# Matching heterogeneous skills demand and supply under limited rationality 

David N. MARGOLIS* and Jaime MONTANA DONCEL ${ }^{\dagger}$

July 16, 2015<br>Preliminary, please do not cite


#### Abstract

This paper models the labor market matching process when skills are multidimensional and workers are naive about the strategic behavior of their competitors. Using supply and demand side data on multidimensional skills from Colombia, the paper numerically solves for the equilibrium allocation of workers to jobs that solves the naive worker problem and finds that the allocation is inefficient, in that workers over-weight job availability at the expense of matching to jobs for which they are over-qualified, leaving less qualified workers to match to jobs with higher skill demands. Two counterfactual simulations suggested that investment subsidies would be a more effective strategy for approaching the efficient allocation than making training available to all unemployed workers.


## Résumé

Ce papier modélise le processus d'appariement sur le marché du travail lorque les compétences sont multidimensionnelles et les travailleurs sont naïfs vis-à-vis le comportement stratégique des autres demandeurs d'emploi. En exploitant des données colombiennes d'offre et de demande de compétences, on trouve numériquement l'allocation d'équilibre des travailleurs aux postes offerts. Cette allocation est inefficace, car les individus mettent trop de poids sur la possibilité á trouver un emploi au frais d'occuper un emploi pour lequel il est sur-qualifiés, laissant les individus moins qualifié occuper les emplois qui nécessitent plus de compétences. Deux simulations contrefactuelles sont étudiées, et on trouve qu'une subvention à l'investissement permettrait à mieux se rapprocher à une allocation efficace qu'une augmentation générale du niveau des compétences parmi les chômeurs.

JEL Codes / Codes JEL: J24, J64, D83, C63
Key Words: Multidimensional Skills, Limited Rationality, Matching, Colombia
Mots Clés: Compétences multiples, Rationalité limitée, Appariement, Colombie

[^0]
## Introduction

Under perfect competition with fully rational agents, and in the absence of frictions (information, transaction costs, or otherwise) and capital constraints, unemployment does not exist. Wages adjust so that the supply and demand of jobs and skills equilibrate, and there are no unfilled vacancies and no unemployed workers. Unemployment occurs when there is a deviation from the frictionless, fully rational and perfectly competitive model.

One particular manifestation of unemployment is skills mismatch. This occurs when the set of skills demanded by firms does not match the set of skills supplied by workers, a situation which arises when individuals and firms do not accurately anticipate the other's actions. Both vacancies and unemployment can simultaneously exist. Models like Mortensen and Pissarides (1994) use a matching function that determines the share of workers that get matched to vacancies at any point in time, but the mechanism is left as a black box and skills differences are most often modeled as being associated with distinct labor markets across which individuals and jobs cannot shift easily. Moreover, heterogeneity in skills, when considered, is often modeled as different levels of a single human capital measure (high skilled versus low skilled workers, for example), and the differences are related to amounts of human capital needed or required in a job.

In the real world, skills are multidimensional, individuals possess different levels of each skill, and each job can require a different combination of skills, and in different amounts (Lazear, 2009). In this context, search frictions on the worker side refer to trying to people trying to find the jobs that provide them with the highest utility given their skill set, while search on the employer side refers to firms trying to find the workers whose skill sets most closely match the technological requirements of the post being offered. Even if workers accurately predict the skills that will be needed by the market and invest accordingly (and thus the supply and demand of skills in the population match), workers and firms can fail to instantly optimally match when worker knowledge of the characteristics of their competition is imperfect, or the degree of sophistication of their reasoning does not allow them to solve the full multidimensional matching equilibrium so that workers only apply to the jobs that will hire them in equilibrium.

This paper theoretically models the worker-side search process when workers have full information about the jobs on offer and the skills available in the population, but whose level of sophistication in their reasoning is limited ${ }^{1}$. It then numerically solves for the

[^1]equilibrium allocation of skills to jobs and the time to job finding using data on cognitive, non-cognitive and technical skills supplied and demanded (as announced in on-line job postings) in Colombia. After establishing that this allocation of workers to jobs is inefficient, in the sense that there are over-qualified workers in medium-skilled jobs and under-qualified workers who require significant skill upgrading in high-skill jobs, the paper introduces a counterfactual simulation in which firm training is subsidized (thereby reducing the cost of hiring workers with skills below the minimum threshold for a job) and a simulation in which all unemployed receive training so that their skills can increase to a level that makes them eligible for jobs that otherwise they would be unable to occupy. The policy of subsidizing firm training is found to do a better job of approaching an efficient allocation of labor than a policy of generalized skill upgrading.

The rest of the paper is divided into four parts. The first part presents the motivation and the relevancy that this document might have for economics. It does so compiling the relevant literature of occupational choice and hiring decision and explaining how the approach presented may be an interesting contribution to the economic debate. The second subsection presents the data used. The third part is devoted to the description of the theoretical model and analyzes the economic implications of this approach. The last part presents the results, conclusions and further extensions to the presented work. An appendix complements the document with considerations on the use of the data and a guide for the implementation of the simulation.

## 1 Relevant Literature

One of the main concerns of policy makers is mismatch between the human capital available in the labor force and the requirements of the firms. Many skilled workers do not possess the technical, cognitive, and socio-emotional skills to fill current vacancies or create new jobs, and those that do possess these skills may not be able to find the jobs for which they are best suited. The World Bank report on skills (Almeida et al., eds, 2012) notes that $45 \%$ of the current employers worldwide claim that they can not fill entry-level jobs, while a similar share of working youth state that their jobs do not use their acquired skills.

The dominant equilibrium framework for explaining the matching of workers to vacancies was proposed by Mortensen and Pissarides (1994). One of the main assumptions of this matching framework is that there exists a complete labor market, comprising a homogeneous pool of unemployed and a homogeneous pool of vacancies. This homogeneity does not allow for the introduction of different kind of jobs or workers. In order
to weaken this assumption, many studies focus their attention on different sectors or groups of jobs, assuming each market as a separated market. One example of this is the work of Stops and Mazzoni (2010), in which the authors analyze the Mortensen and Pissarides model in different occupational groups, considering each occupational group as a family of job types that share the same kind of qualification requirements.

This feature of the Mortensen and Pissarides model, in which is possible to calculate the equilibrium only in a single, complete labor market or with rigidly segmented labor markets, is a binding constraint, since one observes vertical and horizontal mobility of workers in the data. Moreover, the assumption of perfectly segmented markets brings a new challenge since is impossible to model as many unemployment pools for workers as there are groups in the economy. A natural consequence of strictly segmented markets is different prices, even for the same skill set, in different markets with different bargaining settings. This idea is formalized by Moen (1997), in which the bargaining in submarkets of the economy generate different wages. One problem with this approach is that the number of submarkets is determined by the power of the firm to create submarkets, giving the firm the (unrealistic) unilateral ability to segment labor markets. Stops (2014) provide a theoretical and empirical demonstration for which the assumption of separated partial labor markets is not a realistic assumption. His approach is also relevant for this paper since he bases his analysis in what he calls an "occupational topology" for which the proximity between the occupational requirements are take into consideration to form the groups. This is similar to the approach adopted here, where multiple vacancies listed in a single job posting have common occupational requirements, yet these requirements can differ from other vacancies and it is the requirements of the jobs that determine the "market".

In these extensions of the Mortensen and Pissarides (1994) framework, workers are either assigned a sub-market based on exogenous characteristics or endogenously choose the market in which they participate. Moen's (1997) model, in particular, allows for ex-ante posting wages resulting from a trembling hand equilibrium. The mechanism is based on the ability of unemployed workers to search only in a fraction of the jobs offered, so they choose on the basis of the expected wage. In this sense the model can be interpreted as a sort of occupational choice model for which agents choose the market in which they want to participate following an utility maximization approach, but all workers who participate in a particular market receive a given wage and match to jobs with a common probability. The model here presented is based under a similar construction, although in this paper it is the fact that different vacancies have different skill requirements that leads to optimal posted wages that can be heterogeneous across vacancies.

A common thread in the Mortensen and Pissarides (1994) - based literature is that, once a worker is assigned to a market, the probability of matching to a job no longer depends on the worker's specific characteristics. Several authors have also tried to weaken this aspect of the theory. For example, Albrecht et al. (2009) construct an extension of the model allowing for (continuous, unidimensional) heterogeneity across workers and multiple submarkets (in their case, formal and informal employment). Margolis et al. (2012) further extend this model by introducing self employment, but retain the continuous, unidimensional heterogeneity assumption of Albrecht et al. (2009). All these extensions of the Mortensen and Pissarides (1994) framework allow one to explain matching in labor market, but they require the assumption that the determinant of individual productivity is an observable, unidimensional characteristic.

