

# On-the-Job Search, Minimum Wages, and Labor Market Outcomes in an Equilibrium Bargaining Framework<sup>1</sup>

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## Abstract

We look at the impact of a binding minimum wage on labor market outcomes and welfare distributions in both partial and general equilibrium models of matching and bargaining a currently-employed individual can meet other potential employers (on-the-job search). In analyzing and estimating the model, we use two different specifications of the Nash bargaining problem. In one, firms engage in a Bertrand competition for the services of an individual, as in Postel-Vinay and Robin (2002), Dey and Flinn (2005), and Cahuc et al. (2006). In the other, firms do not engage in such competitions, and the outside option used in bargaining is always the value of unemployed search. We estimate both bargaining specifications using a Method of Simulated Moments estimator applied to data from a recent wave of the Survey of Income and Program Participation. Even though individuals will be paid the minimum wage for a small proportion of their labor market careers, we find significant effects of the minimum wage on the *ex ante* value of labor market careers, particularly in the case of Bertrand competition between firms. We carry out a test of which bargaining structure is more consistent with the sample information used in the estimator, and find strong support for the no-renegotiation specification.

# 1 Introduction

The impact of minimum wages on the welfare of agents on the supply and demand sides of the labor market has been at the center of an age-old policy debate on the proper role of the government in the economy. The standard elementary treatment of minimum wage policy views its impact as unambiguously negative. In a competitive market in which unrestricted supply and demand forces combine to determine a unique equilibrium employment and wage level, the imposition of a minimum wage greater than the market clearing wage creates true unemployment, defined as the situation in which individuals who are willing to supply labor at the going wage rate are unable to find jobs. It creates *ex post* inequality as well, in that those individuals fortunate enough to find jobs have a higher welfare level than they would have had in the competitive equilibrium, while those who do not will have lower welfare levels. If individuals are risk-averse, this increased “uncertainty” may be welfare decreasing in an *ex ante* sense, as well.

It has long been appreciated that, for minimum wages to have beneficial effects (at least for the supply side of the market), there must exist labor market frictions and/or multiple equilibria.<sup>1,2</sup> The multiple equilibria case (see van den Berg, 2003) is perhaps the strongest one for the beneficial effects of government-imposed wage policies. There are a large number of labor market models that can produce multiple equilibria, which occurs when the “primitives” of a labor market environment can produce a number of different labor market equilibria.<sup>3</sup> If, for example, two equilibrium outcomes are possible, with the supports of the wage distributions associated with the two equilibria non-overlapping, then a minimum wage placed below the lower bound of the support of the higher wage

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<sup>1</sup>A recent paper by Lee and Saez (2008) looks at optimal minimum wages in a competitive labor market, but one in which there exists heterogeneity on the supply side of the market. They show that optimal minimum wages can improve the distribution of welfare on the supply side from a planner’s perspective. The model considered here and in most of the search frictions literature on the subject assume *ex ante* identical agents, instead, and are capable of delivering Pareto-improvements in welfare on the supply side of the market, and in a general equilibrium setting, even on the demand side of the market.

<sup>2</sup>There are a number of papers in the literature that consider the possibility of welfare-enhancing minimum wage rates (e.g., Drazen (1986), Lang (1987), Rebitzer and Taylor (1996), Swinnerton (1996)), though the frameworks in which the models are set tend to be relatively abstract and the models themselves are typically not estimable. These are important contributions, but in this paper we focus more on models that have been or are capable of being taken to data.

<sup>3</sup>Some examples of labor market models that can easily produce multiple equilibria involve statistical discrimination (see, e.g., Moro (2003)) and the general equilibrium search model with contact rates determined through a non-constant returns to scale matching function (see, e.g., Diamond (1982)).

distribution and above the upper bound of the support of the lower wage distribution can serve as an equilibrium selection device that ensures selection of the preferred equilibrium.<sup>4</sup>

The equilibrium search models of Albrecht and Axell (1984) and Burdett and Mortensen (1998) offer another possible case for positive minimum wage effects on welfare, once again, at least for the supply side of the market. In the Albrecht and Axell model, potential firm entrants into the market are heterogeneous in terms of quality. A high minimum wage prevents low-quality firms from making nonnegative profits, and hence improves the labor market through a selection effect on the demand side. Having higher quality firms competing for their services improves the wage distributions individuals face while searching. As their model is written, there is no adverse impact on employment rates.

The Burdett and Mortensen (1998) equilibrium framework allows for on-the-job (OTJ) search in addition to unemployed search. As do Albrecht and Axell (1984), they assume a wage-posting equilibrium in which firms offer a fixed wage to all potential and actual employees. The Burdett and Mortensen framework does not require heterogeneity in the populations of (potential) firms and workers, in fact their basic specification assumes no heterogeneity. They prove the existence of an equilibrium in which the probability that any two firms offer the same wage is zero. Adding a minimum wage to their model simply shifts the equilibrium wage offer distribution to the right and has no adverse employment effects. As a result, binding minimum wages are beneficial to the supply side of the market.

The framework we use posits search frictions, as do the two we have just discussed. Differently from those two models, an important component of our model is heterogeneous productivity. In particular, when a potential employment opportunity is found after some period of search, the productivity of the match,  $\theta$ , is determined by taking a draw from some fixed distribution  $G(\theta)$ . It is typically assumed that the value of the match for a given possible worker-firm pairing is observed immediately by both sides, an admittedly strong assumption.<sup>5</sup> Once the draw is made, the pair can determine if there exists positive surplus to the potential match. “Positive surplus” is said to exist if there exists any wage rate at which both sides would prefer creating the match to their next best options of continued search. If there exists positive surplus to the match, the worker and firm

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<sup>4</sup>This presumes that the higher wage distribution is associated with the Pareto optimal equilibrium. If this is not the case, implementing a maximum wage policy would select the preferred equilibrium.

<sup>5</sup>Jovanovic (1979) emphasizes the role of learning about match quality in explaining separation decisions and wage progression at a firm. He expands his framework to include unemployment in Jovanovic (1984). The bargaining process is not emphasized in his approach.

bargain over its division. In general, both sides have some degree of bargaining power, because the other side cannot find a perfect replacement for them without spending additional effort and/or resources. In this case, the ultimate source of bargaining power to both derives from the existence of search frictions.

As is common in this literature, we use axiomatic Nash bargaining to determine the division of the surplus between the worker and firm. The minimum wage acts as a side constraint on the Nash bargaining problem. Depending on the form of the equilibrium wage function, the minimum wage may act to preclude the formation of otherwise acceptable matches. This corresponds to the standard negative employment effect found when using a competitive labor market framework. Secondly, for all acceptable matches after the imposition of the minimum wage, the minimum wage affects the bargained wage both directly and indirectly. The direct effect is obvious: for bargained wages less than  $m$  without the constraint, if the match is viable, the wage offer must be increased to  $m$  to comply with the law. The indirect effect is associated with the change in the outside option (in the Nash bargaining problem) associated with the minimum wage change, even when the constraint itself is not directly binding. In the minimum wage empirical literature, this is often referred to as “spillover.”

This paper significantly extends the analysis of minimum wage effects on labor market outcomes and welfare within a search, matching and bargaining environment that was developed and estimated in Flinn (2002, 2006). The most obvious addition is the inclusion of on-the-job (henceforth, OTJ) search. The addition of OTJ search to the bargaining model is a critical extension both in terms of model realism and policy analysis. Most obviously, from descriptive evidence we know that in the U.S. labor market there are a large number of job-to-job transitions that do not involve an intervening spell of unemployment. By ignoring this fact, there exists the potential for a significant degree of model misspecification, leading to inconsistent estimates of model parameters and misleading policy implications drawn from those estimates. The addition of OTJ search is likely to be particularly relevant for purposes of investigating minimum wage effects on labor market outcomes. The work of Leighton and Mincer (1981), and, more recently, by Acemoglu and Pischke (2002), investigates the potential impacts of minimum wage laws on life-cycle wage profiles through reductions in general human capital investment by recent labor market entrants. Under either of the two bargaining specifications investigated here, minimum wages, by truncating the lower tail of the accepted wage distribution, tend to produce less wage growth over employment spells and the life cycle. In the bargaining specification that allows bidding between two competing

employers, minimum wage effects on wage growth are expected to be especially pronounced, since the equilibrium wage function displays a “compensating differential” property, i.e., those jobs offering the highest growth prospects offer commensurately lower wages.<sup>6</sup> Without a minimum wage, low wages are an (imperfect) indicator of high wage growth prospects. The minimum wage limits the extent to which firms can “charge” an employee for this future wage growth potential, thus reducing average wage growth in the market.

In the presence of OTJ search, there are a number of auxiliary assumptions that must be invoked when characterizing bargaining outcomes, in particular with respect to whether firms renegotiate wage contracts or not. The model of renegotiation found in Postel-Vinay and Robin (2002), Dey and Flinn (2005), and Cahuc et al. (2006) has employed workers that have been contacted by a new potential employer conducting an auction for their services between the two bidders, with the lowest value match of the two competing matches generating the outside option used in the final stage wage bargaining between the employee and the winning firm. The renegotiation assumption has been criticized both for depending on a questionable ability to communicate credible offers and counteroffers between the two firms bidding for an employee’s services and for requiring credible commitment by the winning firm to honor the negotiated wage even after the competing firm’s offer has been taken off of the table. On the other hand, models with OTJ search that do not allow renegotiation, such as the wage posting model of Burdett and Mortensen (1997), are subject to the criticism that firms have an incentive to respond to outside offers by “poaching” firms, so that equilibrium in these models requires market discipline. Postel-Vinay and Robin (2004) present an interesting theoretical consideration of this question that applies, with certain modifications, to the framework employed here. In a model in which match productivity between worker  $i$  and firm  $j$  is given by  $y_{ij} = a_i b_j$ , with  $a_i$  denoting the individual’s productivity type and  $b_j$  denoting the firm’s, and search intensity is endogenous, they show that an equilibrium may emerge in which high  $b$  firms renegotiate contracts and low  $b$  firms do not. In our modeling framework, matching heterogeneity is *i.i.d.*, so that there are no permanent differences between agents on the supply side of the market or on the demand side of the market. In such a case, it is reasonable to assume that any pure strategy equilibrium will have all firms renegotiating contracts or no firms doing so. Our approach to the question is empirical. Since the two assumptions regarding renegotiation

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<sup>6</sup>The reason for this is that future firms will have to bid against a high-valued  $\theta$  to attract the worker. This effectively increases the outside option of the worker, and the firm demands compensation for this future bargaining advantage by reducing the current wage offer.

simply represent different mappings from the same set of primitive parameters to the distribution of labor market outcomes observed in the data, we utilize a bootstrap hypothesis testing procedure to determine which mapping generates a distribution of outcomes more in correspondence with the sample distribution of outcomes. Since the results of this test will influence which set of policy experiment results to which we give most credence, the test is of interest from both a purely academic and a practical policy perspective.

The remainder of this paper proceeds as follows. In Section 2 we derive the model and present some analysis of the effects on labor market outcomes of minimum wages with OTJ search. Section 3 contains a discussion of the data used to estimate the equilibrium model, while Section 4 develops the MSM estimator we use. Section 5 presents the empirical results, and Section 6 contains the results of our policy experiments. In Section 7 we conclude.

## 2 Model

In this section we describe the behavioral model of labor market search with matching and bargaining in which the interactions between applicants and firms are constrained by the presence of a minimum wage. The minimum wage,  $m$ , is set by the government and is assumed to apply to all potential matches. We assume that the only compensation provided by the firm is the wage. As a result, there are no other forms of compensation the firm can adjust so as to “undo” the minimum wage payment requirement.

As is common in the search literature, we use a Nash bargaining framework in the wage determination process. We estimate and evaluate the model under two alternative assumptions regarding outside options. The differences and similarities of the two approaches are simple to describe. Say that a firm currently employs a worker who has a certain productivity  $\theta$  and is paid a wage  $w$ . The employee meets another potential employer, and the (potential) productivity at that job is immediately revealed to be  $\theta'$ . In both of the bargaining settings we consider, efficient mobility decisions are made. That is, the employee will change firms if and only if  $\theta' > \theta$ . The models differ in the wage determination process.

We devote most of our attention to the most theoretically interesting case, which allows direct wage competition between firms. This setup has been used by Postel-Vinay and Robin (2002), Dey and Flinn (2005), and Cahuc et al. (2006). Firms, competing for the same individual’s services at a moment in time, engage in a Bertrand competition for the employee, with the firm associated

with the worst productivity level dropping out of the auction at the point at which its profit level is zero. If, for example, the individual's productivity at the new potential employer exceeds that at his current firm, so that  $\theta' > \theta$ , it is assumed that the current firm makes a (doomed) effort to retain his services by making higher and higher wage offers until it drops out of the bidding at a wage offer  $\tilde{w} = \theta$ . In this case, the employee moves to the new firm, and the outside option used to set his wage is the value of being employed at the "losing" firm at a wage equal to  $\tilde{w} = \theta$  (i.e., when he receives all of the surplus of employment match).

The other situation that can arise under renegotiation is when the employee's productivity at the new firm is less than or equal to her current productivity, but greater than her current wage, or  $w < \theta' \leq \theta$ . Though the employee will not leave the current firm, given efficient mobility, she can use the threat of leaving to increase her current wage  $w$ . In this case, the potential employer bids for the individual's services until it reaches the point  $\tilde{w} = \theta'$ , at which point it drops out of the auction. The renegotiated wage at the current employer uses the value of being employed at the match value  $\theta'$  with a wage equal to  $\tilde{w} = \theta'$  as the outside option.

Our second bargaining scenario considers the case in which each labor market participant's outside option value is equal to the value of unemployed search, independent of their current labor market state. Of course, this is the option value for those searching in the unemployment state at the time they encounter a potential employer. The second bargaining scenario posits that the value of unemployed search option serves as the outside option even for employed searchers. This may be due to the fact that employed individuals are not able to credibly convey their current employment conditions (including wage or wage offer) to a new potential employer, while at the same time not being able to credibly reveal current wage offers from potential employers to their current employers.

An alternative justification is one of lack of commitment. If offers must be rejected or accepted at the instant when they are tendered, then a worker loses his or her outside option the moment after it is received. When the option is lost, the only relevant one becomes quitting into the unemployment state, the value of which is always  $V_n(m)$ , where the notation suggests that this value is a function of the statutory minimum wage,  $m$ . Thus wage payments will always be determined using this outside option. Knowing this, a worker may insist that the firm transfer a lump sum amount to them to obtain their services at the moment when they have two employment options. To the extent that this is not recorded as a wage payment, this will have no effect on the wage process. Since all mobility is efficient in any case, such payments will have no impact on the mobility process.



Thus the empirical wage-mobility process should be consistent with model assumptions even in the presence of unobservable (to us) one-time payments associated with the receipt of an offer by an employed individual.

## 2.1 The Model with Renegotiation

The model assumes a stationary labor market environment and is formulated in continuous time. We assume that there exists an invariant, technologically-determined distribution of worker-firm productivity levels which is given by  $G(\theta)$ . To facilitate the numerical solution of the model, we assume that the random variable  $\theta$  is discrete, with the set of values  $\theta$  can assume being given by  $\Omega_\theta = \{\theta_j\}_{j=1}^L$ , where  $0 < \theta_1 < \dots < \theta_L < \infty$ . When a potential employee and a firm meet, the productive value of the match  $\theta$  is immediately observed by both the applicant and the firm. At this point a division of the match value is proposed using a Nash bargaining framework that is described below. The searcher's instantaneous discount rate is given by  $\rho > 0$ . The rate of (exogenous) termination of employment contracts is  $\eta > 0$ .

While unemployed individuals search, their instantaneous utility is given by  $b$ , which can assume positive or negative values. Unemployed workers meet firms at rate  $\lambda_n$ , a value which is taken as exogenous in the partial equilibrium version of the model and treated as exogenous in the general equilibrium version. If both the firm and the worker accept the match, then they split it using a Nash bargaining framework and determine a wage  $w(\theta, U)$ . The acceptance set of matches from the unemployment state is given by  $A(m)$ , with  $\theta^A(m)$  being the minimal  $\theta$  value in  $A(m)$  (further discussion of the decision rule is provided below). It is assumed that labor is the only factor of production and if an individual and a firm meet, but the firm "passes" on the applicant, then the firm receives a value of 0, which is implied by a free entry condition (FEC) in the general equilibrium version of the model, and which is also imposed in the partial equilibrium analysis. This is the firm's disagreement value in the Nash bargaining framework. Analogously, the disagreement value for an unemployed searcher is the value of continued search, which is denoted  $V_n(m)$ .

While employed, workers meet firms at the rate  $\lambda_e$ , which is independent of the employed worker's current match value, and which is also endogenously determined in the general equilibrium version of the model. We denote the current labor market state of an employed individual by  $(\theta, w)$  and any potential new state by  $(\theta', w')$ . We now consider the rent division problem facing a currently employed agent who encounters a new potential employer.

Let there be a currently employed individual with wage  $w$  and match value  $\theta \in A(m)$  who

meets a new potential employer at which the match value is  $\theta'$ . We assume that the potential match value will only be reported to the current employer if the employee has an incentive to do so. One situation in which this will be the case is when  $\theta' > \theta$ . When this occurs, we assume that a bargaining process for the individual's services begins between the current and potential employers and stops when one of the firms' surplus reaches zero, as in Bertrand competition. The loser of the competition will be the employer when  $\theta' > \theta$ . Let the maximal value of match  $\theta$  to the worker be given by  $Q(\theta)$ . Then the objective function for the Nash bargaining problem when  $\theta' > \theta$  is:

$$S(\theta', w', \theta) = \{V_e(\theta', w', \theta) - Q(\theta)\}^\alpha \times \{V_f(\theta', w', \theta) - 0\}^{1-\alpha}$$

where  $V_f(\theta', w', \theta)$  denotes the new firm's value of the match, and  $\alpha \in (0, 1)$  represents the bargaining power of the worker.

