Do Economists and Psychologists Measure Ability In The Same Way? Comparing Individual-Specific Ability Measures Obtained From a Nonlinear Lifecycle Earnings Model to Psychology-Based Cognitive Test Scores

Solomon W. Polachek
Department of Economics
State University of New York at Binghamton
polachek@binghamton.edu

Rewat Thamma-Apiroam
Faculty of Economics
Kasetsart University
Thailand
fecoret@hotmail.com

Tirthatanmoy Das
Department of Economics
State University of New York at Binghamton
tdas1@binghamton.edu

October 2010

VERY PRELIMINARY: NOT TO BE CITED WITHOUT PERMISSION

ABSTRACT

Economists rely on psychology-based IQ and achievement test scores to assess ability. Yet human capital models of lifetime earnings propagation entail human capital production function parameters that incorporate ability parameters which depict an individual's capacity to create new human capital. This paper makes use of human capital theory to derive a highly nonlinear, but empirically tractable, earnings function which when estimated yields a parameter representing cognitive ability. Given that the National Longitudinal Survey (NLS-Y) now has up to 22 years of data on each individual respondent, we estimate these earnings functions for each individual to extract individual-specific estimates of ability. We then compare our estimated ability parameters with independently obtained achievement test scores for these same individuals in the NLS-Y data. We find a significant positive correlation between our measures and the independently obtained psychologically-based cognitive test scores. However, unlike the psychology-based measures, our ability estimates yield a less significant relationship with race, thereby implying the possibility of greater racial biases in the psychologically-based measures than our economics-based measures.

Introduction

Ability reflects an individual's capacity to perform proficiently in intellectual pursuits. Two strands of research measure ability. The first, and most common, is based in psychology. This strand originated in 1904 when the French government commissioned Alfred Binet to develop tests, now known as the IQ (intelligence quotient), to measure various aspects of cognitive skills. The second research strand is based in economics, but is less well known. In this approach one uses lifecycle human capital theory to derive an estimable nonlinear earnings function with parameters that denote ability.

Currently practitioners including social science research scholars, as well as economists, generally ignore the economics-based approach when they measure ability, mostly because the economics-based technique is difficult to implement. In order to identify ability parameters for particular individuals, the technique requires a highly nonlinear specification incorporating sufficiently long panel data, which at least in the past were not readily available. Thus, instead of doing their own estimation, economists relied on psychologists' IQ and achievement tests to identify ability. Typically social scientists used these measures as independent variables in simple earnings function regressions.

Earnings functions that incorporate IQ-type measures as independent variables estimate the *effects* of ability, rather than measure ability itself. The same can be said for fixed-effects regressions which are designed to net out individual specific heterogeneity, including ability. However, with the advent of speedier computers, better optimization programs, and longer panels, measures of ability based on human capital theory can now more easily be retrieved simply as parameters in a nonlinear earnings function. With sufficiently long panels one can estimate ability for specific individuals. Alternatively, one can aggregate the data to estimate ability for selected groups, such as all those of a given level of schooling or all those members of a particular racial group. In either case, one can obtain estimates of ability based on economic principles of optimization which result when individuals accumulate human capital to maximize the present value of lifetime earnings, rather than be based on psychological principles underlying IQ tests.

Obtaining ability measures using economics-based models is important for at least three reasons. First it enables one to compare these new economics based ability measures to previously obtained psychology based measures. If they differ, one can question the validity of one or the other. However, credence is enhanced regarding the reliability of each if both correlate with each other.

Second, and perhaps more controversial, but even more important, one can test whether traditional IQ type tests are racially biased, as many allege is the case. This is a real possibility given that cultures of minority groups are not taken into account when formulating questions for psychologically-based tests. As will be shown, the economics-based ability parameters we estimate are obtained from a model conceptually independent of race. As such, notions of race do not enter the structural definition of our human capital based ability measure, though they do enter the estimation process if racial discrimination affects earnings. Thus at least from a theoretical standpoint, our ability measures are conceptually race neutral. From both pedagogical and policy perspectives this race neutrality is important because finding no racial differences in economics-based ability measures would be strong evidence that IQ type measures are indeed racially biased, since those measures do differ by race, whereas our economics-based measures would not. On the other hand, should our measures differ by race in the same way IQ measures differ by race, one can make either the case that neither measure is racially biased, or alternatively one can make the case that wage discrimination is racially biased in exactly the same way that cultural biases have inculcated the psychologically-based ability measures. In either case, the academic profession will better understand notions of ability and how ability differs by race.

Finally, third, one other important aspect of this paper is how it emphasizes individual heterogeneity. Currently literally hundreds of articles "account" for unobserved individual heterogeneity by "netting out" person-specific effects. Studies that assume person-specific parameters are of three genres. First, random coefficient models and extensions including the correlated random coefficients model (Heckman, et al. 2009) assume individual parameters vary across individuals in accord with a particular distribution but cannot identify each individual's actual parameter values. Nor are they usually certain of the underlying parameter distribution, which are usually assumed to be normal. Second, more sophisticated panel data models (Polachek and Kim, 1994; and Pesaram, 2006) adjust for person-specific slope parameters, but these are often limited to one parameter besides the intercept. Finally non-parametric approaches get at heterogeneity essentially by grouping individuals according to related (neighboring) measured characteristics within optimal "band widths" (Racine, 2004). Rather than individual-specific parameters, they obtain "group-specific parameters." Our approach makes use of long enough panels to obtain individual specific measures for *each* coefficient in the earnings function model we adopt, including parameters specifying ability.

Of course a number of assumptions underlie the economics-based approach. First, the approach assumes individuals plan their human capital investment strategy based on expectations that they seek to work each year of their working life. This is why we concentrate on males who have continuous work histories. Second, the approach assumes a relatively simple human capital production function. We assume individuals use their time and existing human capital to create new human capital, but we ignore other inputs such as books and computers which can also be used to create additional human capital. However, in our model high ability people can create a given amount of human capital with smaller time inputs. Third, the approach assumes labor markets reward individuals based on their existing stock of human capital, and that neither incomplete information nor incentive pay governs worker earnings. Fourth, we assume all human capital production function parameters remain constant throughout each person's life.

Using the Lifecycle Human Capital Model to Estimate Ability

Most empirical studies adopt single equation log-linear Mincer earnings functions to parameterize earnings. The beauty of estimating simple Mincer earnings functions is its computational ease. Assuming schooling and experience are exogenous, Mincer earnings functions are easily estimated by OLS. Although more recent analyses question whether such OLS estimation procedures are free of econometric biases (Heckman, Lochner and

Todd, 2006)[1], in reality the Mincer model is a simplification based on Taylor approximations of a more complex function. Underlying the Mincer model is a lifecycle earnings generating process that yields a highly nonlinear earnings function. From this nonlinear function one is able to identify cognitive ability as an estimable parameter. We define cognitive ability as the ease which an individual can create new human capital from old human capital. This is the individual's human capital production function output elasticity. Cognitive ability is distinct from one's knowledge base, which in our framework is depicted as total factor productivity in the individual's human capital production function.

The derivation entails the typical economics based maximization paradigm. It assumes each economic entity (in our case an individual) invests in human capital to maximize his or her present value of expected lifetime income. Based on this optimization process, one can derive optimal human capital investment, optimal human capital stock, and optimal earnings over a person's lifetime. In the model one's earnings over the lifecycle are directly proportional to one's human capital stock. Each year one's human capital stock is augmented by the amount of new human capital one creates through schooling and on-the-job training, and one's human capital stock is diminished by the amount human capital depreciates. Creating new human capital entails using time and existing human capital to produce new human capital, given one's ability. The greater one's ability the more human capital one can produce, and the more rapidly one can increase his or her earnings power from year-to-year (Ben-Porath, 1967). The result is a nonlinear earnings function with a parameter reflecting cognitive ability.[2]

Whereas not everyone believes in the human capital approach as the basis for one's earnings, the model is surprisingly robust compared to other models in explaining lifecycle earnings patterns. For example, screening models explain why *education* enhances earnings; occupational segregation models explain why women earn less;

[1] There are also some conceptual biases regarding how to interpret such parameters as the schooling and experience coefficients which many take to measure rates of return to school and experience.
[2] Distinct from cognitive ability which we take as one's ability to create new human capital from old is one's stock of knowledge. In this paper, we do not identify this innate stock of knowledge which is reflected by total factor productivity in the individual's human capital production function.

efficiency wage models explain certain wage premiums; and productivity enhancing contract models explain upward sloping (though not necessarily concave) earnings profiles; but none of these theories deal simultaneously with as many aspects of earnings as does the human capital model. Neither do these other models allow one to identify ability from estimated parameters. For this reason we adopt the human capital model from which to approach the problem of measuring ability.

The Ben Porath Model

The Ben-Porath model assumes individuals invest in themselves to maximize expected lifetime income. (Incorporating labor supply enables one to maximize utility, but doing so requires a number of additional assumptions to identify key earnings function parameters. Investment is governed by a production function in which one combines his or her own time and ability along with past human capital investments to create new human capital. At the margin, one equates the marginal cost and marginal gains. The marginal cost of each unit of investment is essentially the cost of the goods as well as the foregone earnings of the time needed to produce a marginal unit of human capital. (as does Mincer we assume the goods components is offset by earnings during the investment process) The marginal gain is the present value of each additional unit of human capital. Ben-Porath's innovation was to realize that the finite life constraint implies the marginal gain declines monotonically over the lifecycle (at least for individuals that work continuously throughout their lives). The equilibrium implies a human capital stock that rises over the lifecycle at a diminishing rate. This yields the commonly observed concave earnings profile.

The closed-form solution to Ben-Porath's earnings function is highly nonlinear. At the time of its discovery in 1967 few computers were fast enough to easily estimate the parameters. However, shortly thereafter, Haley (1976) was able to estimate a version, but even he simplified the estimation because not all parameters were readily identifiable. Given these computational difficulties, most scholars relied on a linearization. This linear-in-the-parameters specification has become known as the Mincer log-linear earnings function, or simply the Mincer earnings function, for short. One problem is that Mincer's simplification does not allow one to identify ability.

Given the advent of faster computers and longer panels of individual data, we feel now is a good time to reexamine Haley's approach. Further, as mentioned above, given sufficiently long panels for particular individuals, the approach enables one to compute ability parameters person-by-person. Obtaining person-specific ability measures addresses one aspect of unobserved heterogeneity, a relatively important issue in micro-based econometric research.

The Haley Model

The human capital model assumes an individual's potential earnings Y_t^* (what a person *could* earn) in time period *t* are directly related to his or her human capital stock E_t. As such,

$$Y_t^* = RE_t \tag{1}$$

where for simplicity *R* is assumed to be the constant rental rate per unit of human capital.[3] Human capital stock is accumulated over one's lifetime by prudent investments in oneself via schooling and on-the-job training (as well as health, job search and other earnings augmenting types of human capital). The rate of change in human capital stock, $\dot{E}_t$, is expressed as the amount of human capital produced, q_t, minus depreciation so that

$$\dot{E}_t = q_t - \delta E_t, \tag{2}$$

where δ is the constant rate of stock depreciation. For simplicity, we assume individuals create human capital using a Cobb-Douglas production function such that

$$q_t = \beta K_t^b \tag{3}$$

[3] Polachek (1981) assumes the rental rate can vary by type of human capital. Polachek and Horvath (1977) assumes the rental rate can vary over time. However, relaxing the assumption about a constant rental rate in these two ways is unnecessary for this application. Nor is it a common practice in the human capital literature. Further, we normalize the value of R to one since its value is dependent upon how currency is nominalized, and hence impossible to identify empirically.

where K_t is the fraction of human capital stock reinvested in time period *t* and parameters β and $b \in [0, 1]$ are production function parameters.[4] The parameter β is the "technology" parameter. It represents "total factor productivity," the amount of human capital not created by other inputs. As such, it is related to one's basic knowledge. The parameter *b* reflects the rate at which current human capital stock is transformed to new human capital. It reflects how one acquires new knowledge from old, and as such reflects how quickly one learns. We denote *b* to depict one's native ability. As such, because it measures how well one transforms past knowledge into new knowledge, it can be construed as related to the intellectual ability, perhaps what should be measured by psychological IQ type tests since it measures how well one transforms past knowledge into new knowledge. In reality IQ and aptitude tests measure a combination of β and *b*.