In reality firms can not hire in base to this single measure, since each firm value different human capital because it depends on the technology it has. This kind of approach has been introduced previously by Lazear (2009) in which each firm weight different each skill in the production function. Firm specific human capital, is opposed to the conventional view of general human capital, for which human capital augments productivity in the same amount in all firms. Instead the firms valuate different characteristic of the individual, valuating his skill endowment differently (i.e the requirement of job postings in which a set of skills are required for filling the position). Firms valuate characteristics such as education, experience among other characteristics. The other characteristics are the skill endowments that the person posses in order to perform the task of the occupation, demographic characteristics and physical factors (Becker, 1962). In this document we are going to focus in the observable characteristics that the firm can observe in the worker: skill and demographic (age, education, experience).

Another separate, and much older, strand of literature in search models focused specifically on the individual-level occupational choice problem (Miller, 1984) and generates outcomes that can vary across job-seekers. In this model, Bayesian job seekers combine prior beliefs about the characteristics of all jobs in the economy with information about jobs which they have previously experienced. Based on these posterior beliefs, the job seeker chooses the best option. The model is based on the construction of an index which represents the expected present value (conditional on beliefs) of the return to each job. This paper adopts a similar approach, basing the decision model on a comparison of the expected values of different jobs which are functions of and index. One key difference in our approach is that the beliefs underlying the expected value calculations are naive, and not Bayesian, and are based on the number of vacancies and the value of each person's index in each job separately, without considering strategic behavior of other job seekers.

The construction of this index depends on the proximity of an individual's skill set to that required by the job. Lazear (2009) formalized this idea that different jobs require different skill sets by allowing heterogeneous skills to provide different levels of specific human capital in different firms, through firm specific weights for each skill. ?'s work is an extension of Becker (1962), and allows for other characteristics that the person possesses that are useful for performing job-related tasks, such as demographic characteristics and physical factors, to also influence an individual's value to the firm. This paper adopts Lazear's approach in considering a vector of characteristics when building the index that underlies the expected value calculation.

One important goal of this paper is use the Miller (1984) occupational choice literature with Lazear's (2009) insight about firm-specific human capital to open the black box of the matching function in the Mortensen and Pissarides (1994) literature. Whereas the Miller (1984) and Lazear (2009) papers are strictly partial equilibrium models, Mortensen and Pissarides (1994) characterizes the full labor market, and this paper also derives the equilibrium allocation of workers to jobs and unemployment.

## 2 The Model

This section details a model of occupational choice on the work side and hiring decisions on the firm side. The model is a sequential in that firms first post their offers (wage and number of vacancies), then workers decide the jobs to which they will apply, firms select among applicants, and unmatched vacancies and workers go back into the pool for another round of applications and hiring. As in Moen (1997), wages are posted and not bargained, but here the worker's utility maximization problem conditions on the skill endowments of each agent and the similarity between the individual's skill set and the occupation requirements. Firms can train workers with inadequate skills, but this cost enters into the firm's profit maximization when making hiring decisions.

This section presents the model in several steps. First, the structure of the model and the objective functions are presented. Then the firms wage posting decision is presented, followed by the worker's occupational choice problem and the hiring mechanism. This section concludes with a presentation of the numerical approach for solving for the equilibrium of the model, as an analytical solutionis intractable.

### 2.1 Matching as a sequential game

The labor market matching process is model as a sequential game with agents who, although fully informed about the distribution of vacancies posted and of skills available, maximize their expected income while (naively) ignoring strategic considerations such as which jobs they should apply to given the optimal application decisions of other agents and the likelihood of their application being accepted given the competition. The players of the game are:

- Job seekers. The number of job seekers in the economy is $I \in\{1,2, \ldots, I\}$. Each job seeker owns a set of non transferable endowments: they can not be exchanged among job seekers, nor do they decrease or increase during the duration of the game. The set of endowments for individual $i$ is characterized by a skill vector $\mathbf{s}_{i}=\left(s_{(1, i)}, . ., s_{(K, i)}\right)$, where $K$ is the number of skills. Skills are heterogeneously distributed among the job seekers, so the vector $\mathbf{s}$ is a multivariate random variable $\mathbf{s} \sim F_{s}\left(s_{1}, . ., s_{K}\right)$ and $I_{s}$ represents the set of skills available in the economy.
- Occupations. The main assumption is that each occupation behaves like a firm and, for ease of exposition, we will refer to occupations in what follows as firms ${ }^{2}$. The number of firms in the economy is $J \in\{1,2, \ldots, J\}$, and each is characterized by a production function of the form:

$$
\begin{equation*}
y_{j}=f\left(\mathbf{s}, \mathbf{r}_{j}, \mathbf{a}_{j}\right) \tag{1}
\end{equation*}
$$

where $\mathbf{r}_{j}=\left(r_{(1, j)}, . ., r_{(K, j)}\right)$ is a vector of the specific requirements of the skill on the production technology of the firm and $\mathbf{a}_{j}=\left(a_{(1, j)}, . ., a_{(K, j)}\right)$ is a vector of the importance of the skill in the technology of production. The vectors $\mathbf{r}_{j}$ and $\mathbf{a}_{j}$ are also multivariate random variables. This specification is a generalization of the firm specific human capital model of Lazear (2009) in which

$$
q_{j}=\sum_{k=1}^{K} a_{(k, j)}\left(\frac{s_{k}}{r_{(k, j)}}\right)
$$

with $\sum_{k=1}^{K} a_{(k, j)}=1$. Each occupation is endowed with a fixed number of vacancies at the beginning of the game. The vector $\mathbf{V}=\left(V_{1}, V_{2}, . ., V_{J}\right)$ characterizes the distribution of vacancies in the economy and $I_{v}$ represents the set of vacancies available in the economy. This information is common knowledge.

The sequence of events is as follows.

[^2]0 . Each firm posts the wage that it will pay in exchange of the supply of one unit of labor (which represents the complete utilization of the skill endowments ${ }^{3}$ ). This happens only once in the beginning of the game. The wages offered by the firms are common knowledge and are determined non-strategically, in that firms do not compete across occupations for workers through wage posting, but rather maximize their own profits (subject to an output constraint). The vector of posted wages can be written as the vector $\mathbf{W}=\left(w_{1}, w_{2}, . ., w_{J}\right)$.

1. In the first stage of the game, job seekers apply for the job that maximize their expected wage given the available information in the market. Each job seeker choose only one vacancy in each iteration, so there is a cost associated in the search process in that individuals cannot diversify their risk by simultaneously applying to several jobs ${ }^{4}$.
2. In the second stage of the game, each firm chooses among the applicants, with the firm that posted the most vacancies choosing first ${ }^{5}$. The firm will select the job seeker whose set of skills $\mathbf{s}_{i}$ that best matches its skill requirements and importance vectors, $\mathbf{r}_{j}$ and $\mathbf{a}_{j}$. Skills in excess of the minimum level of requirements do not generate additional profits for the firm, so other criteria will be used to break ties (see section 2.5). Note that this technological constraint and selection process implies that some skills could go unused, which could be a source of inefficiency for the economy. Following this hiring strategy will maximize the profits of the firm given the production function presented in equation 1 and the posted wage $w_{j}$. Thus if the individual $\hat{i}$ applied to firm $\hat{j}$ (meaning that this firm gives him the highest expected utility), and firm $\hat{j}$ chooses job seeker $\hat{i}$ (meaning that this worker generates the highest profit among the pool of applicants), a match occurs. The new hiring decreases the number of vacancies available to fill in the firm by one, and takes the worker out of the pool of unemployed.
3. The second stage continues until each firm (in order by the number of posted vacancies) has completed its hiring from its iteration-specific applicant pool. Firms that receive more applicants than vacancies will fill all of their vacancies in the first iteration. Other firms will only fill a portion of their vacancies, and the remaining vacancies become available for the next iteration.
4. The game is repeated from step 1 until either all vacancies in the economy are filled or all unemployed workers find jobs. We denote $z$ as an iteration of the process.
[^3]Figure 1: Structure of the game in iteration $z$


The game tree presented in figure 1 shows the sequence of events in iteration $z$.