The firm's value of the current employment contract is defined as follows. Over an infinitesimally small period of time  $\varepsilon$ , the firm earns a profit of  $(\theta - w)\varepsilon$ , which is discounted back to the present with the "infinitesimal" discount factor  $(1 + \rho\varepsilon)^{-1}$ . With "approximate probability"  $\eta\varepsilon$ , the match is exogenously terminated and the firm earns no profit. With approximate probability  $\lambda_e\varepsilon$ , the worker receives a job offer from an alternative firm. If he reports this offer to his current firm, his wage will be renegotiated.<sup>7</sup> With approximate probability  $(1 - \lambda_e\varepsilon - \eta\varepsilon)$ , the worker does not receive another job offer and is not exogenously dismissed over the period  $\varepsilon$ . In this case the status quo is maintained. The term  $o(\varepsilon)$  represents all other states that can occur over the period  $\varepsilon$  that involve two or more events, and which has the property that  $\lim_{\varepsilon \rightarrow 0} \left(\frac{o(\varepsilon)}{\varepsilon}\right) = 0$ . We denote the value to the firm as:

$$\begin{aligned} V_f(\theta', w', \theta) = & (1 + \rho\varepsilon)^{-1} \{(\theta' - w')\varepsilon + (\eta\varepsilon \times 0) + \left( \lambda_e\varepsilon \sum_{\tilde{\theta} \in B(\theta, \theta')} V_f(\theta', w(\theta', \tilde{\theta}), \tilde{\theta}) p(\tilde{\theta}) \right) \\ & + \left( \lambda_e\varepsilon P(\tilde{\theta} \leq \theta) \times V_f(\theta', w', \theta) \right) + (\lambda_e\varepsilon P(\tilde{\theta} > \theta') \times 0) \\ & + \left( (1 - \lambda_e\varepsilon - \eta\varepsilon) \times V_f(\theta', w', \theta) \right) + o(\varepsilon)\}, \end{aligned}$$

where  $V_f(\theta', w(\theta', \tilde{\theta}), \tilde{\theta})$  represents the equilibrium value to a firm of the productive match  $\theta'$  when the worker's next best option has a match  $\tilde{\theta}$ , and where a value  $\theta_j \in B(\theta', \theta)$  if and only if  $\theta_j \leq \theta'$  and  $\theta_j > \theta$ .

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<sup>7</sup>Since the searcher has the option to not report any alternative match value, he will only do so when his employment value at the current firm will increase, which only occurs when the wage increases.

The interpretation of the arguments involving  $\lambda_e$  is as follows. Given a dominant and dominated match value pair  $(\theta', \theta)$ , we can partition  $\Omega_\theta$  into three sets. Since efficient mobility is an implication of our model structure, any new draw of a match value at a prospective employer that is greater than the match value at the current employer,  $\theta'$ , results in an immediate departure. This event implies a value to the firm of 0 under our assumption regarding its outside option. Moreover, if the employee meets a prospective employer at which her productivity is equal to  $\theta'$ , then she will be indifferent regarding which offer to accept since both offer a contract giving her all of the rents.<sup>8</sup> Independent of whether she stays, the firm will receive a value of 0 in this case as well. By our definition of the set  $B$ , we have assumed that she stays in such a case.

The set  $B(\theta', \theta)$  contains all of those values that result in a renegotiation of the contract  $(w)$  at the current firm and that do not result in a departure. Since the total surplus associated with a match value is strictly increasing in the match value, the amount of surplus the individual can appropriate from  $\theta'$  is strictly increasing in the value of the outside option. If the current outside option is  $\theta$ , then any outside option greater than  $\theta$  and less than  $\theta'$  will result in a renegotiation. The value of the job to the firm at that point in time will be  $V_f(\theta', w(\theta', \tilde{\theta}), \tilde{\theta})$ , where  $w(\theta', \tilde{\theta})$  will be the new wage paid and  $\theta < \tilde{\theta} \leq \theta'$ . Finally, all potential matches less than or equal to the current outside option will not be reported to the firm, since the value of the outside option associated with these values is no greater than the current one. When such an offer arrives, the value of the firm's problem does not change. After rearranging terms and taking limits as  $\varepsilon \rightarrow 0$ , we have

$$V_f(\theta', w', \theta) = \left( \rho + \eta + \lambda_e P(\tilde{\theta} > \theta) \right)^{-1} \times \{ \theta - w + \lambda_e \sum_{\tilde{\theta} \in B(\theta, \theta')} V_f(\theta', w(\theta', \tilde{\theta}), \tilde{\theta}) p(\tilde{\theta}) \}.$$

The worker's value of being employed is defined similarly. The value of the worker's problem is given by

$$V_e(\theta', w', \theta) = \left( \rho + \eta + \lambda_e P(\tilde{\theta} > \theta) \right)^{-1} \times \{ w + \eta V_n(m) + \lambda_e [ \sum_{\tilde{\theta} \in B(\theta, \theta')} V_e(\theta', w(\theta', \tilde{\theta}), \tilde{\theta}) p(\tilde{\theta}) + \sum_{\tilde{\theta} \in C(\theta')} V_e(\tilde{\theta}, w(\tilde{\theta}, \theta'), \theta') p(\tilde{\theta}) ] \},$$

where  $V_e(\theta', w(\theta', \theta), \theta)$  is the equilibrium value of employment to a worker with match value  $\theta'$  when his next best option has a match value of  $\theta$  and the set  $C(\theta')$  is the set of all  $\theta_j \in \Omega_\theta$  such that  $\theta_j > \theta'$ . As we saw in the case of the firm, when an employee encounters a firm with a new

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<sup>8</sup>This event has positive probability given our assumption that  $\theta$  is a discrete random variable.

match value  $\tilde{\theta}$  which is lower than his current match  $\theta$  but belongs to the set  $B(\theta', \theta)$ , his new value of employment at the current firm becomes  $V_e(\theta', w(\theta', \tilde{\theta}), \tilde{\theta})$ . Instead, when the match value at the newly-contacted firm is greater than the current match value  $\theta'$ , the employee will change employers, and the new value of employment is given by  $V_e(\tilde{\theta}, w(\tilde{\theta}, \theta'), \theta')$ . Thus, the match value at the current firm becomes the relevant outside option of the worker that is faced by the new firm. Finally, when the match value at the prospective new employer is less than the current dominated match value  $\theta$ , the new contact is not reported to the current firm since it would not improve the terms of the current contract. Because of this selective reporting, the value of employment contracts must be monotonically increasing within a job with at a particular firm. As originally noted by Postel-Vinay and Robin (2002), wage declines can be observed when moving directly between firms, though, the value of the employment match must always be increasing.

With a new match value of  $\theta' > \theta$ , the surplus attained by the individual at the new match value with respect to the value she could attain at the old match value after extracting all the surplus associated with it is

$$V_e(\theta', w(\theta', \theta), \theta) - Q(\theta),$$

where  $Q(\theta) \equiv V_e(\theta, w(\theta, \theta), \theta)$  is the value of employment to the employee if she receives the total surplus of the match  $\theta$ . In this case, the wage function has the property that  $\theta = w(\theta, \theta)$ . Then,

$$Q(\theta) = \left( \rho + \eta + \lambda_e P(\tilde{\theta} > \theta) \right)^{-1} \times \left\{ \theta + \eta V_n(m) + \lambda_e \sum_{\tilde{\theta} \in C(\theta)} V_e(\tilde{\theta}, w(\tilde{\theta}, \theta), \theta) p(\tilde{\theta}) \right\}.$$

The model is closed after specifying the value of nonemployment,  $V_n(m)$ . We will discuss the manner in which minimum wages impact job acceptance, the unemployment rate, and the equilibrium wage offer function in detail below. As we mentioned above, there is a minimal acceptable match value from the unemployment state denoted by  $\theta^A(m)$ , such that for all  $\theta_j \geq \theta^A(m)$  the match is accepted. Define the set of acceptable match values out of the unemployment state by  $D(\theta^A(m))$ . We note that for the firm to earn nonnegative flow profits, it is necessary that  $\theta^A(m) \geq m$ .

The searcher's value of being unemployed is defined as

$$V_n(m) = \left( \rho + \lambda_n P(\tilde{\theta} \geq \theta^A(m)) \right)^{-1} \times \left\{ b + \lambda_n \sum_{\tilde{\theta} \in D(\theta^A(m))} V_e(\tilde{\theta}, w(\tilde{\theta}, U), U) p(\tilde{\theta}) \right\},$$

where the outside option is that associated with remaining in the unemployment state  $U$ .

The equilibrium wage function,  $w(\theta', \theta)$ , is defined as follows. When an employed agent with an (acceptable) outside option  $\theta$  meets a dominating match  $\theta'$ , the Nash-bargained wage is given by

$$w(\theta', \theta) = \arg \max_{w \geq m} S(\theta', w, \theta).$$

When an unemployed agent meets a new potential employer, the solution to the Nash bargaining problem is given by

$$w(\theta, U) = \arg \max_{w \geq m} S_n(\theta, w, U),$$

where  $S_n(\theta, w, U) = \{V_e(\theta, w, U) - V_n(m)\}^\alpha \times \{V_f(\theta, w, U)\}^{1-\alpha}$ . The minimum wage acts as a side constraint in the Nash bargaining problem in both cases.

## 2.2 Analysis of the Model

In this section we consider the nature of the wage function with and without a binding minimum wage function under the renegotiation and no renegotiation bargaining scenarios.

### 2.2.1 The Wage Function with $m = 0$

In this section we describe the method of solution of the model and the properties of the equilibrium wage function. The discrete  $\theta$  assumption facilitates solving and analyzing the model. Recall that the set of values that  $\theta$  can take is contained in the finite set  $\Omega_\theta$ , where the  $L$  elements of the set are ordered

$$0 < \theta_1 < \dots < \theta_L < \infty. \tag{1}$$

The equilibrium wage function is described via the matrix:

**Table 1.1**

*Equilibrium Wage Matrix*

	<i>Dominant Value</i>				
<i>Dominated Value</i>	$\theta_j$	$\theta_{j+1}$	$\dots$	$\theta_{L-1}$	$\theta_L$
$\theta_L$					$\theta_L$
$\theta_{L-1}$				$\theta_{L-1}$	$w(\theta_L, \theta_{L-1})$
$\vdots$				$\vdots$	$\vdots$
$\theta_{j+1}$	$\theta_{j+1}$		$\dots$	$w(\theta_{L-1}, \theta_{j+1})$	$w(\theta_L, \theta_{j+1})$
$\theta_j$	$\theta_j$	$w(\theta_{j+1}, \theta_j)$	$\dots$	$w(\theta_{L-1}, \theta_j)$	$w(\theta_L, \theta_j)$
$U$	$w(\theta_j, U)$	$w(\theta_{j+1}, U)$	$\dots$	$w(\theta_{L-1}, U)$	$w(\theta_L, U)$

(2)

The value  $\theta_j = \theta^A(0)$  in this case is the minimum acceptable match value for an unemployed searcher when there is no (binding) statutory minimum wage. The wage function is not defined for values of  $\theta_j < \theta^A(0)$ . Moreover, the bargaining mechanism always produces efficient mobility, meaning that the current match value (i.e., the “dominant” one) is always at least as large as the “dominated” match value, which generates the outside option value in the Bertrand competition between firms.

An important feature of the Bertrand competition for workers and the discreteness of  $\theta$  is that when the dominated value is equal to the dominant value, a positive-probability event, the worker captures all of the rents from the match. This means that the wage rate in this case is equal to the match value, simplifying the computation of the equilibrium wage function.

The equilibrium wage function computation is conducted in the following recursive manner. We begin by assuming that the only acceptable match value to an unemployed searcher is  $\theta_L$ , which is the largest match in the set  $\Omega_\theta$ , so that  $\theta^A(0) = \theta_L$ . We begin with a guess of the value of unemployment,  $\tilde{V}_n(\theta^A)$ . In terms of an employment spell, the state  $(\theta_L, \theta_L)$  is an absorbing state, since no further job mobility can take place from that state during the current employment spell. The only way such a spell can end is through exogenous termination, which occurs at the constant rate  $\eta$ . The individual’s value of being in such a spell (given the value of unemployment  $\tilde{V}_n$ ) is given by

$$\tilde{V}_e(\theta_L, \theta_L, \theta_L) = \frac{\theta_L + \eta \tilde{V}_n(\theta_L)}{\rho + \eta}, \quad (3)$$

where the second argument in  $\tilde{V}_e$  is the wage rate associated with the state  $(\theta_L, \theta_L)$ , and is equal to  $\theta_L$ . The firm’s value is 0.

An unemployed searcher only accepts a match of  $\theta_L$ , the probability of which is  $p(\theta_L)$ . When the unemployed searcher accepts the one employment contract available to her, it has the associated value

$$\tilde{V}_e(\theta_L, w, U) = \frac{w + \lambda_e p(\theta_L) \tilde{V}_e(\theta_L, \theta_L, \theta_L) + \eta \tilde{V}_n(\theta_L)}{\rho + \lambda_e p(\theta_L) + \eta},$$

while the value to the firm is

$$\tilde{V}_f(\theta_L, w, U) = \frac{\theta_L - w}{\rho + \lambda_e p(\theta_L) + \eta}.$$

The wage associated with this state is then given by

$$\tilde{w}(\theta_L, U) = \arg \max_w (\tilde{V}_e(\theta_L, w, U) - \tilde{V}_n(\theta_L))^\alpha \tilde{V}_f(\theta_L, w, U)^{1-\alpha}.$$

Then the (new) implied value of unemployed search is given by

$$\tilde{V}'_n(\theta_L) = \frac{b + \lambda_n p(\theta_L) \tilde{V}_e(\theta_L, \tilde{w}(\theta_L, U), U)}{\rho + \lambda_n p(\theta_L)}.$$

If  $\tilde{V}'_n(\theta_L)$  is sufficiently “close” to the initial guess  $\tilde{V}_n(\theta_L)$ , then we say that the value of search when only  $\theta_L$  is acceptable is given by  $V_n^*(\theta_L) = \tilde{V}'_n(\theta_L)$ . If not, replace  $\tilde{V}_n(\theta_L)$  with  $\tilde{V}'_n(\theta_L)$ , and repeat the process. We then continue the iterations until convergence.<sup>9</sup>

A similar technique is used for the cases in which we set  $\theta^A = \theta_j$ ,  $j = 1, \dots, L - 1$ . Each different “potential” critical value implies a unique wage distribution associated with it and a value of unemployed search given by  $V_n^*(\theta_j)$ . The optimal acceptance match chosen by the individual is the one that produces that highest value of searching in the unemployment state, i.e.,

$$\theta^A = \theta_j \Leftrightarrow V_n^*(\theta_j) = \max\{V_n^*(\theta_k)\}_{k=1}^L.$$

The equilibrium wage matrix is the one associated with that value of  $\theta^A$ . If, for example,  $L = 10$  and the  $\theta^A = \theta_4$ , then the (lower triangular) wage matrix is  $8 \times 7$ .

This is the algorithmic approach used to compute the wage matrix in Table 1.1. Changes in primitive parameters will of course sometimes change the critical acceptance match  $\theta^A$ , but not always due to the discreteness of the distribution. This will be observed in some of the examples that we turn to now. While the discrete distribution assumption does have some negative aspects, computation of the equilibrium wages/values is simplified and some of the impacts of minimum wages on labor market outcomes and welfare are somewhat more transparent.

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<sup>9</sup>The mapping  $V_n = TV_n$ , while typically not a contraction, is monotone increasing. Subject to existence conditions, there exists a unique fixed point solution.

We now present an example of the wage function computation. We set the parameters of the search environment at  $\alpha = .25$ ,  $\lambda_n = .2$ ,  $\lambda_e = .05$ ,  $\eta = .01$ ,  $\rho = .01$ , and  $b = -5$ . We assume match distribution with six mass points, with  $\Omega_\theta = \{5, 8, 11, 14, 17, 20\}$  and an associated vector of probabilities given by  $(.1 \ .2 \ .25 \ .2 \ .15 \ .1)$ . The equilibrium wage distribution is given below:

**Table 1.2**

*Wage Matrix*

$m = 0$

	<i>Dominant <math>\theta</math> Value</i>					
<i>Dominated <math>\theta</math> Value</i>	5	8	11	14	17	20
20						20
17					17.00	17.35
14				14.00	13.84	14.19
11			11.00	10.27	10.11	10.46
8		8.00	6.70	5.96	5.80	6.15
5	5.00	3.32	2.02	1.80	1.12	1.47
$U$	4.78	3.10	1.79	1.06	0.90	1.25

(4)

At this set of labor market parameters, we note that all elements of  $\Omega_\theta$  are acceptable from the unemployment state. The most striking feature of the matrix is probably the degree of non-monotonicity in the wages observed when the outside option is  $U$ ,  $\theta = 5$ , or  $\theta = 8$ . For individuals coming out of unemployment, the highest wage offer observed is the one associated with the lowest acceptable match value,  $\theta = 5$ . Although the value of the employment contract is strictly increasing in the match value found by the unemployed searcher, wages are not. In fact, were it not for the wage associated with the highest match value, exactly the opposite would be true. A similar pattern is observed in every row of the matrix (of those with more than three entries).

