right)^{2} /\left(4 \lambda_{R}\right) .
$$

Selling mass consumption goods yields market demand $s-\left[\beta \lambda_{P}+(1-\beta) \lambda_{R}\right] p$. The profit maximizing price is $\left[w / a+s /\left(\beta \lambda_{P}+(1-\beta) \lambda_{R}\right)\right] / 2$, which yields profits

$$
\Pi^{M}=\left[s-\left(\beta \lambda_{P}+(1-\beta) \lambda_{R}\right) w / a\right]^{2} /\left[4\left(\beta \lambda_{P}+(1-\beta) \lambda_{R}\right)\right] .
$$

In a symmetric equilibrium we must have $\Pi^{M} \geq \Pi^{E}$ so that no firm has an incentive to deviate and adopt the exclusion strategy. In asymmetric equilibria, mass consumption producers and exclusive producers must earn the same profit $\Pi^{M}=\Pi^{E}$. A situation where $\Pi^{M}<\Pi^{E}$ cannot be an equilibrium: no firm would sell to the poor, which would leave them with idle purchasing power and very high willingness to pay for some goods. Let us now find a condition under which no firm has an incentive to sell exclusively to the rich. For a given wage level $w \geq w^{*}$ we evaluate equilibrium profits in a symmetric equilibrium where all firms have demand $a L^{D}(w) / N$ and denote these profits by $\tilde{\Pi}^{E}$ and $\tilde{\Pi}^{M}$. To eliminate $\lambda_{P}$ and $\lambda_{R}$ note that we can express the marginal utility of income as $\lambda_{i}=s-\Delta_{i} a L^{D}(w) / N$. Since $\gamma \leq \delta$ we must have $\Delta_{P} \leq \delta$. To find a sufficient condition for an asymmetric equilibrium we evaluate the marginal utilities
of income at the lowest level of inequality, i.e., where $\Delta_{P}=\delta$. and we get the critical profits levels $\tilde{\Pi}^{E}$ and $\tilde{\Pi}^{M}$ in terms of $w$ and exogenous parameters
$\tilde{\Pi}^{E}=\frac{w a}{4 N} \frac{(1-\beta) L^{D}(w)^{2}\left(\left(1+\Delta_{R}\right) s N-2 \Delta_{R} a L^{D}(w)\right)^{2}}{\left(s N-a L^{D}(w)\right)\left(s N-2 a L^{D}(w)\right)\left(s N-\Delta_{R} a L^{D}(w)\right)}, \quad$ and $\quad \tilde{\Pi}^{M}=\frac{w a}{N} \frac{L^{D}(w)^{2}}{s N-2 a L^{D}(w)}$.

The symmetric outcome is an equilibrium if, starting from a situation where all firms charge a price that attracts the whole customer base, no single firm has an incentive to deviate and adopt the exclusive good strategy. In other words, the inequality $\tilde{\Pi}^{E}<\tilde{\Pi}^{M}$ must hold strictly. Using equations (12), noting that $\Delta_{R}=(1-\beta \delta) /(1-\beta)$, we get

$$
\begin{equation*}
\beta<\frac{4 \delta(1-z)^{2}}{4 \delta^{2} z(1-z)-4 \delta z+(1+\delta)^{2}} \tag{13}
\end{equation*}
$$

where $a L^{D}(w) /(s N)=(1-w / a) /(2-w / a)=z$ must hold in the labor market equilibrium. The right hand side of (13) is monotonically decreasing in $z$ over the relevant range. Taking the derivative with respect to $z$ gives $-8(1-\delta) \delta(1-z) /(1+\delta(1-2 z))^{3}<0$. As $\zeta=L^{D}(w) z<z$, the condition in proposition 1 is sufficient.
C. Entry with skilled labor input Assume there are skilled and unskilled workers. $G$ units of skilled labor are needed to set up a new firm. Final output is produced with skilled and unskilled labor using the CRS technology $y=a F\left(h_{Y}, l\right)$ where $h_{Y}$ and $l$. Aggregate production employment equals $H_{Y}=N h_{Y}$ for the skilled and $L=N l$ for the unskilled. The production function has an associated marginal cost function which we denote by $m c \equiv$ $\left(w_{H} / a\right) \cdot c\left(w_{L} / w_{H}\right)$, with $c^{\prime}>0$ where $w_{L}$ and $w_{H}$ denote the skilled and unskilled wage, respectively. In equilibrium, all firms choose the same factor intensity $H_{Y} / L$ and the wage ratio must satisfy the condition $w_{L} / w_{H}=F_{L}\left(h_{Y}, l\right) / F_{H}\left(h_{Y}, l\right)=\varphi\left(H_{Y} / L\right)$ with $\varphi^{\prime}>0$. All other elements of the model are unchanged.