### 2.2 Optimal unique wage setting in presence of heterogeneity

The wage posting problem for the firm is non trivial, since the firm does not know ex ante which workers will apply and it has specific technology (in this case represented by the importance and the requirement level) that can make the value of a hire change with the characteristics of the person hired. Firms will target individuals whose skill sets at least meet the requirements of the posted job, so the problem is to find the wage that allows the firm to hire the selected individuals at the lowest cost, while still meeting production requirements for a given workforce size (the number of vacancies is exogenous). To find the unique wage posted, the firm proceeds as if it could compensate each specific skill individually, solving the following cost minimization problem:

$$
\min \sum_{k=1}^{K} \omega_{k}^{j} s_{k}
$$

s.t.

$$
\bar{q}=f\left(\mathbf{s}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)
$$

Where $\omega_{k}^{j}$ is the price of the $k$-th skill for the given technology. The associated Lagrangean of the problem is:

$$
L(\cdot)=\sum_{1}^{K} \omega_{k} s_{k}-\lambda\left(\bar{q}-f\left(\mathbf{s}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)\right)
$$

The $k+1$ first order conditions of the problem are:
F.O.C.

$$
\begin{gathered}
\left.\frac{\partial L(\cdot)}{\partial s_{1}}=\omega_{1}^{j}=\lambda f_{s_{1}}^{\prime}\left(\mathbf{s}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)\right) \\
(. .) \\
\left.\frac{\partial L(\cdot)}{\partial s_{k}}=\omega_{k}^{j}=\lambda f_{s_{k}}^{\prime}\left(\mathbf{s}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)\right) \\
\frac{\partial L(\cdot)}{\partial \lambda}=q=f\left(\overline{\mathbf{s}}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)
\end{gathered}
$$

Given that individuals whose skills exceed the required level do not produce additional output, firms use the distance function $d: \mathbb{R}^{k} \times \mathbb{R}^{k} \rightarrow \mathbb{R}$ to identify how far an individual's skill set is from that required to do the job well. Define the minimal skill set as $\overline{\mathbf{s}}$ such that :

$$
\overline{\mathbf{s}}=\min _{s} \in\left\{s: d\left(\mathbf{s}, \mathbf{r}_{j}\right)=0\right\}
$$

For the fixed level of output $\bar{q}$ and the optimal skill bundle $\overline{\mathbf{s}}$, the optimal wage that ensures the profit maximization is therefore given by ${ }^{6}$ :

$$
\widetilde{W}_{j}=\sum_{1}^{k} \omega_{k}^{j} \bar{s}_{k}=\sum_{1}^{k} \bar{\lambda} f_{s_{k}}^{\prime}\left(\overline{\mathbf{s}}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)
$$

[^4]The firm may, however, not receive any applicants whose available skill set meets the required skill level. In this case, training will be required to bring the individual's skill level up to the minimum requirements. The production function given in equation 1 reflects this cost as a lower net output for individuals whose skill level $\mathbf{s}$ is less than the required amount $\mathbf{r}_{j}$. The firm anticipates this training cost and reduces the offered wage so that, in expectation at the start of the game, the worker pays the full cost of the training. This implies that the final posted wage is reduced from the optimal wage by an amount reflecting the risk of having to make a suboptimal selection. The final wage posted ex-ante by firm $j$ is thus defined as:

$$
W_{j}=\widetilde{W}_{j}-\Delta
$$

where $\Delta$ is defined by:

$$
\Delta=\bar{\lambda} \sum_{1}^{k}\left[\int\left(f_{s_{k}}^{\prime}\left(\overline{\mathbf{s}}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)-f_{s_{k}}^{\prime}\left(\mathbf{s}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)\right) d F_{s}(\mathbf{s})\right]
$$

Where the first part of the equation is the marginal product of skill $s_{k}$ evaluated at the optimal skill level and the second part is the expected marginal product given the skill distribution among job seekers ${ }^{7}$.

There are two important facts to remark to this solution:

- Each firm can value each skill differently. This can be seen in the fact that the marginal productivity for a given skill $s_{k}$ in two firms is going to be different when the technological parameters $\mathbf{r}$ and a differ. This implication is interesting since two persons with the same endowments can have different wages in different jobs.
- Using this setup, even skill supply in the unemployed population was homogeneous, there would be differences in income across jobs. Again, the differences come from the heterogeneity in production technologies across jobs. This last fact has an implication for policy making and planning in that it suggests that training alone cannot eliminate wage inequality, as technological differences would drive wage dispersion even if the skill level of the entire workforce could be increased to the maximum possible skills endowment (through education and training). This is a direct implication of the wage posting assumption, in that firms are allowed to minimize cost through unilateral wage variation, as opposed to being

[^5]price takers on the labor market.

### 2.3 Occupational choice

Given that the wage was posted at the beginning of the game and job seekers know the wage and the associated number of vacancies for each posted job, each job seeker will maximize his utility for his specific skill set by choosing to apply to the occupation with the highest subjective expected wage. Assuming risk neutrality and that there is no savings, workers consume all their income and the utility maximization problem for job seeker $i$ at iteration $z$ comes down to maximizing subjective expected labor income:

$$
\max _{j} E_{(i, z)}\left\{W_{j}^{0} \mid I_{v}, I_{s}\right\}
$$

The vector $\left(E_{(i, z)}\left\{W_{1}^{0} \mid I_{v}, I_{s}\right\}, . ., E_{(i, z)}\left\{W_{j}^{0} \mid I_{v}, I_{s}\right\}\right)$ represents the payoff from each possible strategy for job seeker $i$ at iteration $z$. Define the occupation $\hat{j}$ for iteration $z$ as the one that maximizes utility:

$$
E_{(i, z)}\left\{W_{\hat{j}}^{0} \mid I_{v}, I_{s}\right\}>E_{(i, z)}\left\{W_{j}^{0} \mid I_{v}, I_{s}\right\} \forall j \neq \hat{j}
$$

Note that the expectation is defined with respect to the individuals subjective beliefs about the likelihood of getting a particular job if applying for it. As the worker is naive, he does not consider the application behavior of other workers, and thus from his subjective point of view, the expected value of the job in occupation $j$ in iteration $z$, given the posted wage $W_{j}$, is defined as:

$$
\begin{aligned}
& \left.E_{(\bar{i}, z)}\left\{W_{j} \mid I_{v}, I_{s}\right\}=p_{( } z, v, s\right) W_{j} \\
& \quad=W_{j} \frac{f\left(\mathbf{s}_{\bar{i}}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)}{\frac{v_{i}^{I} f\left(s_{i}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)}{I}} \frac{v_{j}^{z}}{\sum_{j^{\prime}}^{J} v_{j^{\prime}}^{z}}
\end{aligned}
$$

The subjective expected wage as defined above reflects the the fact that the individual assumes his chances of receiving a particular job are related to his relative performance in job $j$ and the number of vacancies available in iteration $z$ of type job type $j$. A higher posted wage also increases the value of job type $j$. The expected wage is thus composed
by three parts ${ }^{8}$ :

- The wage posted by the firm $W_{j}$.
- $\frac{f\left(\mathbf{s}_{i}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)}{\frac{\sum_{i}^{I} f\left(s_{i}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)}{}}$ which is a measure of the productivity of individual $i$ in job $j$ relative to the rest of the population, as measured by the average productivity in the job. It is important to note that this probability reflects the worker's naiveté, in that he does not account for strategic behavior of other job seekers when considering the set of potential competitors for a job. Here, the individual assumes he potentially faces all unemployed workers for each job to which he applies.
- The third part is the share of vacancies in occupation $j$ with respect to the whole economy. This reflects the idea that it is easier to find a job in an occupation with more vacancies than in a particularly rare occupation.


### 2.4 Hiring decision

The objective of the firm is to maximize its profit level. Given the collection of applicants $A \subset I$ for whom the job $j$ maximizes their subjective expected utility, the firm will select the most productive candidate, breaking ties by choosing among individuals using other criteria besides skills. In this way, firm $j$ ensures that profit is maximized by hiring an individual $\tilde{i} \in A$ from among the pool of most qualified indiduals such that.

$$
\tilde{i} \in \hat{A}_{j}=\left\{i \mid f\left(\mathbf{s}_{i}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)=\sup _{i^{\prime} \in A} f\left(\mathbf{s}_{i^{\prime}}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)\right\}
$$

Under this process, a match is thus a stable coalition for which the job seeker $\tilde{i}$ maximizes his utility by choosing firm $\tilde{j}$ and firm $\tilde{j}$ maximize its profit by choosing the job seeker $\tilde{i}$ among the candidates in iteration $z$. This match is a stable coalition since no occupation different that $\tilde{j}$ can provide higher utility (and thus induce a deviation from the worker) and no other applicant beside $\tilde{i}$ can generate higher profits (and thus induce a deviation from the firm).

The number of filled positions in occupation $j$ in period $z$ will be the number of applicants observed if their number is less or equal than the number of vacancies, meaning

[^6]where $\gamma_{i}^{z}$ is the number of people in the STEP survey of type $i$ still unemployed in iteration $z$.
that firms would rather hire from the available applicant pool than forego production and wait until the next iteration. When the number of applicants in $A$ exceed the number of vacancies in occupation $j$, all of the vacancies will be filled and job seekers in later iterations will have to search for a job in a different occupation.

### 2.5 Numerical Resolution of the Equilibrium Allocation

Since the equilibrium of the model is analytically intractable, numerical techniques are used to solve for the equilibrium allocation of workers to jobs. What follows is a brief description of the algorithm used to solve the model.