The individual's objective is to maximize discounted disposable earnings, Y_t, over the working life cycle.[5] This goal is achieved by choosing the amount of human capital K_t to reinvest each year in order to maximize the present value of lifetime earnings

$$\underset{K_t}{Max} \quad J = \int_0^N e^{-rt} Y_t dt \tag{4}$$

where J is the total discounted disposable earnings over the working life cycle, r is the personal discount rate and N is the number of years one works (assumed known with certainly). Disposable earnings are

$$Y_t = R[E_t - K_t] \tag{5}$$

Maximization of (4) subject to equations (2) and (3) can be done by maximizing the following Hamiltonian.

$$H(K, E, \lambda, t) = e^{-rt} R[E_t - K_t] + \lambda_t[\beta K_t^b \ - \delta E_t] \tag{6}$$

with constraints $E_t - K_t \geq 0$, which means one cannot invest using more human capital than one currently has (i.e., no borrowing); and the transversality condition $\lambda_N = 0$,

---

[4] We assume no other inputs other than one's own human capital. Less simplified production functions could entail individuals employing "goods" inputs such as teachers, books, study time. For example, Ben-Porath (1967) assumes $q_t = \beta K_t^{b_1} D_t^{b_2}$ where D_t equals other inputs. Later empirical analysis precludes taking account of these factors of production because no data are available for these inputs. Thus we adopt the above more simplified human capital production function used by Haley (1976).

[5] As already mentioned, we abstract from labor supply.

which indicates a zero (labor market) gain from human capital investing in one's final year at work. The solution involves three phases: (1) Specialization in human capital investment when $K_t = E_t$ which can be defined as being in school since one is spending full-time investing; (2) "Working" which defines the time period when one both works and invests; and (3) Retirement when one ceases investing completely. We are concerned with Phase 2 since this is the only time period one can observe earnings. In school one plows back all one's earnings potential into more human capital investment and hence has no net earnings. Likewise during retirement one does not work so there are no earnings then either.

This maximization yields a nonlinear (in the parameters) earnings function[6]

$$Y_t = A_0 e^{\delta(t^*-t)} + A_1[1 - e^{\delta(t^8-t)}] - A_2[1 - e^{(r+\delta)(t-N)}]^{1/(1-b)} \quad (7)$$

where

$$A_0 = R\beta^{1/(1-b)}\left[\frac{1}{\delta} + \left(\frac{E_0^{1-b}}{\beta} - \frac{1}{\delta}\right)e^{\delta(b-1)t^*}\right]^{1/(1-b)}$$

$$A_1 = R\beta^{1/(1-b)}\left[\frac{b}{r+\delta}\right]^{b/(1-b)}\frac{1}{\delta}$$

$$A_2 = R\beta^{1/(1-b)}\left[\frac{b}{r+\delta}\right]^{1/(1-b)}$$

and where t^* is the age at which one graduates from school (i.e., the age when Phase 1 ends) and N is the anticipated retirement age which we take as 65, a reasonable assumption for this cohort. Finally, given measurement error and other unobservable factors, one need add a time varying error term ε_t for each individual

Haley estimates a version of (7) using income by age data aggregated from the 1956, 1958, 1961, 1964, and 1966 CPS surveys. His estimates can be construed as population averages. However, by employing sufficiently long panel data, equation (7)

---

[6] Appendix A contains the derivation. Note this differs slightly from the Haley specification because in our derivation we assume a one-term Taylor expansion whereas Haley uses a two-term Taylor expansion. Our approach yields a slightly simpler earnings function.

can be estimated person-by-person. To do so one can utilize nonlinear estimation techniques along with data on experience and earnings for each individual.

Equation (7) contains six parameters: R, β, b, r, δ, and E_0. The parameters $r, \delta,$ and b all have no dimension. The parameters r and δ are percents. The parameter b is the output elasticity in the human capital production function (3). It reflects returns to scale of human capital. It also can be construed as an ability parameter since it measures the productivity of old human capital in creating new human capital. These parameters are technically observable. The parameters $R, \beta,$ and E_0 are nominated in terms of units of human capital stock. R and E_0 are dimensioned as dollars per unit of human capital. β is dimensioned as units of human capital to the *1-b* power. However, since only dollar earnings are observable, whereas units of human capital are not, we follow Haley's procedure to estimate the composite term $R\beta^{1/(1-b)}$ since R is in dollars/capital per unit time and β is $(capital)^{1-b}$, making $R\beta^{1/(1-b)}$ dollars per unit of time, for which we have earnings data. Similarly E_0^{1-b} and β are in units of human capital, but combining the two yields $E_0^{1-b}/\beta,$ which is dimensionless. Thus we also treat E_0^{1-b}/β as a single parameter. Finally, to conserve degrees of freedom and quicken convergence we assume a uniform human capital depreciation rate. Based on Haley (1976), we assume this to be 0.01. We also adopt Haley's strategy to assume a value of 0.5 for E_0^{1-b}/β. As a result of these identification restrictions we end up estimating three parameters: $\hat{\omega}_i = R\beta^{\frac{1}{1-b_i}}$, $\hat{b}_i = b_i$, and $\hat{r}_i = r_i$, of which the latter two give individual-specific estimates of b_i and r_i. We estimate parameters $\hat{\omega}_i, \hat{b}_i,$ **and** $\hat{r}_i$ for each individual using nonlinear least squares for those individuals with at least twelve years of data. Experimentation with alternative depreciation rates did not appreciably change the values or rank-ordering of our estimates.

The Data

Nowadays there are a number of panel micro-data sets containing information on schooling, work experience, and earnings over the lifecycle. However, as far as we know, only the National Longitudinal Survey of Youth 1979 also contains extensive independent psychology-based information on ability. For this reason we utilize the NLSY79 data in order to compare our own individual-specific ability parameters to the independent ability measures based on psychological tests.

As is well known, the NLSY79 is a nationally representative sample of 12,686 young men and women aged 14 to 22 years old when first surveyed in 1979. The surveys have been conducted annually until 1994, and then performed every other year. We utilize the 2006 NLSY79, which contains up to 22 years of data for each respondent. The NLSY79 represents various groups such as men, women, Hispanics, blacks, non-Hispanics and non-blacks, as well as the economically disadvantaged. There are three subgroups comprising the NLSY79. The first is a cross-sectional sample of 6,111 representing non-institutionalized civilian youths living in the United States aged 14-22 in 1979. The second sample is a cross-sectional supplemental containing 5,295 youths designed to oversample civilian Hispanic, black, and economically disadvantaged nonblack/non-Hispanics between 14 and 22 in 1979. The third is a cross-sectional military sample of 1,280 youths that represent the population, aged 17-22 in 1979.[7] Since the NLSY79 data set is collected randomly within each group, there are no sampling weights, but this does not matter for us since we are examining each individual separately rather than trying to use each individual's data to build a nationwide mean. To estimate (7) we use data on hourly earnings,[8] age, and years of schooling. From these we compute the experience level when schooling stopped (t^*) and experience (t). Because our earnings function specification is designed for those who work continuously, we concentrate only on the males because females are more likely to have discontinuous labor force participation, making the measurement of experience (t) more difficult and resulting in a highly more nonlinear earnings equation (Polachek, 1975). In addition,

[7] The data and further explanations can be explored from the website

[8] We also estimated (7) using annual earnings data for full-time workers and found very little difference in the results.

current human capital acquisition is affected by future intermittent participation. Not being able to predict when and how long a woman will drop out precludes estimating female earnings functions, at least for the purposes of this paper. Further, we use data only on individuals that have completed school because those working while in school (or those working with the intention of going back to school) earn less than commensurately schooled individuals who completed their education (Lazear, 1977).

As was already mentioned, for the purposes of this study, the main virtue of the NLSY79 data is the information on ability which was obtained independent of economic and demographic variables. For most respondents this consists of at least one of 28 possible intelligence/aptitude tests. Of these we concentrate on using scores of 16 such tests because we drop individuals with less than twelve years of earnings data which we require to estimate the nonlinear earnings functions discussed above.[9] Detailed descriptions of each ability test is given in Appendix B. As already indicated, we compare these psychology-based ability scores respondent-by-respondent to the individual-specific ability parameters we estimated using (7) above.

Estimation Results

For each person with over 12 years of continuous data, we use nonlinear least-squares to evaluate (7). As discussed above, we estimate three crucial parameters. They are the ability parameter (*b*), the discount rate (*r*), and the composite parameter $\hat{\omega}_i = R\beta^{\frac{1}{1-b_i}}$. Mean values across all individuals are given in Table 1 along with mean values for each of the achievement test scores contained in the NLS-Y.

[9] The fifteen tests are: the Armed Forces Qualification Test (AFQT), the American College Test (Math), the American College Test (Verbal), the California Test of Mental Maturity, the Cooperative School and College Ability Test, the Differential Aptitude Test, the Henmon-Nelson Test of Mental Maturity, the Kuhlman-Anderson Intelligence Test, the Lorge-Thorndike Intelligence Test, the Otis-Lennon Mental Ability Test, the Preliminary Scholastic Aptitude Test (Math), the Preliminary Scholastic Aptitude Test (Verbal), the Scholastic Aptitude Test (Math), the Scholastic Aptitude Test (Verbal), and the Wechsler Intelligence Test for Children.

Our ultimate objectives are first to determine how the economics-based individual ability parameters (b_i) are correlated with standardized ability test scores, and second, whether our economics-based ability measures are correlated with race the same way as traditional ability measures reported in the NLS.

To see how our economics-based ability measures (b_i) compare to psychologically-based measures we plot out kernal density functions of both our measures and the psychologically based test scores. To do so, we must scale each test score because each has a different measurement range. Our measure (b) is an exponent in a production function. It ranges from 0.06-0.94. SAT scores are coded between 200 and 800, but for our sample vary between 200 and 750 for math and between 200 and 690 for verbal. Each of the other ability tests also has unique scores. To compare the overall distributions each must be scaled to have the same range of values. To do so, we scale each measure (x_i) of test i by $\frac{x_i - L_i}{H_i - L_i}$ where L is the lowest test score value and h is the highest test score value yields a scaling between zero and one. Figure 1 plots out each test's kernel density function for each test that has over 100 observations. For each test we plot both the distribution of test scores as well as the distribution of our ability measure (b). The most similar distributions are for the SAT and the American College Association tests. The California test, the Otis test, and the Lorge test are each more left skewed than our measures.

The AFQT test scores are given in percentiles ranging from 1 to 99. As such, the AFQT distribution should be uniform as it is in Figure 2. For comparability we also scale our ability measure (b) into percentiles, and as expected, it too is uniform as also illustrated in Figure 2. As expected, both distributions are virtually identical.

Figure 3 plots out test scores for whites and Figure 4 for blacks. Similar patterns emerge as was seen in Figure 1.

Next we plot black and white differences for each test as well and as our economics-based ability measure for the same respondents. Generally blacks fare worse than whites since for each test score the black distribution is further to the left then the white distribution. This pattern holds true for our economics-based (b) measures as well, but not to the same extent. Thus we find smaller ability differences by race in our economics-based measure than is observed in the psychology-based measures.

Clearly, given the number of individuals, we cannot present coefficients for each person. Instead we present aggregate estimates for various groups. These are given in Tables 2 for each achievement test, and in Table 3 for each grade level. Table 2 consists of the average values for individuals taking each specific test. Columns 1-3 give the b, r and w parameters. Column 4 presents the achievement test score. Column 5 has the number of individuals taking the specific achievement test, and column 6 the AFQT test score for those individuals who also took the particular achievement test reported in column 4.