The low wages associated with the high match values, holding constant the dominated value, are the employee’s payment for the future bargaining advantages that the match conveys during the employment spell. Under this set of parameters, the rate of meeting other employers, 0.05, is quite high relative to the rate of exogenous termination of the job (and employment) spell, 0.01. Combined with the relatively low discount rate of 0.01, the wage “compensation” for the future bargaining advantage is high.



Obviously, when there is no OTJ search, a high match value delivers no future bargaining advantage, and there are no compensating differentials observed in the wage function. As an illustration, we determine the equilibrium wage rates for the same parameter values used to generate Table 1.2, except that we set  $\lambda_e = 0$ . The wage function in this case is

**Table 1.3**

*Wage Matrix*

$\lambda_e = 0$

	<i>Dominant <math>\theta</math> Value</i>						
<i>Dominated <math>\theta</math> Value</i>	5	8	11	14	17	20	.
<i>U</i>	—	7.60	8.35	9.10	9.85	10.60	(5)

In this case, the match value of 5 that was previously acceptable is no longer so. The wages are much higher coming out of the unemployment state, since there is no bargaining advantage component of remuneration. Most importantly, for our purposes, the wage function is monotonically increasing in  $\theta$  due to the fact that wages are the only compensation mechanism and the outside option is the same,  $V_n(0)$ , at all jobs.

Before concluding this subsection, we return to the example wage function in Table 1.2 to discuss implications of the model regarding patterns of wage changes over an employment spell. Given the arrival of a “reportable” competing match value, two things can occur. If the new match arrival is larger than the current dominated match value and less than or equal to the current match value associated with the job, there is no job mobility, but there is renegotiation of the wage contract. Since the only “negotiable” element of the employment contract is the wage, this is increased. Thus, while the worker remains at the same firm, all wage changes are positive.

In the second case, where the new match value exceeds the value of the current match, there is job mobility and contract negotiation. The old dominant match value becomes the new dominated match value and generates the outside option value. Though both the dominant and dominated match value increase with a job-to-job move, the wage rate need not. Once again, this is due to the fact that part of the employee’s share of the surplus is generated by the OTJ bargaining option, and this option value may increase to such an extent in the job-to-job move that a wage reduction is required to satisfy the surplus division rule.

### 2.2.2 The Wage Function with $m$ Binding

In the Flinn (2006) analysis without OTJ search, minimum wages could only be binding in one particular manner, which was by constraining the choice set of the worker and the firm. In that analysis, a binding minimum wage always produced an acceptance match value,  $\theta^A$  in our notation, that was greater than what was called the “implicit” acceptable match value. In other words, workers would have accepted lower matches than  $m$ , but were constrained not to by minimum wage law. In that setting, the minimum wage essentially served as a coordination device that enabled workers with little bargaining power to achieve more of the surplus produced by the match. The cost of this gain was a lower probability of finding an acceptable match.

The minimum wage potentially plays an altogether different role in the presence of OTJ search, at least when the wage function displays nonmonotonicities of the type discussed in the previous section. We illustrate the role of the minimum wage with two separate examples. All of the parameters of the labor market environment are the same as they were in the example discussed in the previous subsection. The only state variable that differs from the example above and among the two presented here is the minimum wage rate,  $m$ .

In the first example, the minimum wage is set at \$1.50. Since all match values are acceptable when  $m = 0$ , and since the the minimum match value is 5, clearly all matches are still in the choice set of the bargaining worker-firm pair, by which we mean that the firm can earn nonzero flow profits when paying  $m$  for all  $\theta \in \Omega_\theta$ . Solving the model, we find that the equilibrium wage matrix is given by:

**Table 1.4***Wage Matrix* $m = 1.5$ 

	<i>Dominant <math>\theta</math> Value</i>					
<i>Dominated <math>\theta</math> Value</i>	5	8	11	14	17	20
20						20
17						17.00 17.35
14						14.00 13.84 14.19
11						11.00 10.27 10.11 10.46
8						8.00 6.70 5.96 5.80 6.15
5	5.00	3.40	2.10	1.50	1.50	1.55
<i>U</i>	4.82	3.32	2.02	1.50	1.50	1.50

(6)

The first important thing to note about this example is that the acceptance match value has remained the same at  $\theta^A = 5$ . All matches that were previously accepted in equilibrium still are. However, the minimum wage constraint on the bargaining process has changed the bargained outcomes for most of the wages associated with dominated values of  $\theta = 5$  and  $U$ . The impact has come through the improvement in the wage distribution that resulted from not allowing for the firm to be fully compensated for its contribution to the future bargaining power of the individual during the current employment spell. Where the minimum wage is binding, the implication is that individual is receiving more than their share of the surplus, which in our example is set at 0.25. This is a benefit to the supply side of the market, particularly those in unemployment and who have found employment at the minimal acceptable match value. Firms still earn positive profits whenever they employ an individual who has a dominated match value less than the dominant one, and minimum wages cannot affect the wage payment when  $\theta' = \theta$ .

We conclude this section with an illustration of the wage function when the minimum wage is set at such a high level that an otherwise acceptable match value (to a searching individual) is smaller than the minimum wage, and thus cannot lead to an employment match. Table 1.5 contains the wage function when  $m = 13$ .

**Table 1.5***Wage Matrix* $m = 13$ 

	<i>Dominant <math>\theta</math> Value</i>		
<i>Dominated <math>\theta</math> Value</i>	14	17	20
20	20		
17	17.00 17.35		
14	14.00 13.84 14.19		
$U$	13.00 13.00 13.00		

(7)

In comparison with the relevant rows and columns of Table 1.4, the new, extremely high, value of the minimum wage has no discernible impact on the wages negotiated during OTJ search. The new minimum wage has a large impact on the wage offers to currently unemployed workers, though the probability of getting an acceptable offer has substantially decreased.

### 2.3 Model with No Renegotiation

The model with renegotiation is considerably simpler to describe. As was discussed in the introduction to this section, the outside option in this case is always equal to the value of unemployed search,  $V_n(m)$ . There is efficient mobility, which means that the match value at a new firm in a job-to-job transition is strictly greater than the match value at the firm the individual is leaving. This implies that all job-to-job transitions are associated with an increase in an individual's wage. Since there is no renegotiation, the wage at a given firm is constant over the duration of employment at that firm.<sup>10</sup>

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<sup>10</sup>Given the simplicity of the bargaining process in this case, it is trivial to allow for shocks to match productivity to occur that would change this implication. In particular, say that a current (acceptable) match value of  $\theta_j$  increased to  $\theta_{j+1}$  at rate  $\gamma^+$  for  $j < L$ , while it decreases to  $\theta_{j-1}$  at rate  $\gamma^-$  for  $j > 1$ . This type of process would generate wage increases and decreases on a job spell, and would could obviate the need for an exogenous dismissal rate  $\eta$  if there were a measurable set of unacceptable match values coming out of the unemployment state.

### 2.3.1 The Case of $m = 0$

We now characterize the mobility-wage process in the no renegotiation case. The value of employment at an acceptable match value  $\theta_i$  is given by

$$V_e(\theta_i) = \frac{w(\theta_i) + \eta V_n + \lambda_e \sum_{j>i} p_j V_e(\theta_j)}{\rho + \eta + \lambda_e p_i^+},$$

where  $p_i^+ \equiv \sum_{j>i} p_j$ . Given our assumption regarding the outside option in the bargaining problem, the value of the wage associated with acceptable match value  $\theta_i$  is determined as

$$\begin{aligned} w(\theta_i) &= \arg \max_w \left( \frac{w + \eta V_n + \lambda_e \sum_{j>i} p_j V_e(\theta_j)}{\rho + \eta + \lambda_e p_i^+} - V_n \right)^\alpha \\ &\quad \times \left( \frac{\theta_i - w}{\rho + \eta + \lambda_e p_i^+} \right)^{1-\alpha} \\ &= \arg \max_w (w - (\rho + \lambda_e p_i^+) V_n + \lambda_e \sum_{j>i} p_j V_e(\theta_j))^\alpha \\ &\quad \times (\theta_i - w)^{1-\alpha} \\ &= \alpha \theta_i + (1 - \alpha) \{ (\rho + \lambda_e p_i^+) V_n - \lambda_e \sum_{j>i} p_j V_e(\theta_j) \}. \end{aligned} \quad (8)$$

The wage-setting rule is a simple extension of that associated with the bargaining problem in the absence of on-the-job search (i.e.,  $\lambda_e = 0$ ).

**Proposition 1** *Let  $\theta^A = \theta_i$  denote the minimal acceptable match value.*

$$w(\theta_i) < w(\theta_{i+1}) < \dots < w(\theta_L).$$

Proof: Since

$$\lambda_e p_i^+ V_n - \lambda_e \sum_{j>i} p_j V_e(\theta_j) = \lambda_e \sum_{j>i} p_j (V_n - V_e(\theta_j))$$

is an increasing function of  $i$ , the result is obvious. ■

The monotonicity in the wage function in this case will have important implications in determining the optimal minimum wage rate. In the renegotiation case, monotonicity of the wage in current match value depended on the values of the primitive parameters. In this case, it does not.

The model, without minimum wages, is completed by determining the minimal acceptable match value from the unemployment state. This is accomplished by computing the value of search for each of the  $L$  possible acceptance sets, differentiated by the lowest match value included in the set. Let the value of search be given by  $\tilde{V}_n(\theta_i)$ , when  $\theta_i$  is the minimal acceptable value. Then  $V_n = \max\{\tilde{V}_n(\theta_i)\}_{i=1}^L$ , and  $\theta^A = \theta_j$  is the associated minimal acceptable match value when the  $\arg \max$  of the right hand side is equal to  $j$ .

### 2.3.2 Minimum Wages

Since the wage function is monotonically increasing in the match value, binding minimum wage rates will have qualitatively similar effects to those described in Flinn (2006), where there was no on-the-job search. In particular, we will want to define the value of search in the unemployment state as a function of the minimum wage rate,  $V_n(m)$ . As was true in the model with renegotiation, the set of feasible acceptable match values is truncated from below by the minimum wage, so that the lowest acceptable match value must be at least equal to  $m$ .

Given the value of search, the wage associated with an acceptable match value  $\theta_i$  is given by

$$w_i(m) = \max\{m, \alpha\theta_i + (1 - \alpha)((\rho + \lambda_e p_i^+)V_n(m) - \lambda_e \sum_{j>i} p_j V_e(\theta_j; m))\}.$$

Since the second term in the max function is an increasing in  $\theta_i$ , we see that the match values that yield minimum wages are the lowest ones in the acceptance set, which is not true, in general, under renegotiation.

The value of search in the unemployment state is determined in the same way as described for the case of no binding minimum wage, except that  $\theta^A$  is restricted to the subset of  $\Omega_\theta$  that includes only those values of  $\theta_i \geq m$ .

## 2.4 The Labor Market Participation Decision

Given the data available, and the fact that those earning minimum wages tend to be the youngest labor market participants, we extend the model to allow a labor market participation decision. Although the participation decision is modeled in a somewhat *ad hoc* manner, it will allow us to formulate an estimator of primitive parameters that does not rely on steady state assumptions, assumptions that are particularly troublesome when allowing for OTJ search.<sup>11</sup> Moreover, it is undoubtedly true that there is substantial dispersion among birth cohort members in the age of entry into the labor market. In this model, this distribution will be impacted through changes in the (binding) minimum wage.

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<sup>11</sup>Flinn (2006) used a maximum likelihood-based estimator of primitive parameters based on a likelihood specification that assumed that data was generated by the steady state distribution of wages and unemployment spells. In a model without OTJ search, such as that one, transitions to the steady state from the initial labor market state of  $U$  shared by all labor market participants is rapid. In the case of OTJ search, the transition period can be considerably longer, making the use of a steady state-based estimator even more questionable.

We write the value of being out of the labor force as

$$V_o(\xi, a) = \frac{\xi + r(a)}{\rho},$$

where  $\xi$  is an i.i.d. draw from the distribution  $F$  and  $r(a)$  is a weakly-decreasing function of the individual's (potential labor market) age,  $a$ . The draw  $\xi$  is taken to represent an individual's constant flow value of being out of the labor force, while  $r(a)$  is the (common to all agents) flow value of being out of the labor force as a function of their potential labor market age. Dividing by  $\rho$  gives the function  $V_o$  a present value interpretation.

Since the (flow) value of occupying the out of the labor force state is nonincreasing in the age of the individual, and since the value of the unemployment state is constant, labor force participation is an absorbing state. An individual of potential labor market age  $a$  will be in the labor market if

$$\begin{aligned} V_o(\xi, a) &\leq V_n(m) \\ \Rightarrow \xi &< \rho V_n(m) - r(a), \end{aligned}$$

so that the participation probability of an individual of age  $a$  under minimum wage regime  $m$  is given by

$$\pi(a, m) = F(\rho V_n(m) - r(a)).$$

Given that the function  $r$  is nonincreasing in  $a$ , the participation rate is nondecreasing in  $a$ . It is also immediately obvious that increases in the value of search induced by a change in the minimum wage will result in increases in the participation rate at all ages.

In the estimation section it will be useful to characterize the timing of labor market entry as follows. Given an  $m$ , all individuals with a value of  $\xi$  less than  $\xi_1^*(m)$  will enter the market immediately upon becoming eligible to do so.<sup>12</sup> This critical value is given by

$$\xi_1^*(m) = \rho V_n(m) - r(1).$$

In a similar manner, define the critical value of entry at age  $a$  by

$$\xi_a^*(m) = \rho V_n(m) - r(a).$$

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<sup>12</sup>In introducing the participation decision into the computation of sample moments in the MSM estimation exercise, we will assume that participation decision is made at integer potential labor market ages. The description of the participation rates by integer age in this section follows that convention.

Since  $r$  is a nonincreasing function, this implies that the critical value sequence  $\{\xi_a^*(m)\}$  is non-decreasing. Then the probability that an individual enters the labor market at age  $t$  when the minimum wage rate is  $m$  is given by

$$\kappa(a, m) = F(\xi_a^*(m)) - F(\xi_{a-1}^*(m)), \quad a = 2, 3, \dots,$$

with  $\kappa(1, m) = F(\xi_1^*(m))$ .

## 2.5 Endogenous Contact Rates

We now extend the model to a very simple general equilibrium setting in which contact rates between searchers on both sides of the market are determined endogenously. The standard method to endogenize contact rates is through the use of the “matching” function, in which the number of contacts is (typically) assumed to be a constant returns to scale function having as arguments the measure of unemployed searchers,  $U$ , and the measure of existing vacancies,  $K$ , so that

$$M = M(U, K).$$

The meeting rate per unemployed searcher is given by  $M/U$ , or  $\lambda_u = M(U, K)/U = M(1, K/U)$  under the standard constant returns to scale assumption and no OTJ search.

While it is fairly straightforward to introduce employed search into the matching function framework, the most natural way to proceed yields some empirically unpalatable restrictions. As in Petrongolo and Pissarides (2001), a natural way to proceed would appear to be to define collective search effort as

$$S = U + \tau E,$$

where  $\tau$  denotes the relative search effectiveness of employed searchers, which we might presume to be less than 1. This formulation, however, implies relationships between stocks and flows in the labor market that are not observed in the data. As a result, we have formulated an alternative, directed-search formulation of contact-rate determination.

We assume that a firm creating a vacancy chooses between a vacancy directed toward unemployed searchers or employed searchers. Each type of vacancy is associated with a different flow cost to the firm. For example, in attempting to find unemployed searchers, the firm might place advertisements in newspapers or the internet, while a firm looking for recruits already employed might target their competitors. These types of search activities can be expected to have different costs and success rates in terms of initial contacts. Moreover, given a successful recruitment effort,



it is clear that the expected payoffs can differ greatly. A firm contacting an unemployed searcher will have a successful recruitment effort as long as the match draw is greater than or equal to  $\theta^A(m)$ . The outside option used in the wage bargaining process is  $V_n(m)$  in this case. Conversely, a firm searching among individuals who are already employed at a job with associated match value  $\theta_j \in A(m)$  will have a lower probability of a successful recruitment effort, since only a draw of  $\theta' > \theta_j$  will ultimately lead to positive profits for the firm. In addition, the successfully recruited individual will have an outside option  $Q(\theta_j) > V_n(m)$ , so that a firm making a successful match with an employed individual will earn lower profits, on average. The advantage a firm searching amongst employed individuals has is that there are usually many more of them than there are unemployed searchers in steady state equilibrium.

Let the number of matches in the “unemployed” search market be given by

$$M^U = M^U(U, K_U),$$

and the number of matches in the employed search market by

$$M^E = M^E(E, K_E),$$

and for purposes of estimation we assume that

$$\begin{aligned} M^U &= U^{\omega_U} K_U^{(1-\omega_U)} \\ M^E &= E^{\omega_E} K_E^{(1-\omega_E)}, \end{aligned}$$

with  $\omega_j \in (0, 1)$ ,  $j = U, E$ .