- Construction a similarity matrix. Given the skill demand information of the vacancy and O*NET database and the skill supply information on the STEP survey (see section 3), we define a measure of similarity for each observation and each occupation. The index synthesizes the value of each worker type in each occupation, combining the different skill dimensions as specified by the $\mathbf{r}_{j}$ and $\mathbf{a}_{j}$ vectors for the occupation and the $\mathbf{s}_{i}$ vector for the worker type.
- Estimation of a tiebreaker index. In the case where multiple individuals have equal values of the similarity index, other worker characteristics that can affect the job search process and the firm's selection decision are used to break the tie. This tiebreaking index is calculated by estimating a probit model of the probability of employment as a function of the average skill-based similarity index for the individual, the individual's demographic characteristics and other job-related characteristics.
- Iteration. Match the individual maximizing the utility according to the behavior described in the above model, using the probit index to break ties.
- Stopping. Stop the algorithm when the implied unemployment rate based on unmatched individuals is the same as that in the overall economy.

One of the main advantages of this model is that counterfactual policies can easily be simulated, in particular active labor policies that affect the level of skills. Currently, training in the firm, that in Colombia was part of the approved National Development plan of the second government of the former president of Colombia, has been implemented with the name UVAES. Under this plan, firms provide spaces for learning the task of the company, and the national vocational education training institute - SENA will certificate the competencies of the set of skills learned in the firm for future recognition. This is the reference situation.

Two main counterfactual policies are of interest:

- The first policy is a direct subsidy to firms for technological investment. This is formalized in the context of the model as a reduction in the minimal skill requirements to perform a job well. The simulated policy lowers skill requirements for all firms, extrapolating from any firm decision to take up the policy.
- The second policy is an increase in spending on training for the unemployed. The model allows one to target training specifically to the unemployed, and the simulation is implemented as a shift upward in all skills for those unemployed at the start of the game.


## 3 Data

This paper uses data on the supply and demand sides of the labor market in Colombia. Beyond any intrinsic interest the Colombian labor might posses, it has a distinct advantage in terms of available data that are relevant for this analysis. Three different datasets are used to characterize the Colombian labor market in terms of occupational structure, skill requirements by occupation and skill endowments by job seeker.
0. A vacancy database collected during 2014 in the Colombian Ministry of Labor is used to characterize the structure of job postings on the Colombian labor market. The sources of this data are the two major internet job search portals of Colombia, the data from the employment agency of the national Vocational Training Institution (SENA - Servicio Nacional de Aprendizaje) and the data provided from the Colombian employment service offices (UASPE - Unidad Administrativa especial del Servicio Público de Empleo).

1. The World Bank collected a dataset in Colombia in 2012 using the STEP survey ${ }^{9}$. This survey contains a standard labor force survey component, but has the additional advantage of providing information at the individual level on education and training, health status, employment, job skill requirements, personality, behaviors, preferences, language and family background. In particular, the noncognitive skills collected comprise the "Big 5" and the cognitive skills are both self reported and directly measured on site.
2. $\mathrm{O}^{*}$ NET provides a taxonomy of occupations, covering (among other dimensions) the skill content of occupations. The levels and importance of various skills for

[^7]each occupation of the $\mathrm{O}^{*}$ NET classification are available, and the analysis undertaken here used 2010 revision of the data at the six-digit level of precision of occupations (770 occupations). This , in order to have a decomposition level that was enough to give idea of a detailed structure of the economy. This data was collected using the API, on-line information services and documentation. This information was merged into the vacancy dataset to characterize job offers in terms of skills.

As the data analyzed here are drawn from multiple sources, coverage and representativity are key concerns. In order for the analysis to be relevant for a sample broader than that covered by the data collected, the distributions of characteristics of jobs offered has to be common across sampled and non-sampled jobs, and individuals must apply the same decision process to the unsampled jobs as they would to the sampled jobs. In that case, one could treat the data as a random sample from the underlying distribution and conclude for an absence of bias in the estimated allocations of workers to job types. Unfortunately, there exists no definitive reference for comparison. However, some indices concerning representativity can be drawn from available data, and the analysis data are reweighted to replicate employment shares for recent hires from a representative labor force survey.

The first sign that the data might be representative is shown in table A. 6 in the appendix. This table shows the channels used for job searching in the Colombian market drawn from the Household Income Survey (GEIH). The sources of data collected here cover more than the $50 \%$ of the channels used for job searching. The use of informal networks for job finding is common, even in developed countries (Montgomery, 1991), but there is no consensus as to whether the jobs found through networks are qualitatively different from those found through formal channels (see also Mortensen and Vishwanath (1994), Margolis and Simonnet (2002)).

A second issue that arises in developing countries is the presence of informality. Informality in Colombia represents a large share of overall employment, although much of it is self-employment (Perry et al., 2007). For wage employment, it is unlikely that informal employers use the search channels collected in this data. Nevertheless, informal employers tend to offer lower productivity jobs than formal employers (due to the need to cover payroll taxes), and thus the wages they propose are often lower (Perry et al., 2007). In this case, if informal opportunities are available with comparable frequency to formal opportunities but pay less, they will be dominated in the naive individual's decision problem and their absence from the data will not affect the allocation found here.

Figure 2: Occupation by sector 2010-2014


Source: DANE - Household Survey (GEIH)

A final issue arises from the fact that the data collected do not all correspond to the same year. As figure 2 shows, however, the distribution of occupations in the Colombian economy was remarkably stable across the set of years from which the data are drawn, suggesting that there are unlikely to have been major shifts in the types of jobs offered from year to year.

### 3.1 Colombian Vacancy Data

The Colombian vacancy dataset is a sample of the registries in 2014, collected by the Colombian Ministry of Labor with the objective of monitoring jobs and job requirements. The idea behind taking only a subsample for the year is to have the monthly seasonality of job posting during the course of the year, while still maintaining a large number of job postings. The data contain information for $1,892,219$ vacancies. These vacancies correspond to roughly 1 million registries, the difference being due to the fact than a single job posting can correspond to more than one vacancy opened. All of the analysis undertaken here is done at the level of the vacancy, not the posting.

The structure of the data allows one to recover information on the characteristics of the job offered by place, wage, sector, occupation, etc. Table 1 shows the variables and the
description of data.
Table 1: Database Content - Variables and description

| Variable | Description |
| :--- | :--- |
|  | Number of the job vacancy (Requisition ID in the data warehouse. <br> ID <br>  <br> This number is unique and the role is to identify the vacancy within <br> the warehouse <br> "Title" of the vacancy, i.e., the name given to the occupation. This <br> provides information for categorization, clustering and the basis for |
| Title | splitting the identification of skills and competencies of occupations |
| Company Name | Company name |
| Sector | Sector of the company |
| Position | Area where the person performs |
| Total years of experience | Total experience required |
| Experience in the position offered | Total required experience in the position |
| City | Location of the vacancy |
| Professional title | Title of the person requesting the vacancy i.e. economist |
| Wage | Wage proposed for work |
| Level of education | Degree (i.e. Technical, University, Bachelor) |
| Type of contract | Type of contract |
| Language | Language requirements for the position |
| Number of vacancies per offer | Number of vacancies that the job posting offers. |
| Publication date | Date of publication of the vacancy |
| Expiring Date | Expiration date of the vacancy |
| Description | Description of the occupation |
| Occupation CIUO | ISCO 08 classification of occupation |
| Occupation O*NET | O*NET classification of occupation |

One of the main advantages of these newly-collected data is that they allow one to identify the human capital needs of employers in Colombia. Importantly, descriptive statistics calculated on these data fit many of the measures that are observed in other surveys and previous studies. For example, the mode of the wage distribution is between 500.000 pesos and 1.000 .000 pesos, also observed in the household survey data (GEIH). The data also replicate specific characteristics of differences in the wage distribution by schooling level required within occupation, in that they show more gradual wage increases with education for non-professional occupations relative to professional occupations ${ }^{10}$. This is especially true for specialized workers (one year post secondary education), master and doctorate levels.

Is also relevant that the years of experience required for the job are concentrated in the "one year or less" category. Along with the fact that the most common required level of education for the job is completed high school, this suggests that an important

[^8]Figure 3: Vacancy distribution in Colombia

share of job offers targets low skill occupations. Regarding the education level required to enter particular job, the data shows that $42.3 \%$ of jobs require at least completion of high school, while $24.8 \%$ and $19.5 \%$ require technical and technological level education respectively. It is worth noting that levels below high school are not in high demand, in that overall demand for these levels represents $2.59 \%$ of jobs offered, while levels of university and postgraduate added $10.1 \%$.