The model has four endogenous variables: $m c, n, H_{Y}$, and $M$. The equilibrium conditions are (7) (where $w$ is now replaced by $m c$ ), the resource constraint which now changes to $a F\left(H_{Y},(1-u) L\right)=s(N+M)[n(1-m c) /(2-m c)+(1-n)(1-\beta)(g(m c)-m c) /(2 g(m c)-m c)]$ the zero-profit condition now given by $w_{H} G=\Pi(m c)$, and the feasible number of entrants $M=G\left(H-H_{Y}\right)$. We get rid of $w_{H}$ by using $w_{H}=a \cdot m c / c\left(\varphi\left(H_{Y} / L\right)\right)$ to obtain $a \cdot m c \cdot G$. $c\left(\varphi\left(H_{Y} / L\right)\right)^{-1}=s(1-m c)^{2} /(2-m c)$.

We can now easily solve the model by focusing on two equations (the free-entry condition and the resource constraint) in the two unknowns $m c$ and $H_{Y}$. The right-hand-side of the free-entry condition decreases in $m c$ and the left-hand-side decreases in $H_{Y} / L$ which defines a monotonically increasing curve in the $\left(H_{Y}, m c\right)$ space. The resource constraint defines a (non-monotonic or upward sloping) curve in $\left(H_{Y}, m c\right)$ space.

Proposition 6 There exists an equilibrium with $H_{Y} \geq 0$. a) If the elasticity of substitution $\varepsilon$ between production factors is between zero and one, $0<\varepsilon \leq 1, H_{Y}>0$ and there is no unemployment among the low skilled. b) If $\varepsilon>1$, there may be unemployment. Unemployment arises if $s G H(1+\vartheta-\beta) / 2<a F(0, L)$.

Proof. Existence. The slope of the equilibrium curves is discussed above. The labor demand curve crosses the $H_{Y}$ - axis at $a F\left(H_{Y}, L\right) /\left(s G\left(H-H_{Y}\right)\right)=(1-\beta) / 2$ which must occur at a $\widehat{H}_{Y}<H$. If the curves do not cross, $H_{Y}=0$ in equilibrium.
a. When intercept of the resource constraint at the $m c$-axis exceeds that of the free entry curve, $H_{Y}>0$ must hold in equilibrium. To see this note first, if $\varepsilon \leq 1$, the resource constraint only holds true for $H_{Y}=0$ when $m c=1$. On the other hand, both factors are necessary in production or $F(0, L)=0$. In that case $c(\phi(0))=c(0)=0$, hence the value of marginal costs $m c$ satisfying the free-entry condition goes to zero when $H_{Y}$ approaches zero.
b. If $\varepsilon>1$, positive production can be achieved using one factor only $F(0, L)>0$. In a possible unemployment equilibrium, mark-ups are infinite and aggregate demand for low skilled labor equals $G H(1+\vartheta-\beta) s / 2$. If this number falls short of $a F(0, L)$, there is unemployment with a zero wage.

If skilled workers are a necessary input in the production of final output ( $\varepsilon \leq 1$ ) unemployment cannot arise when the labor market is perfect. If skilled workers are not a necessary input, however, unemployment can arise even if the unskilled wage goes to zero. In that case, the productivity of the unskilled is $a F(0, L)$. The maximum number of entrants is pinned down by the stock of high skilled workers, $M=G H$, and the total number of firms is given by $G H+N$. In that case, we are back in the model studied in section 6.
D. More general preferences We show the following: With HARA preferences and $v^{\prime}(0)$ finite, more inequality raises markups in a unique asymmetric equilibrium.

When preferences are HARA, $v($.$) is given by v^{\prime}(c)=(c / \sigma+s)^{-\sigma}$ with $s>0$ and $\sigma \in \Re$. Note that we get for $\sigma=-1$ the quadratic utility function used above. The assumption of $s>0$ guarantees that $v^{\prime}(0)$ is finite. The elasticity of substitution equals $c /(c / \sigma+s)$ which is monotonically increasing in $c$.

We will consider an asymmetric equilibrium with full employment. Note that unemployment equilibria are possible when the demand curve exhibits a revenue maximum when the elasticity of substitution $-v^{\prime}(c) /\left(v^{\prime \prime}(c) c\right)$ falls below unity at some finite $c$ (which occurs if and only if $\sigma<1$ ). The generalized Stone-Geary with $\sigma<1$ and negative consumption requirement satisfies this property, for example.