Firms may create vacancies in either type of search market, so that the standard FEC applies to both. Let the flow cost associated with a vacancy in market be given by  $\psi_j$ ,  $j = U, E$ . Then the expected value of holding a vacancy in the unemployed search market,  $V_{K_U}$  is expressed as

$$\rho V_{K_U} = -\psi_U + \frac{M^U(U, K_U)}{K_U} \sum_{\tilde{\theta} \in A(m)} p(\tilde{\theta}) [V_f(\tilde{\theta}, w(\tilde{\theta}, U), U) - V_{K_U}],$$

while the expected value of a holding a vacancy in the employed agent search market satisfies

$$\rho V_{K_E} = -\psi_E + \frac{M^E(E, K_E)}{K_E} \sum_{\tilde{\theta} \in A(m)} p_{SS}(\tilde{\theta}) \sum_{\theta' > \tilde{\theta}} p(\theta') [V_f(\theta', w(\theta', \tilde{\theta}), \tilde{\theta}) - V_{K_E}],$$

where  $p_{SS}(\cdot)$  is the steady state distribution of match values across jobs. Imposing the FEC, we

have

$$\psi_U = \frac{M^U(U, K_U)}{K_U} \sum_{\tilde{\theta} \in A(m)} p(\tilde{\theta}) V_f(\tilde{\theta}, w(\tilde{\theta}, U), U) \quad (9)$$

$$\psi_E = \frac{M^E(E, K_E)}{K_E} \sum_{\tilde{\theta} \in A(m)} p_{SS}(\tilde{\theta}) \sum_{\theta' > \tilde{\theta}} p(\theta') V_f(\theta', w(\theta', \tilde{\theta}), \tilde{\theta}). \quad (10)$$

Although it slightly anticipates our consideration of identification issues, it is useful to note the relationship between estimable quantities from the supply side data available to us and the costs of holding vacancies in the two search markets, which are treated as primitive parameters. The rate of arrival of job offers to unemployed searchers is given by

$$\begin{aligned} \lambda_n &= \frac{M^U(U, K_U)}{U} \\ &= \left( \frac{K_U}{U} \right)^{1-\omega_U}. \end{aligned}$$

If  $\omega_U$  and  $\lambda_n$  is known, then  $K_U = U \lambda_n^{1/(1-\omega_U)}$ . Since all of the terms in the summand on the right hand side of (9) are known, then all terms on the right hand side are known, and  $\psi_U$  is determined. In an exactly analogous fashion we can determine  $\psi_E$  if  $\lambda_E$  and  $\omega_E$  are known. The values of  $\psi_U$  and  $\psi_E$  are determined through the use of the FEC, and thus are consistent with all of the general equilibrium assumptions of the model.

### 3 Data

The data used to estimate the model contain information on individuals from the 1996 panel of the Survey of Income and Program Participation (SIPP). A main objective of the SIPP is to provide accurate and comprehensive information about the principal determinants of the income of individual households in the United States. The SIPP collects monthly information regarding individual's labor market activity including earnings, average hours worked, and whether the individual changed jobs during the month, making it an attractive data set with which to study employment dynamics of job seekers and workers.

Although the size of the SIPP's target sample is quite large, our sample size has been greatly reduced through the imposition of several restrictions. We only consider individuals ages 16 to 30 who do not participate in the armed services or in any welfare program (e.g., TANF, Food Stamps, WIC) during the sample period. We focus on this age group because minimum wage earners are

typically young. The parameter estimates, as well as the results from policy simulations presented in subsequent sections, should be interpreted within this population context.

In addition to these general selection criteria, we impose a restriction that is particular to estimating a stationary on-the-job search model with minimum wages. The minimum wage changed from \$4.75 to \$5.15 on September 1, 1997, and remained at \$5.15 for the remainder of the SIPP survey period. Although the SIPP interviews individuals every four months for up to twelve times from 1996 to 2000, we use data only from February 1998 to February 2000 in order to allow adequate time for the labor market to adjust to the policy change and to avoid minimum wage changes within the survey period. A drawback of defining a sample window close to the end of the panel, however, is that discontinuities in respondents' employment histories become increasingly present as individuals approach the end of the survey period. Because our econometric specification relies heavily on identifying transitions between labor market states, it is essential that individuals have complete labor market histories. After excluding individuals with incomplete histories, our final sample consists of 3048 individuals.

As we discuss in the next section, we use a moment-based estimation procedure to estimate the model. The set of sample moments is estimated using cross-sectional data from February 1998 and February 1999, as well as data that describes individuals' labor market dynamics between these two points in time. The cross-sectional moments include the proportion of the sample that is unemployed, the mean and standard deviation of wages, and the proportion of workers who earn the minimum wage. Other moments describe employment transitions and wage changes between the two points in time, and include the proportion of individuals employed in February 1998 who lose their job before February 1999, and the mean wage change among workers who have a job-to-job transition during the year.

Table 2.a contains descriptive statistics generated from the data. In February 1998, 4.3 percent of the sample was unemployed. Among employed workers, the mean and standard deviation of wages were \$9.47 and \$4.67, with 3.3 percent of workers earning the minimum wage. Our moment-based estimation strategy allows us to include workers in the sample with wages below the minimum wage, although they comprise a very small proportion of the sample.

Turning to measures of employment and wage dynamics, we find that 29 percent of workers in the sample who are employed in February 1998 transition directly to another job before February 1999 (i.e., there is no intervening spell of unemployment). The mean wage in February 1998 for these workers is \$8.43, about one dollar less than the mean wage of the full sample. While the mean

change in wages across jobs for these workers of \$0.90 is fairly small, the distribution of changes is considerably dispersed (the standard deviation of the change is \$4.32). The size of the mean wage change is due partially to the existence of wage decreases across consecutive jobs, as 32.2 percent of employed workers accept new jobs with lower wages (see Figure 1). However, the nature of the wage-bargaining process between individuals and firms in our model does not predict that wages have to be at least as large at the destination job as at the current job in order for the individual to leave his current job. Workers may leave their current jobs to accept new, lower wage jobs if the option value of doing so is large enough, at least under the model that includes renegotiation.

The transition rate from employment to unemployment is also included in the set of moments that measure employment dynamics. The percentage of individuals in the sample who are employed in February 1998 who exit into unemployment before February 1999 is 6.4 percent. The group of individuals who make this transition consists of both those who voluntarily leave their job and those who are “involuntarily” dismissed. Among those individuals who make the opposite transition, from unemployment to employment, the mean wage at the first job is \$8.48. This is about one dollar less than the mean of the cross-sectional wage distribution in February 1998 for the full sample. The distribution of initial wages for the individuals making this transition is also slightly less dispersed than the cross-sectional distribution.

Table 2.b contains the age-specific labor market participation rates from the data. Our simple model of the labor market participation decision, described in Section 2.4, implies that participation rates should be nondecreasing. This is the pattern that broadly emerges, with a few relatively minor exceptions. We note that by age 29, approximately 85 or 86 percent of the sample are in the labor market. Of course, there is some entry and exit from the labor market associated with fertility and health shocks, for example, and these are not included in this model. Abstracting from these reasons for nonparticipation, the sample proportions are consistent with “full participation” by age 30.

## 4 Estimation Method

In this section we discuss the simulation-based method used to estimate the primitive parameters of the model developed above. Given the rather rich patterns of wage mobility and turnover that the model generates, and the fact that the wage function is not (in general) monotonic in the value of the current match under the bargaining model that allows renegotiation, the use of a maximum

likelihood estimator is problematic. We have opted to use a Method of Simulated Moments (MSM) estimator instead. This also allows us to directly compare the performance of the two models of the bargaining process that are “nonnested.”

The procedure used is similar to that employed by Dey and Flinn (2008) in their estimation of a model of household (husband and wife) labor market search, which was also used implemented using SIPP data. The panel data we have access to is used to construct an event-history data set, in which the labor market status of each sample member is considered known at each point in time during their sample participation period. Let  $M_{98}$  denote a set of sample characteristics from the point sample constructed in February 1998, and let  $M_{99}$  denote the analogous sample characteristics computed from the February 1999 point sample (recall that these are the same set of individuals). Finally, let  $P_{99|98}$  denote a set of sample characteristics computed from February 1998 to February 1999 transitions of the sample members. For example, one such characteristic could be the probability of observing the individual in the unemployment state in February 1999 given that they were in the employment state in February 1998. Another might be the average wage of employed individuals at the 1999 survey date who were unemployed at the time of the 1998 survey.

Virtually all estimators of equilibrium search models assume that the population is in the steady state when the point sampling takes place. The Dey and Flinn (2008) analysis involved OTJ search, as in the model considered here, though sample members were considerably older and the assumption that initial conditions had “worn off” by the time of the measurement of the sample member’s labor market states was defensible. The Flinn (2006) minimum wage analysis included only young workers, making it harder to argue that the distribution of observed outcomes was generated by the steady state distribution of the model. A mitigating factor, however, was the fact that OTJ search was not allowed, making convergence to the steady state considerably more rapid than would typically be the case in the presence of OTJ search.

In our case, it is reasonably straightforward to test whether the steady state assumption is appropriate by comparing sample characteristics from the 1998 and 1999 point samples. If the model is in the steady state, then  $\text{plim}_{N \rightarrow \infty} M_{98} = \text{plim}_{N \rightarrow \infty} M_{99} = M_{SS}$ , where  $M_{SS}$  is the vector of steady state values of the measured characteristics. From a cursory look at Table 2.a, the steady state assumption looks extremely questionable. Since all individuals begin their actual labor market career in the unemployment state, on the transition path to the steady state we expect to observe a monotonically decreasing proportion of active labor market participants in the unemployment

state, which is what is indicated. Moreover, along the transition path, we also expect to observe increases in the average wage of the employed, which is also consistent with sample moments. A formal test of the equality of a subset of sample characteristics at the two points in time leads to an overwhelming rejection of the null hypothesis of equality.<sup>13</sup> This has prompted us to allow for a very simple type of age-dependence of outcome distributions in the model, which is implemented through the use of the participation decision that was discussed in Section 2.4.

#### 4.1 The Participation Decision

Our sample consists of individuals aged 16 through 30, inclusive, which corresponds to the labor market ages interval 1 through 15. In ‘estimating’ the participation component, we proceed as follows. Given the use to which we put the estimated sequence, we decided to estimate the parameters characterizing the participation decision in a separate step in the estimation process. From the 1998 sample, we use the participation rates for individuals aged 16 through 25, which represent (potential) labor market ages of 1 through 10. We denote these participation proportions by  $\tilde{\pi}_o(a)$ ,  $a = 1, \dots, 10$ . Under the model we had, after slightly adjusting notation,

$$\pi(a, V_n(m)) = F(\rho V_n(m) - r(a); \delta),$$

where  $\delta$  is a parameter vector that completely characterizes  $F$ . Now if  $\delta$  is known, as well as  $V_n(m)$ , we can form an estimator of  $r(a)$  at each of the ages 1 through 10 by setting

$$\begin{aligned} \tilde{\pi}_o(a) &= F(\rho V_n(m) - r(a); \delta) \\ \Rightarrow \hat{r}(a) &= F^{-1}(\tilde{\pi}_o(a); \delta) - \rho V_n(m), \quad a = 1, \dots, 10. \end{aligned}$$

It is immediately apparent that for any choice of cumulative distribution function  $F$ , values of the  $r$  sequence can be found that perfectly “fit” the participation rates over the first 10 years of potential labor market age, subject to appropriate restrictions on the support of  $F$ . We assume that the distribution  $F$  is negative exponential with parameter  $\delta$ , so that

$$F(\xi; \delta) = 1 - \exp(-\delta\xi).$$

We normalize the value of  $\delta$  to 1, so that

$$\hat{r}(a) = \ln(1 - \tilde{\pi}_o(a)) + \rho V_n(m), \quad a = 1, \dots, 10.$$

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<sup>13</sup>This test was described and implemented in an earlier version of this paper.

If the sequence  $\tilde{\pi}_o(a)$  is nondecreasing in  $a$ , then the sequence  $\hat{r}(a)$ ,  $a = 1, \dots, 10$ , is nonincreasing in  $a$ . To complete the labor market career, we assume that  $r(j) = r(10)$ ,  $j = 11, 12, \dots$

We conclude this brief section with a few comments. We have restricted the estimation process to utilize only the first 10 potential labor market ages because ages 11 through 15 displayed some nonmonotonicity, as can be seen from Table 2.b. It does appear that participation rates are stable after age 10, so that assuming that all values of  $r$  after age 10 are equal to  $r(10)$  does not appear to be too unreasonable.

We include the participation rates of individuals in their first ten years of labor market age in the set of sample moments included in the MSM estimator. As the discussion above makes clear, it is in principle possible to perfectly fit these participation rates given the flexible specification of  $r(a)$  that is employed. In practice, the fit of these sample participation rates is not perfect, since the estimated participation rate functions play an important role in forming the other simulated sample moments, and hence some “trade off” in the fit of the participation rates and the other sample characteristics is observed. Nonetheless, the implied participation rates are reasonable given the simplicity of our characterization of the participation decision.

Finally, we note that the estimator of the  $r$  process proposed here seems to be more appropriate for a discrete time representation of the labor market process. We think of the observed  $\tilde{\pi}_o(a)$  values as being time aggregated measures of the underlying continuous time (and continuous age) labor market process, though we do not formally account for time aggregation in the estimation of the  $r$  process nor in our estimates of the other parameters of the model.

## 4.2 Estimation of the Remaining Supply-Side Parameters

In terms of the other parameters of the model, and the simulation aspect of the estimator, we proceed as follows. Our sample is comprised of individuals 16-30 years of age. Given a minimum wage  $m$ , our model implies a distribution of times of entry into the labor market by age. For individuals actually in the market at the time of the sample, the longest period of participation an agent could have would be 15 years (if they were currently 30 and entered the market at age 16). We generate  $15 \times R$  sample paths,  $R$  of which are truncated at 6 months, another  $R$  at 1.5 years, up to the last set of  $R$  replications, which are truncated at 14.5 years. In terms of the  $R$  generated sample paths for experience level  $e$ , we denote the sample path characteristics at  $e - .5$  by  $M(e; \varphi)$ , where  $\varphi$  denotes the primitive parameters of the model under the partial equilibrium

specification (in which the contact rates  $\lambda_n$  and  $\lambda_e$  are considered to be predetermined).<sup>14</sup> Denote the age distribution of the sample by  $B(a)$ . Then the simulated vector of characteristics are given by

$$M(\varphi) = \sum_{a=1}^{15} \sum_{e=1}^{a-15} M(e; \varphi) \kappa(e, a; m, \varphi) B(a), \quad (11)$$

where

$$\kappa(e, a; m, \varphi) = \frac{\kappa(e; m, \varphi)}{\sum_{1 \leq j \leq a} \kappa(j; m, \varphi)}, \quad 1 \leq e \leq a.$$

where the  $\kappa(e; m, \varphi)$  function was defined above (without the explicit conditioning on  $\varphi$ ). The function  $\kappa(\cdot, a; m, \varphi)$  then is the labor market experience distribution for individuals of (potential) labor market age  $a$  and who are in the labor market at age  $a$ . For example, for individuals of labor market age 1 (i.e., calendar age 16) who are in the labor market at age 16, the probability that they are experience level 1 is unity. For individuals of calendar age 17 who are in the labor market, their experience-specific distribution has probability mass at 1 and 2, only.

Let the value of the sample characteristics used be given by  $X = (M'_{98} P'_{99|98})'$ , which is a column vector containing  $B$  sample characteristics. Let the corresponding model counterparts, conditional on the parameter vector  $\varphi$ , be given by  $\tilde{X}(\varphi) = (M(\varphi)' \pi(\varphi)')'$ , where the  $\pi(\varphi)$  transition characteristics are defined in an exactly analogous way to how we defined the average of the point sample characteristics in (11). Then the minimum distance estimator of  $\varphi$  is given by

$$\hat{\varphi} = \arg \min_{\varphi} (X - \tilde{X}(\varphi))' A (X - \tilde{X}(\varphi)),$$

where  $A$  is an  $B \times B$  symmetric, positive definite weighting matrix. We compute the weighting matrix by resampling the SIPP data matrix from which the sample characteristics,  $X$ , are computed. The matrix  $A$  is the inverse of this cross-products matrix.

We have computed estimates of the standard errors by bootstrapping; this involved resampling the original individual data to compute new values of  $X$ . For each bargaining structure, we generated over 100 resamples.

### 4.3 Identification of Supply-Side Parameters

We estimate two specifications of the equilibrium model that vary in terms of the possibility of renegotiation. The same parameters characterize both specifications, and there are no identification

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<sup>14</sup>In the general equilibrium specification,  $\lambda_n$  and  $\lambda_e$  are not primitive parameters, but we can estimate them as equilibrium outcomes. These estimates are used in the second estimation stage in which a subset of demand-side parameters are estimated.



issues that are particular to either. Therefore we consider just the simpler (in terms of characterizing the equilibrium of the model) case of no-renegotiation.