A few patterns can be noted in the data. First AFQT scores are lower for blacks than whites. The same is true for the economics-based ability (*b*) measures, but slightly less so. Second, *b* and AFQT are positively correlated. These patterns are also presented in Table 4 which contains specific regressions. Row (1) indicates a lower ability score for blacks in all cases except for column (3) when examining *b* while holding AFQT constant. Third, also noteworthy in Table 2 is that *b* and the achievement test scores are positively correlated as is AFQT and the achievement test scores.

Table 3 contains coefficient averages by years of school. Again the table is divided between whites (left panel) and blacks (right panel). Again, columns 1-3 contain the estimated coefficients. Column 4 gives the number of individuals in each schooling group. Finally column 5 gives the AFQT score of these individuals. Here, too, a few patterns are noteworthy. First *b* values are higher for whites than blacks, but only marginally so. Second, the *b* values rise significantly with years of school. AFQT scores are significantly higher for whites and also rise with years of school. These results are given in Table 5. Of course the results from Tables 4 and 5 are based on weighted regressions using the aggregate b and AFQT values. For that reason we next examine results using individual-specific values rather than aggregates.

As described above, each NLS-Y respondent could have taken any of sixteen IQ-type tests. We divide the population into sixteen groups, each representing all respondents who took that particular test. For each respondent we have the IQ score as well as our own estimate of ability. A positive correlation between b and IQ is consistent with our interpretation that the parameter b measures ability.

Table 6 contains three sets of columns. The first column in each set is the un-weighted correlation and the second the weighted correlation. Weights are the inverse of the standard error of the b coefficient. The first set of columns depicts the correlation for the population of test takers independent of race. The second and third sets depict correlations separated by race. In virtually all cases the computed b ability parameter and the psychologically-based ability tests are positively correlated. This means that despite being computed completely independently (the psychologically tests based on achievement test scores and the b ability based on a nonlinear estimation of individual-specific human capital production functions), the two sets of ability measures correlate well. This positive association adds credence to our ability measures computed based on the lifecycle earnings model.

It is generally well known that IQ-type measures correlate with race (Herrnstein and Murray, 1994). Typically black scores are lower than white scores. Table 7 gives correlations between each IQ-type test scores and race. With the exception of the Henmon scores, all correlations are negative indicating lower black scores (the first column in each double column). Nine are significantly so. One would expect the correlation between race and b to follow a similar pattern if b depicts ability. These latter correlations are given in column 2 of each double column. With the exception of only four cases they mirror the correlations of the traditional ability measures. Further, the correlation between b and race is positive for the respondents taking the Henmon test.

Based on Tables 4-7 the *b* coefficients behave comparably to the traditional ability measures. Both are positively correlated (Tables 2-6) and both exhibit lower values for blacks than whites, though b somewhat less than traditional ability measures (Tables 2-6), though slightly less so for *b* than the other ability measures.

Conclusion

Utilizing psychology-based test scores has the advantage of measuring ability early in one's lifetime. On the other hand, there is controversy regarding how well test scores really reflect cognitive ability. Further, there is controversy regarding race differences in these measures.

Based on an entirely different paradigm, economists utilize concepts of ability in a number of earnings propagation models. Perhaps the most prominent of the models employs ability to define an individual's proclivity to create human capital. In these models individuals create earnings power by producing human capital. At least some of the parameters underlying an individual's human capital production function reflect ability.

Lifecycle models of earnings propagation use these human capital production functions to generate nonlinear earnings functions some of the parameters of which denote an individual's ability to create human capital. These ability parameters can be taken to constitute cognitive ability because these ability parameters indicate an individual’s effectiveness in creating new human capital from old.

The purpose of this paper is to identify such an ability parameter obtainable from a nonlinear lifecycle earnings function. The parameter we identify (*b*) constitutes a (human capital) output elasticity of one’s own time in creating new human capital based on the individual’s “technology,” or in other words an individual’s ability to create new human capital. We do so for each individual (having sufficient earnings information) by utilizing panel data from the National Longitudinal Survey of Youth. Not only does this dataset contain sufficient earnings data but it also contains independent measures of ability from a wide range of psychologically-based achievement tests. As such, we are able to compare our economics-based ability measures with the psychologically based measures contained in the data.

Comparing both measures is important for a number of reasons. First, it serves as an independent check of the underlying paradigms upon which each is based. Second, it enables one to gain some insight into the question of possible racial biases inherent in psychologically-based tests, as is often alleged. Third, it serves as an example of how one can use newer and longer panels to measure aspects of the individual heterogeneity.

The results indicate an uncanny parallel between our economics-based ability measures and psychology-based measures from the data's various achievement test scores. For example, we find both ability measures to be higher for those greater levels of schooling. Second, we find both measures to be positively correlated. Third, we find both to indicate higher levels of measured ability for whites compared to blacks, though the correlation is weaker for our measure than for the standardized tests. From this we infer the possibility of greater racial biases in the psychologically-based measures than our economics-based measures.

Of course, employing an economics-based model is not a panacea for measuring ability. Even if the economics-based approach provides a viable alternative to the psychologically based achievement tests, it is not informative early in one's life because it requires earnings data for a period of time long after one terminates school. Further, both discrimination in the availability of high quality schooling, as well as discrimination in the labor market itself can cause racial biases in estimating an ability parameter using earnings data. In this case racial discrimination in the labor market manifests itself in a similar way to cultural biases which might have inculcated psychologically-based models.

Technical simplifications could also mar interpretation of our results. Underlying our approach are the typical assumptions incorporated in lifecycle models. Obviously, our results may be suspect if earnings are determined by other frameworks such as incentive contracts or deferred compensation schemes. In addition, for computational simplicity, we utilize a relatively simple human capital production function, which in our case only has one ability measure. We envision more complicated versions incorporating a human capital production function with both knowledge-based as well as cognitive-based ability. These latter models yield more complex earnings functions than even the ones we already use. On the other hand, we feel strongly that our results are not simply verifying the well-known fact that high ability people simply earn more. Our ability measures are unrelated to earnings level. Instead, they arise from the curvature of the earnings profile.

Our results are promising enough to warrant pursuing the approach further. For example, identifying various types of ability might enable one to gain insights into occupational choice decisions including answering questions relating to gender differences in one's inclination to go into scientific professions.