Denote by $x^{E}$ and $x^{M}$ consumption of mass and exclusive goods, respectively. Instead of the price of mass consumption goods, we now normalize marginal costs $w / a=1$ and get the
following Lerner indices

$$
\begin{equation*}
\frac{\bar{p}-1}{\bar{p}}=\frac{x^{E}(1-\beta)}{x^{E}(1-\beta) / \sigma+s} \tag{A1}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\underline{p-1}}{\underline{p}}=\frac{x^{M}}{x^{M} / \sigma+s} . \tag{A2}
\end{equation*}
$$

The profit arbitrage condition is given by

$$
\begin{equation*}
(\underline{p}-1) x^{M}=(\bar{p}-1) x^{E} . \tag{A3}
\end{equation*}
$$

For simplicity, we consider a full employment equilibrium, hence the aggregate resource constraint reads

$$
\begin{equation*}
n x^{M}+(1-n) x^{E}=1 \tag{A4}
\end{equation*}
$$

Now consider a rise in inequality. In a unique equilibrium, more inequality leads to more exclusion, i.e., a decrease in $n$. Assume to the contrary that $\bar{p}$ falls. By (A1), $x^{E}$ must also decrease. (A3) then implies that $(\underline{p}-1) x^{M}$ falls. From (A2) we know, however, that $\underline{p}$ and $x^{M}$ are positively related. Therefore, both $\underline{p}$ and $x^{M}$ must decrease. Taken together $n x^{M}+(1-n) x^{E}$ must fall (recall that $x^{M}>x^{E}$ ). But this contradicts the aggregate resource constraint (A4). Hence, we conclude $\bar{p}$ must increase. By the same reasoning, $x^{E}$ and therefore $\underline{p}$ and $x^{M}$ must increase. Thus, markups rise.

Figure 1: Aggregate Demand and Monopolistic Pricing Decision


Figure 2: Equilibrium with Unemployment


Figure 3: Impact of More Inequality


Figure 4: Full Employment Equilibrium


Figure 5: Multiple Equilibria



[^0]:    ${ }^{*}$ University of Zurich, Institute for Empirical Research in Economics, Bluemlisalpstrasse 10, CH- 8006 Zürich, Tel: $++41-1-63437$ 26, Fax: $++41-1-6344907$, e-mail: rfoellmi@iew.unizh.ch.
    ${ }^{\dagger}$ University of Zurich, Institute for Empirical Research in Economics, Bluemlisalpstrasse 10, CH- 8006 Zürich, Tel: $++41-1-6343724$, Fax: ++41 -1-634 49 07, e-mail: zweim@iew.unizh.ch. Josef Zweimüller is also associated with CESifo, IZA, and CEPR.
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[^1]:    ${ }^{1}$ For a recent summary of the state of research on Engel-curves, see Lewbel (2006).

[^2]:    ${ }^{2}$ Other papers that incorporate non-homothetic preferences into a general equilibrium framework are Falkinger (1994), Chou and Talmain (1996), Li (1996), Galor and Moav (2004), and Foellmi and Zweimüller (2006) in the context of economic growth and Flam and Helpman (1987), Matsuyama (2000), and Mitra and Trindade (2005) in the context of international trade.

[^3]:    ${ }^{3}$ The resulting Lorenz-curve is piecewise linear, with slope $\delta \ell+\gamma(1-\ell)$ over the range $(0, \beta)$ and with slope $(1-\beta \delta) /(1-\beta) \cdot \ell+(1-\beta \gamma) /(1-\beta) \cdot(1-\ell)$ over the range $(\beta, 1)$, where $\ell$ is the economy-wide share of wages in total income. While $\beta, \gamma$, and $\delta$ are exogenous parameters of the model (that will be the focus of comparative static analysis later on) the labor share $\ell$ will be endogensouly determined. This means that the size distribution of income is endogenously determined as well.

[^4]:    ${ }^{6}$ Notice that $\bar{w}$ is a real wage (more precisely, the real consumption wage of the poor). This minimum real wage could be implemented by indexing wages to the cost of living of the poor. This fixes the mark-up for mass consumption goods and, via the equal-profit condition, also the mark-up for exclusive goods.

[^5]:    ${ }^{7}$ Notice that our discussion in the text assumes that government authorities set a minimum wage per efficiency

[^6]:    ${ }^{8}$ Note that the properties of a quadratic subutility function are quite different from those of the standard Dixit-Stiglitz formulation. In that case, $v^{\prime}(0)=\infty$, so that even the poorest consumers purchases all goods that are supplied (albeit in tiny amounts); and the elasticity of demand $\eta(c)$ is the same for all consumers, i.e. does not depend on consumed quantities.

[^7]:    ${ }^{9}$ Foellmi and Zweimüller (2004) analyze the impact of inequality on mark-ups in the context of a symmetric full employment equilibrium. It turns out that it depends on the curvature of the coefficient of absolute risk aversion, $-v^{\prime \prime}(c) / v^{\prime}(c)$, whether higher inequality in the size distribution of income increases or decreases the mark-up.