From the Flinn and Heckman (1982) identification analysis of the partial-partial equilibrium<sup>15</sup> search model, we know that the c.d.f.  $G$  is not identified except under a class of parametric assumptions if there are rejected wage offers (or match values, in our analysis). We utilize a discrete distribution of  $\theta$ , so that the results of Flinn and Heckman are not strictly applicable. The distribution of  $\theta$  contains  $L$  points in its support, and it is assumed that this set is known. The distribution of  $\theta$  is then characterized in terms of  $L - 1$  unknown probabilities, which can be a large number of parameters with the value  $L = 30$  that we utilize in the estimation. To eliminate this overparameterization, we define grid points in terms of quantiles of the distribution  $G$ . We assume that the distribution that generates our discrete distribution  $G$  is lognormal, with  $\ln z \sim N(\mu_\theta, \sigma_\theta^2)$ . Then given that there are  $L$  points, define

$$\varpi_l = \frac{l - 0.5}{L}, \quad l = 1, 2, \dots, L.$$

Then define the “representative” values associated with each of the  $L$  equiprobable intervals under the distribution of  $\ln z$  to be

$$\theta_l(\mu_\theta, \sigma_\theta) = \exp(\sigma_\theta \Phi^{-1}(\varpi_l) + \mu_\theta).$$

Changes in  $\mu_\theta$  or  $\sigma_\theta$  yield new values of  $\theta_l$ ,  $l = 1, \dots, L$ . Since the intervals that each  $\theta_l$  represents are all considered to be equiprobable, we assume that the matching distribution is discrete uniform, with each  $\theta_l$  having probability  $1/L$ . The entire discrete distribution is described in terms of the two parameters  $(\mu_\theta, \sigma_\theta)$ . We can think of this method of constructing the discrete  $G$  distribution as an approximation to the continuous lognormal distribution in the sense that

$$\text{plim}_{L \rightarrow \infty} (G(x; \mu_\theta, \sigma_\theta) - \Phi\left(\frac{\ln x - \mu_\theta}{\sigma_\theta}\right) < \varepsilon) = 1$$

for all  $x > 0$  and for all  $\varepsilon > 0$ . This makes the Flinn and Heckman results somewhat applicable, at least asymptotically. Since the lognormal distribution with lower support point 0 is recoverable in the sense in which Flinn and Heckman define it, if the problem involved directly estimating a lognormal wage offer distribution, identification could be claimed. In our case, the matching

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<sup>15</sup>This corresponds to the case of an exogenous wage offer distribution (and contact rates). In this case,  $G$  corresponds to the wage offer distribution itself. This can be viewed as a special case of the bargaining set up used here in which  $\alpha = 1$ .

distribution is filtered through the Nash bargaining process to deliver an observed wage distribution, and the correspondence between wage offers and the underlying match distribution is less immediate. However, using the likelihood function with Nash bargaining and no OTJ search, Flinn (2006) provided some sufficient conditions for the (continuous) matching distribution to be identified, and the lognormal distribution satisfies his sufficient condition. The fact that individuals are also searching on the job makes the mapping between observed wage distributions (at a point in time, for example) and the underlying match distribution even less immediate, though in other similar applications (e.g., Dey and Flinn (2005)), a likelihood-based estimator performed well in recovering lognormal parameters in a simulation exercise. We have no reason to believe that our simulation-based estimator performs significantly worse in the present application.

Flinn and Heckman (1982) also showed that the parameters  $b$  and  $\rho$  were not individually identified. As is commonly done, we fix the parameter  $\rho$  at a value reasonably commensurate with the interest rate. The value we use (denominated in months) is  $\rho = 0.05/12$ . With  $\rho$  fixed, it is in principle straightforward to point identify  $b$  if the model is parameterized in terms of the critical match value given when the match value distribution is absolutely continuous. With a discrete match distribution, there is no unique critical match value. Since most estimates of  $b$  using micro-data find a negative value, we have simply fixed  $b = -2$ . We found that the estimates of the other identified parameters were not sensitive to even relatively large changes in the value of this parameter.

While we do not use event history data in defining the estimator, the steady state distributions and transition probabilities have been found sufficient to yield reasonable and precise estimates of the transition rate parameters  $(\lambda_n, \lambda_e, \eta)$  using the SIPP data. For example, Dey and Flinn (2008) estimate a joint husband-wife labor market search model using a similar estimation strategy, and obtain precise estimates of the transition rate parameters for *both* husbands and wives. While their model is not set in an equilibrium framework, the transition dynamics are more complex than those generated by ours. We are confident that a MSM approach based on reasonable sample characteristics can yield “good” estimates of these parameters, which is borne out in our empirical results.

The most vexing problem we face is the estimation of the bargaining power parameter  $\alpha$  using only information from the supply side of the market. Flinn (2006) showed that, for a continuously distributed  $\theta$ , a sufficient condition for identification of  $\alpha$  was the  $G$  not belong to a location-scale family. He assumed that  $G$  was lognormal, which belongs to a log location-scale family, but not a

location-scale family. Monte Carlo experiments showed that  $\alpha$  could be recovered when extremely large samples were available under this functional form assumption.

In our application, the situation is worse on one hand and better on the other. We do not have a continuously-distributed  $\theta$ , so his results are not directly applicable here. As we argued above, thinking of our discrete distribution as an approximation to an underlying continuous lognormal ameliorates this problem. On the positive side, our model with on-the-job search provides a richer mapping from a fixed-population wage-offer distribution, a set of outside options (determined by primitive parameters), and the bargaining power parameter than did the one he faced in the no OTJ case. Wage changes across firms during the same employment spell provide a rich potential source of identifying information about primitive parameters, including  $\alpha$ , that is not present when all employment spells consist of one job spell.

#### 4.4 Estimation of Demand-Side Parameters

In our discussion of the general equilibrium version of the model, which incorporated endogenously-determined contact rates, we showed how one could “back out” the measure of vacancies created both for unemployed and employed searchers with access to  $\lambda_n$ ,  $\lambda_e$ , and the Cobb-Douglas parameters characterizing the matching functions in the two search markets,  $\omega_U$  and  $\omega_E$ , if the values associated with all of the states in the model were known. We have consistent estimators of the equilibrium values of the contact rates from the supply side portion of the estimation, as well as consistent estimates of the other elements of  $\varphi$ , which are required to form consistent estimates of the value functions and steady state distributions appearing in the summands on the right hand sides of (9) and (10). In the unemployed search market, we saw that

$$K_U = U\lambda_n^{1/(1-\omega_U)}.$$

With consistent estimators available for  $\lambda_n$  and  $U$ , this relationship contains two unknowns. If we fix  $\omega_U$  at a given value, then  $K_U$  can be consistently estimated. This is the route we follow. By fixing  $\omega_U = \omega_E = 0.5$ , we are able to consistently estimate the equilibrium values  $K_U$  and  $K_E$ . Given these consistent estimates, we can consistently estimate the flow costs of vacancies in the two markets,  $\psi_U$  and  $\psi_E$ . If only one market is observed in equilibrium, these parameters are the only ones that are estimable. Estimation of  $\omega_U$  and  $\omega_E$  would require observations on the same market in equilibrium at two points in time in which binding (in at least one period) minimum wages were set at different values.

## 5 Results

Table 3 contains estimates from the models with and without renegotiation. The estimates in the first column indicate that, without renegotiation, the average time between contacts for unemployed searchers is  $(0.505)^{-1}$ , slightly less than 2 months. Once a job offer is received, the acceptance probability is  $\tilde{G}(m) = 0.49$ , resulting in a mean unemployment duration of  $(\lambda_n \tilde{G}(m))^{-1} = 4.1$  months. An employed individual receives an alternative job offer from a competing firm approximately every  $(0.309)^{-1} = 3.24$  months. Given an offer, the probability that the worker accepts it and changes employers depends on his or her current wage and match value. The estimated exogenous dissolution rate of jobs is 0.013, implying that workers are exogenously terminated from their jobs every  $(0.013)^{-1} = 6.4$  years, on average.

The estimates for the model with renegotiation in the second column of Table 3 indicate that the average time between contacts for unemployed searchers is  $(0.505)^{-1}$ , which is identical to the estimate in the model without renegotiation. For all other parameters, the estimates differ, sometimes noticeably, across the two bargaining specifications. When a job offer is received by an unemployed searcher, the probability he accepts it is  $\tilde{G}(m) = 0.99$ . Thus, the estimated length of an unemployment spell is  $(\lambda_n \tilde{G}(m))^{-1} = 2$  months, about half the duration implied by the model without renegotiation. Once employed, an individual receives an alternative job offer from a competing firm every  $(0.152)^{-1} = 6.4$  months, approximately half as frequently as in the model without renegotiation. This is expected, since in order to generate the same amount of wage dispersion as in a model without renegotiation, offers must arrive less frequently. The estimated exogenous dissolution rate of jobs is 0.010, which implies that workers are exogenously terminated from their jobs every 8.3 years, on average.

In the model with renegotiation, the average ln match draw in the population is 2.621 with a standard deviation of 0.175 (the implied mean and standard deviation of the match draw  $\theta$  in levels are 7.19 and 7.35). Without renegotiation, these values are 1.615 and 0.846 (with the mean and standard deviation of the match draw  $\theta$  in levels being 13.96 and 2.46). The differences in these parameters across models stem from differences in the estimates of the bargaining power parameter. The estimate of the bargaining power parameter for workers is 0.32 when renegotiation is allowed and is 0.38 when workers are not allowed to renegotiate. The lower bargaining power in the model with renegotiation causes the same set of observed wages to be mapped to a match distribution that is centered to the right of the match distribution in the model without renegotiation. This was

apparent in the different transition rates out of unemployment implied by the two models given the same job offer arrival probability—the larger mean and smaller standard deviation of the match distribution in the model with renegotiation leaves no mass of the distribution near the minimum wage, leading all unemployed searchers to accept any job offer they receive.

In Table 4 we compare the sample and simulated moments used in the estimation procedure at the solution vector. The most striking result is that the value of the criterion function is almost half as large for the model without renegotiation. Given the same set of moments (and the same weighting matrix) were used to estimate each model, this suggests that this model has a much better fit to the data. In particular, the unemployment rate and proportion of workers that earn minimum wage in the model without renegotiation are close to their sample values. This is also true of the mean and standard deviation of the wage distribution. We also observe that neither model produces any mass in the upper tail of the wage distribution (the proportion of workers earning wages greater than \$20 is zero in each case).

Figures 2.a and 2.b plot the wage function against workers' current match value. Figure 2.a depicts the case in which workers are not allowed to renegotiate their wage at their current job. The wage function is increasing in the current match value, as expected. Figure 2.b depicts the case in which workers are allowed to renegotiate their wage at their current job. Each wage function plotted in this figure corresponds to a different outside option. The wage functions corresponding to the outside options of unemployment,  $U$ , and the minimal acceptable match value,  $\theta_9$ , are nonmonotone.<sup>16</sup> Workers coming out of unemployment, for example, accept lower initial wages at jobs with higher match values as payment for future bargaining advantages. The wage function loses this feature as the outside option increases. Outside options of at least  $\theta_{14}$ , for example, produce a wage function that is increasing in the dominant match value.

The demand-side parameter estimates are shown in Table 5. Setting the Cobb-Douglas parameter equal to 0.5 in the model without renegotiation results in vacancy rates for unemployed and employed searchers of 0.009 and 0.092 and corresponding flow costs of a vacancy of \$92.45 and \$54.26, respectively. The vacancy rates are over half as large in the model with renegotiation, equal to 0.004 for vacancies for unemployed searchers and 0.024 for vacancies for employer workers. The corresponding flow costs are larger for vacancies for the unemployed than in the model without renegotiation, \$137.85, but smaller for vacancies for employed searchers in the model with

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<sup>16</sup>The model is estimated using a 30-point discrete match distribution. Given our parameter estimates, the first eight match values are not accepted in equilibrium.

renegotiation, \$30.75.

## 5.1 Testing Between the Bargaining Protocols

Given the estimates of the primitive parameters of the model that characterize the supply side of the market, we proceed to conduct a bootstrap-based test between the renegotiation and no-renegotiation specifications of the model. The idea behind the test is the following. Given the actual data and the weight-matrix  $A$  we computed by resampling the original data set a large number of times, we found what seems to be a large difference between the value of the distance function by which the estimator is defined between the two bargaining protocols. Under the no renegotiation assumption, the value of the distance function was 743.77, while under the renegotiation assumption the value of the distance function was 1216.34, indicating a substantially better fit between the moments and the model, given the weighting matrix  $A$ , under the no renegotiation version of the model.

In order to assess the “significance” of the difference, we resampled the original data set 50 times,<sup>17</sup> recomputed the weighting matrix  $A$  each time, and then reestimated the model with the resampled data characteristics vector and weighting matrix under both bargaining assumptions. More formally, define

$$D_z^r = \min_{\varphi} (X^r - \tilde{X}_z(\varphi))' A^r (X^r - \tilde{X}_z(\varphi)),$$

where  $z = 1$  denotes the case of renegotiation and  $z = 2$  denotes the case of no renegotiation, and  $r$  denotes the bootstrap replication,  $r = 1, \dots, 50$ . We then define the difference

$$\Delta D^r \equiv D_1^r - D_2^r.$$

We then look at the distribution of the (50) values of  $\Delta D^r$ . If less than two of the values were negative, say, we would conclude that there was strong support for the no renegotiation assumption, given the data characteristics we have included in defining the estimator and all of the other usual caveats that apply.

We found that all 50 of the values were positive. From the actual data, we found the difference,  $\Delta D$ , to be 472.57. Over the 50 replications, the average value of  $\Delta D^r$  was 468.93, with the minimum value being 417.08 and the maximum value being 544.66. While the number of replications is fairly small, there is no indication that performing more replications would alter the finding that within

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<sup>17</sup>We were limited to 50 replications because of the time it takes to estimate the model with renegotiation, which on our quad-core Sun workstation was about 35 hours per replication.

our modeling structure, the no renegotiation bargaining protocol is significantly more consistent with the data features that are used to define our estimator.

## 6 Policy Experiments and Comparative Statics Results

In this section we look at the impact of the minimum wage on labor market outcomes and welfare, using the estimates of primitive parameters we obtained under our two bargaining specifications. As we will see, our determination of the “optimal” minimum wage level, defined below, depends critically on which specification of the bargaining environment we assume, and whether we perform our policy evaluation in partial or general equilibrium. We begin our analysis by looking at the impact of the minimum wage on wage growth over the life cycle and over the course of an employment spell (i.e., a sequence of job spells with no intervening unemployment spell).

### 6.1 Minimum Wage Effects on Wage Profiles

The work of Leighton and Mincer (1981), and, more recently, by Acemoglu and Pischke (2002), investigates the potential impact of minimum wage laws on life-cycle wage profiles through reductions in general human capital investment by recent labor market entrants. While we do not consider human capital investment in our model, the potential for minimum wage impacts on the shape of lifetime wage profiles exists due to effects on the bargaining environment. Under either of the two bargaining specifications investigated here, minimum wages, by truncating the lower tail of the accepted wage distribution, tend to produce less wage growth over employment spells and the life cycle.

We perform two simulations. The first examines how the minimum wage affects workers’ wage profiles in their first employment spell, while the second examines how the minimum wage affects workers’ age-earnings profiles (i.e., over a labor market career). In both cases, we simulate profiles using models with and without renegotiation at the worker’s current job using minimum wages of \$5.15, \$7.15, \$9.15, and \$11.15. For each value of the minimum wage, we resolve the model (using the point estimates of all estimated parameters).

Figures 3.a and 3.b plot the average wage as a function of the time elapsed in individuals’ first employment spells for various minimum wage values for the model without and with renegotiation. All individuals begin the simulation in unemployment and the average wage is obtained at each month of the first employment spell. This spell can consist of one job or many jobs and ends when

the worker is exogenously dismissed from employment. For both models and for all four minimum wages, the wage profiles increase with time. In the model without renegotiation at the current job, the average wage increases solely due to job-to-job transitions. While this effect is also present in the model in which renegotiation is allowed, intrafirm competition generates increasing, and potentially decreasing, wages at workers' current job as well. Comparing the wage profiles across models given a minimum wage of \$5.15, we observe that average wages increase more quickly in the early months of the workers' first employment spells when renegotiation is not allowed. This may be attributed to workers experiencing wage decreases when renegotiating their wage at their current firm in the model with renegotiation.

The effect of increasing the minimum wage is the same across models, with higher minimum wages flattening the wage profiles in each figure. In both models, higher minimum wages increase the minimally acceptable match value for unemployed workers. Thus, workers that find jobs earn higher wages initially, but the smaller set of viable (larger) match values decreases the chances of a worker finding an alternative firm that will offer him a higher wage. There is less wage growth as a result. In the bargaining specification that allows bidding between two competing employers, the equilibrium wage function displays a "compensating differential" property, which we have commented upon extensively. In this case, those jobs offering the highest growth prospects offer commensurately lower wages due to future firms having to bid against a high-valued match value to attract the worker. This effectively increases the outside option of the worker, and the firm demands compensation for this future bargaining advantage by reducing the current wage offer. The minimum wage limits the extent to which firms can "charge" an employee for this future wage growth potential, thus reducing average wage growth in the market.

Figures 4.a and 4.b depict the effect of the minimum wage on workers' age-earnings profiles. The labor market career of any individual can consist of many labor market cycles, defined as sequences of labor market states beginning with an unemployment spell and ending with the last job prior to the following unemployment spell for a given individual. Thus, throughout their labor market career, individuals can become employed at a job, renegotiate their wage at their current job (in one of two bargaining specifications), change jobs to receive a higher wage, and become exogenously terminated and return to unemployment.

Figures 4.a and 4.b show that individuals' age-earnings profiles are increasing in age. Most wage growth occurs early in their labor market careers. There are two effects of increasing the minimum wage on these age-earnings profiles. The first is that average wages are higher at any point in time



in the labor market career. By shrinking the set of viable matches available to the searcher while unemployed (the standard (negative) employment effect), the minimum wage impacts workers' wage profiles by delaying the start of the firm competition process during which significant wage gains occur. Thus, higher minimum wages delay entry into employment, but tend to produce better job offers once employed. The second effect is that wage profiles flatten out earlier in individuals' labor market careers when minimum wages are higher. Workers receive better offers once employed, but these offers arrive less frequently when the minimum wage is higher. This is similar to the effects observed in Figures 3.a and 3.b.

## 6.2 Optimal Minimum Wages

In this subsection we determine the optimal minimum wage using several welfare measures for each of our bargaining environment assumptions. We compute optimal minimum wages for both the general and partial equilibrium settings under each of the bargaining assumptions.

We confine our attention to four distinct sets of agents. Agents on the supply side of the market consist of those who do not participate and those who are unemployed and employed. On the demand side are firms that either choose not to create a vacancy or that create a vacancy and have it unfilled or filled in the steady state. Because the FEC implies that the welfare of members of these first two groups on the demand side is zero, the only set of agents on the demand side that enter explicitly into the welfare analysis is the set of firms with filled vacancies.