Figure 1

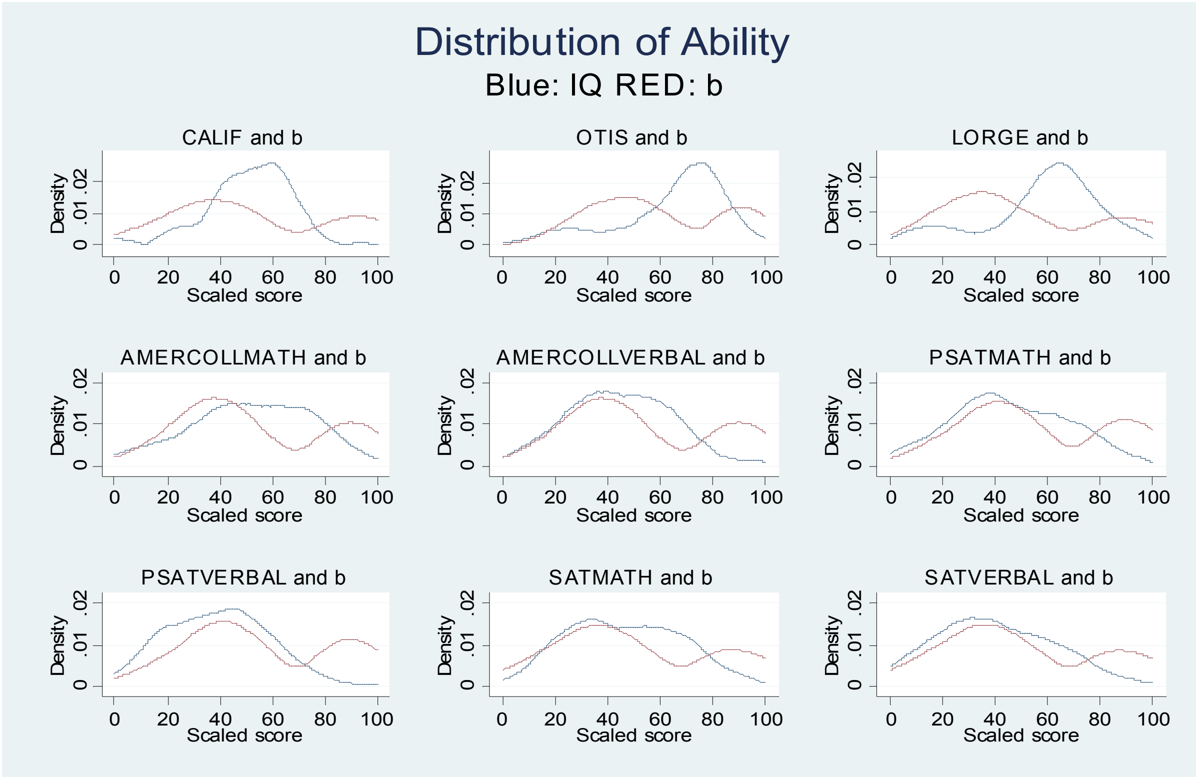


Figure 2
Distribution of AFQT and b Measured as a Percentile

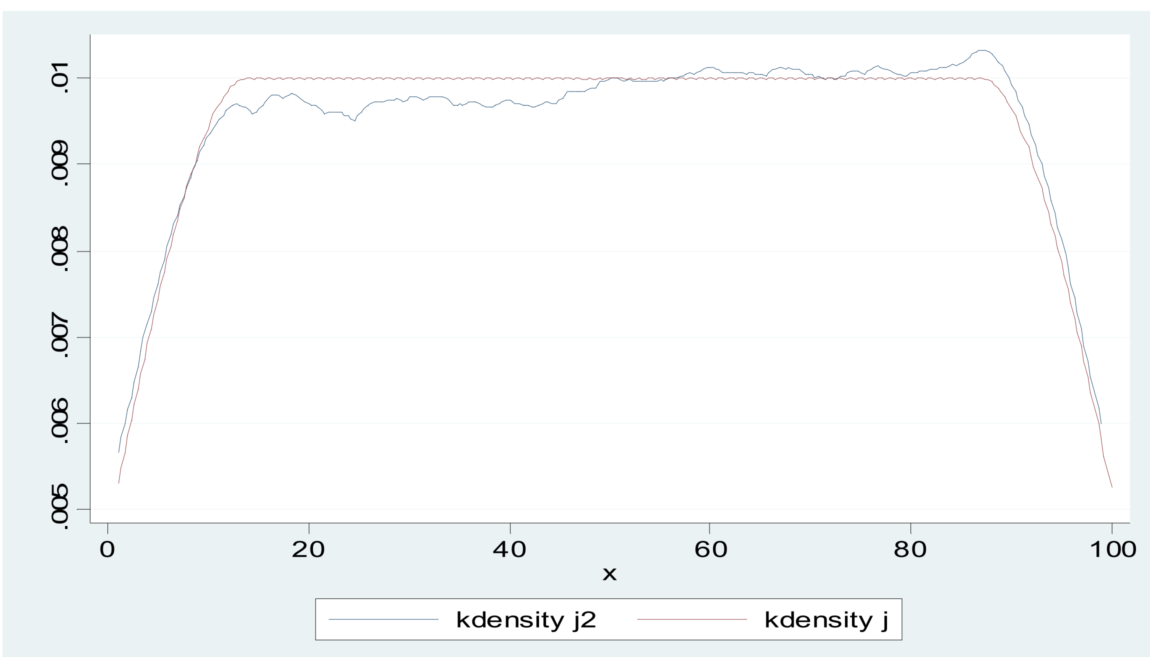
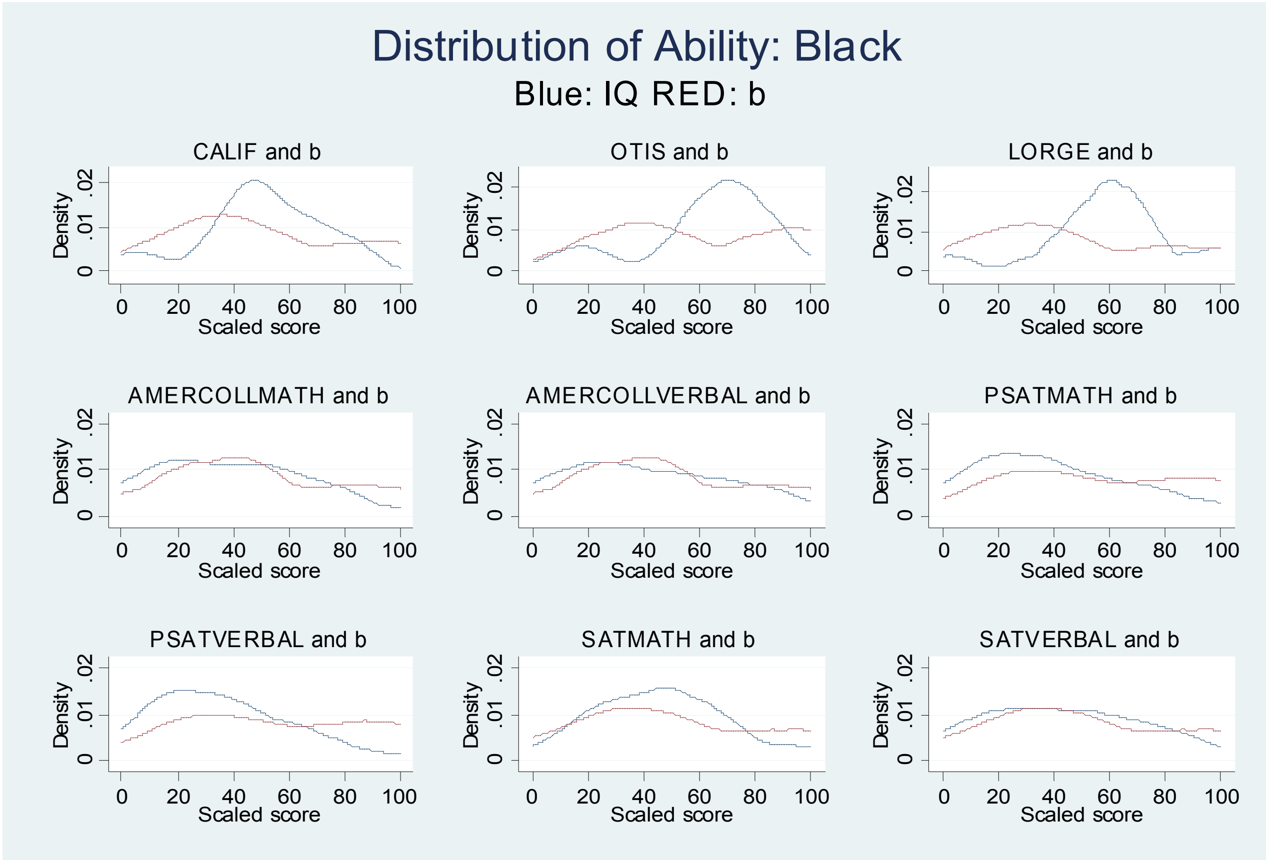


Figure 3


Distribution of Ability: Black
Blue: IQ RED: b
CALIF and b
OTIS and b
LORGE and b
AMERCOLLMATH and b
AMERCOLLVERBAL and b
PSATMATH and b
PSATVERBAL and b
SATMATH and b
SATVERBAL and b
Density
Scaled score
0
.01
.02
0
20
40
60
80
100

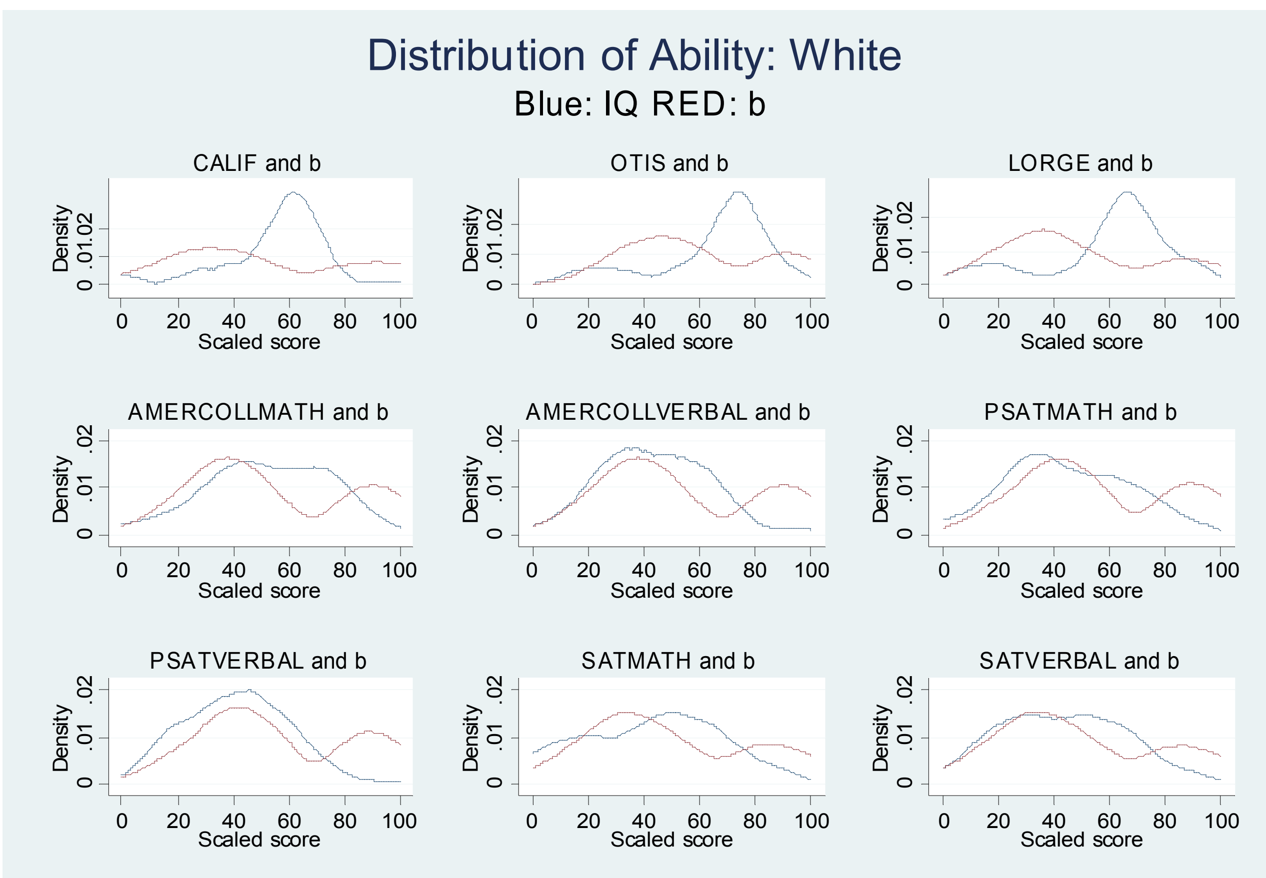
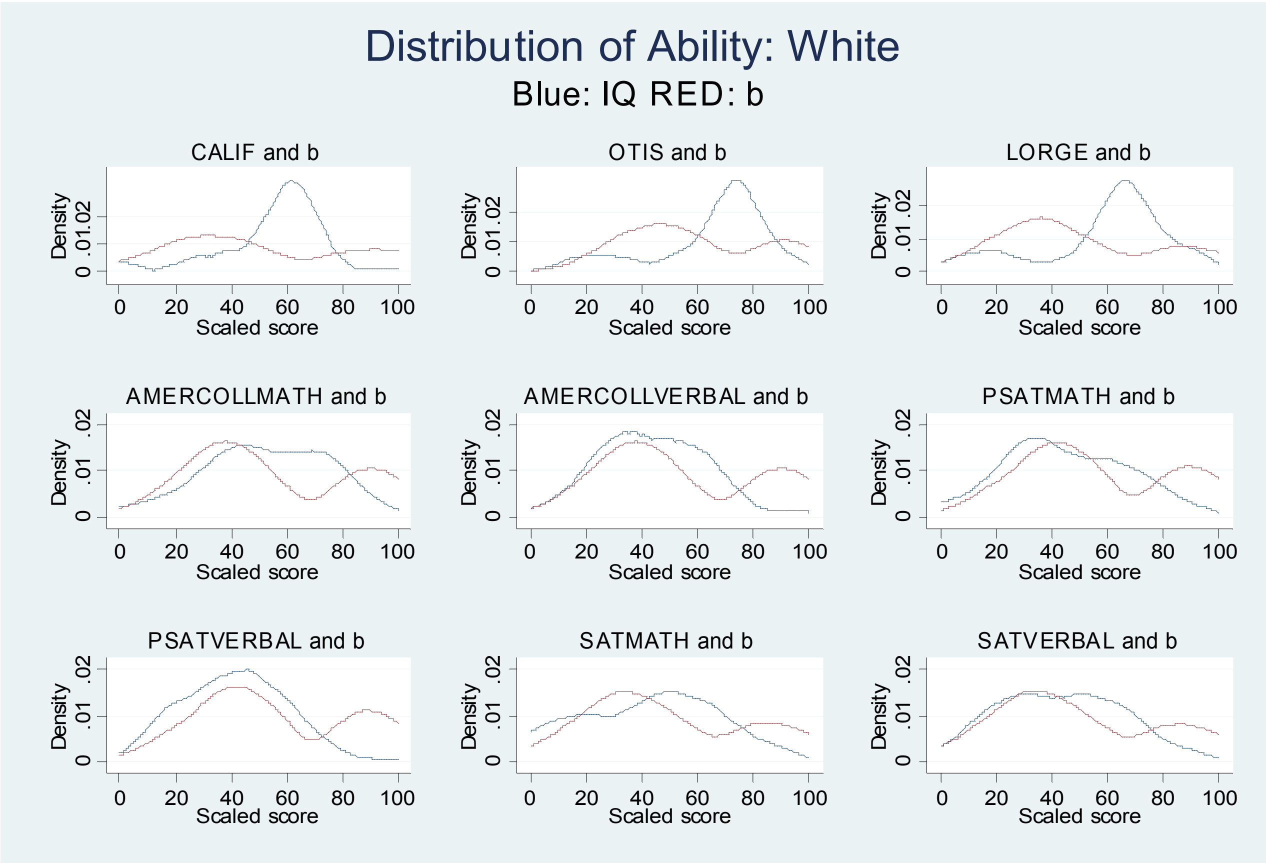

Distribution of Ability: White
Blue: IQ RED: b
CALIF and b
OTIS and b
LORGE and b
AMERCOLLMATH and b
AMERCOLLVERBAL and b
PSATMATH and b
PSATVERBAL and b
SATMATH and b
SATVERBAL and b
Density
Scaled score
0
.01
.02
0
20
40
60
80
100

Figure 4


Distribution of Ability: White
Blue: IQ RED: b
CALIF and b
OTIS and b
LORGE and b
AMERCOLLMATH and b
AMERCOLLVERBAL and b
PSATMATH and b
PSATVERBAL and b
SATMATH and b
SATVERBAL and b
Density
Scaled score