Regarding gender-specific job postings, the data shows that there is no gender preference specified in $81.18 \%$ of the vacancies . This is also in line with the Colombian legislation ${ }^{11}$, for which there must be no gender discrimination at work and in wage levels. However, $3.46 \%$ of the vacancies target women only while $8.83 \%$ of the job vacancies are targeted exclusively to men. Only the $6.53 \%$ indicate that the firm and occupation are indifferent considering gender. ${ }^{12}$.

Under the assumptions about the comparability of sampled and unsampled jobs presented at the start of this section, a reweighting scheme was designed so as to render the distribution of sampled job offers across occupations representative of recent hires. A table describing the occupational structure of labor demand for the Colombian Economy was built, based on the O *NET Code at the 6 -digit level, the number of vacancies per occupation, the mean wage offered by occupation and a vector that assigns a weight to each occupation to match the number of recent hires in the STEP survey. The weight was calculated so that the occupational distribution of vacancies matched the share of people with one year of seniority or less in the STEP survey. Differences between the distribution of posted vacancies and the weights reflect differences in the probability of a given occupation being filled. Table 2 presents the top 10 most frequent occupations by number of posted vacancies, their wages and the associated weights.

### 3.2 STEP Survey 2012 - Colombia

The World Bank STEP survey for Colombia is used to capture the supply side of the labor market. The Skills Towards Employability and Productivity (STEP) program includes a household survey which is a tool designed to generated internationally comparable, quantitative data on employment skills. The main objective of the STEP survey is to provide measures of the human capital stock of the country, including cognitive, non-cognitive and technical measures, in order to provide a baseline for policy implementation and international comparison. The choice of specific skills measured is based on the relevance on the skill for employers and employability, as determined by refer-

[^9]Table 2: Structure of the Vacancy Final Database

| O*NET | Occupation Title | Wages | Number of vacancies | Weight |
| :--- | :--- | :--- | :--- | :--- |
| $41-2031$ | Retail Salespersons | 843525.8 | 324494 | 4.552409 |
| $43-4051$ | Customer Service Representatives | 856134.3 | 130709 | 4.77578 |
| $41-9011$ | Demonstrators and Product Promoters | 734387.8 | 92029 | 5.392699 |
| $43-5081$ | Stock Clerks | 749143 | 63231 | 5.573521 |
| $51-9198$ | Helpers - Production Workers | 745251.9 | 47480 | 4.96724 |
| $15-1152$ | Computer Network Support Specialists | 1246871 | 33200 | 4.882156 |
| $41-2011$ | Cashiers | 821517.2 | 32066 | 3.648313 |
| $15-1131$ | Computer Programmers | 1121887 | 30627 | 4.594952 |
| $43-3031$ | Bookkeeping, Accounting, and Auditing Clerks | 922857.4 | 24903 | 4.935306 |
| $43-5021$ | Couriers and Messengers | 743303.8 | 18867 | 5.924564 |
| $51-6052$ | Tailors, Dressmakers, and Custom Sewers | 720115.6 | 17817 | 4.733252 |

ring to the academic literature (see for example Felstead et al. (2007); Heckman et al. (2006); John and Srivastava (1999)).

The specific skills that are measured include cognitive skills (reading, writing and numeracy), socioemotional skills (personality, behavior and preferences) and certain skills related to work (a subset of transversal skills). The survey samples the working age population (between the ages of 15 and 64), active and inactive. Data collection for the survey began on March 2012, the results were processed and cleaned, and the final database was published officially in February 2013.

The survey instrument is comprised of several different modules. The first part of the first module collects household level information, including basic roster information for all household members such as relationship to the household head, characteristics (academical and level of literacy) and labor market status (employed, unemployed or inactive). The second part of the first module contains information about household assets such as household size, materials, facilities, appliances, number of books and income sources for the household. The later modules gather information on a randomlyselected individual respondent and covers education and training (quantity and type of education), health status, employment status, job skill requirements, personality and behavior measures, family background, and test to directly measure cognitive skills (reading and numeracy).

The methodology for collecting the data of the survey is based on a random representative sample of households in rural and urban areas of the country. The information of the first module is collected by asking questions to the main household respondent (a proxy for other household members at this step), and collects the income, size and characteristic situation of the household. The second step of the data collection is based
on direct questioning of a randomly selected household member, and thus the detailed information, in particular concerning skills, is not the result of proxy response.

This paper does not exploit the proxy responses from module 1, and focuses on the active population ${ }^{13}$. The main descriptive statistics for the underlying data are shown in table 3. The results show that most variables are have reasonable variation when coded on a 4-point scale, although these variables were rescaled to be between 0 and 100 for comparability. Survey weights are used in all calculations.

### 3.3 O*NET Occupations: minimum skill level requirements and importance

The O*NET taxonomy of occupations is used to quantify the demand for specific skills in each occupational job posting. $\mathrm{O}^{*}$ NET is a database that contains detailed information for 965 occupations in the United States ${ }^{14}$ and was developed to replace the Dictionary of Occupational Titles (DOT). The project started in 1991 and the idea was to collect detailed information on the different aspects of occupations, in order to be able to describe and analyze them with a quantitative approach. The methodology for collecting the information is based on continuous surveys to employers, research studies by sector and occupation, continuous revision of the estimates and updating of the information and occupational analysis. The database has information on many occupational dimensions including: tasks, generalized work activities, knowledge, education and training, work styles, work context, skills and abilities.

O *NET is a publicly-accessible on-line database, so all the available dimensions of occupations can be accessed trough the web. The data for the analysis was obtained through systematically structured queries of the database, which allowed for the collection of the information on the skills in table 4 for each occupation. The skills in the O*NET database are grouped into two broad categories, basic skills and crossfunctional skills. The basic skills are the ones that facilitate the acquisition of knowledge, while the cross-functional skills are the ones that facilitate the performance in activities, and thereby the performance of specific tasks inherent to each occupation.

[^10]Table 3: Descriptive Statistics for skills of STEP survey - Active Population

| Variable | Mean | Std. Dev. | Min | Max |
| :--- | :--- | :--- | :--- | :--- |
| Read | 1.889447 | 1.005517 | 0 | 3 |
| Write | 1.223411 | 0.838479 | 0 | 3 |
| Numeric | 1.779516 | 0.830061 | 0 | 3 |
| Interpersonal | 2.05339 | 1.174125 | 0 | 3 |
| Presentation | 0.233002 | 0.422858 | 0 | 1 |
| Supervise | 0.338616 | 0.473367 | 0 | 1 |
| Computer | 1.340207 | 1.354008 | 0 | 3 |
| Computer type | 0.559322 | 0.850601 | 0 | 2 |
| Drive | 0.106101 | 0.308051 | 0 | 1 |
| Repair | 0.053435 | 0.224959 | 0 | 1 |
| Operate | 0.100263 | 0.300431 | 0 | 1 |
| Think | 1.289892 | 1.176287 | 0 | 3 |
| Learn | 1.820946 | 1.207189 | 0 | 3 |
| Cognitive Challenge | 1.557281 | 0.940956 | 0 | 3 |
| Autonomy | 2.015327 | 0.86067 | 0 | 3 |
| Physical | 1.901545 | 1.013278 | 0 | 3 |
| Extroversion | 3.047863 | 0.640609 | 1 | 4 |
| Conscientiousness | 3.326227 | 0.498628 | 1.666667 | 4 |
| Openness | 3.238473 | 0.513277 | 1 | 4 |
| Emotional Stability | 2.543818 | 0.726405 | 1 | 4 |
| Agreeableness | 3.176563 | 0.554637 | 1.333333 | 4 |
| Grit | 2.990806 | 0.613184 | 1 | 4 |
| Decision making | 3.118844 | 0.599811 | 1.25 | 4 |
| Hostile bias | 1.710988 | 0.603815 | 1 | 4 |
| Risk | 1.640442 | 1.080305 | 1 | 4 |
| Discount | 0.543931 | 0.498192 | 0 | 1 |
| Gender | 34.96111 | 13.16419 | 15 | 64 |
| Age | $S T E P$ |  | 4 |  |
|  |  |  |  |  |

Source: WB STEP Survey Colombia 2012

The O *NET skill content of the broad categories is divided in 35 skills. The basic skills are subdivided in content skills (reading comprehension, active listening, writing, speaking, mathematics and science) and process skills (critical thinking, active learning, learning strategies, monitoring). The cross-functional skills are subdivided in social skills (social perceptiveness, coordination, persuasion, instructing, service orientation), complex problem solving, technical skills (operation analysis, technology design, equipment selection, installation, programming, operations monitoring, operations and control, equipment maintenance, troubleshooting, repairing, quality control), system skills (judgment and decision making, system analysis, system evaluation) and resource management skills (time management, management of financial resources, management of material resources, management of personnel resources).