Whether an individual participates in the labor market depends on his or her age and the value of being out of the labor force,  $V_o(\xi, a) = \frac{\xi + r(a)}{\rho}$ , as well as the value of unemployed search. We denoted the participation probability for an individual of potential labor market age  $a$  by  $\pi_o(a) = F(\rho V_n(m) - r(a); \delta)$  where  $F$  is the distribution function of the idiosyncratic, permanent flow value of being out of the labor force. Although the age-specific participation probabilities were used in the estimation procedure to weight age-specific moments according to labor market entry times, the welfare analysis is based on the steady state distribution of outcomes. In the steady state, the size of the set of individuals who are out of the labor force,  $p_O(m)$ , is equal to  $\tilde{F}(\rho V_n(m) - r(a_{15}); \delta) \equiv 1 - F(\rho V_n(m) - r(a_{15}); \delta)$  where  $a_{15}$  is the age of the oldest labor market entrants in the sample. The average welfare for this group,  $V_o(m)$ , is the expected value of  $V_o(\xi, a_{15})$  conditional on not participating:

$$E(V_o(m) | \xi > \rho V_n(m) - r(a_{15})) = \int_{\rho V_n(m) - r(a_{15})} V_o(\xi, a_{15}) \frac{dF(\xi)}{\tilde{F}(\rho V_n(m) - r(a))}.$$

All members of the set of individuals who are unemployed at a moment in time share the same value,  $V_n(m)$ . The proportion of labor market participants who are unemployed in the steady state when the minimum wage is  $m$  is given by

$$\pi_U(m) = \frac{\eta}{\eta + \lambda_n p_U(m)},$$

where  $p_U(m)$  is the probability of drawing a match value in the acceptable set under the minimum wage of  $m$ .

The proportion of participating employed individuals in the labor market is  $1 - \pi_U(m)$ , obviously. The average welfare of employed individuals is a complicated expression under OTJ search, since the wage distribution is a mapping from the steady state distribution of dominant and dominated match values,  $(\theta', \theta)$  under the minimum wage  $m$ , which is denoted by  $p_{SS}(\theta', \theta|m)$ . We can represent this expected value as

$$EV_e(m) = \sum_{\theta'} \sum_{\theta} V_e(\theta', w(\theta', \theta|m), \theta) p_{SS}(\theta', \theta|m).$$

We can obtain an arbitrarily good approximation to  $EV_e(m)$  through simulation.

Turning to the firms' side of the market, a similar situation prevails. For firms with vacancies, the value of the vacancy is zero. Outside of general equilibrium, where the number of vacancies is determined, the measure of firms with vacancies is indeterminate. However, for each worker participating in the labor market, there is a firm with a filled vacancy. Therefore the measure of firms with employees is  $1 - p_U(m)$ , of those firms participating in the labor market. The expected value of firms with an employee under minimum wage  $m$  is

$$EV_f(m) = \sum_{\theta'} \sum_{\theta} V_f(\theta', w(\theta', \theta|m), \theta) p_{SS}(\theta', \theta|m).$$

Putting all of these terms together, an egalitarian social welfare function can be written

$$W(m) = p_O(m)EV_o(m) + (1 - p_O(m)) (p_U(m)V_n(m) + (1 - p_U(m))EV_e(m) + (1 - p_U(m))EV_f(m)).$$

In this expression, the steady state probabilities of unemployment and employment conditional on participating in the labor market have been multiplied by the participation probability,  $(1 - p_O(m))$ . This function is egalitarian in the sense that each individual and firm is given the same weight in determining aggregate welfare.

We first examine the impact of the minimum wage on the equilibrium steady state probability of unemployment. The situation for the two bargaining environments is presented in Figures 5.a

and 5.b for the partial and general equilibrium cases, respectively. Due to the discreteness of the match distribution, we see that the unemployment probability function is a step function in  $m$ . We see discontinuities in these curves due to the discreteness of the match distribution and the reduction in the feasible set of match values following an increase in the minimum wage. These eliminations can be “voluntary” or “involuntary,” loosely speaking. In the case of no renegotiation, all of these discontinuities occur when the minimum wage is increased above an otherwise acceptable match value, which we label “involuntary,” since it arises directly from the increase in an external constraint. In the case of renegotiation, there also exist “voluntary” eliminations, when the equilibrium wage function is non-monotone. In this case, an increase in the minimum wage can improve the value of unemployed search to such a degree that agents eliminate a match value from their acceptance set even if it still feasible (i.e., yields non-negative flow profits to the firm).

We also note that, at each value of  $m$ , the unemployment probability is greater under the no renegotiation case than under renegotiation in the partial equilibrium case only (see Figure 5.a). This ordering is not pre-ordained by the theory, since we use different sets of parameter estimates to compute these functions for the two bargaining situations. What should be true is that, at the same set of parameter values, the equilibrium unemployment rate would be no less under no renegotiation, since the value of employment across all states can be no greater than under renegotiation. For the general equilibrium case, the equilibrium unemployment rate in the model with renegotiation jumps considerably starting at around a minimum wage of \$13.00 and eventually exceeds the equilibrium unemployment rate in the model without renegotiation. The main thing to note is that high minimum wages can induce a substantial increase in the steady state unemployment rate under either bargaining situation and in either equilibrium setting.

Tables 6 and 7 present the values of the minimum wage that maximize the welfare measure for each set of agents and for the aggregate welfare measure  $W(m)$ . It also contains the percent change in average welfare when moving from the baseline minimum wage of \$5.15 to the optimal minimum wage, as well as the unemployment rate at the optimal minimum wage. If we just focus attention on the aggregate welfare measure in these tables.<sup>18</sup> Under exogenous contact rates (Table 6), we see that the minimum wage has a much less beneficial effect in the case without renegotiation. While a binding minimum wage does improve aggregate welfare, the maximizing level is at \$12.05

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<sup>18</sup>The average welfare functions for each set of agents, as well as the aggregate measure, have a roughly concave shape, once the minimum wage starts to bind (and ignoring the discontinuities due to the discreteness of the match distribution). This was found in Flinn (2006) as well for the case of no OTJ search.

an hour. When we allow for renegotiation between workers and firms, the situation changes, and the optimal minimum wage in terms of maximizing  $W$  is \$14.05. This is mainly due to the fact that adverse employment effects are essentially absent in the model with renegotiation until we get to very high levels of the minimum wage.

The optimal minimum wages in both models are dramatically higher than the baseline value of \$5.15. Average aggregate labor market welfare is maximized at these values, despite the associated rise in the unemployment rate. In the model without renegotiation, the unemployment rate increases from 5.1 to 14.0 percent when the minimum wage is set to maximize aggregate labor market welfare ( $m = 12.05$ ). When renegotiation is allowed, the increase in the minimum wage leads to an increase in the unemployment rate from 2.2 percent to 4.2 percent.

There are gains in the average welfare of unemployed and employed workers of 1.7 percent and 5.0 percent, respectively, in the model with no renegotiation. When renegotiation is allowed, the average welfare gain is significantly greater for the unemployed (4.8 percent) and slightly less for the employed (4.0 percent). While a different set of parameters governs the search environment in each model, setting a high minimum wage impacts unemployed worker welfare less in the model with renegotiation. While this is also the case in the aggregate labor market, consisting of all workers and firms, the difference in the percent change in welfare is mitigated slightly by the effect of the minimum wage on the welfare of firms with filled vacancies. In the model without renegotiation, firms that employ workers experience a 34.8 percent increase in average welfare at the optimal minimum wage, whereas there is no increase in the model with renegotiation (the average welfare begins decreasing when the minimum surpasses \$5.15). Increases in the minimum wage render match values less than the minimum wage unacceptable in both models, but they prevent the firm from being fully compensated for its contribution to the future bargaining power of the individual in the model in which renegotiation is allowed. Though the parameter estimates differ across simulations of each model, we attribute the absence of a welfare gain for firms with filled vacancies to this second effect.

Allowing the contact rates to be determined endogenously leads to a dramatic decrease in the optimal minimum wage in the model without renegotiation and a small decrease in the model with renegotiation (Table 7). A minimum wage of \$4.70 maximizes welfare for all agents on the supply side of the market, as well as aggregate welfare for all agents, when workers are not allowed to renegotiate on the job. For the model with renegotiation, this value is \$13.30.

Which set of policy experiment results should be given the most credibility? Under our model

structure and the MSM estimator employed, there was strong empirical evidence in favor of the no renegotiation specification. Other things equal, the general equilibrium version of the model should be preferred, in assessing policy impacts, to the partial equilibrium specification. The general equilibrium results for the no renegotiation case suggest an optimal minimum wage below the statutory minimum wage in effect at the time of the survey, which was \$5.15. Then the model predicts that minimum wage increases above \$5.15 which have recently occurred will reduce the average level of welfare in the population. There is one strong caveat that applies here, and that is that crucial parameters characterizing the matching technology were not estimable using the data available to us, and were set, arbitrarily, at 0.5 for both the unemployed and employed search markets. Variations in these matching parameters can lead to dramatically different policy conclusions. Until a convincing way is found to estimate these parameters, the policy predictions in the general equilibrium case must be interpreted cautiously.

## 7 Conclusion

In a matching model with search frictions and on-the-job search, minimum wages may “bind” in different ways depending on the nature of the worker-firm bargaining problem. We found that in a model that allowed for “bidding wars” between firms, minimum wages may bind at relatively high match values, somewhat counterintuitively. The reason is that high match values, particularly when the individual is coming from the unemployment state, may have high value in terms of the future bargaining advantages they convey over the course of the current employment spell. To “pay” for this advantage, workers obtain a lower wage rate. Hence, over a certain range, minimum wages may merely impact the degree to which firms can charge employees for the bargaining advantage associated with the match value. Only at high values of  $m$  does the minimum wage eliminate otherwise advantageous match values and result in actual deadweight loss.

The situation is different in the no renegotiation case, where the value of unemployed search, given  $m$ , always serves as the outside option in the Nash bargaining problem. In that case, as was true in the no OTJ framework investigated in Flinn (2006), binding minimum wages always eliminated otherwise acceptable match values. This leads to “nonadvantageous” increases in the equilibrium unemployment rate, and increases the costs of imposing high values of  $m$ .

In their search, matching, and Nash bargaining frameworks, Dey and Flinn (2005) and Cahuc et al. (2006) found that allowing for OTJ search substantially reduced the estimate of the worker’s

bargaining power parameter in comparison with the case in which OTJ search was not introduced (e.g., Flinn, 2006). Our results here show that this is to some degree a result of allowing for Bertrand competition. When competition between firms is introduced, substantial wage gains over an employment spell can be generated simply from this phenomenon, even when the individual possesses little or no bargaining power in terms of  $\alpha$ .<sup>19</sup> When we allow for Bertrand competition, the estimated value of  $\alpha$  is 0.32. When we assume no wage competition between firms, the estimate increases to 0.38. In order to fit the observed wage distributions, lower values of  $\alpha$  are typically associated with wage offer distributions that have more mass in the right tail. This means that minimum wages are “strictly” binding at higher values of  $m$  in the case of renegotiation, which is what we found.

A small, but potentially important, contribution of this research involved the extension of the matching function framework to separate search markets for the unemployed and the employed. As discussed in Petrongolo and Pissarides (2001), the usual approaches to introducing OTJ search into the matching function framework are problematic, in that they imply contact rates between unemployed and employed searchers with firms that are broadly inconsistent with estimates of these contact rate parameters. As is the case with the “unified” search market assumption, the Cobb-Douglas parameters characterizing the two matching functions are not estimable using the data available to us, though the flow cost parameters characterizing the two search markets are estimable conditional on assumed values for the matching function parameters. This arbitrariness of the general equilibrium specification makes the policy experiments using it much more difficult to interpret. In future work we plan to attempt to devote more attention to the important task of generating credible estimates of matching function parameters.

We saw that parameter estimates varied markedly depending upon the nature of the bargaining protocol assumed, as did results of the policy experiment we conducted. Since the different bargaining protocols generate different mappings from the (same) set of primitive parameters into the features of the data included in the definition of the MSM estimator, standard nested hypothesis testing procedures are not applicable. We formulated a bootstrap-based test that is based on the distribution of the differences in the distance metrics associated with the two bargaining protocols across data resamples. Our test results indicated that the no renegotiation case significantly outperformed the renegotiation case. These results provide some guidance as to the appropriate model

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<sup>19</sup>The (approximately) limiting case of this is that considered by Postel-Vinay and Robin (2002), in which workers possessed no bargaining power whatsoever.

specification when theoretical considerations alone are of limited usefulness.

## References

- [1] Acemoglu, D., and J.-S. Pischke. 2002. "Minimum Wages and On-the-Job Training." CEP Discussion Paper no. 527, Centre for Economic Performance, LSE, London.
- [2] Albrecht, J., and B. Axell. 1984. "An Equilibrium Model of Search Employment." *Journal of Political Economy* 92: 824-840.
- [3] Burdett, K., and D. Mortensen. 1998. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review* 39: 257-273.
- [4] Cahuc, P., F. Postel-Vinay, and J.-M. Robin. 2006. "Wage Bargaining with On-the-Job Search: Theory and Evidence." *Econometrica* 74(2), 323-64.
- [5] Dey, M., and C. Flinn. 2005. "An Equilibrium Model of Health Insurance Provision and Wage Determination." *Econometrica* 73: 571-627.
- [6] Dey, M., and C. Flinn. 2008. "Household Search and Health Insurance Coverage." *Journal of Econometrics* 145: 43-63.
- [7] Diamond, P. 1982. "Wage Determination and Efficiency in Search Equilibrium." *Review of Economic Studies* XLIX: 217-227.
- [8] Drazen, A. 1986. "Optimum Minimum Wage Legislation." *Economic Journal* 96: 774-784.
- [9] Flinn, C. 2006. "Minimum Wage Effects on Labor Market Outcomes under Search, Bargaining, and Endogenous Contact Rates." *Econometrica* 74: 1013-1062.
- [10] Flinn, C. 2002. "Interpreting Minimum Wage Effects on Wage Distributions: A Cautionary Tale." *Annales d'Économie et de Statistique* 67-68: 309-355.
- [11] Flinn, C., and J. Heckman. 1982. "New Methods for Analyzing Structural Models of Labor Force Dynamics." *Journal of Econometrics* 18: 115-68.
- [12] Jovanovic, B. 1979. "Job Matching and the Theory of Turnover." *Journal of Political Economy* 87: 972-990.
- [13] Jovanovic, B. 1984. "Matching, Turnover, and Unemployment." *Journal of Political Economy* 92: 108-122.



- [14] Leighton, L., and J. Mincer. 1981. "The Effects of the Minimum Wage on Human Capital Formation." In *The Economics of Legal Minimum Wages*, edited by S. Rottenberg. Washington D.C.: American Enterprise Institute for Public Policy Research, pp. 155-173.
- [15] Lang, K. 1987. "Pareto Improving Minimum Wage Laws." *Economic Inquiry* 25: 145-158.
- [16] Moro, A. 2003. "The Effect of Statistical Discrimination on Black-White Wage Inequality: Estimating a Model with Multiple Equilibria." *International Economic Review* 44: 467-500.
- [17] Petrongolo, B. and C. Pissarides. 2001. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature* 39: 390-431/
- [18] Pissarides, C. 2000. *Equilibrium Unemployment Theory*, 2nd edition. Cambridge, MA: MIT Press.
- [19] Postel-Vinay, F., and J-M Robin. 2002. "Equilibrium Wage Dispersion with Heterogeneous Workers and Firms." *Econometrica* 70: 2295-1350.
- [20] Postel-Vinay, F., and J-M Robin. 2004. "To Match or not to Match? Optimal Wage Policy with Endogenous Search Intensity." *Review of Economic Dynamics* 7: 297-330.
- [21] Rebitzer, J., and L. Taylor. 1996. "The Consequences of Minimum Wage Laws: Some New Theoretical Ideas." *Journal of Public Economics* 56: 245-256.
- [22] Swinnerton, K. 1996. "Minimum Wages in an Equilibrium Search Model with Diminishing Returns to Labor in Production." *Journal of Labor Economics* 14: 340-355.
- [23] Van den Berg, G. 2003. "Multiple Equilibria and Minimum Wages in Labor Markets with Informational Frictions and Heterogeneous Production Technologies." *International Economic Review* 44: 1337-1357.