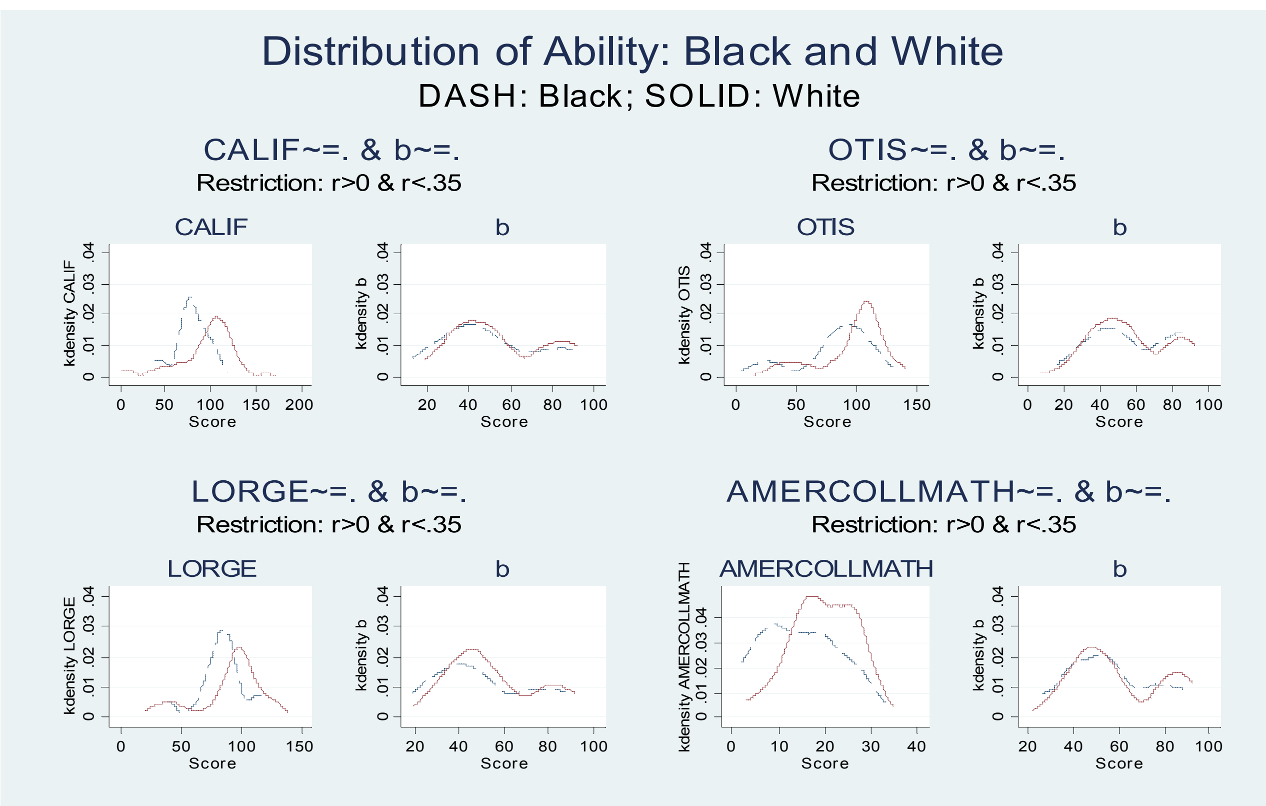
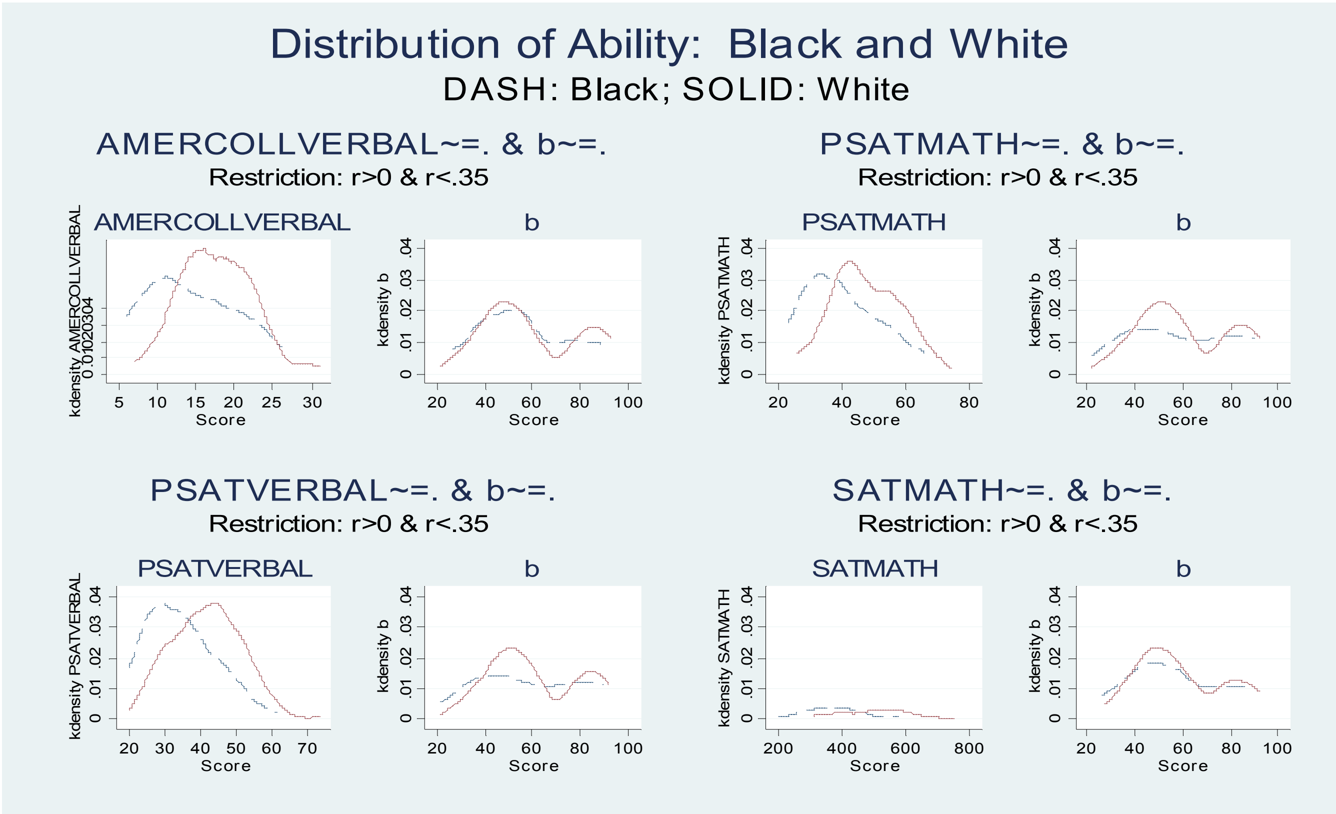

Distribution of Ability: Black and White
DASH: Black; SOLID: White
CALIF~=. & b~=.
Restriction: r>0 & r<.35
OTIS~=. & b~=.
Restriction: r>0 & r<.35
CALIF
b
OTIS
b
kdensity CALIF
kdensity b
kdensity OTIS
Score
LORGE~=. & b~=.
Restriction: r>0 & r<.35
AMERCOLLMATH~=. & b~=.
Restriction: r>0 & r<.35
LORGE
b
AMERCOLLMATH
b
kdensity LORGE
kdensity AMERCOLLMATH

Figure 4 (continued)



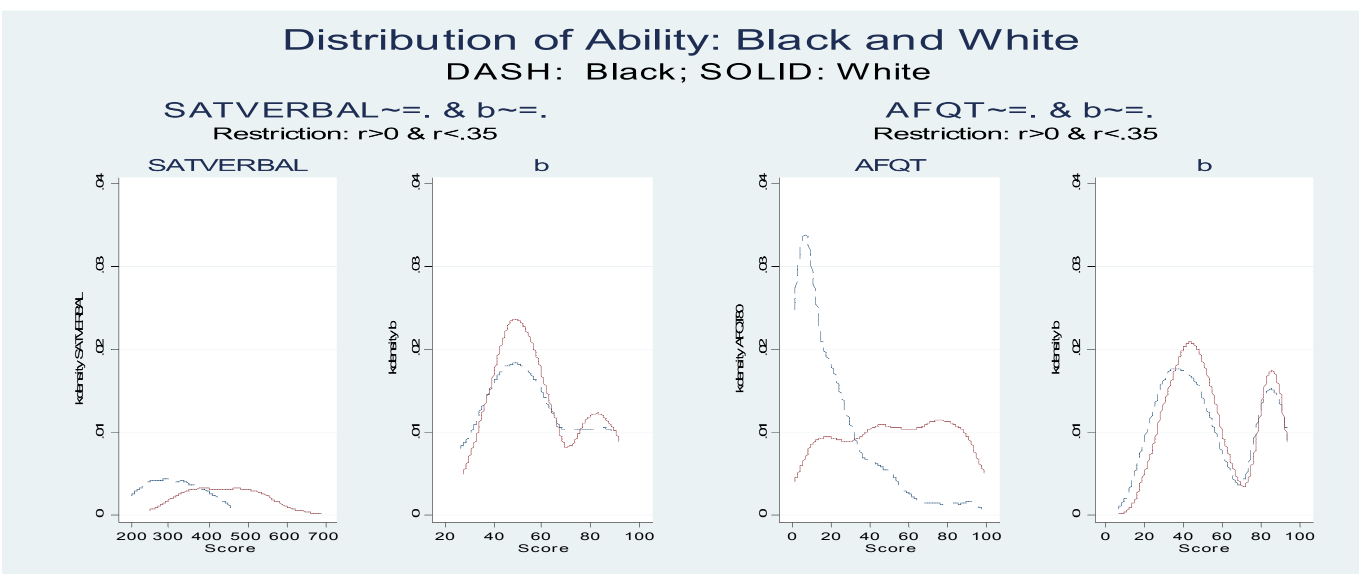


Table 1
Descriptive Statistics:
Individual Specific Parameters and Ability Test Scores

Variable	Obs	Mean	Std. Dev.	Min	Max
b	1521	0.54	0.22	0.06	0.94
W	1521	13.79	13.09	0.00	188.98
r	1521	0.11	0.10	0.00	0.35
DBLACK	1521	0.35	0.48	0.00	1.00
AFQT80	1494	41.12	29.55	1.00	99.00
CALIF	105	88.64	28.42	2.00	172.00
OTIS	192	91.75	29.11	4.00	141.00
LORGE	113	88.25	26.79	20.00	139.00
HENMON	33	67.91	29.48	18.00	129.00
KUHLMAN	22	89.82	24.07	16.00	116.00
DIFFEREN	85	49.02	36.87	8.00	208.00
COOP	30	115.40	143.76	20.00	465.00
STANFORD	18	72.94	38.58	5.00	149.00
WECHSLER	29	81.72	19.99	40.00	131.00
AMERCOLLVE~L	156	17.12	4.89	6.00	31.00
AMERCOLLMATH	156	19.07	7.54	2.00	35.00
SATMATH	107	454.67	116.46	200.00	750.00
SATVERBAL	106	394.81	108.86	200.00	690.00
PSATVERBAL	165	40.64	10.09	20.00	73.00
PSATMATH	165	46.23	11.11	23.00	74.00

Source: computed from NLS-Y. The values of b, w, and r are averages parameter values obtained by estimating equation (7) separately for each individual. DBLACK is the proportion of blacks in the sample. The remaining variables refer to specific achievement/ability test scores contained in the NLS-Y.