The skill taxonomy of $\mathrm{O}^{*}$ NET presupposes that skills are the characteristics that an individual has to have in order to perform a task well, and thus the presence of a certain skill level in an individual can make him able to perform the different activities associated with a particular occupation. An important implication of this assumption is that employers value skills in the hiring decision: they do not decide solely on whether a worker can already perform a particular set of tasks, but rather whether the person possesses the skills needed to perform those tasks.

The $\mathrm{O}^{*}$ NET occupations were aggregated to the 6 -digit level, in order to match the vacancy data. The values of each different occupation at the 8 -digit level were averaged, reducing the dimension to 770 titles. The $\mathrm{O}^{*}$ NET database characterizes skill requirements along two dimensions: the level, referring to the minimum amount of the skill level required by the employer to perform the tasks associated with a specific occupation, and the importance, referring to the relative mix of skills needed in order to perform an occupation well. The analysis undertaken here only uses 29 of the 35 listed skills, since these were the only skills present in both the STEP survey and the O*NET taxonomy ${ }^{15}$.

## 4 Results

Numerical resolution of the equilibrium for the model was attained after 120 iterations (see figure 4) Table 5 summarizes the results in terms of the average skill index value of individuals and of the average productivity of workers in occupations. Clearly, the model generates an equilibrium allocation of workers to jobs that, although individually optimal at the point in time the match occurs, id socially inefficient. Workers whose av-

[^11]Table 4: Skill Requirements: Descriptive Statistics

| Variable |  | Mean | Std. Dev. | Min | Max |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Active Learning | Importance | 50.83777 | 12.53559 | 19 | 78 |
|  | Level | 44.0941 | 11.09776 | 16 | 80 |
| Active Listening | Importance | 64.34634 | 11.05332 | 35 | 97 |
|  | Level | 49.09316 | 9.422747 | 29 | 84 |
| Critical Thinking | Importance | 61.95727 | 10.81199 | 31 | 94 |
|  | Level | 49.8022 | 9.003765 | 29 | 80 |
| Learning Strategies | Importance | 42.46033 | 14.45521 | 3 | 85 |
|  | Level | 39.17115 | 12.08091 | 0 | 77 |
| Mathematics | Importance | 36.97865 | 14.25698 | 0 | 100 |
|  | Level | 34.6804 | 13.40057 | 0 | 87 |
| Monitoring | Importance | 57.17222 | 8.987114 | 31 | 85 |
|  | Level | 47.40881 | 8.207129 | 27 | 70 |
| Reading Comprehension | Importance | 59.54293 | 13.79717 | 25 | 97 |
|  | Level | 50.38899 | 12.08755 | 20 | 86 |
| Science | Importance | 23.13736 | 21.57817 | 0 | 91 |
|  | Level | 19.71977 | 19.99701 | 0 | 84 |
| Speaking | Importance | 62.90062 | 12.26975 | 31 | 94 |
|  | Level | 47.85948 | 10.49566 | 25 | 77 |
| Writing | Importance | 52.37814 | 15.2853 | 10 | 97 |
|  | Level | 45.54867 | 12.31633 | 7 | 75 |
| Coordination | Importance | 53.01419 | 9.221817 | 25 | 81 |
|  | Level | 44.71346 | 7.161652 | 27 | 68 |
| Instructing Others | Importance | 44.86891 | 15.03995 | 0 | 91 |
|  | Level | 40.71048 | 11.30251 | 0 | 70 |
| Negotiation | Importance | 40.20755 | 11.79493 | 13 | 91 |
|  | Level | 35.91504 | 9.609598 | 12 | 71 |
| Persuasion | Importance | 43.46775 | 11.7508 | 16 | 81 |
|  | Level | 39.0232 | 9.774566 | 14 | 68 |
| Service Orientation | Importance | 47.74338 | 13.21695 | 0 | 91 |
|  | Level | 40.089 | 9.048359 | 2 | 73 |

Table 4 (continued)

| Variable |  | Mean | Std. Dev. | Min | Max |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Social Perception | Importance | 54.30171 | 10.96683 | 0 | 94 |
|  | Level | 43.31731 | 9.596944 | 5 | 84 |
| Complex problem Solving | Importance | 53.91795 | 11.38924 | 22 | 81 |
|  | Level | 43.72311 | 8.949525 | 21 | 73 |
| Equipment Maintenance | Importance | 18.32409 | 21.64172 | 0 | 81 |
|  | Level | 15.28671 | 18.32055 | 0 | 68 |
| Equipment Selection | Importance | 18.45868 | 17.77134 | 0 | 75 |
|  | Level | 14.67052 | 15.38731 | 0 | 57 |
| Installation | Importance | 6.146651 | 12.17319 | 0 | 78 |
|  | Level | 4.656793 | 10.86593 | 0 | 60 |
| Operations and control | Importance | 30.94551 | 22.46198 | 0 | 97 |
|  | Level | 25.40269 | 18.22881 | 0 | 80 |
| Operations and monitoring | Importance | 39.58697 | 19.1671 | 0 | 94 |
|  | Level | 32.42439 | 14.51982 | 0 | 70 |
| Operation Analysis | Importance | 27.12225 | 15.9832 | 0 | 75 |
|  | Level | 24.67528 | 16.02521 | 0 | 73 |
| Programming | Importance | 12.34139 | 11.81543 | 0 | 88 |
|  | Level | 9.484008 | 11.4184 | 0 | 68 |
| Quality control | Importance | 35.28261 | 17.34819 | 0 | 78 |
|  | Level | 30.56915 | 14.99334 | 0 | 57 |
| Repairing | Importance | 17.49324 | 21.80392 | 0 | 85 |
|  | Level | 14.75675 | 18.47062 | 0 | 61 |
| Tech Design | Importance | 15.93318 | 9.947358 | 0 | 60 |
|  | Level | 12.72916 | 10.517 | 0 | 60 |
| Troubleshooting | Importance | 26.1647 | 19.6714 | 0 | 81 |
|  | Level | 22.34657 | 16.78092 | 0 | 75 |
| Judgment Decision Making | Importance | 55.50743 | 10.28359 | 25 | 85 |
|  | Level | 44.5131 | 9.250787 | 23 | 71 |

Source: O*NET
erage skill level is high can be found allocated to jobs that only need a low level skills, while some high-skilled occupations undertake costly investment to increase the skills of their low-skilled recruits. The main reason for this result comes from the means by which workers calculate the subjective probability of obtaining a job. Although being more skilled than average increases the likelihood of hiring in any job and occupations with higher skill requirements pay more, it appears that the availability of vacancies is a more dominant factor in determining an individual's occupational choice. If the amount of vacancies for a particular occupation is really large in comparison to other occupations, the results suggest that the choice will be towards that occupation regardless of variation in wages or any job-specific productivity advantage an individual might have. In this data analyzed here, nearly the $10 \%$ of all vacancies are in retail or related occupations. The easy availability of such jobs make these occupations particularly appealing.

Figure 4: Convergence of the simulation to the Colombian 2014 unemployment rate 9.6\%

Matching simulation-120 iterations


Another interesting aspect of the model is its ability to produce a declining hazard in job finding rates (figure 5). This effect is due in part to the fact that the most common vacancies are the ones to which there are more applications, leading many workers to match to jobs quite rapidly. Once those vacancies are filled, the opportunities for employment in the remaining occupations are more homogeneous, in which case wage variation and relative productivity advantages both have the potential to become more important. The fact that workers still find jobs at a relatively high rate from iterations

Table 5: Results for the base scenario and policy impact evaluation

|  |  | Baseline simulation <br> Training in the firm | Technological change | Training for unemployed |
| :---: | :---: | :---: | :---: | :---: |
| High Skill Job | High skill worker | 33.1 | 37.5 | 33 |
|  | Medium Skill worker | 54.4 | 13.7 | 45.7 |
|  | Low Skill Worker | 12.5 | 48.8 | 21.3 |
| Medium Skill Job | High skill worker | 58.3 | 46.6 | 58.3 |
|  | Medium Skill worker | 35.5 | 43 | 29.4 |
|  | Low Skill Worker | 6.2 | 10.4 | 12.3 |
| Low skill Job | High skill worker | 44.3 | 41.3 | 44.3 |
|  | Medium Skill worker | 30.1 | 29.7 | 29.1 |
|  | Low Skill Worker | 25.6 | 29 | 26.6 |

10-30 may suggest that, once the easy-to-find jobs are gone, workers do sort to the types of jobs where they have a productivity advantage. As the number of simulations increases, however, wage variation becomes more important. The relatively high paying remaining jobs have few vacancies but will be oversubscribed, leading to even lower hazard rates into employment.