**Table 2a**  
**Descriptive Statistics**  
Individuals Aged 16-30

	Feb 1998	Feb 1999
Proportion unemployed	0.043	0.030
Mean wage	\$9.47	\$10.39
Standard deviation of wages	\$4.67	\$5.09
Proportion of minimum wage earners	0.033	0.018
Proportion earning wage greater than \$20	0.043	0.053
Proportion employed in Feb 1998 that exit into unemployment within 12 months	0.064	
Proportion employed in Feb 1998 with at least one job change within 12 months	0.291	
Mean wage at initial job among individuals employed in Feb 1998 who have job-to-job transition before Feb 1999	\$8.43	
Mean wage change among individuals employed in Feb 1998 who have job-to-job transition before Feb 1999	\$0.90	
Standard deviation of wage change among individuals employed in Feb 1998 who have job-to-job transition before Feb 1999	\$4.33	
Mean wage at first job for those unemployed in Feb 1998 who become employed within 12 months	\$8.48	
Standard deviation of wages at first job for those unemployed in Feb 1998 who become employed within 12 months	\$4.43	

**Table 2b**  
**Descriptive Statistics**  
Individuals Aged 16-30

Participation Rate for Individuals Age 16	0.209
Participation Rate for Individuals Age 17	0.253
Participation Rate for Individuals Age 18	0.392
Participation Rate for Individuals Age 19	0.480
Participation Rate for Individuals Age 20	0.593
Participation Rate for Individuals Age 21	0.672
Participation Rate for Individuals Age 22	0.671
Participation Rate for Individuals Age 23	0.803
Participation Rate for Individuals Age 24	0.801
Participation Rate for Individuals Age 25	0.833
Participation Rate for Individuals Age 26	0.890
Participation Rate for Individuals Age 27	0.844
Participation Rate for Individuals Age 28	0.864
Participation Rate for Individuals Age 29	0.853

**Table 3**  
**Model Estimates**

	Without Renegotiation	With Renegotiation
$m$	5.15	5.15
$\lambda_n$	0.505 (0.006)	0.505 (0.055)
$\lambda_e$	0.309 (0.004)	0.156 (0.005)
$\eta$	0.013 (0.001)	0.010 (0.001)
$\mu_\theta$	1.615 (0.048)	2.621 (0.058)
$\sigma_\theta$	0.846 (0.026)	0.175 (0.059)
$\alpha$	0.380 (0.010)	0.320 (0.012)
Distance Function Value	743.77	1216.34

Note: Instantaneous value of search,  $b$ , set equal to -2 in both estimations

**Table 4**  
**Sample and Simulated Moments**  
Individuals Aged 16-30

	Sample	Model Without Renegotiation	Model With Renegotiat
Proportion unemployed	0.043	0.046	0.017
Mean wage	\$9.47	\$9.01	\$8.67
Standard deviation of wages	\$4.67	\$3.48	\$2.99
Proportion of minimum wage earners	0.033	0.031	0.024
Proportion earning wage greater than \$20	0.043	0.000	0.000
Proportion employed in Feb 1998 that exit into unemployment within 12 months	0.064	0.084	0.071
Proportion employed in Feb 1998 with at least one job change within 12 months	0.291	0.141	0.147
Mean wage at initial job among individuals employed in Feb 1998 who have job-to-job transition within 12 months	\$2.34	\$1.01	\$1.48
Mean wage change among individuals employed in Feb 1998 who have job-to-job transition within 12 months	\$0.25	\$0.614	\$0.54
Standard deviation of wage change among individuals employed in Feb 1998 who have job-to-job transition within 12 months	\$2.31	\$1.60	\$1.45
Mean wage at first job for those unemployed in Feb 1998 who become employed within 12 months	\$1.18	\$0.83	\$0.54
Standard deviation of wages at first job for those unemployed in Feb 1998 who become employed within 12 months	\$3.18	\$2.06	\$1.40

Note: Wage moments conditional on transition within 12 months defined using size of full sample.

**Table 5: Point Estimates of Demand-Side Parameters**

	Without Renegotiation	With Renegotiation
Vacancy Rate		
Unemployed	0.009	0.004
Employed	0.092	0.024
Flow Vacancy Cost		
Unemployed	\$92.45	\$137.85
Employed	\$54.26	\$30.75

**Table 6**  
**Policy Experiments: Exogenous Contact Rates**

	Unemployed Workers	Employed Workers	Firms with Filled Vacancies	Out of the Labor Force Workers	Aggregate Labor Market
<b><u>Without Renegotiation</u></b>					
Optimal $m$	\$12.05	\$17.50	\$17.55	\$12.05	\$12.05
Percent Change With Respect to Baseline ( $m = \$5.15$ )	0.017	0.050	0.348	0.184	0.017
Unemployment Rate (Baseline = 0.051)	0.140	0.246	0.303	0.140	0.140
<b><u>With Renegotiation</u></b>					
Optimal $m$	\$14.05	\$14.30	\$5.15	\$14.05	\$14.05
Percent Change With Respect to Baseline ( $m = \$5.15$ )	0.048	0.040	0.00	0.247	0.035
Unemployment Rate (Baseline = 0.022)	0.042	0.047	0.058	0.042	0.042

**Table 7**  
**Policy Experiments: Endogenous Contact Rates**

	Unemployed Workers	Employed Workers	Firms with Filled Vacancies	Out of the Labor Force Workers	Aggregate Labor Market
<b><u>Without Renegotiation</u></b>					
Optimal $m$	\$4.70	\$4.70	\$17.55	\$4.70	\$4.70
Percent Change With Respect to Baseline ( $m = \$5.15$ )	0.006	0.005	0.338	0.066	0.005
Unemployment Rate (Baseline = 0.051)	0.046	0.046	0.404	0.046	0.046
<b><u>With Renegotiation</u></b>					
Optimal $m$	\$13.30	\$13.30	\$17.85	\$13.30	\$13.30
Percent Change With Respect to Baseline ( $m = \$5.15$ )	0.046	0.037	0.224	0.240	0.033
Unemployment Rate (Baseline = 0.022)	0.035	0.035	0.809	0.035	0.035



Figure 1  
Sample Wage Changes  
for Individuals Making a Job-to-Job Transition

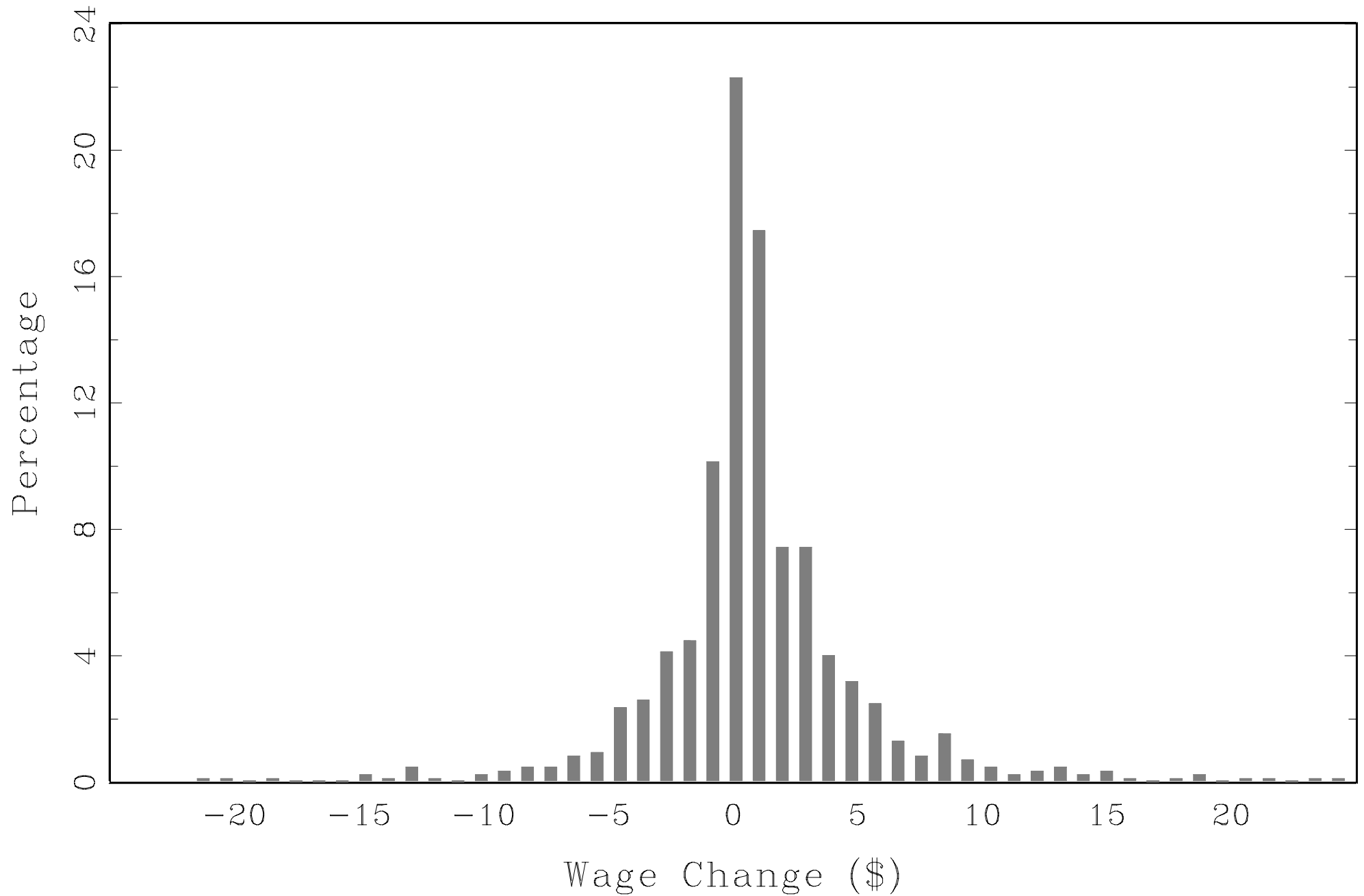


Figure 2.a  
Estimated Wage Function  
(Without Renegotiation)

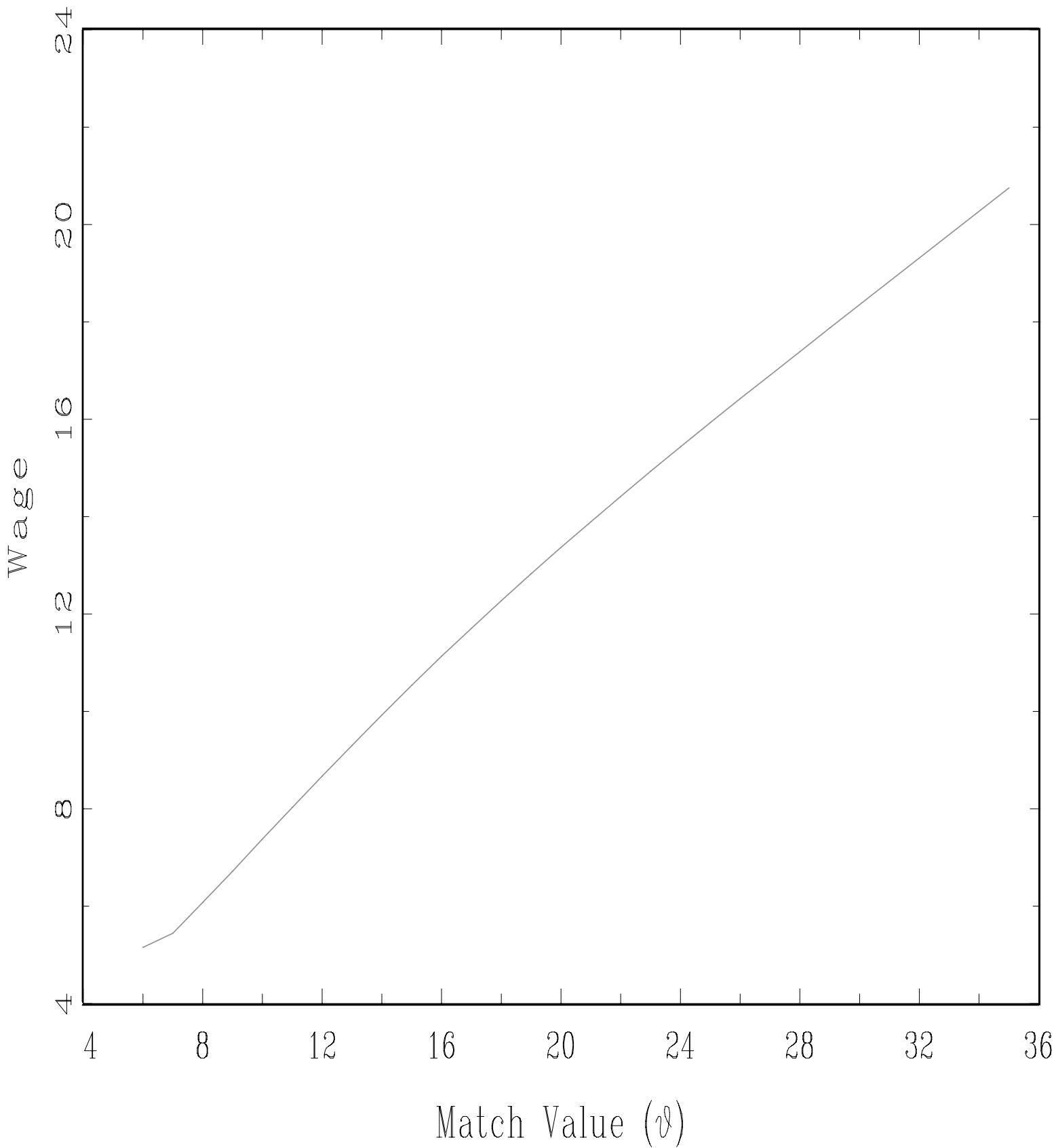


Figure 2.b  
Estimated Wage Function  
(With Renegotiation)

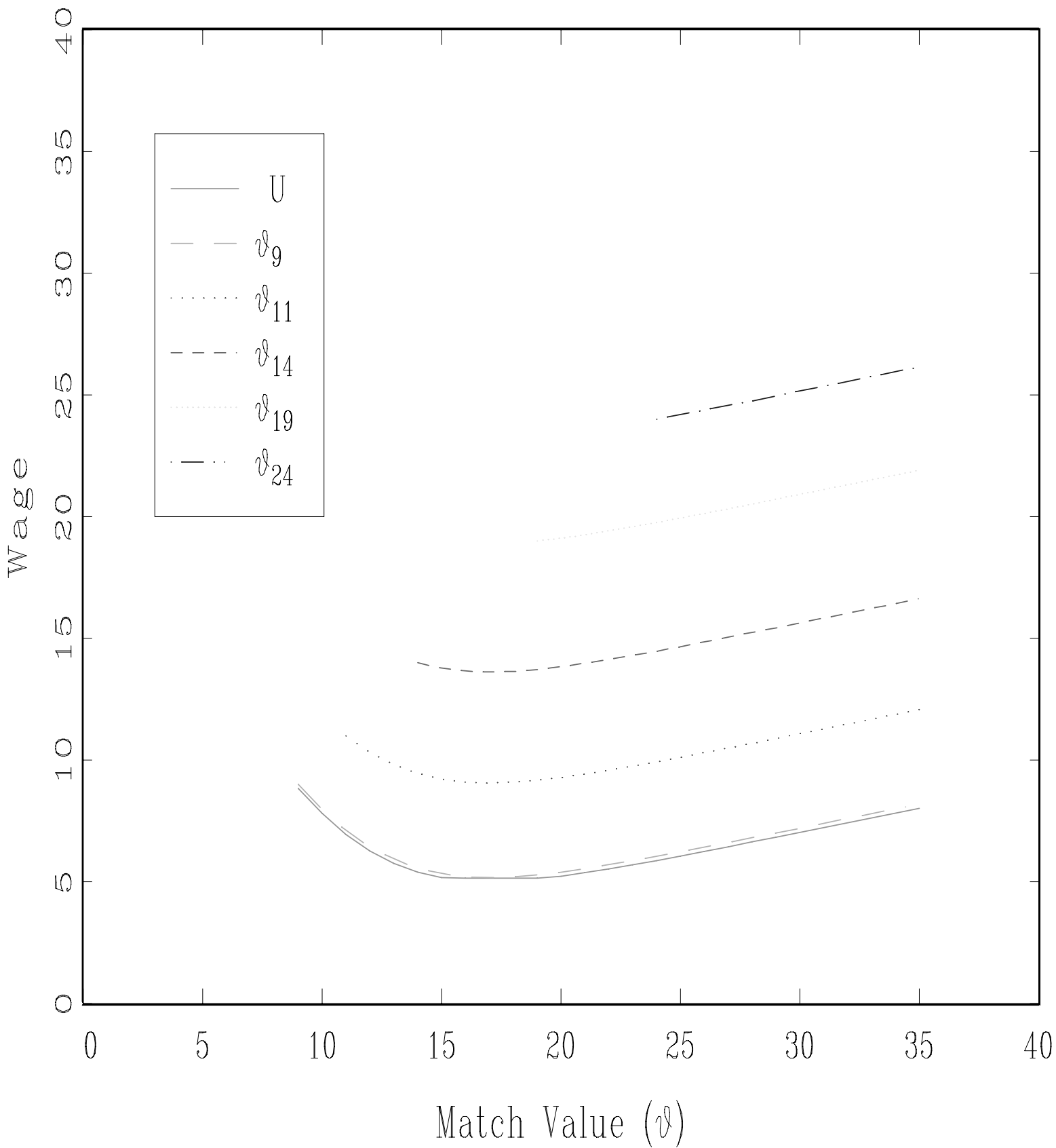


Figure 3.a  
Average Wage over First Employment Spell  
(No Wage Renegotiation Allowed)