Table 2

Parameter Values by Race and Type Ability Test

	White							Black						
TEST	b	w	r	IQ	N	AFQT	N	b	w	r	IQ	N	AFQT	N
CALIF	0.56	14.9	0.11	95.3	61	56.1	60	0.52	12.6	0.11	79.3	44	18.5	44
OTIS	0.56	13.7	0.1	95.9	132	62.2	125	0.59	10.4	0.12	82	59	27	56
LORGE	0.55	15.2	0.12	89.5	89	55.7	87	0.54	16.1	0.12	83.5	24	23.8	21
HENMON	0.5	17	0.08	66.6	21	41.1	20	0.57	12.2	0.08	70.3	12	34.4	12
KUHLMAN	0.52	12.6	0.13	96.5	12	45.9	10	0.49	13.7	0.08	81.8	10	27.8	9
DIFFEREN	0.53	15.6	0.1	50.1	70	61.3	69	0.42	19.3	0.085	43.2	14	26.8	14
COOP	0.51	15.4	0.11	134	23	50.4	20	0.59	12.4	0.12	54.6	7	27.4	7
STANFORD	0.62	11.6	0.15	69.8	8	39.4	7	0.32	21.1	0.06	75.5	10	11.9	10
WEXLER	0.57	12.8	0.16	90.9	12	16.7	11	0.45	12.9	0.13	75.2	17	3.1	17
AM COL VERBAL	0.59	12.7	0.12	17.5	135	73.1	129	0.56	12.4	0.11	14.6	21	51.6	20
AM COL MATH	0.59	12.7	0.12	19.7	135	73.1	129	0.56	12.4	0.11	15	21	51.6	20
SAT MATH	0.59	13.9	0.11	496.7	69	74.6	68	0.58	12.4	0.11	374.6	37	50.6	37
SAT VERBAL	0.59	14.1	0.11	439	68	74.2	67	0.58	12.4	0.11	311.1	37	50.6	37
PSAT VERBAL	0.6	13.5	0.12	41.7	139	74.3	138	0.6	13.9	0.14	34.5	25	52.4	25
PSAT MATH	0.6	13.5	0.12	47.3	139	74.3	138	0.6	13.9	0.14	40	25	52.4	25
AFQT	0.55	14.6	0.11	52	965	52	965	0.52	12.3	0.11	21.1	527	21.1	527

$0<r<.35$

Source: Computed from the NLS-Y. Average values of b, w, r and ability test scores.

Table 3

Parameter Values by Race and Schooling Level

	White					Black				
School	b	w	r	N	AFQT	B	w	r	N	AFQT
8	0.43	19.4	0.099	22	12.54	0.41	19.9	0.096	12	4.5
9	0.47	15.95	0.096	28	18.32	0.51	12.6	0.11	28	6.96
10	0.46	17.23	0.093	39	26.38	0.51	11.77	0.1	42	10.55
11	0.5	16.57	0.107	74	29.6	0.46	14.28	0.099	58	11.21
12	0.53	15.02	0.109	430	46.63	0.51	13.44	0.109	237	17.72
13	0.59	12.99	0.11	73	54.85	0.55	11.53	0.11	40	31.13
14	0.57	15.8	0.106	79	61.91	0.58	11.59	0.13	28	31.36
15	0.66	8.4	0.14	23	69.65	0.59	8.96	0.12	32	26.41
16	0.61	12.98	0.11	159	75.52	0.65	8.16	0.15	41	47.8
17	0.61	12.32	0.12	19	81.22	0.61	9.09	0.12	11	50.27
18	0.65	11.13	0.12	19	83.53	0.7	5.75	0.17	2	39

Source: Computed from the NLS-Y. Average values of b, w, r and ability test scores.

Table 4
Ability and Race: How Human Capital Production Function Measured Ability and AFQT Are Related to Each Other and Race

VARIABLES	b	b	b	AFQT	AFQT	AFQT
Black	-.0318**		0.0344**	-32.99***		-24.12***
	(0.01)		(0.01)	(4.44)		(3.33)
AFQT		0.00133***	0.00201***			
		(0.00)	(0.00)			
B					425.3***	279.3***
					(68.07)	(45.94)
Constant	0.532***	0.488***	0.478***	27.03***	-185.7***	-121.6***
	(0.01)	(0.01)	(0.01)	(3.71)	(37.81)	(24.58)
Observations	32	32	32	32	32	32
R-squared	0.192	0.565	0.645	0.648	0.565	0.845

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Weighted by number of observations in each race-test group from Table 1.
OLS regressions of b on AFQT and race (columns 1-3) and AFQT on b and race (columns 4-6).

Table 5

VARIABLES	b	B	b	b	AFQT	AFQT	AFQT	AFQT	AFQT
Black	-.0215		-.00861	0.0730***	-30.59***		-26.57***	-24.69***	-25.98***
	(0.02)		(0.01)	-0.0133	(7.13)		(1.99)	-2.909	-1.97
School		0.0228***	0.0226***			7.832***	7.047***		5.503***
		(0.00)	(0.00)			(1.41)	(0.45)		-1.119
AFQT				0.00309***					
				-0.0003					
B								274.6***	68.41
								-26.66	-45.56
Constant	0.527***	0.252***	0.250***	0.463***	20.91***	-58.42***	-65.64***	-123.9***	-82.75***
	(0.02)	(0.03)	(0.03)	-0.00992	(5.73)	(18.08)	(5.79)	(14.25)	(12.7)
Observations	22	22	22	22	22	22	22	22	22
R-squared	0.039	0.846	0.852	0.854	0.479	0.607	0.962	0.921	0.966

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Weighted by number of observations in each race-test group from Table 2.

Table 6
Correlations Between b and Psychology-Based Ability

	"b"					
	All		White		Black	
	Unwgt	Wgt	Unwgt	Wgt	Unwgt	Wgt
CALIF	0.1924	0.1251	0.3582	0.235	-0.1567	-0.2804
sig	0.1133	0.3057	0.0252	0.1498	0.4082	0.1335
Obs	69	69	39	39	30	30
OTIS	0.3479	0.4122	0.3285	0.4316	0.2883	0.241
sig	0.0001	0	0.0015	0	0.0982	0.1697
Obs	125	125	91	91	34	34
LORGE	0.2044	0.1708	0.2229	0.1816	0.0007	-0.1269
sig	0.0726	0.1349	0.0816	0.1578	0.9979	0.6397
Obs	78	78	62	62	16	16
HENMON	-0.1661	0.1038	-0.062	0.1105	-0.3864	-0.0179
sig	0.4379	0.6292	0.8196	0.6837	0.3443	0.9665
Obs	24	24	16	16	8	8
KUHLMAN	0.3588	0.6342	0.6376	0.8359	0.2391	0.0414
sig	0.1723	0.0083	0.089	0.0097	0.5685	0.9225
Obs	16	16	8	8	8	8
DIFFEREN	0.1019	0.1221	0.125	0.1873	0.0024	-0.2183
sig	0.4268	0.3404	0.3821	0.1881	0.9941	0.4955
Obs	63	63	51	51	12	12
COOP	0.0403	-0.3091	0.003	-0.3867	0.4646	0.781
sig	0.8624	0.1728	0.9908	0.1252	0.5354	0.219
Obs	21	21	17	17	4	4
STANFORD	0.35	0.468	0.8164	0.6375	0.1237	0.463
sig	0.201	0.0785	0.0918	0.2472	0.7335	0.1778
Obs	15	15	5	5	10	10
WECHSLER	0.2648	0.4595	0.0823	0.0847	0.3488	0.6752
sig	0.3044	0.0635	0.8768	0.8732	0.2931	0.0226
Obs	17	17	6	6	11	11
AMERCOLLVE~L	0.2307	0.0121	0.3171	0.1207	-0.1385	-0.2938
sig	0.0174	0.9018	0.0022	0.2545	0.6225	0.2879
Obs	106	106	91	91	15	15
AMERCOLLMATH	0.2192	0.1332	0.3887	0.3151	-0.4465	-0.5068
sig	0.024	0.1735	0.0001	0.0023	0.0952	0.0538
Obs	106	106	91	91	15	15

SATMATH	0.1106	0.2745	0.0323	0.269	0.0897	0.295
sig	0.3481	0.0179	0.8255	0.0617	0.6699	0.1523
Obs	74	74	49	49	25	25
SATVERBAL	0.1607	-0.0568	0.1295	-0.1171	0.0354	0.1538
sig	0.1713	0.6308	0.3752	0.4228	0.8664	0.463
Obs	74	74	49	49	25	25
PSATVERBAL	0.2696	0.1299	0.2467	0.1252	0.1983	0.2485
sig	0.0048	0.1802	0.0165	0.2293	0.4968	0.3915
Obs	108	108	94	94	14	14
PSATMATH	0.1952	0.0172	0.1929	0.0156	-0.0013	-0.0414
sig	0.043	0.86	0.0625	0.8812	0.9966	0.8882
Obs	108	108	94	94	14	14
AFQT80	0.4052	0.3594	0.4085	0.3744	0.2993	0.3213
sig	0	0	0	0	0	0
Obs	993	993	659	659	334	334

Correlation tables with b (wt=1/bse)
.1<b<.75; 0<r<.15
Source: betacorrected new.xls

Table 7

Correlations Between Race (Black) and Ability by IQ Test

	Correlation Coefficient	
IQ Test	IQ Test	b coefficient
CALIF (ρ)	-0.1097	-0.0758
Sig	0.2952	0.4704
nobs	93	93
OTIS	-0.1534	-0.0935
	0.0472	0.228
	168	168
LORGE	-0.0735	-0.087
	0.4889	0.4121
	91	91
HENMON	0.3321	0.439
	0.068	0.0135
	31	31
KUHLMAN	-0.0224	0.2971
	0.9276	0.2167
	19	19
DIFFEREN	-0.1031	-0.152
	0.3819	0.196
	74	74
COOP	-0.4389	0.1842
	0.0249	0.3676
	26	26
STANFORD	-0.1658	-0.5677
	0.5395	0.0218
	16	16

	Correlation Coefficient	
IQ Test	IQ Test	b coefficient
WECHSLER	-0.1811	-0.2095
	0.4321	0.3621
	21	21
AMERCOLLVE~L	-0.8674	-0.1624
	0	0.0648
	130	130
AMERCOLLMATH	-0.8266	-0.1624
	0	0.0648
	130	130
SATMATH	-0.4643	0.1715
	0	0.1061
	90	90
SATVERBAL	-0.3676	0.1966
	0.0004	0.0648
	89	89
PSATVERBAL	-0.7522	-0.3667
	0	0
	132	132
PSATMATH	-0.8373	-0.3667
	0	0
	132	132
AFQT80	-0.6962	-0.2903
	0	0
	1273	1273

pwcorr dblack bcalif-bafqt80 CALIF- AFQT80 [aw=wt] if b>=.1 & b~=. & b<.95 & r>0 & r<.25
weighted by inverse of standard error of estimate of b

Appendix A: Derivation of the Human Capital Earnings Function

The individual's objective is to maximize discounted disposable earnings, Y_t, over the working life cycle.[10] The objective is achieved by choosing the amount of human capital K*t* to reinvest each year in order to maximize the present value of lifetime earnings

$$\underset{K_t}{Max} \quad J = \int_0^N e^{-rt} Y_t dt$$

where J is the total discounted disposable earnings over the working life cycle, r is the personal discount rate and N is the number of years one works, assumed known with certainly. Disposable earnings are

$$Y_t = R[E_t - K_t]$$

Maximization of (4) subject to equations (2) and (3) can be done by maximizing the following Hamiltonian.

$$H(K, E, \lambda, t) = e^{-rt} R[E_t - K_t] + \lambda_t [\beta K_t^b - \delta E_t]$$

with constraints $E_t - K_t \geq 0$. and making use of the transversality condition $\lambda_N = 0$.

The function λ_t is the marginal contribution to the total discounted disposable earnings if there is one more unit of human capital investment. Assuming that no corner solutions are binding, the necessary conditions are as follows.

$$\frac{\partial H}{\partial K_t} = 0$$

$$\frac{\partial H}{\partial E_t} = -\dot{\lambda}_t$$

$$\frac{\partial H}{\partial \lambda_t} = \dot{E}_t$$

From equation (9), after solving for the differential equation, the explicit solution for E_t can be expressed as

$$\dot{E}_t = q_t - \delta E_t$$

$$E_t = \left[\frac{\beta}{\delta} + \left(E_0^{1-b} - \frac{\beta}{\delta} \right) e^{\delta(b-1)t} \right]^{1/1-b}, \text{ for } t > t^*$$

Note that for $t \leq t^*$, $E_t = K_t$. E_0 is the initial human capital stock and t^* is the age at which the individual stops schooling. During the first phase (while in school), an individual chooses to invest in human capital full time, implying that no human capital stock can be rented in the market, thus resulting in zero earnings in the first phase.