Figure 5: Variation of the hazard into employment rate

## Hazard rate for 120 iterations



Analyzing the actual matching process, the most skilled individuals have the shorted unemployment spells, while low skilled workers have the longest spells. High skilled workers apply to, and are matched with, the very common medium and low skilled
vacancies first. Medium and low skilled individuals match in the middle and latter iterations of the game, once the competition from high-skilled workers clears out. This is a direct implication of job seekers behaving non-strategically. If high skilled workers considered that they would beat all other workers whose skill sets are less well adapted for the high productivity jobs, then they would not apply for the medium skilled jobs. Likewise, medium skilled workers would see the competition from high skilled workers evaporate for medium productivity jobs, and would thus apply there, leaving the low productivity jobs to the low skilled workers.

Among the two policy evaluated, the technological change counterfactual does a better job at reducing the allocative inefficiency of the labor market. This is mainly because the effect opens up the more common occupation to a larger set of individuals, making the more skilled individuals compete along dimensions other than skills for the medium productivity jobs. As they lose out on these jobs they are more likely to apply for higher skill jobs, for which they have a comparative advantage. This suggests that although technological change can improve the efficiency of the allocation, it does so at the expense of longer unemployment spells for high skilled workers.

The second policy simulation is less effective at inducing reallocation, since the high skilled workers already have enough skills to meet the criteria for high-productivity jobs, while training allows low skilled workers to be competitive for the better-paid high productivity jobs that were previously inaccessible. This highlights the role of dispersion in skills for the matching process. When the lower tail of the distribution shifts upward, workers become more similar and it becomes less likely that workers endogenously match in an assortative manner with firms. However, although sorting is less present, total output increases as less output is wasted in training underqualified individuals to reach a skill level sufficient to perform higher productivity jobs well.

## 5 Conclusion

This paper has presented a model in which skills are multidimensional and skills mismatch occurs as a result of optimizing behaviors of workers and firms. Workers apply for jobs in a way that maximizes their subjective expected utility, although they behave in a naive manner by not taking into account the strategic job application decisions of their competitors. Firms post wages for a given number of vacancies and select among applicants based (initially) on the appropriateness of their skill sets for the job on offer in a manner that maximizes profits. Numerically solving for the equilibrium allocation shows that although each agent behaves optimally, the socially optimal allocation of workers to jobs is not reached, primarily due to the naive behavior of workers. This re-
sult helps explain why job search assistance is among the most effective type of active labor market program, as it allow workers to better assess their chances of finding a job and to better target vacancies for job applications.

Although the model studied here relies on limited rationality and relatively straightforward behavior on behalf of all labor market participants, it does a reasonably good job of reproducing certain stylized facts (higher wages for more skill-intensive occupations, decreasing hazard rates into employment, longer unemployment durations for less skilled workers). Moreover, the model specification makes implementation of skillsbased policy simulations straightforward. The main drawback of the model, however, is the lack of an analytical solution to the equilibrium allocation of workers to firms. Introducing fully rational agents could also extend the model, although it is less clear whether such an extension would render the model more realistic. On the data side, estimating the model for Colombia has some major advantages (availability of data), but also so disadvantages (coverage of job offers). Nevertheless, this paper has shown that one can gain interesting insights into labor market behavior and outcomes even when introducing the complexity of multidimensional skills, and many extensions of the model can be envisioned to both make it more realistic and more easy to solve analytically.

## References

Albrecht, James, Lucas Navarro, and Susan Vroman, "The Effects of Labour Market Policies in an Economy with an Informal Sector," Economic Journal, 07 2009, 119 (539), 1105-1129.

Almeida, Rita, Jere Behrman, and David Robalino, eds, The Right Skills for the Job? Rethinking Training Policies for Workers, Washington, DC: The World Bank, 2012.

Becker, Gary S., "Investment in Human Capital: A Theoretical Analysis," Journal of Political Economy, 1962, 70, 9.

Card, David, Jochen Kluve, and Andrea Weber, "Active Labour Market Policy Evaluations: A Meta-Analysis," Economic Journal, November 2010, 120 (548), F452F477.

Dupuy, Arnaud and Alfred Galichon, "Personality Traits and the Marriage Market," Journal of Political Economy, 2014, 122 (6), 1271-1319.

Felstead, Alan, Duncan Gallie, Francis Green, and Ying Zhou, "Skills at Work, 1986 to 2006," 2007.

Heckman, James J., Jora Stixrud, and Sergio Urzua, "The Effects of Cognitive and Noncognitive Abilities on Labor Market Outcomes and Social Behavior," Journal of Labor Economics, July 2006, 24 (3), 411-482.

John, Oliver P and Sanjay Srivastava, "The Big Five trait taxonomy: History, measurement, and theoretical perspectives," Handbook of personality: Theory and research, 1999, 2 (1999), 102-138.

Lazear, Edward P., "Firm-Specific Human Capital: A Skill-Weights Approach," Journal of Political Economy, October 2009, 117 (5), 914-940.

Margolis, David, Lucas Navarro, and David A Robalino, Unemployment Insurance, Job Search, and Informal Employment, June, 2012.

Margolis, David N. and Véronique Simonnet, "Technical/professional versus general education, labor market networks and labor market outcomes," International Journal of Manpower, August 2002, 23 (5), 471 - 492.

Miller, Robert A, "Job Matching and Occupational Choice," Technical Report 6 December 1984.

Moen, Espen R, "Competitive search equilibrium," Journal of Political Economy, 1997, 105 (2), 385-411.

Montgomery, James D, "Social Networks and Labor-Market Outcomes: Toward an Economic Analysis," American Economic Review, December 1991, 81 (5), 1407-18.

Mortensen, Dale T and Christopher A Pissarides, "Job Creation and Job Destruction in the Theory of Unemployment," Review of Economic Studies, July 1994, 61 (3), 397-415.

Mortensen, Dale T. and Tara Vishwanath, "Personal contacts and earnings : It is who you know!," Labour Economics, March 1994, 1 (2), 187-201.

Perry, Guillermo, William Maloney, Omar Arias, Pablo Fajnzylber, Andrew Mason, and Jaime Saavedra-Chanduvi, Informality: Exit and Exclusion, Washington, DC: The World Bank, 2007.

Stops, Michael, "Job matching across occupational labour markets," Oxford Economic Papers, 2014, p. gpu018.

- and Thomas Mazzoni, "Matchingprozesse auf beruflichen Teilarbeitsmärkten/Job Matching on Occupational Labour Markets," Jahrbücher für Nationalökonomie und Statistik, 2010, pp. 287-312.


## A Tables and figures

Table A.1: Gender Preferences by Colombian Firms

| Gender | $\%$ |
| :--- | :--- |
| Female | $3.46 \%$ |
| Both Genders | $6.53 \%$ |
| Male | $8.83 \%$ |
| Does not specify | $81.18 \%$ |

Table A.2: Experienced required by vacancies

| Experience required | $\%$ |
| :--- | :--- |
| Less than a year | $48.34 \%$ |
| At least 1 year | $37.23 \%$ |
| At least 2 year | $7.94 \%$ |
| At least 3 year | 2.49 |
| At least 4 year | $60.0 \%$ |
| At least 5 year | $135.0 \%$ |
| At least 6 year | $166.0 \%$ |
| At least 7 year | $10.0 \%$ |
| At least 8 year | $9.0 \%$ |
| At least 9 year | $1.0 \%$ |
| At least 10 year | $16.0 \%$ |
| More than ten years | $3.0 \%$ |

Table A.3: Wages offered and educational attainment required

| Wages Offered | None | Elementary | Mid school | High-school | Vocational Education | Advanced Vocational education | Bachelor | Specialization | Advanced Master | PhD |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| less than $\$ 550.000$ | 0.0\% | 0.9\% | 3.3\% | 2.5\% | 2.4\% | 1.9\% | 1.9\% | 0.4\% | 0.2\% | 0.0\% |
| 550.001-1.000.000 | 89.9\% | 80.2\% | 84.0\% | 83.1\% | 69.6\% | 61.0\% | 32.0\% | 2.6\% | 9.4\% | 11.3\% |
| 1.000.001-1.500.000 | 9.2\% | 16.0\% | 6.6\% | 12.0\% | 20.2\% | 26.8\% | 29.3\% | 4.3\% | 8.0\% | 4.2\% |
| 1.500.001-2.000.000 | 0.0\% | 2.9\% | 2.7\% | 1.7\% | 4.5\% | 6.2\% | 15.0\% | 14.0\% | 9.9\% | 1.4\% |
| 2.000.001-2.500.000 | 0.0\% | 0.0\% | 0.7\% | 0.2\% | 1.5\% | 2.0\% | 7.9\% | 21.8\% | 13.3\% | 2.8\% |
| $2.500 .001-3.000 .000$ | 0.0\% | 0.0\% | 0.0\% | 0.2\% | 1.0\% | 0.9\% | 4.4\% | 13.4\% | 20.3\% | 5.6\% |
| $3.000 .001-3.500 .000$ | 0.0\% | 0.0\% | 0.0\% | 0.2\% | 0.3\% | 0.4\% | 4.2\% | 8.7\% | 10.2\% | 2.8\% |
| $3.500 .001-4.000 .000$ | 0.0\% | 0.0\% | 0.0\% | 0.0\% | 0.3\% | 0.3\% | 2.9\% | 11.4\% | 8.5\% | 4.2\% |
| 4.500.001-5.500.000 | 0.0\% | 0.0\% | 2.2\% | 0.0\% | 0.1\% | 0.2\% | 0.9\% | 9.4\% | 4.4\% | 2.8\% |
| $5.500 .001-\mathbf{6 . 0 0 0 . 0 0 0}$ | 0.0\% | 0.0\% | 0.5\% | 0.0\% | 0.0\% | 0.1\% | 0.4\% | 4.7\% | 4.6\% | 38.0\% |
| $6.000 .001-\mathbf{8 . 0 0 0 . 0 0 0}$ | 0.0\% | 0.0\% | 0.0\% | 0.0\% | 0.1\% | 0.1\% | 0.7\% | 5.2\% | 4.4\% | 4.2\% |
| \$8.000.001 and more | 0.9\% | 0.0\% | 0.0\% | 0.0\% | 0.1\% | 0.1\% | 0.4\% | 4.2\% | 6.8\% | 22.5\% |