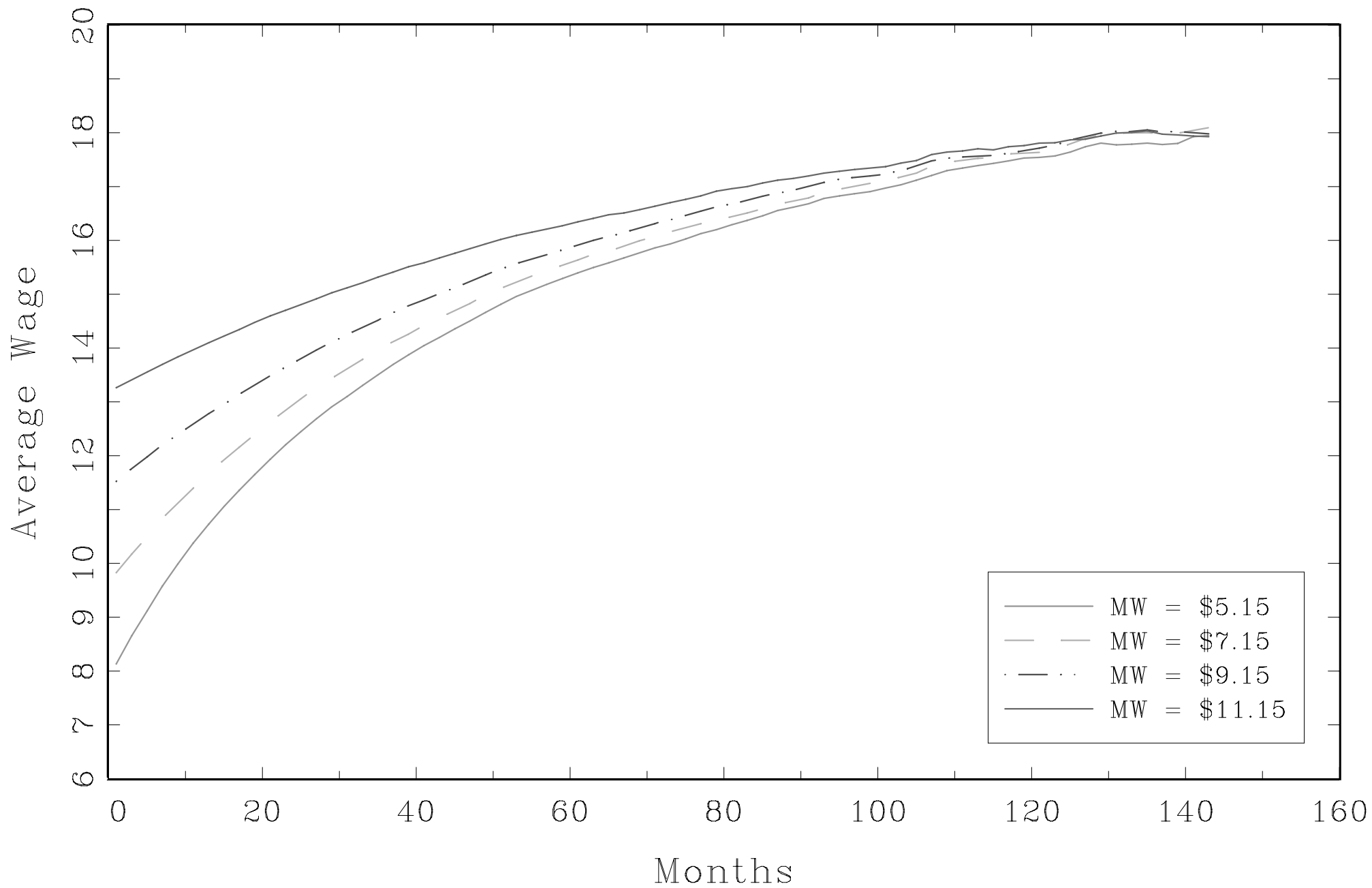


Figure 3.b  
Average Wage over First Employment Spell  
(Wage Renegotiation Allowed)

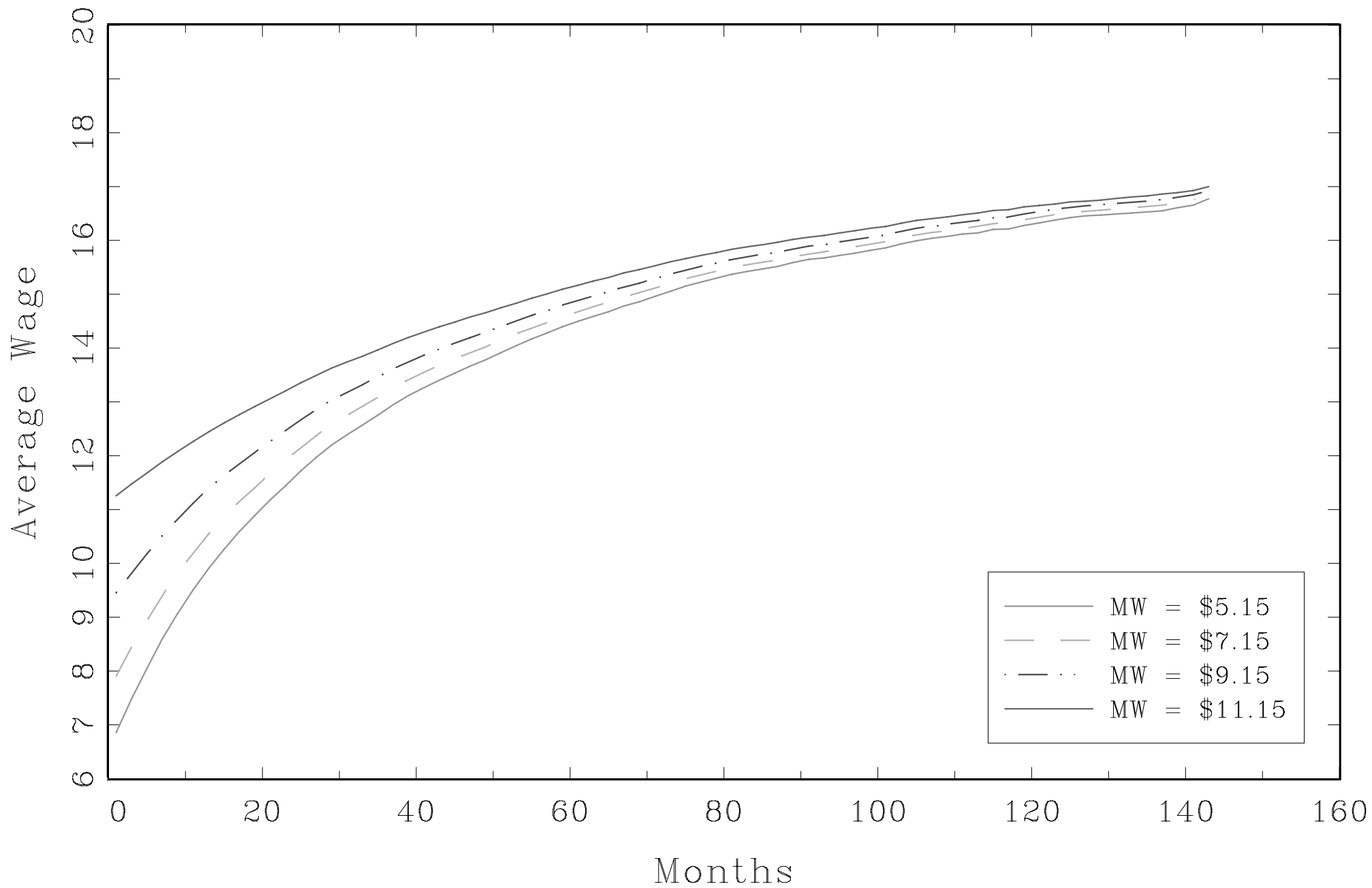


Figure 4.a  
Average Wage over Labor Market Career  
(No Wage Renegotiation Allowed)

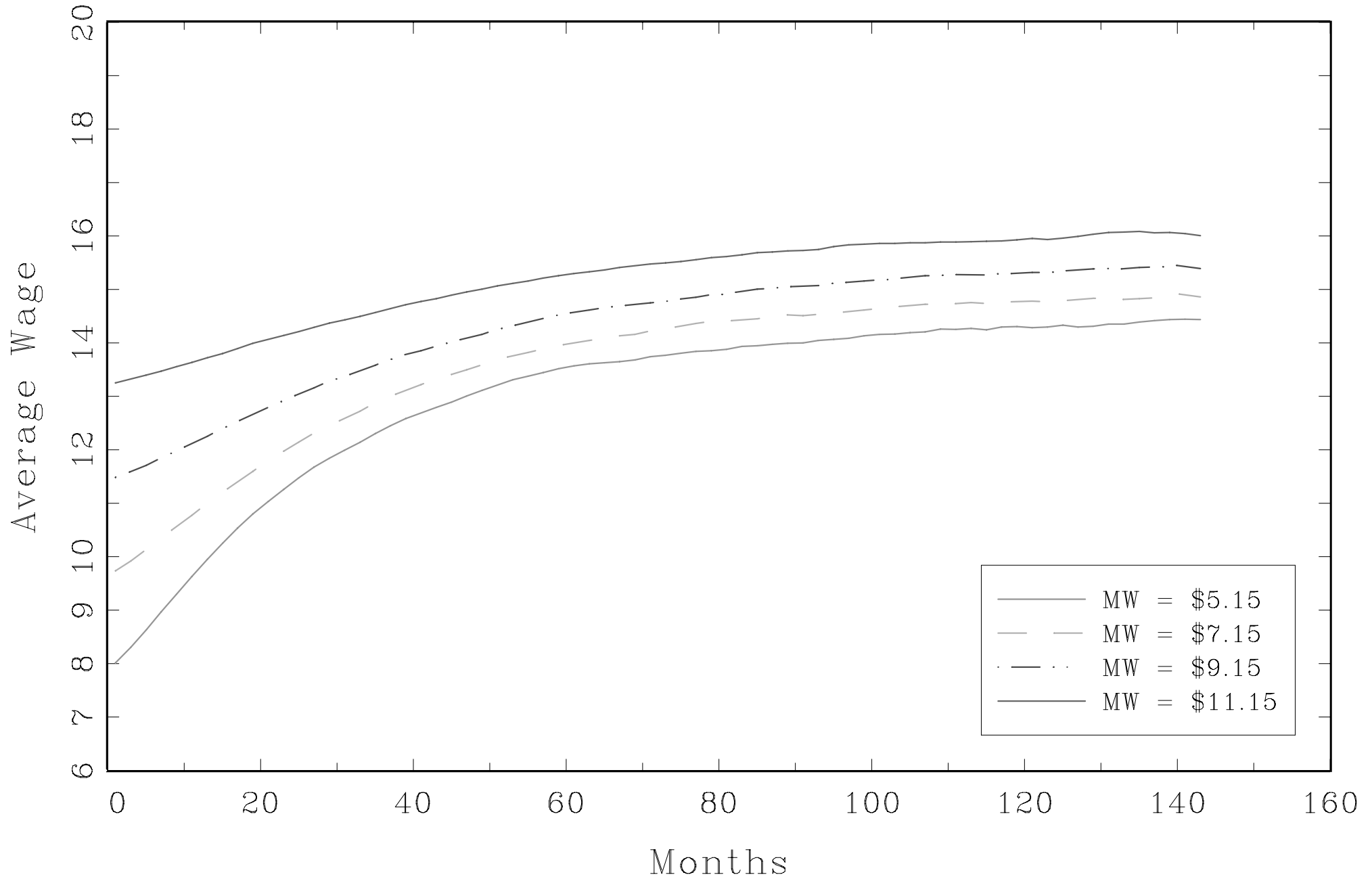


Figure 4.b  
Average Wage over Labor Market Career  
(Wage Renegotiation Allowed)

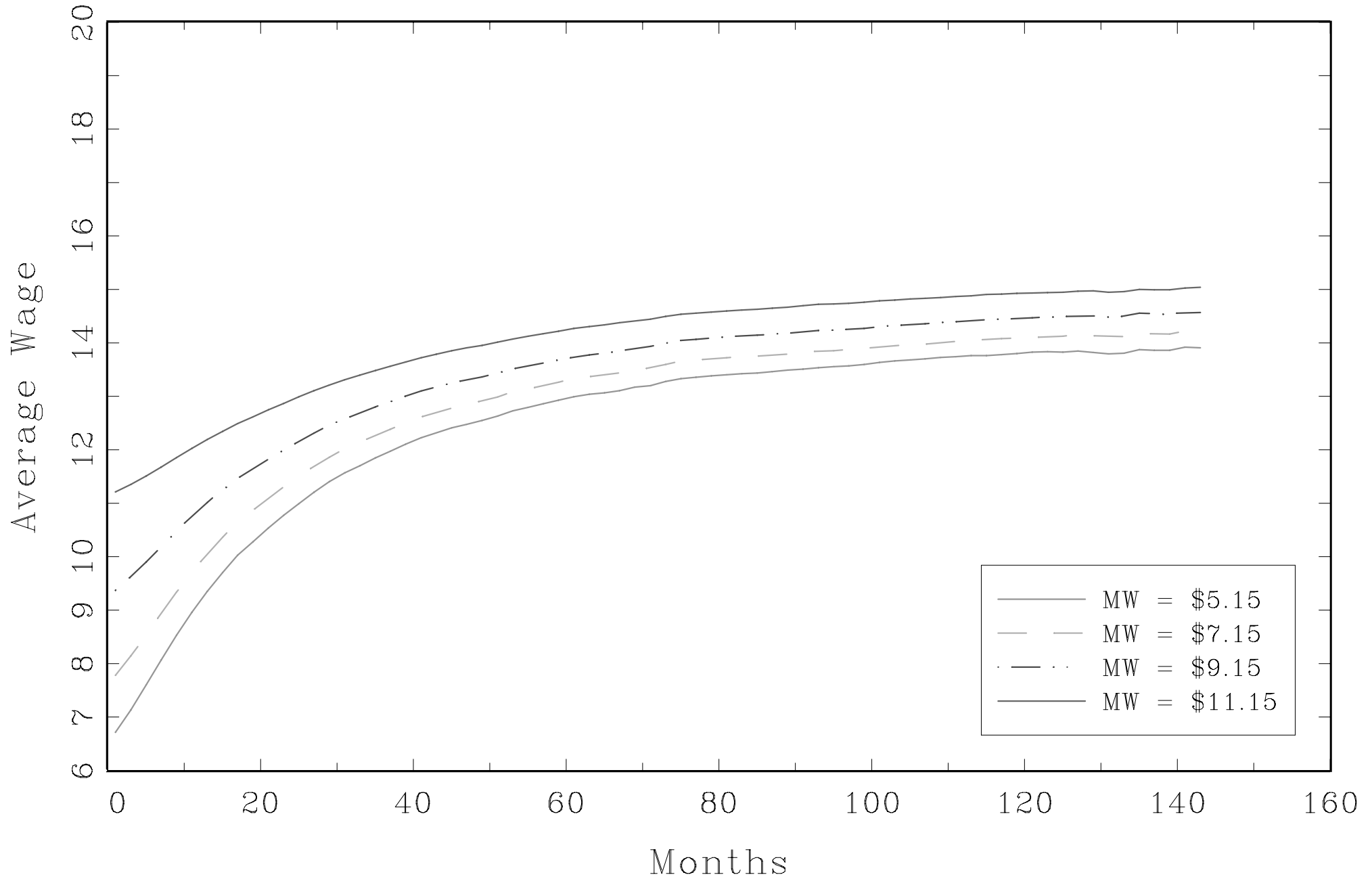


Figure 5a  
Steady State Proportion of Unemployed  
(Exogenous Contact Rates)

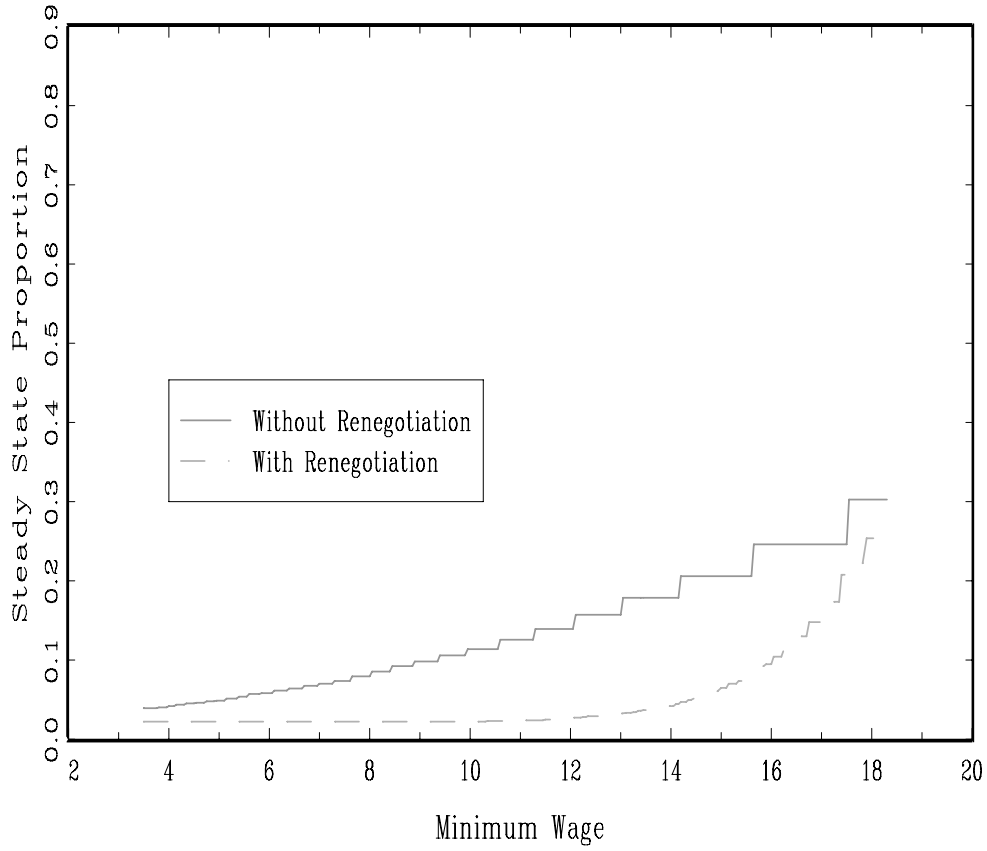


Figure 5b  
Steady State Proportion of Unemployed  
(Endogenous Contact Rates)