According to equation (8) and using the tranversality condition, $\lambda(N) = 0$, implying that at the third phase, the earnings would be zero. Then we have

$$\dot{\lambda}_t = -\left(\mathrm{Re}^{-rt} - \delta \lambda_t \right)$$

$$\lambda_t = \frac{R}{r + \delta} e^{-rt} \left[1 - e^{-(r+\delta)(N-t)} \right].$$

[10] As noted in the text, we abstract from labor supply considerations.

This $\dot{\lambda}_t$ indicates that if there is one more unit of human capital investment, the marginal contribution to the total discounted disposable earnings will diminish over time.
Equation (7) can be solved as the following.

$$\frac{\partial H}{\partial K_t} = 0 = -\mathrm{Re}^{-rt} + \lambda_t \beta b K_t^{b-1}$$

$$K_t = \left[\frac{\mathrm{Re}^{-rt}}{\lambda_t \beta}\right]^{1/b-1}$$

$$= \left[\frac{b\beta e^{rt}\lambda_t}{R}\right]^{1/1-b}$$

After substituting equation (11) into equation (12), we have

$$K_t = \left(\frac{b\beta}{r+\delta}\right)^{1/1-b}\left(1-e^{-(r+\delta)(N-t)}\right)^{1/1-b}, \text{ for } t > t*$$

For the second phase, solve for E_t because $E_t \neq K_t$.

Let $F_t = \left(1-e^{-(r+\delta)(N-t)}\right)^{1/1-b}$.

After plugging this equation in the production function, $\dot{E}_t$ can be written as

$$\dot{E}_t = \beta K_t^b - \delta E_t$$

$$= AF_t^b - \delta E_t$$

$$\dot{E}_t + \delta E_t = AF_t^b$$

$$\left(e^{\delta t}E_t\right)^{\bullet} = e^{\delta t}AF_t^b$$

$$e^{\delta t}E_t = A\int_{t*}^{t} e^{\delta s}F(s)^b\,ds + c$$

$$E_t = e^{-\delta t}A\int_{t*}^{t} e^{\delta s}F(s)^b\,ds + ce^{-\delta t}$$

The equation (14) is the function we want to get so that we proceed with letting

$E_t^* = E_* = K_t^*$ or K_t at $t*$ and letting

$E_* = ce^{-\delta t*}$, we have

$$E_* = \left[\frac{\beta}{\delta} + \left(E_0^{1-b} - \frac{\beta}{\delta}\right)e^{\delta(b-1)t*}\right]^{1/(1-b)} - \left[\frac{b\beta}{r+\delta}\right]^{1/(1-b)}\left[\left(1-e^{-(r+\delta)(N-t*)}\right)\right]^{1/(1-b)} e^{-\delta t*}$$

Solving for a part of E_t equation, we have

$$e^{\delta s}F(s)^b = \left(1-e^{-(r+s)(N-s)}\right)^{b/(1-b)} e^{\delta s}$$

$$= \sum_{j=0}^{\infty}\binom{b/(1-b)}{j}(-1)^j e^{-(r+\delta)j(N-s)+\delta s}$$

Therefore,

$$Ae^{-\delta t}\int_{t*}^{t} e^{\delta s}F(t)^b\,dt = A\sum_{j=0}^{\infty}\binom{b/(1-b)}{j}(-1)^j \frac{e^{-(r+\delta)(N-t)j}}{j(r+\delta)+\delta}$$

$$= A\frac{1}{r+\delta}\sum_{j=0}^{\infty}\binom{b/(1-b)}{j}(-1)^j \frac{e^{-(r+\delta)(N-t)j}}{j+(\delta/(r+\delta))}$$

$$= \left(\frac{\beta}{r+\delta}\right)^{1/(1-b)} b^{b/(1-b)} \sum_{j=0}^{\infty} \binom{b/(1-b)}{j} (-1)^j \frac{e^{-(r+\delta)(N-t)j}}{j+(\delta/(r+\delta))}$$

where $A = \beta\left(\frac{b\beta}{r+\delta}\right)^{b/(1-b)}$

Adding equations (15) and (16) together gives us the E_t.

Following the disposable earnings function, $Y_t = R[E_t - K_t]$, Y_t is equal to the rental rate times the result of equation (15) + equation (16) – equation (13). This is the same earnings function hypothesized by Haley (1976), which shows that the age-earnings profile can be explicitly in a closed form using the following two essential equations.

$$K_t = \left(\frac{b\beta}{r+\delta}\right)^{1/(1-b)} \left[1 - e^{(r+\delta)(t-N)}\right]^{1/(1-b)}, \quad t \in [t^*, N],$$

and

$$E_t = Be^{\delta(t^*-t)} + \left(\frac{\beta}{r+\delta}\right)^{1/(1-b)} b^{b/(1-b)} \sum_{j=0}^{\infty} \binom{b/(1-b)}{j} \frac{e^{j(r+\delta)(t-N)}}{j+(\delta/(r+\delta))}(-1)^j, \quad t \in [t^*, N],$$

where

$$B = \left[\frac{\beta}{\delta} + \left(E_0^{1-b} - \frac{\beta}{\delta}\right)e^{\delta(b-1)t^*}\right]^{1/(1-b)} - \frac{\beta^{1/(1-b)}}{r+\delta}\left(\frac{b}{r+\delta}\right)^{b/(1-b)} \sum_{j=0}^{\infty} \binom{b/(1-b)}{j} \frac{e^{j(r+\delta)(t^*-N)}}{j+(\delta/(r+\delta))}(-1)^j.$$

Haley shows that the infinite hyper-geometric series converges to a particular value from the second term. In Haley's derivation the convergence criterion is set for 6 decimal points. A simpler form can be obtained by setting the convergence at 4 decimal points level. Note that Haley's convergence table shows that it converges from the first term (i.e. j=0) at 4 decimal points. We use this slightly less stringent convergence criterion to construct the earnings function.

VALUE OF THE INFINITE SUM FOR j = 0,...,34 AND b = 1/8, , = 75, EO= 50, R = .38, yj = .08, 5 = .03, N = 65, AND t = 1

j	Sum
0	.537558
1	.537544
2	.537544
.	
.	
34	.537544

At j = 0, the infinite sum of the hyper-geometric series becomes

$$\sum_{j=0}^{\infty} \binom{b/(1-b)}{j} \frac{e^{j(r+\delta)(t^*-N)}}{j+\frac{\delta}{(r+\delta)}}(-1)^j = \frac{(r+\delta)}{\delta}$$

Thus,

$$B = [\frac{\beta}{\delta} + \left(E_0^{(1-b)} - \frac{\beta}{\delta}\right) e^{\delta(b-1)t^*}]^{1/(1-b)} - \frac{\beta^{a/(a-b)}}{(r+\delta)} \left(\frac{b}{(r+\delta)}\right)^{\frac{b}{a-b}} \frac{(r+\delta)}{\delta}$$

or,

$$B = [\frac{\beta}{\delta} + \left(E_0^{(1-b)} - \frac{\beta}{\delta}\right) e^{\delta(b-1)t^*}]^{1/(1-b)} - \frac{\beta^{a/(a-b)}}{(\delta)} \left(\frac{b}{(r+\delta)}\right)^{\frac{b}{a-b}}$$

Thus the stock of human capital at time *t* can be expressed as

$$E_t = B * e^{\delta(t^*-t)} + \left(\frac{\beta}{r+\delta}\right)^{\frac{a}{a-b}} b^{\frac{b}{a-b}} \frac{(r+\delta)}{\delta}$$

or

$$E_t = B * e^{\delta(t^*-t)} + \frac{\beta^{a/(a-b)}}{(\delta)} \left(\frac{b}{(r+\delta)}\right)^{\frac{b}{a-b}}$$

or

$$E_t = \{ [\frac{\beta}{\delta} + \left(E_0^{(1-b)} - \frac{\beta}{\delta}\right) e^{\delta(b-1)t^*}]^{1/(1-b)} - \frac{\beta^{a/(a-b)}}{(\delta)} \left(\frac{b}{(r+\delta)}\right)^{\frac{b}{a-b}} \} * e^{\delta(t^*-t)} + \frac{\beta^{a/(a-b)}}{(\delta)} \left(\frac{b}{(r+\delta)}\right)^{\frac{b}{a-b}}$$

or

$$E_t = \beta^{\frac{a}{a-b}} [\frac{1}{\delta} + \left(\frac{E_0^{(a-b)}}{\beta} - \frac{1}{\delta}\right) e^{\delta(b-1)t^*}]^{1/(1-b)} e^{\delta(t^*-t)} + \frac{\beta^{a/(a-b)}}{\delta} \left(\frac{b}{(r+\delta)}\right)^{\frac{b}{a-b}} \left(1 - e^{\delta(t^*-t)}\right)$$

Observed earnings can be expressed as following

$$Y_t = R[E_t - K_t]$$

where, *R* is the rental rate of human capital. Thus,

$$Y_t = A_0 e^{\delta(t^*-t)} + A_1 [1 - e^{\delta(t^*-t)}] - A_2 [1 - e^{(r+\delta)(t-N)}]^{1/(1-b)} \qquad (7)$$

where

$$A_0 = R\beta^{\frac{1}{1-b}} [\frac{1}{\delta} + \left(\frac{E_0^{(1-b)}}{\beta} - \frac{1}{\delta}\right) e^{\delta(b-1)t^*}]^{1/(1-b)}$$

$$A_1 = R\beta^{\frac{a}{a-b}} \left(\frac{b}{(r+\delta)}\right)^{\frac{b}{(a-b)}} \left(\frac{1}{\delta}\right)$$

$$A_2 = R\beta^{\frac{1}{1-b}} \left(\frac{b}{(r+\delta)}\right)^{\frac{1}{1-b}}$$

.

Appendix B: Description of Ability Measures Contained in the NLSY79

1. Armed Forces Qualification Test (AFQT)

Armed Forces Qualification Test scores are calculated from some portions of Armed Services Vocational Aptitude Battery (ASVAB) which is administered by the Ministry of Defense. The main purpose of the AFQT is to determine the enlistment eligibility for branches of the Armed Services. The test itself comprises two chief parts which are the Math and the Verbal. The Verbal part contains Word Knowledge and Paragraph Comprehension and the Math contains Arithmetic Reasoning and Mathematics Knowledge. However, to calculate the score of AFQT, which is reported as a percentage, the Math sections are counted only once, whilst the Verbal parts are counted twice. In practice, for example, the percentiles below the 30th generally are not eligible for being part of any branches of the Armed Forces.[1]

2. American College Test (Math)

The American College Test (ACT) is used to assess high school students' performances in general education development and their abilities to complete a degree at the college level. This multiple-choice test consists of four parts: English, Mathematics, Reading, and Science. Also, there is an optional part of the test that is basically supposed to measure ability in planning and writing a short essay.

As for the Mathematics part, the test contains Pre-Algebra, Elementary Algebra, Intermediate Algebra, Coordinate Geometry, Plane Geometry, and Trigonometry. The total number of questions for this part is 60, which is almost one-fourth of the 215 questions on the entire test.[2]

3. American College Test (Verbal)

Another section of the American College Test is the Verbal. This part measures the English ability in the areas of Usage/Mechanics, Rhetorical Skills, etc. The total number of questions is 75. This section is weighted the most heavily. Additionally, the score of the whole test, every section combined, can range from 1 to 36, and the raw scores, the number of correct answers, would be converted to the scale scores before the final scores are reported.[3]

4. California Test of Mental Maturity

The California Test of Mental Maturity (CTMM), administered by California Test Bureau, was primarily designed for students from Grades 7-14; its main objective is to gauge the mental abilities of students. This diagnostic evaluation is closely related to student success in a wide range of school activities, so that the teacher can be directly informed of who has learning difficulties (Carroll, 1982). Moreover, it provides comprehensive measurement of the functional capabilities essential to learning, problem-solving, and responding to new situations.[4]

5. Cooperative School and College Ability Test

This ability test was designed to assess both verbal and mathematical abilities, primarily for students Grades 4-12. Rather than diagnosing individuals, its focus is on predicting student success in related areas of activity. There are two forms of the test, A and B, which have been proven equivalent in terms of ability measurement and reliability. In terms of scores, percentiles and converted scores are reported for each grade level (Kaya, 1969).