Table A.4: Wages offered and years of experience required

| Wages | Less than 1 year of experience | At least 1 year of experience | At least 2 years of experience | At least 3 years of experience | At least 4 years of experience | 5 years of experience or more |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Less than \$550.000 | 1.5\% | 0.5\% | 0.0\% | 0.0\% | 0.0\% | 0.0\% |
| $550.001-\mathbf{1 . 0 0 0 . 0 0 0}$ | 38.6\% | 25.7\% | 2.4\% | 0.4\% | 0.1\% | 1.9\% |
| 1.000.001-1.500.000 | 7.7\% | 7.4\% | 1.9\% | 0.5\% | 0.1\% | 0.3\% |
| 1.500.001-2.000.000 | 1.3\% | 2.2\% | 1.1\% | 0.4\% | 0.1\% | 0.1\% |
| 2.000.001-2.500.000 | 0.4\% | 0.7\% | 0.7\% | 0.4\% | 0.1\% | 0.1\% |
| 2.500.001-3.000.000 | 0.2\% | 0.4\% | 0.4\% | 0.2\% | 0.0\% | 0.1\% |
| $3.000 .001-\mathbf{3 . 5 0 0 . 0 0 0}$ | 0.1\% | 0.5\% | 0.2\% | 0.1\% | 0.1\% | 0.1\% |
| $3.500 .001-4.000 .000$ | 0.0\% | 0.2\% | 0.1\% | 0.1\% | 0.1\% | 0.1\% |
| $4.500 .001-\mathbf{5 . 5 0 0 . 0 0 0}$ | 0.0\% | 0.1\% | 0.0\% | 0.0\% | 0.0\% | 0.1\% |
| $5.500 .001-\mathbf{6 . 0 0 0 . 0 0 0}$ | 0.0\% | 0.0\% | 0.0\% | 0.0\% | 0.0\% | 0.0\% |
| 6.000.001-8.000.000 | 0.0\% | 0.0\% | 0.0\% | 0.0\% | 0.0\% | 0.1\% |
| \$ 8.000.001 and more | 0.0\% | 0.0\% | 0.0\% | 0.0\% | 0.0\% | 0.1\% |

Table A.5: Share of working population by activity

|  | $\mathbf{2 0 1 2}$ | $\mathbf{2 0 1 3}$ | $\mathbf{2 0 1 4}$ |
| :--- | :--- | :--- | :--- |
| Agriculture | $0.92 \%$ | $0.88 \%$ | $0.80 \%$ |
| Mining | $0.32 \%$ | $0.32 \%$ | $0.33 \%$ |
| Manufacturing | $16.98 \%$ | $16.28 \%$ | $16.10 \%$ |
| Energy | $0.51 \%$ | $0.53 \%$ | $0.55 \%$ |
| Construction | $6.59 \%$ | $6.19 \%$ | $6.29 \%$ |
| Comerce | $30.55 \%$ | $30.69 \%$ | $30.58 \%$ |
| Transport | $9.69 \%$ | $9.48 \%$ | $9.29 \%$ |
| Finance Intermediation | $2.02 \%$ | $2.21 \%$ | $2.18 \%$ |
| House rent and other | $10.29 \%$ | $10.79 \%$ | $11.17 \%$ |
| activities |  |  |  |
| Services (personal and social) | $22.15 \%$ | $22.61 \%$ | $22.71 \%$ |

Souurce: DANE household Income Survey (GEIH)

Table A.6: Channels for job searching

| Means for searching personal | Industry | Trade | Services |
| :---: | :---: | :---: | :---: |
| Informal networks | 23.80\% | 26.90\% | 18.00\% |
| Databases / own records | 17.40\% | 18.30\% | 18.70\% |
| Web job boards | 16.70\% | 13.70\% | 20.20\% |
| National Apprenticeship Service (SENA ) Public Employment Service | 12.30\% | 13.70\% | 10.40\% |
| Advertising on media | 12.20\% | 10.80\% | 10.40\% |
| Job Boards of Universities and other organizations | 8.40\% | 6.90\% | 10.80\% |
| Headhunters / Job boards | 6.70\% | 6.50\% | 6.70\% |
| Contact with other educational institutions | 2.10\% | 2.70\% | 4.00\% |
| Job Fairs | 0.50\% | 0.50\% | 0.80\% |

Source: Households Income Survey. DANE (2014)


[^0]:    *Paris School of Economics, CNRS and IZA. Mailing address: Centre d'Economie de la Sorbonne, 106-112 boulevard de l'Hôpital, 75647 Paris Cedex 13, France. E-mail: David.N.Margolis@gmail.com. Telephone: +33 (0)1 440782 20. Fax: +33 (0)1 44078247.
    ${ }^{\dagger}$ Université Paris 1 Panthéon-Sorbonne. E-mail: jaimem.montana@ gmail.com.

[^1]:    ${ }^{1}$ Solving for the multidimensional matching equilibrium is only starting to attract attention in the economics literature (see Dupuy and Galichon (2014)), and the large literature demonstrating the important significant role of job search assistance and placement for the unemployed (Card et al., 2010) suggests that the complexity of the decision problem is non-trivial.

[^2]:    ${ }^{2}$ An alternative, equivalent interpretation could be that if multiple firms post vacancies for the same occupation, they behave in the same manner.

[^3]:    ${ }^{3}$ Individuals are assumed to be unable to split the use of their skill endowment across jobs.
    ${ }^{4}$ Note, however, that the time between iterations is assumed to tend to zero, so individuals can appear to have applied to multiple jobs, although the process is not, in fact, simultaneous.
    ${ }^{5}$ This could reflect worker beliefs that the most visible firms are the ones in highest demand.

[^4]:    ${ }^{6}$ Note that $\bar{\lambda}$ is fixed since the restriction will not change given the fixed level of production.

[^5]:    ${ }^{7}$ Recall that firms do not set wages strategically so as to attract a specific set of workers, and thus they assume ex ante that the skill distributiojn of applicants they will receive is the same as the skill distribution in the overall unemployed population.

[^6]:    ${ }^{8}$ In the proposed simulation, the second part is considered in the form of :

    $$
    \frac{\gamma_{i}^{z} f\left(\mathbf{s}_{i}^{-}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)}{\frac{\sum_{i}^{I} \gamma_{i}^{z} f\left(\mathbf{s}_{i}, \mathbf{r}_{j}, \mathbf{a}_{j}\right)}{I}}
    $$

[^7]:    ${ }^{9} \mathrm{http}: / /$ microdata.worldbank.org/index.php/catalog/step/about

[^8]:    ${ }^{10}$ The wage distributions by level of education required are presented in table A.3.

[^9]:    ${ }^{11}$ See Law 1496 of 2011.
    ${ }^{12}$ See table A.1.

[^10]:    ${ }^{13}$ Labor force participation decisions are not modeled here, and are thus treated as exogenous with respect to the worker's decision problem.
    ${ }^{14}$ There is no comparable taxonomy relating skill requirements to occupations for Colombia. Using this data for the analysis undertaken here requires the additional assumption that the relative skill content of occupations is comparable between Colombia and the United States. This does not imply that the same technologies are necessarily used in each country, which would be particularly unrealistic given the different levels of development. It does, however, require that technological differences across countries result in a homogeneous shift in skill requirements between countries, and that the relative importance of each skill type for each occupation is preserved.

[^11]:    ${ }^{15}$ The unused skills were related to resource management, a skill type not quantified by the STEP survey.