6. Differential Aptitude Test

Differential Aptitude Test (DAT) was designed to measure an individual's ability to learn or to succeed in various areas. This test consists of 8 areas: verbal reasoning, numerical ability, abstract reasoning, perceptual speed and accuracy, mechanical reasoning, space relations, spelling, and language usage. All of the DATs are multiple-choice, with time limits ranging from 12 to 25 minutes.[5] In addition, one of the benefits of this test over others is that it provides a ranking for the student against national averages in the respective areas. The DAT results can be interpreted as an indicator of student progress with an identified future plan in pursuing a vocational program or college.[6]

7. Henmon-Nelson Test of Mental Maturity

Henmon-Nelson Test of Mental Maturity was fundamentally designed to measure a variety of areas of mental abilities that are crucial for success both in academic work and outside the classroom. In detail, this test can be identified as four different levels: appropriate for Grades 3-6, Grades 6-9, Grades 9-12, and college level.[7] It would be most accurate if the test taker's age is between 12 and 18. The 90 multiple-choice questions are divided into three parts: word problems, number problems, and graphical representation. The overall score is believed to adequately represent the individual's general cognitive abilities.[8]

8. Kuhlman-Anderson Intelligence Test

Similar to other intelligence tests, Kuhlman-Anderson Intelligence Test was specifically designed to measure an individual's academic potential by assessing general cognitive skills pertaining to the learning process.[9] This test is a well-known standardized intelligence group test that can be given to Grades K-12. Originally developed in the 1920's, it has been updated several times as the number of test-takers has increased. There are verbal and nonverbal items in this test whose scores can indicate performances among children by both chronological age and grade level.[10]

9. Lorge-Thorndike Intelligence Test

The Lorge-Thorndike Intelligence test is another standardized, group-administered test suitable for Grades K-8 students. Its average score can be representative of the nationwide school population. According to the manual for this test, it was primarily intended to measure reasoning abilities, not the proficiency in particular skills taught in school. The test in general consists of two parts, which are verbal and nonverbal. Furthermore, it has been acknowledged that the Lorge-Thorndike Test is one of the best paper-and-pencil general intelligence tests (Jensen, 1973).

10. Otis-Lennon Mental Ability Test

The Otis-Lennon Mental Ability Test is the fourth generation of Otis series, which dates back to 1918. This revised edition is a substantial improvement on its predecessors but still focuses on educational settings. Raw scores are easily converted to various types of normative scores, and normative data are reported both by age- and grade-based reference groups (Grotelueschen, 1969). There are three types of abilities that are meant to be measured by this test: comprehension of verbal concepts, quantitative reasoning and reasoning by analogy. Suitable for students in Grades 8-9, this test is also a group intelligence test whose norms can be updated annually.[11]

11. Preliminary Scholastic Aptitude Test (Math)

Administered by the College Board and National Merit Scholarship Corporation, the Preliminary Scholastic Aptitude Test (PSAT) is a standardized test that is usually given to high school juniors. Not only does this test provide an opportunity to practice for the SAT, but it can also pinpoint the test-taker's weaknesses. Furthermore, if the scores are high enough, they might qualify for a scholarship from the National Merit Scholarship competition. Like other standardized aptitude tests, PSAT is designed to measure a variety of skills. This multiple-choice test is comprised of three primary parts: Critical Reading, Mathematics, and Writing Skills.

Focusing on the Math part, it contains 28 multiple choice and 10 "grid-in" questions that aim at testing skills in basic math, algebra, geometry, measurement, data analysis, and statistics, as well as basic probability.[12] There are five possible answers provided for the multiple choice questions, while the grid-in questions require the test takers to determine their own answers. Generally, the strategies used for PSAT – Math are same as for SAT – Math, but the allotted time for PSAT is somewhat shorter than SAT for Math and the other sections.

12. Preliminary Scholastic Aptitude Test (Verbal)

As for the Verbal part, this test contains 48 multiple questions that focus on both sentence completions and critical reading skills (passage-based reading). The Verbal questions are arranged in random order; however, the test structure has 13 questions for sentence completions and 35 questions for the passage-based reading. The total amount of time allowed to complete this section is 50 minutes.[13]

13. Scholastic Aptitude Test (Math)

The Scholastic Aptitude Test (SAT) is perhaps the nation's most widely accepted college-entrance exam, and is administered by the College Board. The SAT is typically taken by high-school juniors and seniors. It can reflect how well students are in terms of skills and knowledge they have acquired in and outside of the classroom, as well as how they think, communicate, and solve problems. This test is used by most schools as one of the best predictors of how successful the students are in college. Similar to the PSAT, the SAT comprises three parts: Critical Reading, Mathematics, and Writing. Each section of the SAT is scored on a scale of 200-800.[14]

The types of Math questions are five-choice multiple-choice and student-produced responses. In further detail, this section aims at testing skills of students in the following areas: exponential growth, absolute value, functional notation, linear functions, manipulations with exponents, properties of tangent lines, estimation, and number sense.[15]

14. Scholastic Aptitude Test (Verbal)

The SAT Verbal part, currently known as the critical reading section, is also similar to the PSAT Verbal, assessing critical and sentence-level reading. More specifically, it tests students reading comprehension, sentence completions, and paragraph-length critical reading. Questions may be based on one or two reading passages. Some questions, on the other hand, are not based on passages; instead, students need to complete sentences.[16]

15. Wechsler Intelligence Test for Children

The Wechsler Intelligence Test for Children was originally developed by David Wechsler in 1949 to measure the individual's intelligence, especially for children aged 6 years to 16 years and 11 months. Theoretically, it is believed that human intelligence is complex and multifaceted, so this test is designed to reflect this belief through testing both verbal and nonverbal (performance) abilities. The verbal IQ score is derived from scores on 6 subtests: information, digit span, vocabulary, arithmetic, comprehension, and similarities. The nonverbal score is from 6 subtests: picture arrangement, block design, object assembly, coding, mazes, and symbol search. In addition to its uses in intelligence assessment, this test is also used in neuropsychological evaluation, specifically with regard to brain dysfunction. Substantial differences in verbal and nonverbal scores may indicate some potential problems of brain damage.[17]

16. Armed Services Vocational Aptitude Battery (ASVAB)

The Armed Services Vocational Aptitude Battery (ASVAB) was first instituted in 1976. Also, it was used as a special survey administered in 1980 to the 1979 sample of NLSY79 respondents. Administered by the United States Military Entrance Processing Command, it has been utilized to enlist people into all branches of the United States Armed Forces. In terms of its components, this test is comprised of 10 subtests measuring General Science, Arithmetic Reasoning, Word Knowledge, Paragraph Comprehension, Numerical Operations, Coding Speed, Automobile and Shop Information, Mathematics Knowledge, Mechanical Comprehension, and Electronics Information.[18] A narrower scale based on the ASVAB is often called AFQT, in which only the verbal and mathematics parts count (Evans, 1999). ASVAB is presently offered in only 8 fields, having dropped Numerical Operations and Coding Speed.[19]

---

[1] Kaplan Prep Test and Admissions, http://www.kaptest.com/Military/ASVAB/Learn-ASVAB/ML_afqt_testoverview.html

[2, 3] American College Testing Programs,

[4] York University, http://www.yorku.ca/psycentr/tests/iq_test.html

[5] Society for Human Resource Management, http://www.shrm.org/testing/products/PsychCorp/DAT.asp

[6] Swartz Creek Community Schools, http://www.swartzcreek.org/Docs/Differential%20Aptitude%20Test%20Interpretation%20Meeting.ppt2_files/frame.htm

[7] York University, http://www.yorku.ca/psycentr/tests/iq_test.html

[8] See Hoge (1999).

[9] Department of Psychology, The College of New Jersey, http://psychology.department.tcnj.edu//documents/Test_InventoryList.001.doc

[10] Family Education Network, http://school.familyeducation.com/gifted-education/educational-testing/40939.html

[11] Educational Research Centre, http://www.erc.ie/index.php?p=39

[12] Study Guide Zone Company, http://www.studyguidezone.com/psattest.htm

[13] About.com Company, http://testprep.about.com/od/psat/a/PSAT_CR.htm
[14] The College Board, http://www.collegeboard.com/student/testing/sat/about/SATI.html
[15] The College Board, http://www.collegeboard.com/student/testing/sat/about/sat/math.html
[16] The College Board, http://www.collegeboard.com/student/testing/sat/about/sat/reading.html
[17] Encyclopedia of Mental Disorders, http://www.minddisorders.com/Py-Z/Wechsler-Intelligence-Scale-for-Children.html
[18] National Longitudinal Survey of Youth, http://www.nlsinfo.org/pub/usersvc/NLSY79/NLSY79%202004%20User%20Guide/79text/achtests.htm
[19] ASVAB Prep Information, http://asvabprepinfo.com/

References

Ben Porath, Yoram. (1967). "The Production of Human Capital and the Life Cycle of Earnings," *Journal of Political Economy*, 75 (4), pp. 352-365.

Haley, William J. (1976). "Estimation of the Earnings Profile from Optimal Human Capital Accumulation," *Econometrica*, 44 (6), pp. 1223-1238.

Heckman, James (1975) "Estimates of Aging Capital Production Function Embedded in a Lifecycle Model of Labor Supply," in Nester Terleckyji, ed. *Household Production and Consumption*, (New York: Columbia University press for the National Bureau of Economic research), pp. 227-58.

Heckman, James (1976) "A Life Cycle model of Earnings, Learning and consumption," *Journal of Political Economy*, 84(4): S11-S44.

James Heckman, Lance Lochner, and Petra Todd (2006) "Earnings Functions, Rates Of Return And Treatment Effects: The Mincer Equation And Beyond," in Eric A. Hanushek and Finis Welch, eds. Handbook of the Economics of Education, Volume 1, pp. 307-458.

Heckman, James J., Lochner, Lance J. and Petra, Todd E. (2008). "Earnings Functions and Rates of Return," Discussion Paper No. 3310, IZA.

Heckman, J. J., D. A. Schmierer, and S. S. Urzua. (2009). Testing the Correlated Random Coefficient Model. NBER Working Paper Number 15463.

Herrnstein, Richard J. and Charles Murray (1994) *The Bell Curve* (New York: the Free Press).

Lazear, Edward (1977). "Schooling as a Wage Depressant," *Journal of Human Resources* 12(2): 164-76.

Pesaran H. (2006). Estimation and Inference in Large Heterogeneous Panels with a Multifactor Error Structure. *Econometrica* 74: 967-1012.

Polachek, Solomon (1981). "Occupational Self Selection: A Human Capital Approach to Sex Differences in Occupational Structure," *Review of Economics and Statistics*, 63(1) 60-69.

Polachek S.W., Kim M-K. (1994). Panel Estimates of the Gender Wage Gap: Individual-Specific Intercept and Individual Specific Slope Models. *Journal of Econometrics* 61: 23-42.

Solomon Polachek and Francis Horvath (1977) "A Life Cycle Approach to Migration: Analysis of the Perspicacious Peregrinator," *Research in Labor Economics* 1:103-49.

Racine J and Q. Li. (2004). Nonparametric Estimation of Regression Functions with Both Categorical and Continuous Data. *Journal of Econometrics* 119: 99-130.

Ryder, Harl, Frank Stafford and Paula Stephan (1976). "Labor, Leisure and Training Over the Life Cycle," *International Economic Review*, 651-674.